

## HS: STATISTICS & PROBABILITY- MAKING INFERENCES & JUSTIFYING CONCLUSIONS

**Cluster Statement:** A: Understand and evaluate random processes underlying statistical experiments

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

<b>Standard Text</b>	<b>Standard for Mathematical Practices</b>	<b>Students who demonstrate understanding can:</b>
<p>HSS.IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.</p>	<p>SMP 2: Students reason abstractly and quantitatively using statistics from random samples to make inferences about populations.</p> <p>SMP 3: Students can construct viable arguments about populations parameters based on statistics from random samples and can critique the inferences of studies about a population by examining the process for creating a random sample.</p> <p>SMP 4: Students can model population parameters based on statistics from random samples.</p>	<p><b>Students who demonstrate understanding can:</b></p> <ul style="list-style-type: none"> <li>• Explain the difference between bias and unbiased sampling.</li> <li>• Explain why the statistical measures of a random sample should be roughly the same as the statistical measures of the population.</li> <li>• Make inferences about a population based on a random sample.</li> </ul>
		<b>Webb's Depth of Knowledge: 2-3</b>
		<b>Bloom's Taxonomy:</b> Understand and Analyze
<p><b>Standard Text</b></p> <p>HSS.IC.A.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. <i>For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?</i></p>	<p><b>Standard for Mathematical Practices</b></p> <p>SMP 2: Students can reason abstractly and quantitatively by proposing sampling techniques and exploring their nature to produce bias and unbiased results.</p> <p>SMP 5: Students can use appropriate tools strategically (e.g., spinner, die, coin, cards, computer simulation) to analyze a model.</p> <p>SMP 8: Students can look for and express regularity in repeated</p>	<p><b>Students who demonstrate understanding can:</b></p> <ul style="list-style-type: none"> <li>• Explain the difference between a rare event and an ordinary event.</li> <li>• Use tools to analyze results from data-generating process, i.e., create a simulation.</li> <li>• Make inferences about a model to decide if a model is consistent with the result given.</li> </ul>
		<b>Webb's Depth of Knowledge: 3-4</b>

	reasoning to determine if events are consistent or inconsistent through simulation.	<b>Bloom's Taxonomy:</b> Analyze and Evaluate
<p><b>Previous Learning Connections</b></p> <ul style="list-style-type: none"> <li>Connect to the work students have done to determine mean, median, mode, range, IQR, minimum, maximum.</li> </ul>	<p><b>Current Learning Connections</b></p> <ul style="list-style-type: none"> <li>Connect to work throughout the Statistics and Probability domain around interpreting and making inferences about populations based upon sample quantitative data.</li> </ul>	<p><b>Future Learning Connections</b></p> <ul style="list-style-type: none"> <li>Connect to future work with Statistics and Probability in college level courses and careers.</li> </ul>
<p><b>Clarification Statement</b></p> <p>Students move beyond analyzing data to making sound statistical decisions based on probability models. The reasoning process is as follows: develop a statistical question in the form of a hypothesis (supposition) about a population parameter; choose a probability model for collecting data relevant to that parameter; collect data; compare the results seen in the data with what is expected under the hypothesis.</p>		
<p><b>Common Misconceptions</b></p> <p>Students may struggle with the difference between rare and impossible.</p> <p>Students may struggle with recognizing the difference between random and non-repeating events. Humans tend to believe random means an outcome will not repeat but in large data sets of random outcomes it common to have strings of the same outcome (i.e., flipping a coin and 100 time and finding a string of 5 heads in a row).</p>		
<p><b>Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies</b></p> <p><b>Pre-Teach</b></p> <p>Pre-teach (targeted): <i>What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?</i></p> <p>For example, some learners may benefit from targeted pre-teaching that previews new contexts for tasks within the unit when studying the units in understanding and evaluating random processes underlying statistical experiments because a preview often piques learner curiosity and knows that there is a reason for learning this material.</p> <p>Pre-teach (intensive): <i>What critical understandings will prepare students to access the mathematics for this cluster?</i></p> <p>7.SP.C.7: This standard focuses on developing a probability model and using it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. This standard provides a foundation for work with units in Understand and Evaluate Random Processes Underlying Statistical Experiments cluster because misconceptions with probabilities will impact a learner's ability to evaluate random processes. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.</p> <p><b>Core Instruction</b></p> <p><i>Access</i></p> <p>Perception: <i>How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?</i></p>		

- For example, learners engaging with understanding and evaluating random processes underlying statistical experiments benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as different representations of the data, auditory descriptions of data, electronic methods of manipulating the data into various formats because learners effectively access information in different modes. Providing more modes of access empowers more students to be successful in the task.

#### *Build*

*Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understanding and evaluating random processes underlying statistical experiments benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because this can encourage students to engage in the problem from both a logical and mathematical perspective.

*Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with understanding and evaluating random processes underlying statistical experiments benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf because these problems tend to be in paragraph form with detailed descriptions. We want to be sure we are assessing student ability to show what they know about the mathematical concepts rather than their ability to read detailed information in English.

*Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding and evaluating random processes underlying statistical experiments benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches, strategies, skills, etc.) because students may express their understanding in a variety of ways. Allowing for multiple solution methods to be shown can illuminate errors as well as connections between the different methods.

#### *Internalize*

*Comprehension: How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with understanding and evaluating random processes underlying statistical experiments benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a

process for meaning making of new learning; and, applying learning to new contexts such as using multiple examples and non-examples to emphasize critical features because students may initially struggle to connect logical reasoning with mathematical justification. Providing several examples of statistical claims that can and cannot be mathematically supported will model for students how to look for evidence to support or refute claims.

**Re-teach**

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

For example, students may benefit from re-engaging with content during a unit on understanding and evaluating random processes underlying statistical experiments by critiquing student approaches/solutions to make connections through a short mini-lesson because misconceptions about the nature of random events is common and may lead to incorrect conclusions. For example, many people believe that luck is real and can affect the outcome of random events.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

For example, some students may benefit from intensive extra time during and after a unit in the understanding and evaluating random processes underlying statistical experiments by addressing conceptual understanding because small subtle misconceptions about random processes can lead to invalid or inaccurate decisions. For example, many people believe that if they get cancer once, they are less likely to get it again.

**Extension**

*What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?*

- For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when studying evaluating random processes underlying statistical experiments because many learners want or need to know how this cluster will relate to future career or college interests.

**Culturally and Linguistically Responsive Instruction:**

**Validate/Affirm:** How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

**Build/Bridge:** How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying HS.IC.B: Summarize, represent, and interpret data on two categorical and quantitative variables cluster goal setting is critical because statistics are often used to help describe how a specific ethnic or cultural group is doing and what needs they may need. Knowing how to interpret and present data in meaningful ways helps develop a more robust model of the community. The 10 year Census is the largest statistical tool the United States uses to help with this objective.

**Standards Aligned Instructionally Embedded Formative Assessment Resources:**

CollegeBoard		Question ID 4169753					
Assessment SAT	Test Math	Cross-Test and Subscore Problem Solving and Data Analysis	Difficulty Medium	Primary Dimension Problem Solving and Data Analysis	Secondary Dimension Evaluating statistical claims: Observational studies and experiments	Tertiary Dimension 1. With random samples, describe which population the results can be extended to.	Calculator Calculator

A sample of 40 fourth-grade students was selected at random from a certain school. The 40 students completed a survey about the morning announcements, and 32 thought the announcements were helpful. Which of the following is the largest population to which the results of the survey can be applied?

**Question Difficulty:** Medium

- A. The 40 students who were surveyed
- B. All fourth-grade students at the school
- C. All students at the school
- D. All fourth-grade students in the county in which the school is located

Choice B is correct. Selecting a sample of a reasonable size at random to use for a survey allows the results from that survey to be applied to the population from which the sample was selected, but not beyond this population. In this case, the population from which the sample was selected is all fourth-grade students at a certain school. Therefore, the results of the survey can be applied to all fourth-grade students at the school.

Choice A is incorrect. The results of the survey can be applied to the 40 students who were surveyed. However, this isn't the largest group to which the results of the survey can be applied. Choices C and D are incorrect. Since the sample was selected at random from among the fourth-grade students at a certain school, the results of the survey can't be applied to other students at the school or to other fourth-grade students who weren't represented in the survey results. Students in other grades in the school or other fourth-grade students in the country may feel differently about announcements than the fourth-grade students at the school.

This type of assessment question requires students to analyze a scenario and determine what a potential population could have been. Students will engage with SMP 6 as they must carefully read the information to pick out a relevant population.

**Relevance to families and communities:**  
During a unit focused on evaluating random processes underlying statistical experiments consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, how statistics are used to describe how the risk of different cultural and ethnic groups for developing breast cancer and how this might affect medical breast cancer screening frequency recommendations.

**Cross-Curricular Connections:**  
Social Studies: In high school the New Mexico Social Studies Standards state students should "explain how to use technological tools to research data, verify facts and information, and communicate findings." Consider providing a connection for students to determine the best fit of a function for a set of data and explain their choice.