NEW MEXICO
Public Education Department

HS: STATISTICS & PROBABILITY- MAKING INFERENCES & JUSTIFYING CONCLUSIONS

Cluster Statement: B: Make inferences and justify conclusions from sample surveys, experiments, and observational studies

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSS.IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	SMP 3: Students can construct viable arguments and critique the reasoning of others by understanding how decisions based on sample data are related to probability, and that this decision process does not guarantee a correct answer to the underlying statistical question	• Recognize that randomization is necessary to making accurate statistical inferences. • Compare and contrast the differences between a sample survey, experiment, and observational study and the advantages to their uses. • Explain how to use random sampling techniques and the importance of random sampling.
		Webb's Depth of Knowledge: 3
		Bloom's Taxonomy: Understand, Apply
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSS.IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	SMP 3: Students can construct viable arguments and critique the reasoning of others by using simulation and the collection of data to make inferences. SMP 5: Students can use appropriate tools strategically to decide which kind of sampling technique to use in different situations (experiment, survey, observational study).	• Estimate a population mean and calculate margin of error given a simulation. • Use real world data to determine the population mean and margin of error.
		Webb's Depth of Knowledge: 3

	SMP 8: Students can look for and express regularity in repeated reasoning by examining simulation data and recognizing the proportion of times the simulation rejected the hypothesis.	**Bloom's Taxonomy:** Understand, Apply
Standard Text HSS.IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	**Standard for Mathematical Practices** SMP 3: Students can construct viable arguments and critique the reasoning of others by using simulation and the collection of data to make inferences. SMP 5: Students can use appropriate tools strategically to decide which kind of sampling technique to use in different situations (experiment, survey, observational study). SMP 8: Students can look for and express regularity in repeated reasoning by examining simulation data and recognizing the proportion of times the simulation rejected the hypothesis.	**Students who demonstrate understanding can:** • Identify differences between parameters. • Compare two treatment groups in an experiment and decide if the difference in parameters is significant. **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy:** Analyze, Evaluate
Standard Text HSS.IC.B.6 Evaluate reports based on data.	**Standard for Mathematical Practices** MP.6; MP.7; MP.8	**Students who demonstrate understanding can:** • Explain the parameters of data and their significance • Define the characteristics of experimental design (control, randomization, and replication). • Evaluate the experimental study design, how the data was gathered, what analysis (numerical or graphical) was used. • Draw conclusions based on graphical and numerical summaries. • Evaluate reports based on data. **Webb's Depth of Knowledge:** 3

		Bloom's Taxonomy: Analyze, Evaluate
Previous Learning Connections • Connect to work previous math courses, where students have learned to determine mean, median, mode, range, IQR, minimum, maximum. Students have also learned how to graph data distributions (e.g., histograms, box plots).	**Current Learning Connections** • Connect to students work with evaluating the randomness of a sample and use this to determine if a specified model is consistent with the results. **(HSS.IC.A)**	**Future Learning Connections** • Connect to work in subsequent statistics course (AP or college level).

Clarification Statement

Once students see how **probability** intertwines with data collection and analysis, students use this knowledge to make **statistical inferences** from data collected in **sample surveys** and in **designed experiments**, aided by **simulation** and the technology that affords it

Students should be able to explain the reasoning in a **statistical decision** and the nature of the error that may have been made.

Student will look at quality of a statistical question to be answered, question clarity, quantization of responses and error calculations, as absolute value of difference of the sample values from the mean.

Common Misconceptions

Students may struggle with distinguishing between the difference between estimates for means and proportions.

Students may have difficulty in relating the margin of error from a simulation model to the inference about a population.

Students may have the misconception that when the margin of error increases that the statistic does not contain the true population parameter.

Students may struggle with using statistical language that includes the possibility of error in measurement rather than absolute language such as always, never, guaranteed.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying the Making Inferences And Justifying Conclusions cluster because learners are often more ready to engage the material at a deeper level when they feel frustrated and confused just enough to have questions that need answered during the upcoming units.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.SP.A: This cluster of standards provides a foundation for work with the units in the HS.S-IC.B: Making Inferences And Justifying Conclusions Cluster because the 7th

grade standard provides a foundation for valid random sampling techniques, providing validity for conclusions based on the data. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with making inferences and justifying conclusions from sample surveys, experiments, and observational studies benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because students may have collected data and made a simple comparison. When students have data that is geared towards their interest, they analyze the data and can interpret the marginal error, population mean, or graph.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with making inferences and justifying conclusions from sample surveys, experiments, and observational studies benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as prompting or requiring learners to explicitly formulate or restate learning goals because keeping the end in mind, students will keep their goal in mind as to what they are working towards. Too little data may not support their conclusion, graph, or create a difference in the parameters.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with making inferences and justifying conclusions from sample surveys, experiments, and observational studies benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) because data can be organized with an organizational map. Data can be analyzed, graphed, or used to compare data to another set of data.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with making inferences and justifying conclusions from sample surveys, experiments, and observational studies benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) because social media can be used to collect data by setting up a survey and people can randomly decide to participate all over the world. Or the

student can set it up to where only a specific group can participate. Online graphic organizers and graphing software can be used to analyze the data.

Internalize

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with making inferences and justifying conclusions from sample surveys, experiments and observational studies benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as posting goals, objectives, and schedules in an obvious place because students may forget that they are drawing and supporting inferences mathematically. By posting explicit goals/objectives in an obvious place, teachers can remind students to focus their attention on the mathematics that can support their justifications rather than getting lost in the logical argument.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on HS.S-IC.B: Making Inferences And Justifying Conclusions cluster by revisiting student thinking through a short mini-lesson because this conversation can serve as a diagnostic tool so the teacher can prescribe the needed review needed to get the learner moving.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit in the HS.S-IC.B: Making Inferences And Justifying Conclusions cluster by helping students move from specific answers to generalizations for certain types of problems because learners often benefit from seeing the work of more experienced problem solvers.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics because advanced or gifted learners often need or want to explore more into how data is used. For example, a learner could look at how Big Data is being used to make life better but not without potential risks.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of

low expectations and low achievement. For example, when studying HS.IC: Making Inferences & Justifying Conclusions cluster goal setting is critical because interpreting statistical results are often used to infer how a specific ethnic or cultural group is doing and what needs they may need. The census is the largest statistical tool the United States uses to help with this objective.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

CollegeBoard **Question ID 4789744**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Problem Solving and Data Analysis	Medium	Problem Solving and Data Analysis	Inference from sample statistics and margin of error	1. Use sample mean and sample proportion to estimate population mean and population proportion. Utilize, but do not calculate, margin of error.	Calculator

A bag containing 10,000 beads of assorted colors is purchased from a craft store. To estimate the percent of red beads in the bag, a sample of beads is selected at random. The percent of red beads in the bag was estimated to be 15%, with an associated margin of error of 2%. If r is the actual number of red beads in the bag, which of the following is most plausible?

Question Difficulty: Medium

A. $r > 1,700$

B. $1,300 < r < 1,700$

C. $200 < r < 1,500$

D. $r < 1,300$

Choice B is correct. It was estimated that 15% of the beads in the bag are red. Since the bag contains 10,000 beads, it follows that there are an estimated $10,000 \times 0.15 = 1,500$ red beads. It's given that the margin of error is 2%, or $10,000 \times 0.02 = 200$ beads. If the estimate is too high, there could plausibly be $1,500 - 200 = 1,300$ red beads. If the estimate is too low, there could plausibly be $1,500 + 200 = 1,700$ red beads. Therefore, the most plausible statement of the actual number of red beads in the bag is $1,300 < r < 1,700$.

Choices A and D are incorrect and may result from misinterpreting the margin of error. It's unlikely that more than 1,700 beads or fewer than 1,300 beads in the bag are red. Choice C is incorrect because 200 is the margin of error for the number of red beads, not the lower bound of the range of red beads.

This type of assessment question requires students to select a reasonable answer using an estimated measure of center and given margin of error. Solutions are provided as inequalities. Students will engage with SMP 1 and SMP 4 as they apply knowledge of margin of error to solve the problem and model their solution using an inequality.

Relevance to families and communities:
During a unit focused on HS.IC: Making Inferences & Justifying Conclusions, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, how statistics are used to describe how the risk of different cultural and ethnic groups for developing breast cancer and how this might affect medical

Cross-Curricular Connections:

Social Studies: Connection to the difference between correlation versus causation when reading data.

breast cancer screening frequency recommendations. During a unit focused on making inferences, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about how data might allow us to infer how an infection might be slowed in a large extended family living in one house.	