NEW MEXICO
Public Education Department

# HS: NUMBER AND QUANTITY- THE COMPLEX NUMBER SYSTEM

**Cluster Statement:** C: Use complex numbers in polynomial identities and equations.

| **Standard Text** | **Standard for Mathematical Practices** | **Students who demonstrate understanding can:** |
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| HSN.CN.C.7<br>Solve quadratic equations with real coefficients that have complex solutions. | SMP 2<br>Students reason abstractly and quantitatively by explaining their solutions to quadratics and their representations relative to roots of the quadratic.<br>SMP 3<br>Students construct viable arguments and critique the reasoning of others by justifying solutions and appropriate techniques for solving different quadratic equations. | • Determine the number and nature of quadratic solutions;<br>• Solve a quadratic equation using various methods (e.g., factoring, completing the square, quadratic formula) |
| | | **Webb's Depth of Knowledge:** 1-2 |
| | | **Bloom's Taxonomy:** understand, apply |
| **Previous Learning Connections**<br>In Algebra 1, students solved quadratic equations using a variety of methods. Their solutions were limited however to real solutions. | **Current Learning Connections**<br>Students learn to solve polynomial equations that have complex answers. | **Future Learning Connections**<br>Students will connect this knowledge of complex numbers to solving rational equations, trigonometric equations and trigonometric form in subsequent math courses (Pre-Calculus, AP Calculus, College Algebra, etc). |

**Clarification Statement**
Students will be able to use multiple methods to solve quadratic equations with complex solutions.

**Common Misconceptions**
Students may confuse non-real, imaginary and irrational numbers.

**Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies**

**Pre-Teach**

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying using complex numbers in polynomial identities and equations because students will have to recall prior knowledge from previous grade levels.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.EE.A.1: This standard provides a foundation for work using complex numbers in polynomial identity and equations because students should be able to apply properties of operation strategies. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

**Core Instruction**

*Access*

Interest: How will the learning for students provide multiple options for recruiting student interest?

- For example, learners engaging with using complex numbers in polynomial identities and equations benefit when learning experiences include ways to recruit interest such as setting personal academic goals because students who are involved in setting their own goals are vested in their learning. Students tend to take more pride in learning the material and become more connected to the learning. Students should be involved in setting goals for the current content as well as for looking forward to future content connections. Students need to be offered choices in how to obtain their goals (such as how to practice their skills solving polynomials and using complex numbers).

*Build*

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with using complex numbers in polynomial identities and equations benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because supporting students to build individual skills in self-determination and not giving up can improve their learning potential. When students are motivated and encouraged to be successful, they generally are in the face of adversity and difficulty.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with using complex numbers in polynomial identities and equations benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) because <students are able to find resources quickly to aid in

any misconceptions of the content when referencing what a complex number is for example or how to distinguish between polynomial identities and polynomial equations.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with using complex numbers in polynomial identities and equations benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because students are presented with a variety of strategies may find one strategy easier to comprehend than another which will provide students with a better understanding of the content of polynomials and complex numbers.

*Internalize:*

Executive Functions: How will the learning for students support the development of executive functions to allow them to take advantage of their environment?

- For example, learners engaging with using complex numbers in polynomial identities and equations benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as posting goals, objectives, and schedules in an obvious place because students will be able to focus on the objectives that have been presented and set goals for achieving the learning associated with those objectives and posting them in an obvious place makes it easy for students to refer back to them throughout the learning.

**Re-teach**

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on using complex numbers in polynomial identities and equations by providing specific feedback to students on their work through a short mini-lesson because students who are having difficulty or who may be struggling will be able to get immediate feedback which will help them to better understand possible misconceptions..

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit using complex numbers in polynomial identities and equations by offering opportunities to understand and explore different strategies because students will be able to visual different perspectives with the different strategies and may get a better understanding of the content being presented..

**Extension**

*What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?*

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying using complex numbers in polynomial

identities and equations because students will be able to direct their studying to the specific areas that they need further clarification in.

**Culturally and Linguistically Responsive Instruction:**
**Validate/Affirm:** How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
**Build/Bridge:** How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?
Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying using complex numbers in polynomial identities and equations, goal setting is critical because students are able to make connections to their learning and prior knowledge can be accessed when goals are clearly identified.

**Standards Aligned Instructionally Embedded Formative Assessment Resources:**
**Source: Illustrative mathematics**
http://tasks.illustrativemathematics.org/content-standards/HSN/CN/C/7/tasks/1690
The goal of this task is to solve quadratic equations with complex roots by completing the square. Students could of course directly use the quadratic formula, but going through the process of completing the square helps reinforce the mathematics behind the quadratic formula. The teacher may wish to have students graph the solutions so that they can see that the imaginary solutions are reflections of one another about the real axis. In the case of the equation x2+x+1=0 these two solutions also lie on the unit circle and they are third roots of unity, that is, the roots of this equation equal 1 when they are raised to the third power.
Additional Assessment:
https://algebra-equation.com/solving-quadratic-equation.html
https://member.mathhelp.com/courses/middle_and_high_school/14/chapter/11/lesson/4085?tab=test&tabItem=1

**Relevance to families and communities:**
During a unit focused on using complex numbers in polynomial identities and equations, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different complex numbers and making reference to how they could be used at home or in the community can be a great way to connect the tasks to their own personal tasks.

**Cross-Curricular Connections:**
For centuries there were quadratic equations that were deemed not to have solutions. Equations like x^2 = -1 and x^2 -2x + 2 = 0 have no solutions among the positive and negative numbers. The problem in seeking solutions to equations like these two is that the squares of positive and negative numbers are both positive. Solutions for equations like these can be found, however, if we decide to invent a completely new number whose square is -1; of course, it is not a number that we have seen before. We name this number "i". The square of -i is also -1.
http://mathforum.org/library/drmath/view/55747.html
Reactant particles sometimes collide with one other and yet remain unchanged by the collision. Other times, the collision leads to the formation of products. The state of

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| | the particles that is in between the reactants and products is called the activated complex. An activated complex is an unstable arrangement of atoms that exists momentarily at the peak of the activation energy barrier. Because of its high energy, the activated complex exists for an extremely short period of time (about 10−13 s). There is equal likelihood that the activated complex either reforms the original reactants or goes on to form products.<br><br>**Activated Complex** |