New Mexico Mathematics Instructional Scope for Algebra 1

June 2020

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Overview

This mathematics instructional scope was created by a cohort of New Mexico educators and the New Mexico Public Education Department. This document is organized into three sections. Section 1 describes how to use this document to support equitable and excellent mathematics instruction. Section 2 contains planning support for each cluster of mathematics standards within the grade level or course. Section 3 provides additional resources, references, and glossary.

The intention of this document is to act as companion during the planning process alongside High Quality Instructional Materials (HQIM). A sample template is presented to show a quick snapshot of planning supports provided within each cluster of standards in section 2.

During the creation of this document, we leveraged the work of other states, organizations, and educators from across country and the world. This work would not have been possible without all that came before it and we wish to express our sincerest gratitude for everyone that contributed to the resources listed within our references. This document is a work in progress and in some circumstances, our team of New Mexico educators may have embedded content from resources that have yet to be cited, as these elements are discovered in the use of this tool the references in section 3 will be updated.

Section 1: New Mexico Instructional Scope for Supporting Equitable and Excellent Mathematics Instruction

To better understand the planning supports provided in section 2, for each cluster of standards, this section provides a brief description of each planning support including: *what* support is provided; *why* the planning support is critical for equitable and excellent mathematics instruction; and, *how* to use the planning support with HQIM.

Cluster Statement

What: The New Mexico Mathematics Standards are grouped by Domains with somewhere between 4 to 10 domains per grade level. Within each domain the standards are arranged around clusters. Cluster statements summarize groups of related standards. The cluster statement planning support also indicates if the clusters is major, supporting, or additional work of the grade.

Why: The New Mexico Mathematics Standards require a stronger *focus*[1] on the way time and energy are spent in the mathematics classroom. Students should spend the large majority of their time (65-85%) on the major clusters of the grade/course. Supporting clusters and, where appropriate, additional clusters should be connected to and engage students in the major work of the grade.

How: When planning with your HQIM consider the time being devoted to major versus additional or supporting clusters. Major Work of each grade should be designed to provide students with strong foundations for future mathematical work which will require more time than additional or supporting clusters. Consider also the ways the HQIM makes explicit for students the connections between additional and supporting clusters and the major work of the grade.

Standard Text

What: Each cluster level support document contains the text of each standard within the cluster.

Why: The cluster statement and standards are meant to be read together to understand the structure of the standards. By grouping the standards within the cluster the connectedness of the standards is reinforced.

How: The text of the standards should always ground all planning with HQIM. Reading the standards within a cluster intentionally focuses on the connections within and among the standards.

Standards for Mathematical Practice

What: The Standards for Mathematical Practice describe the varieties of expertise and habits of mind that mathematics educators at all levels should seek to develop in their students.

Why: Equitable and excellent mathematics instruction supports students in becoming confident and competent mathematicians. By engaging with the standards for mathematical practice students are engaging in the practice of doing mathematics and development of mathematical habits of mind—the ability to think mathematically, analyze situations, understand relationships, and adapt what they know to solve a wide range of problems, including problems they may not look like any they have encountered before.[2]

How: When planning with HQIM it is critical to consider the connections between the content standards and the standards for mathematical practice. The planning supports highlight a few practices in which students could engage when learning the content of the standard. Note it is not necessary or even appropriate to engage in all of the practices every day, rather choosing a few and spending time intentionally supporting students in learning both the what (content standards) and the how (standards for mathematical practice) will create a stronger foundation for ongoing learning.

Students Who Demonstrate Understanding Can (Webb's Depth of Knowledge and Bloom's Taxonomy)

What: The New Mexico Mathematics Standards include each aspect of mathematical rigor: conceptual understanding, procedural skill and fluency, and application to the real world.[3] This planning support considers which aspect(s) of rigor are within each standard and then identifies academics skills students need to demonstrate comprehension of

[1] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

[2] Seeley, C. L. (2016). Math is Supposed to Make Sense. In *Making sense of math: How to help every student become a mathematical thinker and problem solver*. Alexandria, VA, USA: ASCD. (P. 13)

[3] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

the standard and associated mathematical practices. The statements also highlight both the receptive (listening and reading) and expressive (speaking and writing) parts of language by considering the types of mathematical representations (verbal, visual, symbolic, contextual, physical) within the standard and what students need to do with them. The planning supports also provide information about two common classifications on cognitive complexity, Webb's Depth of Knowledge and Bloom's Taxonomy.

Why: Analyzing standards alongside the standards for mathematical practice provide a fuller picture of the mathematical competencies demanded in the standard.

How: When planning for a cluster of standards with your HQIM a critical first step is to analyze the content and language demands of the standards and standards for mathematical practice. The analysis can be used to inform formative assessment, or it can be used to plan/design appropriate formative assessment.[4] The planning supports provide a possible break-down of the standard that can serve as the basis for this sort analysis.

Connections

What: The New Mexico Mathematics Standards are designed around coherent progressions of learning. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but an extension of previous learning.[5] The connections to previous, current and future learning make this coherence visible.

Why: Students build stronger foundations for learning when they see mathematics as an inter-connected discipline of relationships rather than discrete skills and knowledge. The intentional inclusion of connections to previous, current, and future learning can support a more inter-connected understanding of mathematics.

How: When planning with HQIM use the connection planning supports to find ways to support students in making explicit connections within their study of mathematics.

Clarification Statement

What: The clarification statement provides greater clarity for teachers in understanding the purpose of the standards within a cluster.

Why: The New Mexico Mathematics Standards illustrate how progressions support student learning within each major domain of mathematics. The clarification statement provides additional context about the ways each cluster of standards supports student learning of the larger learning progression.

How: When planning with HQIM use the clarification statement to support an understanding of how the materials use specific types of representations or change the learning sequence from instructional approaches not grounded in progressions of learning.

Common Misconceptions

What: This planning support identifies some of the common misconceptions students develop about a mathematical topic.

Why: Students create misconceptions based on an over generalization of patterns they notice or an over reliance on rules rather than underlying mathematics. Rules in mathematics expire[6] over time (e.g., you can't subtract 1-3) as students expand their knowledge of mathematics (e.g., from whole numbers to rational numbers). It is critical to understand some of the common misconceptions students can develop so we can address them directly with students and continue to build a strong foundation for their mathematical learning.

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[4] English Learners Success Forum. (2020). ELSF | Resource: Analyzing Content and Language Demands. Retrieved from https://www.elsuccessforum.org/resources/math-analyzing-content-and-language-demands
[5] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics
[6] Cardone, T. (n.d.). Nix the Tricks. Retrieved from https://nixthetricks.com/

How: When planning with your HQIM look for ways to directly address with students some common misconceptions. The planning supports in this document provide some possible misconceptions and your HQIM might include additional ones. The goal is not to avoid misconceptions, they are a natural part of the learning process, but we want to support students in exploring the misconception and modifying incorrect or partial understandings.

Multi-Layered System of Supports/Suggested Instructional Strategies

What: The section on Multi-Layered Systems of Supports(MLSS)/Suggested Instructional Strategies is designed to support teachers in planning for the needs of all students. Each section includes options for pre-teaching, reteaching, extensions and core instructional supports for students. Targeted pre-teaching and reteaching support student's acquisition of the knowledge and skills identified in the New Mexico Mathematics Standards to support student success with high-quality differentiated instruction. Intensive supports may be provided for a longer duration, more frequently, smaller groups, or otherwise be more intensive than targeted supports. Progress monitoring should occur to assess students' responses to additional supports, see Standards Aligned Instructionally Embedded Formative Assessment Resources.

Why: MLSS is a holistic framework that guides educators, those closest to the student, to intervene quickly when students need additional supports. The framework moves away from the "wait to fail" model and empowers teachers to use their professional judgement to make data-informed decisions regarding the students in their classrooms to ensure academic success with the grade level expectations of the New Mexico Mathematics Standards.

How: When planning with your HQIM use the suggestions for pre-teaching as a starting point to determine if some or all of the students in your classroom may need targeted or intensive pre-teaching at the start of unit to ensure they can access the grade level material with the unit. The core-instruction and reteach sections work together to support planning within a unit, look for the ways the materials are supporting greater access for all students and providing options to revisit materials based on formative assessments. The planning supports for each cluster are grounded in the Universal Design Learning (UDL) Framework, additional planning supports based on this framework can be found in Section 3 of this document in the part titled, Planning Guidance for Multi-Layered Systems of Support: Core Instruction.

Culturally and Linguistically Responsive Instruction

What: Culturally and Linguistically Responsive Instruction (CLRI), or the practice of situational appropriateness, requires educators to contribute to a positive school climate by validating and affirming students' home languages and cultures. Validation is making the home culture and language legitimate, while affirmation is affirming or making clear that the home culture and language are positive assets. It is also the intentional effort to reverse negative stereotypes of non-dominant cultures and languages and must be intentional and purposeful, consistent and authentic, and proactive and reactive. Building and bridging is the extension of validation and affirmation. By building and bridging students learning to toggle between home culture and linguistic behaviors and expectations and the school culture and linguistic behaviors and expectations. The building component focuses on creating connections between the home culture and language and the expectations of school culture and language for success in school. The bridging component focuses on creating opportunities to practice situational appropriateness or utilizing appropriate cultural and linguistic behaviors.[7]

Why: The mathematical identities of students are shaped by the messages they receive about their ability to do mathematics and the power of mathematics in their lives outside of school.[8] Mathematics educators must intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages. In addition, create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to supports students in creating mathematical identities as capable mathematicians within school and society.

[7] Hollie, S. (2011). *Culturally and linguistically responsive teaching and learning*. Teacher Created Materials.
[8] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics. (P. 14)

How: When planning instruction is critical to consider ways to validate/affirm and build/bridge from your students cultural and linguistic assets. The planning supports for each cluster provide an example of how to support equity-based teaching practices. Look for additional ways within your HQIM to ensure all students develop strong mathematical identities.

Standards Aligned Instructionally Embedded Formative Assessment Resources

What: Formative Assessment is the planned, ongoing process used by all students and teachers during learning and teaching to elicit and use evidence of student learning to improve student understanding of the outcomes and support students to become directed learners. All New Mexico educators have access to standards aligned instructionally embedded formative assessments: iStation at K-2; Cognia at 3-8, and the SAT Suite Question Bank at 9-12. These are intended to be used during instruction for each at each grade alongside assessments within your HQIM.

Why: When student thinking is made visible the teacher can examine the progression of learning towards the goals of the standards and adjust instruction as necessary. By including students in the assessment and analysis process students become strategic and goal-directed with their learning.

How: The planning supports at each cluster provide an example of a task that addresses one more aspect of the cluster of standards. This example can be used to discuss possible responses by students and next steps for instruction. A similar process can then be used to identify additional items from one of the formative assessment resources provided by NM PED and your HQIM.

Relevance to Families and Communities

What: Relevance to families and communities requires finding the relevance of mathematics outside of the classroom by connecting to families and communities and learning about varied and often unexpected ways they use mathematics.

Why: When school mathematics is connected to the mathematics outside of school students can build a bridge between their ways of thinking about quantities outside and inside school created a bridge between home and school.

How: When planning at the year and unit level with you HQIM find ways to intentionally learn from your families and communities the cultural and linguistic ways they use mathematics outside of school.

Cross-Curricular Connections

What: New Mexico defines cross-curricular connections as connections between two or more areas of study made by teachers or students within the structure of a subject.

Why: The purpose of planning cross-curricular connections in an instructional sequence is to ensure that students build connections and recognize the relevance of mathematics beyond the mathematics classroom.

How: When planning with HQIM look for opportunities to make explicit connections to other content areas such as the examples provided for each cluster.

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Template of the New Mexico Cluster Level Planning Support for the New Mexico Mathematics Standards

<GRADE/COURSE/DOMAIN ABBREVIATION: DOMAIN NAME>

Cluster Statement: Statement from New Mexico Mathematics Standards summarize a group of related standards.

Major/Additional/Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)
Identifies if the cluster is major, additional or supporting work of the grade.

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
Full text of the standard	The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.	The cognitive skills students perform to demonstrate to comprehension of a standard. **Depth Of Knowledge:** Correlation of standard to Webb's Depth of Knowledge **Bloom's Taxonomy:** Correlation of standard to Bloom's Taxonomy
Connections to Previous Learning: Supports student connections to learning from previous grade levels.	**Connections to Current Learning** Supports student connections to learning within the grade level.	**Connections to Future Learning** Supports student connections to learning in a future grade.

Clarification Statement: Clarifies the language of the standard.

Common Misconceptions: Guidance on where a student misconception or misunderstanding could potentially occur.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach
Pre-teach (targeted): Guidance for how to activate students' knowledge to support their learning.
Pre-teach (intensive): Guidance for how to use earlier grade standards to build a strong foundational understanding upon which to build grade level concepts.

Core Instruction
Access: Guidance for optimizing universal access to learning experiences.
Build: Guidance for supporting students build their understanding of the cluster.
Internalize: Guidance for ensuring student internalization of the learning goal.

Re-teach
Re-teach (targeted): Guidance for adjusting instruction during a unit by using formative assessment data.
Re-teach (intensive): Guidance for analyzing assessment data to identify content that would benefit from more intensive reteaching.
Extension Ideas: Suggestions that offer additional challenges to 'broaden' students' knowledge of the mathematics within the cluster.

Culturally and Linguistically Responsive Instruction: Provides equity based instructional suggestions aligned to the cluster of standards

Standards Aligned Instructionally Embedded Formative Assessment Resources: Includes reference to high-quality formative assessment resources, including examples from New Mexico's formative assessment banks.

Relevance to Families and Communities:	**Cross Curricular Connections:**
Connecting with families and communities to create relevant connections between mathematics inside and outside of school.	Includes examples of how the cluster provides opportunities to connect to other disciplines such as literacy, science, social studies, and the arts.

Section 2: Cluster Level Planning Support for the New Mexico Mathematics Standards

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[9] Appendix A of the Common Core State Standards was used to determine the standards within each high school course.

Linear, Quadratic, & Exponential Models

HSF.LE.A

HSF.LE.B

Strand: Statistics & Probability

Interpreting Categorical and Quantitative Data

HSS.ID.A

HSS.ID.B

HSS.ID.C

HS: NUMBER AND QUANTITY- QUANTITIES

Cluster Statement: A: Reason quantitatively and use units to solve problems.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSN.Q.A.1: Use units to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. *Foundation for work with expressions, equations and functions*	SMP 1: Students can make sense of problems and persevere in solving them by utilizing appropriate units and/or quantities in the context of the problems. SMP 2: Students can reason abstractly and quantitatively by applying appropriate units and/or quantities in the context of the problems.	• Choose the units in a formula. • Correctly scale a graph with unit increments and identify a quantity from a graph with a scale in unit increments of a specified measurement. • Use units to guide the solution of a familiar multi-step problem with scaffolding. • Make measurement conversions between compound units. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSN.Q.A.2: Define appropriate quantities for the purpose of descriptive modeling. *Foundation for work with expressions, equations and functions*	SMP 1: Students can make sense of problems and persevere in solving them by defining different quantities in a problem and its solution. SMP 4: Students can model mathematically using appropriate quantities in the context of the problems.	• Identify important information, plan, and develop strategies to solve a problem in a context. • Define appropriate quantities to construct a model **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply

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Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSN.Q.A.3: Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *Foundation for work with expressions, equations and functions*	SMP 5: Use appropriate tools strategically by experiencing different measurement tools digitally and concretely to observe measurement error. SMP 6: Students can attend to precision by using the measurement of the same object multiple times to determine an acceptable level to report.	• Determine whether a measurement is appropriate in each context. (e.g., measuring the length of a desk in inches versus yards). • Determine the appropriate level of precision of measurement in each context. • Write solutions using appropriate units and rounding techniques based on the context of the problem. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
Previous Learning Connections • Connect to rounding. **(4.NBT.3, 5.NBT.4)** • Connect to finding unit rates. • Connect to labeling x- and y-axes with appropriate scales and units. **(8.F.4-5)**	**Current Learning Connections** • Connect to application problems using linear, quadratic, and exponential models. **(HSF.IF.4- 6)**	**Future Learning Connections** • Connect will continue to use and expand upon the use of units to make sense of problems and use the context of a problem to create and label graphs using appropriate scales. **(HSF.IF.4- 7) -** *Focus on using key features to guide selection of appropriate type of model function***)**

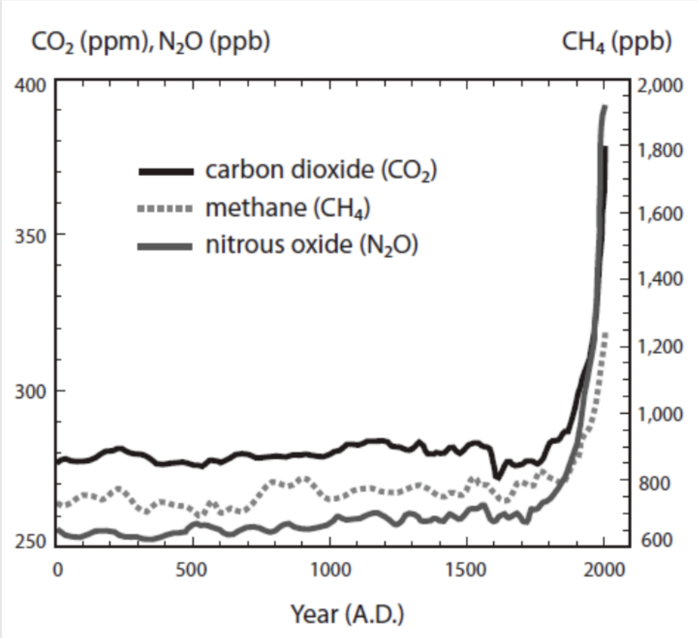
Clarification Statement

- HSN.Q.A.1: **Reasoning quantitatively** includes knowing when and how to **convert units** in **computations**, such as when adding and subtracting **quantities** that measure the same **attribute** but are expressed in different units and other computations with **measurements** in different units or converting units for **derived quantities** such as **density** and **speed**. Reasoning quantitatively can also include **analyzing** the units in a calculation to reveal the units of the answer. This can help reveal a mistake if, for example, the answer comes out to be a distance when it should be a speed (MP.2).

  Students should specify units when defining variables and attend to units when writing **expressions** and **equations** (MP.6).

  In **applications**, **formulas** are often used, and errors can occur in the use of the formulas if units are not attended to carefully. The formula d=vt notwithstanding, a car driving at 25 mph for 3 minutes does not cover 25 x 3 miles. Conversely, if the student does attend carefully to units, the result can be a deeper understanding of a formula or a situation.

  A good **quantitative understanding** of [a real-life situation] helps a student make sound choices for the **scale** and **origin** of a **graph** or a **display**. In a map of arable land area, for example, there is no sense in having a scale that extends to **negative values**, in a graph showing the concentration of atmospheric carbon dioxide over the past 2000 years, the choice of origin in the **vertical scale** is an important editorial decision. These considerations apply to graphs, **data tables**, **scatter plots**, and other visual displays of **numerical data**. It should go without saying that graphs and displays must be properly **labeled**, or else they are meaningless (MP.6)



Concentration of carbon dioxide and other gases in the atmosphere over the past 2,000 years. Source: Forster et al., 2007, Changes in Atmospheric Constituents and in Radiative Forcing. In Solomon et al. (Eds.), *Climate Change 2007: The Physical Science Basis*, Figure 1, p. 135, `http://www.ipcc.ch/pdf/assessment-report/ar4/wg1/ar4-wg1-chapter2.pdf`.

- HSN.Q.A.2: In modeling situations (MP.4), defining the **key quantity of interest** might be part of the task. For example, in a situation that involves crop productivity, a student might choose to examine the number of tons of fertilizer per acre as the variable of interest. In a situation that involves content development for a web site, a choice might arise as to whether the number of posts per day or the number of words per day is the key productivity **variable**.
- HSN.Q.A.3: Quantitative reasoning includes choosing an **appropriate level of accuracy** when reporting quantities. For example, if the doctor measures your height as 73 inches and your weight as 210 pounds, then your Body Mass Index (BMI) is (weight in pounds)/(height in inches ^{2}) x 703 = (210)/(73^2) x 703 ≈ 27.7031 ≈ 28. There is no point in reporting a value more **precise** than 28 here, because any value between 25 and 30 is considered overweight. * (See reference under Figure #).

Common Misconceptions

- Students may have difficulty with multi-step problems.
- Students frequently confuse precision with accuracy.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying using units to understand problems and to guide the solution of multi-step problems; Using descriptive modeling and choosing a level of accuracy appropriate to the limitations because students need time to determine relevant information and the unit's importance of the units given in the context to help guide their approach. Students also need to use reasoning skills to determine the level of accuracy appropriate to the limitations of their problem. Students need to make sense of the problem, use reasoning to create a plan and use precision to develop a solution that makes sense in the context of the problem. Students should be given multiple opportunities to apply these skills.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.RP.A.1 This standard provides a foundation for work with using units as a way to understand problems and to guide the solution of multi-step problems because understanding the concept of a ratio and use ratio language to describe a ratio relationship between two quantities is the building blocks for proportional reasoning and graphs. Students can gain confidence in their problem-solving ability by attempting a problem based on prior learning. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with reasoning quantitatively and use units to solve problems benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features <give an example connected to this standard such as the size of text, images, graphs, tables, or other visual content; contrast between background and text or image; color used for information or emphasis; volume or rate of speech or sound; speed or timing of video, animation, sound, simulations, etc.; layout of visual or other elements; font used for print materials> because this domain is embedded throughout algebra. Students use units to understand problems, guide their solutions, justify solutions as viable/non-viable, and understand the accuracy as well as the limitations of their solutions. Emphasis should be placed on the core of this domain when displays such as graphs, tables, anchor charts, videos, etc. For example, showing a video modeling volume is cubic units. Showing a video modeling the formula distance equals rate time time. Displaying an anchor chart that connects linear, square, and cubed units to a physical model will make connections for students.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with benefit when learning to reason quantitatively and use units to solve problems experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that emphasizes effort, improvement, and achieving a standard rather than on relative performance because students need to understand the importance of their approach not just their solution. Providing students with feedback that allows improvement will build their reasoning and problem-solving abilities. Students should understand what part of their approach viable and which part of their approach needs improvement in order to persevere in solving a contextual problem.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with reason quantitatively and use units to solve problems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making relationships between elements explicit (e.g., highlighting the transition words in an argument, links between ideas in a concept map, etc.) because students need a structure when reasoning and persevering in solving contextual problems. Using strategies such as highlighting important information, crossing out irrelevant information, and using a graphic organizer to organize relevant information help students make sense of the information and bring understanding on how the information is related.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with reason quantitatively and use units to solve problems benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because students will gain an understanding of the best approach for a given contextual problem and also see some contextual problems offer many different approaches. Students need to be provided opportunities to see the limitations of an approach for a given context.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with reason quantitatively and use units to solve problems benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as offering devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the mathematical practices for the purpose of monitoring and improving because students need their learning recorded in an organized manner to access the learning in the future. Students need access to devices such as, anchor charts, concept maps, and charts displaying the mathematical practices and problem-solving approaches to use as a reference. These resources allow students the opportunity to learn from their mistakes and not repeat the same mistakes. Problem solving should be a fluid concept in all classrooms. Students should increase their "bag of tricks" to approaching problems through their ability to improve their work by using multiple approaches.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on using units as a way to understand problems and to guide the solution of multi-step problems; Using descriptive modeling and choosing a level of accuracy appropriate to the limitations by critiquing student approaches/solutions to make connections through a short mini-lesson because providing students with feedback not only on their solution but on their approach will engage students in discussions that will lead to clarifying the best approach for a given context.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit using units as a way to understand problems and to guide the solution of multi-step problems; Using descriptive modeling and choosing a level of accuracy appropriate to the limitations by offering opportunities to understand and explore different strategies because students with unfinished learning need ample opportunities to explore different strategies to determine the validity of each strategy given a specific context. Students need opportunities to solve contextual problems that involve using units to understand and solve problems.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when studying using units as a way to understand problems and to guide the solution of multi-step problems; Using descriptive modeling and choosing a level of accuracy appropriate to the limitations because through open ended tasks linking multiple disciplines students begin to understand the relationship between mathematics and other disciplines. Students engage in using problem solving approaches to address problems in a context other than mathematics. Students will extend their thinking to contextual situations to reinforce their understanding of using units to understand and persevere through all problems.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying using units as a way to understand problems and to guide the solution of multi-step problems, choosing and interpreting units consistently in formulas, choosing and interpreting the scale and the origin in graphs and data displays, defining appropriate quantities for the purpose of descriptive modeling, and choosing a level of accuracy appropriate to limitations on measurement when reporting quantities the use of mathematical representations within the classroom is critical because students approaches as well as their solutions need to be validated. For example, multi-entry tasks allow students to choose the tools and approaches best suited for the situation. Allowing for discourse regarding the tools and approach selected provides students' knowledge that there are limitations to tools and approaches. When selecting an approach or tools to attempt a mathematical task, students use their reasoning skills to determine if their approach is valid for the situation and whether there are limits to the approach. Also, students' awareness that many approaches or tools may be accurate for the situation, but some are more precise than others.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 5207812

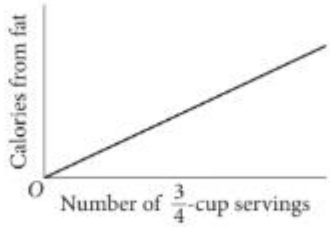
Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Analysis in Science	■■□	Heart of Algebra	Linear functions	4. Make connections between verbal, tabular, algebraic, and graphical representations of a linear function, by a. deriving one representation from the other;	Calculator

Questions 18-20 refer to the following information.

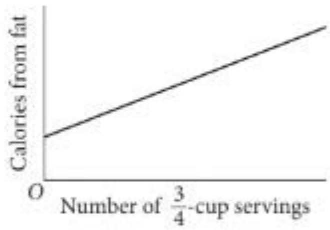
Jennifer bought a box of Crunchy Grain cereal. The nutrition facts on the box state that a serving size of the cereal is $\frac{3}{4}$ cup and provides 210 calories, 50 of which are calories from fat. In addition, each serving of the cereal provides 180 milligrams of potassium, which is 5% of the daily allowance for adults.

Which of the following could be the graph of the number of calories from fat in Crunchy Grain cereal as a function of the number of $\frac{3}{4}$-cup servings of the cereal?

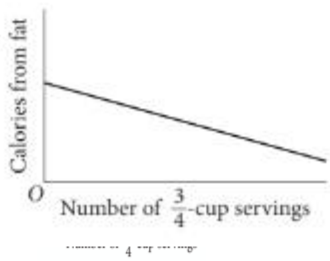
A.



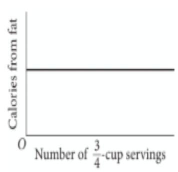
B.



C.



D.



Rationale

Choice A is correct. There are 0 calories in 0 servings of Crunchy Grain cereal so the line must begin at the point $(0,0)$. Point $(0,0)$ is the origin, labeled O. Additionally, each serving increases the calories by 250. Therefore, the number of calories increases as the number of servings increases, so the line must have a positive slope. Of the choices, only choice A shows a graph with a line that begins at the origin and has a positive slope.

Choices B, C, and D are incorrect. These graphs don't show a line that passes through the origin. Additionally, choices C and D may result from misidentifying the slope of the graph.

Weed Killer (HSN.Q.A.1, HSN.Q,A.2, HSN.Q.A.3): http://tasks.illustrativemathematics.org/content-standards/HSN/Q/A/2/tasks/81

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on using units as a way to understand problems and to guide the solution of multi-step problems, choosing and interpreting units consistently in formulas, choosing and interpreting the scale and the origin in graphs and data displays, defining appropriate quantities for	Science: In high school the NGSS state students should “carefully format data displays and graphs, attending to origin, scale, units, and other essential items.” Consider providing a connection for students to choose and interpret the scale and the origin in graphs and data displays that they are working with in science.

the purpose of descriptive modeling, and choosing a level of accuracy appropriate to limitations on measurement when reporting quantities, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, discussing how different cultures eat food will show that although certain cultures choose different tools, such as, forks, chopsticks, tortillas, they are all possible approaches, but some may be more precise than others. Also, with practice other approaches can be useful. The connection can be made that although trying something new, as in a new approach to a mathematical task, may be uncomfortable, but with practice it becomes more useful.	Social Studies: In high school the New Mexico Social Studies Standards state students should “explain how to use technological tools to research data, verify facts and information, and communicate findings.” Consider providing a connection for students to look at the accuracy/precision of measurement data.

HS: NUMBER AND QUANTITY- THE REAL NUMBER SYSTEM

Cluster Statement: A: Extend the properties of exponents to rational exponents.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSN.RN.A.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*	SMP 3: Students can construct viable arguments by explaining the meaning of rational exponents using the properties of exponents. SMP 7: Students can look for and make sure of structure by extending the properties of integer exponents to multiply and divide expressions with rational exponents that have common denominators.	• Explain how integer exponent properties apply to rational exponent properties. • Show how a rational exponent (whose numerator is not one) can be expanded as a whole number multiplied by a fraction. • Justify that raising the base to power and then taking the root is equivalent to taking the root and then raising the base to a power. **Webb's Depth of Knowledge:** 2 **Bloom's Taxonomy:** Apply, Analyze

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSN.RN.A.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.	SMP 1: Students can make sense of problems and persevere in solving them by making meaning of rational exponents in terms of exponent and root and applying that meaning to various problems SMP 3: Students can construct viable arguments by explaining the meaning of rational exponents in terms of radicals and roots. SMP 7: Students can look for and use patterns and structures in exponential expressions by applying properties of exponents to rational exponents.	• Apply properties of exponents to simplify algebraic expressions with fractional exponents. • Apply power of zero, negative exponent, product, quotient, power to a power, product to a power, and quotient rules of exponents to simplify or write equivalent expressions. • Convert radical expression to expressions with rational exponents and vice versa. • Identify the exponent property used when rewriting expressions and recognize when laws of exponents cannot be used to rewrite an expression. **Webb's Depth of Knowledge:** 1-2

		Bloom's Taxonomy: Understand, Apply, Analyze
Previous Learning Connections • Connect to using square root and cube root symbols. **(8.EE.2)** • Connect to understanding and applying the properties of integer exponents. **(8.EE.1**)	**Current Learning Connections** • Connect to applying the properties of exponents to rewrite exponential functions. **(HSA.SSE.3)**	**Future Learning Connections** • Connect to solving equations using rational exponents and radical operations. **(HSA.REI.2)** • Connect to continuing to use exponent properties as they arise in various situations. **(HSF.IF.8, HSF.LE)**

Clarification Statement

- RN.A.1: **Exponent notation** is a remarkable success story in the expansion of mathematical ideas. It is not obvious at first that a number such as sqrt(2) can be represented as a **power of 2**. But reflecting that $(\text{sqrt}(2))^2=2$ and thinking about the **properties of exponents**, it is natural to define $2^{(1/2)}=\text{sqrt}(2)$ since if we follow the rule $(a^b)^c=a^{(bc)}$ then $(2^{(1/2)})^2=2^{((1/2)*2)}=2^1=2$.
- RN.A.2: Because **rational exponents** have been introduced in such a way as to preserve the **laws of exponents**, students can now use those laws in a wider variety of situations. For example, they can rewrite the **formula** for the **volume** of a **sphere** of **radius** r, $V=(4/3)(pi)(r^3)$ to express the radius **in terms of** the volume, $r=((3/4)(V/pi))^{(1/3)}$.

Common Misconceptions

- Struggle to connect rational exponents to its radical form. Students tend to multiply the number by the exponent.
- When using the Power of a Power Property some students may forget to multiply the entire quantity by the exponent and only multiply the variable.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying the rewriting of expressions involving radicals and rational exponents by applying the properties of exponents because students may have unfinished learning simplify expressions with exponents using the exponent properties and would benefit from the access of that prior learning. Re-visiting expanded form and connecting to exponent properties as well as anchor charts would be beneficial to provide students with access to this content.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.EE.A.1 This standard provides a foundation for work with extending exponent properties to rewriting expressions with radicals and rational exponents because this was the first-time students were introduced to applying the exponent properties in simplifying and generating equivalent expressions. Students will benefit from time to access and apply this prior knowledge. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with extending the properties of exponents to rational exponents and using properties of rational and irrational numbers benefit when learning experiences include ways to recruit interest such as providing choices in their learning <give an example such as the sequence or timing of task completion because students must see the connection of the properties of exponents to rational exponents. Students have difficulty with the exponent properties because they have the flexibility to simplify expressions involving integer exponents multiple ways. Students must be provided choices when learning the exponent properties, communicate and discuss multiple ways to simplify the same expression, and make the connection between the exponent properties to expressions with rational exponents.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with extending the properties of exponents to rational exponents and using properties of rational and irrational numbers benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities because for this domain it is imperative students work in cooperative groups to strengthen their understanding of simplifying an expression using the exponent properties provides students with multiple entry points that end in the same result. Students must know and understand the role of their group and their own individual role of the group. Students must be provided many opportunities to simplify expressions using the exponent properties and rewriting expressions with rational exponents for them to persevere and solidify their knowledge and understanding of this concept. When students gain the confidence of simplifying expressions with the exponent properties, they can extend that knowledge to rewrite expressions with rational exponents into radical form.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with extending the properties of exponents to rational exponents and using properties of rational and irrational number benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because accessing the prior knowledge of simplifying exponential expressions using the exponent properties is essential for students to rewrite expressions with rational exponents in radical form. Understanding the entry point of students' prior knowledge of this concept will allow for the level of reteaching before students rewrite expressions with rational exponents.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with extending the properties of exponents to rational exponents and using properties of rational and irrational number benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches, strategies, skills, etc.) because students have a difficult time when mathematical problems provide students with multiple entry points. Students must be provided with multiple ways to simplify an expression using the exponent properties to understand their connection to rewriting expressions with rational exponents. Providing students examples of expanded form and having the students make the connection to the properties <i.e. product rule -keep the base and add the exponent, quotient rule- keep the base and subtract the exponent, etc..> is important for students to gain conceptual understanding of the concept. This also allows all students access to the content, the opportunity to reason and make connections.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with extending the properties of exponents to rational exponents and using properties of rational and irrational number benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as addressing subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") because students struggle with the flexibility of this domain and might need to have more focused learning with this standard. For instance, many students have great difficulty with the negative exponent property of exponents. Approaching this standard by ensuring all exponents are positive as the first step in simplifying helps students manage their anxiety. Focusing on problems with just one or two properties will help some learners be able to persevere in this domain instead of becoming overwhelmed and giving up.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on rewriting radical and rational exponent expressions by clarifying mathematical ideas and/or concepts through a short mini-lesson because students must build a conceptual understanding of the meaning of a rational exponent by decomposing the exponent into parts. Students must understand the parts of the rational exponent ⅔; ⅔ can be rewritten as 2 times ⅓; the numerator of 2 is the power and the denominator 3 of (⅓) is the cube root.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit of rewriting radical expressions and expressions with rational exponent by offering opportunities to understand and explore different strategies because students must make the connection that a rational exponent can be broken down to

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the root and the power. Students need time to explore performing different operations first to understand the mathematical relationships between inverse operations. <i.e. $(5^3)^{(1/3)}$ is 5>

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying how to rewrite radical expressions and expressions with rational exponents because students extend this thinking by using the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time." For example, when studying how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents and rewriting expressions involving radicals and rational exponents using the properties of exponents eliciting and using student thinking is critical because students need to experience mathematics that allow students to use different approaches to find the same end result. When rewriting expressions with rational exponents in order to simplify expressions, by using student thinking, students will be provided the opportunity to see the order in which you perform the root and the power does not make a difference with the end result. Also, by using student thinking about simplifying exponential expressions using the exponent properties, students are provided the opportunity to reason, communicate their reasoning and justify their solution. Students will have the opportunity to build on the knowledge that mathematics is a powerful tool and all approaches should be validated.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 1053933

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math	■■■	Passport to Advanced Mathematics	Equivalent expressions	1. Make strategic use of algebraic structure and the properties of operations to identify and create equivalent expressions, including b. rewriting expressions with rational exponents and radicals;	No Calculator

If $\frac{\sqrt{x^5}}{\sqrt[3]{x^4}} = x^{\frac{a}{b}}$ for all positive values of x, what is the value of $\frac{a}{b}$?

Rationale

The correct answer is $\frac{7}{6}$. The value of $\frac{a}{b}$ can be found by first rewriting the left-hand side of the given equation as $\frac{x^{\frac{5}{2}}}{x^{\frac{4}{3}}}$. Using the properties of exponents, this expression can be rewritten as $x^{\left(\frac{5}{2}-\frac{4}{3}\right)}$. This expression can be rewritten by subtracting the fractions in the exponent, which yields $x^{\frac{7}{6}}$. Thus, $\frac{a}{b}$ is $\frac{7}{6}$. Either 7/6, 1.16, or 1.17 can be entered as the correct answer.

Evaluating Exponential Expressions: http://tasks.illustrativemathematics.org/content-standards/HSN/RN/A/1/tasks/1866

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents and rewriting expressions involving radicals and rational exponents using the properties of exponents, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about what systems are in place at home to organize and simplify the household can provide students' with a powerful connection on why we simplify expressions.	Science: Rational exponents can be applied to scientific notation. Consider providing a connection for students to explore rational exponents in this context, such as to determine the maximum distance of each planet from the sun. Music: The frequencies in the musical range of a various instruments can be modeled using rational exponents. Consider providing a connection for students to find the highest and lowest frequencies.

HS: NUMBER AND QUANTITY- THE REAL NUMBER SYSTEM

Cluster Statement: B: Use properties of rational and irrational numbers.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSN.RN.B.3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	SMP 3: Students can construct viable arguments by explaining the result of computations with rational and irrational numbers by providing examples and counterexamples as justification and explain why their conjectures work. SMP 7: Students can look for and make sure of structure by examining the underlying structure of number arithmetic and applying it to real numbers.	• Identify the difference between a rational and an irrational number. • Perform operations on rational and irrational numbers. • Explain that the sum and product of two rational numbers is rational. • Explain that the sum and product of a rational number and a nonzero irrational number are irrational. **Webb's Depth of Knowledge:** 2 **Bloom's Taxonomy:** Apply, Analyze
<u>**Previous Learning Connections**</u> • Connect to identifying and comparing rational and irrational numbers. **(8.NS.2)** • Connect to computing rational and irrational values when working with volume, surface area, and circles. **(7.G.4,6)**	<u>**Current Learning Connections**</u> • Connect to using the same strategies as classifying one number as rational or irrational to classify sums and products.	<u>**Future Learning Connections**</u> • Connect to rationalizing denominators using an understanding of products of irrational numbers. **(HSN.CN.5)** • Connect to working with irrational numbers when solving equations. **(HSN.CN.7)** • Connect to simplifying radicals using an understanding of irrational numbers. **(HSA.REI.2)** • Connect to calculating and interpreting measurements using irrational numbers. **(HSF.TF.1-3)**

Clarification Statement

HSN.RN.B.3: An important difference between **rational** and **irrational numbers** is that rational numbers form a **number system**. If you **add**, **subtract**, **multiply**, or **divide** two rational numbers, you get another rational number (provided the **divisor** is not 0 in the last case). The same is not true of irrational numbers.

Although in applications of mathematics the distinction between rational and irrational numbers is irrelevant, since we always deal with **finite decimal approximations** (and therefore with rational numbers), thinking about the **properties** of rational and irrational numbers is good practice for mathematical reasoning habits such as constructing viable arguments and attending to precision. (MP.3, MP.6).

Common Misconceptions

- Students may think that the quotient of two rational numbers isn't always rational because some quotients do not appear to terminate or repeat.
- Students may wrongly believe that a single explanation is an explanation or proof of a property.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying properties of rational and irrational numbers because some students have either unfinished learning or misconceptions such as confusing repeating with non-terminating regarding rational and irrational numbers. Assessing students prior learning of rational and irrational numbers and addressing student misconceptions is imperative to avoid further misconceptions on classifying sums and products of rational and irrational numbers.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.NS.A.1 This standard provides a foundation for work with the classification of sums and products of rational and irrational numbers because this standard introduces the concept that a number can't be both rational and irrational simultaneously. In prior grades, students were presented with only the rational number system. Students must understand the difference between a rational and irrational number before they can classify expressions. Allowing time for those discussions and addressing misconceptions regarding the real number system will diminish further misconceptions from developing. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with use properties of rational and irrational numbers benefit when learning experiences include ways to recruit interest such as providing time for self-reflection about the content and activities because students need to communicate their thinking verbally and in writing to solidify the outcomes of sums and products of rational and irrational numbers. Students need time to reflect on their learning to solidify their understanding of this domain.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with use properties of rational and irrational numbers benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as constructing communities of learners engaged in common interests or activities because students need to work in a community of learners with a common goal to understand how to identify sums and products of rational and irrational numbers. For example, students should be given specific questions, such as, Is the outcome of the sum of two irrational numbers always irrational? What about the products of rational and irrational numbers? Does your conjecture hold true for all cases? Can you find a counterexample to disprove your claim? to target the learning goals and allow time within a community to make a conjecture and see if it holds true in all cases.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with use properties of rational and irrational numbers benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because students must understand how to classify numbers as rational or irrational before they can classify the sums and products of rational and irrational numbers. Pre-teaching rational and irrational numbers by providing students with several rational and irrational numbers and having students collaboratively come up with the definition allow students to use reasoning skills and communicate their thinking. Then by allowing students the opportunity to classify sums and products of rational and irrational numbers in this same manner, solidifies the students conceptual understanding of rational and irrational expressions.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with use properties of rational and irrational numbers benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because for students to gain conceptual knowledge of classifying expressions as rational or irrational, calculators are useful as an efficiency tool. Students can use the calculator to visually see when a decimal terminates, has repeating value (leads to discuss why 1.6666… shows as 1.666666667 in the calculator), or is non-terminating, non-repeating. The use of a calculator when determining if an expression is rational or irrational can be used as a tool to prove a student's conjecture or disprove their conjecture by finding a counterexample. Students can then apply their conceptual understanding of the characteristics of a rational or irrational sum or product and use it to classify expressions using reasoning skills without the use of technology.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with use properties of rational and irrational numbers benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as offering opportunities over time to revisit key ideas and linkages between ideas because students need to build on the concept of rational and irrational outcomes when dealing with precision in future mathematics. Students need to understand when a mathematical relationship calls for an exact answer versus an approximate answer. In solving quadratics using the quadratic formula, students are presented with irrational solutions and need to understand the exact and approximate solution when identifying the zeros of the function. The irrational number square root of 17 is close to 4 since the square root of 16 is 4. so, an approximate solution of square root of 17 is about 4.1.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on Classifying Expressions as Rational or Irrational by revisiting student thinking through a short mini-lesson because students cannot classify expressions as rational or irrational before they have developed conceptual understanding of rational and irrational numbers. Revisiting student thinking before the presentation of this concept will show student's unfinished learning, student misconceptions, and students' level of reasoning. Understanding student thinking is essential to present this concept in a way that students can extend their thinking.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit Classifying Sums and Products as Rational or Irrational by helping students move from specific answers to generalizations for certain types of problems because students need time to reason and apply their thinking using generalizations to develop conceptual understanding. For example, students should be given time and tools (calculators) to investigate whether the sum of two irrational numbers are always, sometimes, or never rational. Students can explore sums of different irrational numbers to determine if they are always irrational or can a counterexample be found. Students then should be allowed time to communicate their thinking verbally and in writing to write a general statement regarding the posed question.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying the classification of sums and products of rational and irrational Numbers because students use reasoning skills to make conjectures and provide counterexamples to disprove conjectures and develop deep understanding of the concept. For example, Does the product of a rational and irrational number always produce an irrational product? If not, can you provide a case where it does NOT hold true.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational the types of mathematical tasks are critical because students must develop conceptual understanding of this concept making conjectures regarding the sum and products of rational and irrational numbers , conduct investigations by exploring many cases, providing counter examples if possible to refute the conjecture, and justifying their claims through verbal and written communication. Students who are given rules, do not remember them unless they make a personal connection to the rule. Discovery is the connection students need to truly understand and remember the outcomes of sums and products of rational and irrational numbers.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/
Operations with Rational and Irrational Numbers http://tasks.illustrativemathematics.org/content-standards/HSN/RN/B/3/tasks/690

Relevance to families and communities:

During a unit focused on why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the student interests, how they are able to explore their interests and make connections that provide meaning to their interests can help students learning occurs from the ability to make connections and making sense of how it connects to their world.

Cross-Curricular Connections:

Science: Two irrational numbers that are of great importance in physics are e and π. Consider providing a connection for students to explore irrational numbers in this context, and the fact that whenever we compute a number answer we must use rational numbers to do it, most generally a finite-precision decimal representation.

Social Studies: In high school the New Mexico Social Studies Standards state students should explain and analyze "tension and cooperation between religion and new scientific discoveries". Consider providing a connection for students to learn about Hippias's who was rumored to have been murdered for divulging the existence of irrational numbers.

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HS: ALGEBRA- SEEING STRUCTURE IN EXPRESSIONS

Cluster Statement: A: Interpret the structure of expressions.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.
Note, the A-SSE domain is especially important in the high school content standards overall as a widely applicable prerequisite.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.SSE.A.1: Interpret expressions that represent a quantity in terms of its context.* • **HSA.SSE.A.1.A: Interpret parts of an expression, such as terms, factors, and coefficients.** • **HSA.SSE.A.1.B: Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*** *Note: Algebra 1 focuses on linear, exponential, and quadratic.*	SMP 4: Students can model with mathematics by identifying the meaning of the terms, factors, and coefficients of linear, exponential and quadratic expressions in context. SMP 7: Students can look for and make use of structure in expressions by seeing how the structure of an algebraic expression reveals properties of the function it defines.	• Identify parts of an expression, such as terms, factors, coefficients, exponents, etc. • Interpret simple compound expressions by viewing one or more of their parts as a single entity. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Remember, Understand, Analyze

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.SSE.A.2: Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* *Note: Algebra 1 focuses on linear, exponential, and quadratic.*	SMP 3: Students can construct viable arguments by explaining whether expressions are equivalent using mathematical justifications. SMP 8: Students look for and express regularity in repeated reasoning by connecting exponents to repeated multiplication.	• Recognize equivalent forms of expressions. • Use the structure of an expression to identify ways to rewrite it. • Make generalizations about the possible equivalent forms expressions can have (e.g., a quadratic expression can always be represented as the product of two factors containing its roots). • Rewrite expressions to identify important components, such as where zeros may occur or end behavior.

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		Webb's Depth of Knowledge: 1-2 **Bloom's Taxonomy:** Remember, Understand, Apply
Previous Learning Connections • Connect to identifying and interpreting slope and y-intercept for linear representations. (8.F.3-4) • Connect to rewriting standard linear equation to slope-intercept form for systems of equations. (8.EE.8)	**Current Learning Connections** • Connect to rewriting quadratic functions to find specific key features. (HSA.SSE.B.3) • Connect to rewriting formulas to highlight quantities of interest. (HSA.CED.4)	**Future Learning Connections** • Connect to work with expressions of all function types. (HSA.SSE.A.1-2 - *polynomial and rational*)

Clarification Statement

- HSA.SEE.A.1: The middle grades standards in Expressions and Equations build a ramp from **arithmetic expressions** in elementary school to more sophisticated work with **algebraic expressions** in high school. As the complexity of expressions increases, students continue to see them as being built out of **basic operations**; they see expressions as **sums** of **terms** and **products** of **factors**. In "Animal Populations" students compare P + Q and 2P by seeing 2P as P + P. They distinguish between (Q–P)/2 and Q – P/2 by seeing the first as the **quotient** where the **numerator** is a **difference** and the second as a difference where the second term is a quotient. [This example] illustrates how students are able to see complicated expressions as built out of simpler ones.

Animal Populations

Suppose P and Q give the sizes of two different animal populations, where $Q > P$. In 1–4, say which of the given pair of expressions is larger. Briefly explain your reasoning in terms of the two populations.

1. $P + Q$ and $2P$
2. $\dfrac{P}{P+Q}$ and $\dfrac{P+Q}{2}$
3. $(Q - P)/2$ and $Q - P/2$
4. $P + 50t$ and $Q + 50t$

Task from Illustrative Mathematics. For solutions and discussion, see http://www.illustrativemathematics.org/illustrations/436.

- HSA.SEE.A.2: Seeing structure in expressions entails a dynamic view of an algebraic expression, in which potential rearrangements and manipulations are ever present. An important skill for college readiness is the ability to try possible manipulations mentally without having to carry them out, and to see which ones might be fruitful and which not.

Common Misconceptions

- Students may confuse the parts of an expression, such as counting variables and not terms and therefore misidentifying the number of terms an expression has.
- Students may not have a conceptual basis for patterns, such as an area model for difference of squares, and therefore struggle to recognize and apply them to new situations.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying interpreting the structure of expressions because when student feel comfortable with the vocabulary being used, they are more likely to use it and using the correct terminology when discussing the structure of an equation allows everyone (bother students and teachers) to communicate their ideas and understanding more clearly.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 5.OA.A.2: This standard provides a foundation for work with interpreting the structure of expressions because students write out the numerical expression without the calculation. Students become comfortable with using the vocabulary words: difference, greater than, multiple, etc. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
- 6.EE.A.4: This standard provides a foundation for work with interpreting the structure of expressions because being able to tell if two expressions are equivalent is the building blocks for being able to construct and deconstruct expressions to use their structure. Being able to tell if what you have done to an expression essentially changes it or not leads to the understanding of how to use these changes to manipulate the expressions and equations to better understand their structure. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because using the structure of expressions is mostly about making sense of the problems that they have been given an using different parts of the problem to be able to be creative and solve the problem.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics representations and scaffolds because seeing a variety of different representations allows the students to more easily see the connections between the different parts of the expressions and their possible meanings.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to the structure because the more that students can see the building block of expressions and equations and how they can be combined to give a larger amount of information they will be more prepared to make their own sense of the structure by combining the building blocks in their own way.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners) because students might see the various parts of the structure differently or approach them differently so they will need feedback that helps them to access the other ways of seeing the structure and this will depend greatly on the individual student.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as offering opportunities over time to revisit key ideas and linkages between ideas because learning how to use the structure of an expression in one setting doesn't mean that the students will understand it in all settings so offering the opportunity to revisit the idea in a different setting will allow students to better understand the concept as a whole with regards to the various branches of mathematics.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on interpreting the structure of expressions by clarifying mathematical ideas and/or concepts through a short mini-lesson because the structure of an expression can be looked at in many different ways and you don't students to get locked into one way of thinking about equations, like understanding that slope-intercept, point-slope, and standard form are all useful ways of looking at linear equations and can tell you different things about the equation.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, students may benefit from re-engaging with content during a unit on interpreting the structure of expressions by clarifying mathematical ideas and/or concepts through a short mini-lesson because the structure of an expression can be looked at in many different ways and you don't students to get locked into one way of thinking about equations, like understanding that slope-intercept, point-slope, and standard form are all useful ways of looking at linear equations and can tell you different things about the equation.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying interpreting the structure of expressions because looking at the structure of the different types of equations and disciplines will help reinforce concepts such as the inverse relationship between logarithmic and exponential functions.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying interpreting the structure of expressions the types of mathematical tasks are critical because the conceptual part of interpreting the structure of expressions is foundational for being able to build and understand equations later on and is not something that is going to be culturally relevant to most students' home lives.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

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Question ID 19789

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra	■■□	Heart of Algebra	Linear functions	3. For a linear function that represents a context a. interpret the meaning of an input/output pair, constant, variable, factor, term, or graph based on the context, including situations where seeing structure provides an advantage;	Calculator

The average number of students per classroom at Central High School from 2000 to 2010 can be modeled by the equation $y = 0.56x + 27.2$, where x represents the number of years since 2000, and y represents the average number of students per classroom. Which of the following best describes the meaning of the number 0.56 in the equation?

A. The total number of students at the school in 2000

B. The average number of students per classroom in 2000

C. The estimated increase in the average number of students per classroom each year

D. The estimated difference between the average number of students per classroom in 2010 and in 2000

Rationale

Choice C is correct. In the equation $y = 0.56x + 27.2$, the value of x increases by 1 for each year that passes. Each time x increases by 1, y increases by 0.56 since 0.56 is the slope of the graph of this equation. Since y represents the average number of students per classroom in the year represented by x, it follows that, according to the model, the estimated increase each year in the average number of students per classroom at Central High School is 0.56.
Choice A is incorrect because the total number of students in the school in 2000 is the product of the average

Equivalent Expressions: http://tasks.illustrativemathematics.org/content-standards/HSA/SSE/A/2/tasks/87

Relevance to families and communities:

During a unit focused on interpreting the structure of expressions, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, ask them about structures that they encounter in their everyday lives that help them parse information, such as knowing which gaming platform a video game is operating on or how to substitute ingredients in a recipe based on someone's food allergies.

Cross-Curricular Connections:

Science: Many science formulas take on linear, exponential and quadratic forms. For example, $F = ma$. Consider providing a connection for students to explore these formulas and identify their structure and how knowing that structure helps them make sense of the context.

Social Studies: In high school the New Mexico Social Studies Standards state students should “understand basic economic principles.” Consider providing a connection for students to use expressions to model cost and revenue.

HS: ALGEBRA- SEEING STRUCTURE IN EXPRESSIONS

Cluster Statement: A: Interpret the structure of expressions.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.
Note, the A-SSE domain is especially important in the high school content standards overall as a widely applicable prerequisite.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.SSE.B.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. * • HSA.SSE.B.3.A: Factor a quadratic expression to reveal the zeros of the function it defines. • HSA.SSE.B.3.B: Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. • HSA.SSE.B.3.C: Use the properties of exponents to transform expressions for exponential functions. *For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* *Note: Algebra 1 focuses on quadratic and exponential.*	SMP3: Students can construct viable arguments to explain whether two expressions are equivalent or not. SMP 7: Students can look for and make use of structure in expressions by using equivalent forms of expressions identify important components of functions, such as where zeros may occur or end behavior.	• Write a quadratic expression with rational coefficients in an equivalent form by factoring and by completing the square. • Identify and use the zeros to solve or explain familiar problems. • Use properties of exponents to write equivalent forms of exponential functions with one or more variables, integer coefficients, and nonnegative rational exponents involving operations of addition, subtraction and multiplication, including distributing an exponent across terms within parentheses. • Find the maximum or minimum values of a quadratic function. • Choose an appropriate equivalent form of an expression in order to reveal a property of interest when solving problems. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
Previous Learning Connections • Connect to knowing and apply the properties of integer exponents to generate equivalent, simplified numerical expressions using the properties of exponents. **(8.A.1)**	**Current Learning Connections** • Connect to recognizing and flexibly writing expressions (or rewriting) to use that expression and solve the problem at hand. **(HSA.SSE.A.2)**	**Future Learning Connections** • Connect to factoring polynomial functions of varying degrees. **(HSA.APR.3)** • Connect to completing the square to solve quadratic equations with imaginary solutions. **(HSN.CN.7)**

		• Connect to rewriting exponential equations as logarithmic equations. **(HSF.LE.4)**

Clarification Statement

HSA.SSE.B.3: The Standards emphasize purposeful **transformation of expressions** into **equivalent forms** that are suitable for the purpose at hand. The Standards avoid talking about **simplification**, because it is often not clear what the simplest form of an expression is, and even in cases where that is clear, is in not obvious that the simplest form is desirable for a given purpose.

There are three commonly used forms for a quadratic expression:

- **Standard form**, e.g., $x^2 - 2x - 3$
- **Factored form**, e.g., $(x + 1)(x - 3)$
- **Vertex form** (a **square** plus or minus a **constant**), e.g. $(x - 1)^2 - 4$

Rather than memorize the names of these forms, students need to gain experience with them and their different uses. The traditional emphasis on simplification as an automatic procedure might lead students to automatically convert the second two forms to the first, rather than convert an expression to a form that is useful in each context.

The introduction of **rational exponent**s and systematic practice with the **properties of exponents** in high school widen the field of operations for manipulating expressions.

Common Misconceptions

- When factoring a quadratic where $a > 0$, students may look at c only when determining which factors to use, rather than looking for the factors of the product a and c.
- When completing the square, students may forget to subtract the number that was added inside the parentheses.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying writing expressions in equivalent forms to solve problems because they will be asked to combine several skills that they had previously learned independently of each other, like rewriting exponents, into one larger problem so reviewing these individual skills will help them be more confident in the larger problem.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.EEA.1: This standard provides a foundation for work with writing expressions in equivalent forms to solve problems because they need to be able to manipulate expressions in a basic sense of linear equations if they are going to be successful at manipulating more complex expressions. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features like the font or the format in which the problem is presented (e.g. worksheet, whiteboard, grouping cards because the variety will relieve some of the monotony of accessing problems in the same way all the time and stimulate different aspects of their brains as they encounter the material in the different formats like up on the wall verses always being on a paper in front of them.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that is substantive and informative rather than comparative or competitive because there is not one best right way to write and equation to solve a problem and the nuance need to be highlighted so that students can improve their work over time.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) because students are often asked to interpret a scenario with information given in one form to answer a question and if they don't understand the context of what is happening, it makes it nearly impossible to complete the task.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because the concept is to be able to look at the different forms, not the incidental calculations needed to do so.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because the path forward is not always clear when trying to find the best way to solve a problem, with the most obvious path not always being the most efficient path to the solution.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on writing expressions in equivalent forms to solve problems by critiquing student approaches/solutions to make connections through a short mini-lesson because there are a variety of ways to solve problems and looking at the ways that other students are solving the problems can help the students to make connections between their preferred methods and another that could help them become more efficient at solving similar problems in the future.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit writing expressions in equivalent forms to solve problems by offering opportunities to understand and explore different strategies because different strategies of looking at the equivalent forms are more efficient for certain tasks and exploring when it is most appropriate to use a particular form will help them become more flexible in their problem solving skills. ...

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying writing expressions in equivalent forms to solve problems because some students will pick up on the technical mechanics of a particular technique quickly, so having them go more deeply into why it works will help them gain a better understanding of the overall intricacies of the method. For example, factoring using a variety of methods, like factoring by grouping and how it relates to factoring a traditional trinomial into two binomials.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

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Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. “Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence.” Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or “warm-demander” requires a strong relationship with students and an understanding of the culture of the students. For example, when studying writing expressions in equivalent forms to solve problems supporting productive struggle is critical because students will come to this cluster with a variety of knowledge about how to manipulate equations and there are a lot of correct ways to do so, therefore, they need to be encouraged to work through the process to find the ways that are more effective on their own instead of being asked to memorized rote procedures for a given situation.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 1474142

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math	■■□	Passport to Advanced Mathematics	Nonlinear equations in one variable and systems of equations in two variables	1. Make strategic use of algebraic structure, the properties of operations, and reasoning about equality to d. solve polynomial equations in one variable that are written in factored form;	No Calculator

$x^2+x-12=0$

If a is a solution of the equation above and $a>0$, what is the value of a ?

Rationale
The correct answer is 3. The solution to the given equation can be found by factoring the quadratic expression. The factors can be determined by finding two numbers with a sum of 1 and a product of -12. The two numbers that meet these constraints are 4 and -3. Therefore, the given equation can be rewritten as $(x+4)(x-3)=0$. It follows that the solutions to the equation are $x=-4$ or $x=3$. Since it is given that $a>0$, a must equal 3.

Question Difficulty: ■■□

Profit of a company (HSA.SSE.B.3): http://tasks.illustrativemathematics.org/content-standards/HSA/SSE/B/3/tasks/434

Relevance to families and communities:

During a unit focused on writing expressions in equivalent forms to solve problems, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, asking students to share about structures that they encounter in their everyday that they manipulate, such as changing the formatting of a picture or video so that it is sharable on different device platforms.

Cross-Curricular Connections:

Science: Finding the zeros and maximum for a model that created a projectile motion equation may also require students to rewrite a quadratic in an equivalent form. Consider providing a connection for students to experiment with projectile motion by tossing objects themselves, possibly using technology, and then exchanging equations with another classmate or group to identify key components.

Social Studies: In high school the New Mexico Social Studies Standards state students should “understand basic economic principles.” Consider providing a connection for students to rewrite the model $P(1+r)^t$ for compound interest to identify the quarterly, monthly or weekly interest rate.

NEW MEXICO
Public Education Department

HS: ALGEBRA - ARITHMETIC WITH POLYNOMIALS & RATIONAL EXPRESSIONS

Cluster Statement: A: Perform arithmetic operations on polynomials.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.APR.A.1" Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. *Note: Algebra 1 focuses on linear and quadratic*	SMP 4: Students can model with mathematics by using algebra tiles to model polynomial operations. SMP 7: Students can look for and make use of structure by seeing how the structure of arithmetic is also used when working with polynomials.	• Relate integer arithmetic to polynomial arithmetic. • Demonstrate polynomials are closed under addition, subtraction, and multiplication (but not division). • Add, subtract, and multiply multi-variable polynomials of any degree. **Webb's Depth of Knowledge:** 1 **Bloom's Taxonomy:** Remember, Understand

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• Connect to combining like terms and simplifying expressions using the distributive property **(6.EE.3)**	• Connect to using the properties of operations to write expressions in different but equivalent forms. **(HSA.SSE.A.2)**	• Connect to dividing polynomials. **(HSA.APR.2-3)** • Connect to performing operations with rational expressions **(HSA.APR.7)**

Clarification Statement

HSA.APR.A.1: The development of **polynomials** and **rational expressions** in high school parallels the development of numbers in elementary and middle grades. In elementary school, students might initially see expressions for the same numbers 8 + 3 and 11, or 3/4 and 0.75, as referring to different entities: 8 + 3 might be seen as describing a calculation and 11 is its answer; 3/4 is a fraction and 0.75 is a decimal. They come to understand that these different expressions are different names for the same numbers, that properties of operations allow numbers to be written in different but **equivalent forms**, and that all of these numbers can be represented as points on the number line. In middle grades, they come to see numbers as forming a unified system, the number system, still represented by points on the number line. The whole numbers expand to the integers—with extensions of addition, subtraction, multiplication, and division, and their properties. Fractions expand to the rational numbers—and the four operations and their properties are extended. A similar evolution takes place in algebra. At first algebraic expressions are simply numbers in which one or more letters are used to stand for a number which is either unspecified or unknown. Students learn to use the properties of operations to write expressions in different but equivalent forms. At some point they see equivalent expressions, particularly polynomial and rational expressions, as naming some underlying thing.

Common Misconceptions

- Students might think polynomials are only monomial, binomial, or trinomial.

- Students may not confuse the impact of adding and subtracting polynomials on the degree of the variable.
- Students may not fully distribute the multiplication of polynomials and only multiply like terms.
- When adding and multiplying like terms students may initially confuse $x + x$ as x^2 instead of $2x$.
- Students may not think $x^2 \cdot x = x^3$ is not an example of closure for polynomial multiplication since the result has a different exponent than the factors.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when performing arithmetic operations on polynomials, understanding the relationship between zeros and factors of polynomials, using polynomial identities to solve problems and rewriting rational expressions. because students may have unfinished learning when identifying and combining like terms, understanding the relationship between a zero and a factor, and division of numerical expressions. Students need to understand the connection between numerical and variable expressions. Also, students may have unfinished learning on identifying the parts, such as, coefficient, variable, constant of a variable expression and would benefit from targeted pre-teaching.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.EE.A.4 provides a foundation for work with performing arithmetic operations of polynomials because students learned to identify two expressions as equivalent written in the form y + y + y and 3y by substituting a fixed value for y. Students should use this same reasoning to add and subtract polynomials. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with performing arithmetic operations on polynomials, understanding the relationship between zeros and factors of polynomials, using polynomial identities to solve problems, and rewriting rational expressions benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because this cluster is the building blocks for future mathematical content. Students have difficulty performing operations with polynomials using the algorithms. It is beneficial to link operations with polynomials back to operations with numerical expressions to help build conceptual understanding. Students will benefit from novel and relevant problems using multiple entry points and various strategies, colored pencils, area models, algebra tiles, technology, etc. It is essential that students gain a conceptual understanding of polynomials to include all operations adding, subtracting, multiplying, and dividing as well as a conceptual understanding of zeros of a polynomial and how the zeros help provide a sketch of the graph.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with performing arithmetic operations on polynomials, understanding the relationship between zeros and factors of polynomials, using polynomial identities to solve problems, and rewriting rational expressions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success because students need to view errors and wrong answers as a tool for learning how to validate solutions as viable outcomes. When students can make sense of the problem and understand solutions as viable/non-viable they are able to use reasoning to justify their solution or use strategies to improve their solution.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with performing arithmetic operations on polynomials, understanding the relationship between zeros and factors of polynomials, using polynomial identities to solve problems, and rewriting rational expressions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as highlighting structural relations or make them more explicit because making connections to prior learning is essential for conceptual understanding of this domain. Connections such as linking perimeter to a variable with an exponent of one, which is a linear unit, area to a variable with an exponent of two, which is a square unit, long division of numerical expressions to long division of polynomials, and zeros of a polynomial function to the x-intercept (0,y) helps students make those relevant connections which solidifies their understanding of performing operations on polynomials and identifying zeros.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with performing arithmetic operations on polynomials, understanding the relationship between zeros and factors of polynomials, using polynomial identities to solve problems, and rewriting rational expressions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks, 3D models, base-ten blocks) because making connections to area models through algebra tiles help students understand adding and subtracting like terms, multiplying and dividing polynomials. Providing students with the flexibility to use actual algebra tiles, virtual algebra tiles, or sketching area models using colored pencils allows all students to access the content and build conceptual understanding, so the algorithms make sense when applied.

Internalize

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with performing arithmetic operations on polynomials, understanding the relationship between zeros and factors of polynomials, using polynomial identities to solve problems, and rewriting rational expressions benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as using templates that guide self-reflection on quality and completeness because students must have time for self-reflection in order to understand how to self-assess work and determine if their solution makes sense and is a viable solution. It is imperative students check work for quality and completeness because students will make errors when performing operations with polynomials. Also, by allowing students time to communicate and discuss solutions with peers provides students strategies to improve their solution.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on performing arithmetic operations on polynomials, by examining tasks from a different perspective through a short mini-lesson because students need to understand the parts of the expression are related to the outcome <i.e. Sum, difference, product, quotient>. Given the outcome and one of its parts, students can find the other part. Example: (4x + 6) + ? = 8x -10

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit performing arithmetic operations on polynomials by offering opportunities to understand and explore different strategies because some students may need support strategies, such as using colored pencils to color code like terms, using algebra tiles to perform operations on polynomials, or the use of calculators to assist in the adding or subtracting of integers. ...

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students. when studying performing arithmetic operations on polynomials because students need to expand their algebraic thinking to gain a deeper understanding of polynomials by generating their own equivalent expressions. Students' understanding of integer sums, differences, products and quotients will be reinforced when students are asked to use reasoning to generate their own equivalent expressions.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying understanding that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; adding, subtracting, and multiplying polynomials the types of mathematical tasks are critical because students need to build procedural fluency by practice that is embedded in tasks that build their conceptual understanding. Students need to understand conceptually like terms, how and why the result of adding and multiplying polynomials is different, how multiplying polynomials is connected to an area model, and how adding polynomials connects to a linear model like perimeter. Algebra tiles, area models, tasks involving perimeter and area will help students build conceptual understanding while improving their procedural fluency.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 19948

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	**Math**	**Passport to Advanced Math**	■■■	**Passport to Advanced Mathematics**	**Equivalent expressions**	**2. Fluently add, subtract, and multiply polynomials.**	**Calculator**

19948

$(-3x^2+5x-2)-2(x^2-2x-1)$

If the expression above is rewritten in the form ax^2+bx+c, where a, b, and c are constants, what is the value of b ?

Rationale

The correct answer is 9. To rewrite the difference $(-3x^2+5x-2)-2(x^2-2x-1)$ in the form ax^2+bx+c, the expression can be simplified by using the distributive property and combining like terms as follows:

$(-3x^2+5x-2)-(2x^2-4x-2)$

$(-3x^2-2x^2)+(5x-(-4x))+(-2-(-2))$

$-5x^2+9x+0$

Non-Negative Polynomials:
https://drive.google.com/drive/u/0/folders/1QLXJMWDXpb4MRU4Oj7QDVPkypf67usG9

Powers of 11: https://drive.google.com/drive/u/0/folders/1QLXJMWDXpb4MRU4Oj7QDVPkypf67usG9

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on understanding that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; adding, subtracting, and multiplying polynomials, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, by discussing how their	Industrial Arts: Often construction makes use of multiplying polynomials in deciding how to design various aspects of a house or office to fit a predetermined area. Consider providing a connection for students to design something like a sliding door given a specific frame to height ratio and surrounding framework and then plugging in to find the total area for different input values.

family culture celebrates an event. Use that event to show some elements of each families' celebration may be similar and different but each is valid and link it to learning. Some students may need to use area models, some may need colored pencils, some students may prefer to add horizontally, and some will prefer to add vertically. Although we learn in different ways, our learning is valid.	Art: Often students like to "play" with manipulatives. Consider having students make a work of art using algebra tiles and then create a polynomial expression to represent their artwork.

NEW MEXICO
Public Education Department

HS: ALGEBRA – CREATING EQUATIONS

Cluster Statement: A: Create equations that describe numbers or relationships.
Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

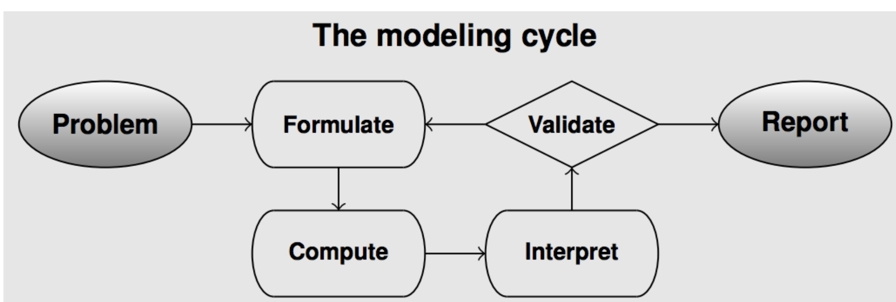
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSA.CED.A.1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* * *Note: Algebra 1 focuses on linear, quadratic, and exponential (integer inputs only)*	SMP 1: Students can make sense of problems and persevere in solving them by appropriately choosing from among linear, exponential and quadratic functions when creating an equation or inequalities in one variable to solve a problem. SMP 4: Students can model with mathematics by creating equations or inequalities with one variable when given a problem with context.	• Write a linear equation in one variable based on a given context and use their equation to solve problems. • Write quadratic equations in one variable based on a given context and use their equation to solve problems. • Write an exponential equation in one variable based on a given context and use their equation to solve problems. • Write inequalities in one variable based on a given context and use their inequality to list possible solutions for the problem. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSA.CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.* *Note: Algebra 1 focuses on linear, quadratic, and exponential (integer inputs only)*	SMP 1: Students can make sense of problems and persevere in solving them by appropriately choosing from among linear, exponential and quadratic functions when creating a system of equations or inequalities in two or more variables to solve a problem. SMP 4: Students can model with mathematics by creating a system equations or inequalities in two or more variable when given a problem with context.	• Write equations in two or more variables based on a given context. • Write equations in two or more variables combining linear, quadratic and/or exponential equations based on a given context. • Graph equations on coordinate axes with scales clearly labeling the axes, defining what the values on the axes represent and the unit of measure. • Select intervals for the scale that are appropriate for the context and display adequate information about the relationship.

		• Analyze points on and off a graph and interpret them in context.
		Webb's Depth of Knowledge: 1-2
		Bloom's Taxonomy: Understand, Apply, Analyze
Standard Text **HSA.CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.** *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. ** *Note: Algebra 1 focuses on linear only*	**Standard for Mathematical Practices** SMP 2: Students can reason abstractly and quantitatively by contextually, analytically and graphically checking a solution set of inequalities to determine the viability of each solution. SMP 4: Students can model with mathematics by representing constraints using equations or inequalities and systems of equations and/or inequalities when given a problem with context.	**Students who demonstrate understanding can:** • Identify constraints of equations, inequalities, and systems of equations and inequalities given a context • Interpret solutions of equations, inequalities, and systems of equations and inequalities as viable or non-viable given a context. • Interpret solutions analytically and graphically to answer questions about the quantities in context.
		Webb's Depth of Knowledge: 1-3
		Bloom's Taxonomy: Understand, Apply, Analyze, Evaluate
Standard Text **HSA.CED.A.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's***	**Standard for Mathematical Practices** SMP 4: Students can model with mathematics by applying literal when given a problem in context. SMP 7: Students can reflect and recognize the various structures in mathematic formulas and use them	**Students who demonstrate understanding can:** • Make connections between solving equations and rearranging formulas. • Apply inverse operations to rearrange formulas for a specified variable.

law V = IR to highlight resistance R.* *Note: Algebra 1 focuses on linear, quadratic, and exponential (integer inputs only)*	when solving problems requiring those formulas.	**Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply
Previous Learning Connections • Connect to creating and solving equations in one variable. **(7.EE.4)** • Connect to reasoning with inequalities. **(7.EE.4)** • Connect to solving real-world problems involving two linear equations in two variables. **(8.EE.8)**	**Current Learning Connections** • Connect to graphing equations and inequalities. **(HSF.IF.7)** • Connect to graphing systems of equations and inequalities. **(HSA.REI.7)** • Connect to solving equations in one variable including those equations with coefficients represented by variables. **(HSA.REI.3-4)** • Connect to communicating relevant domain and range for linear, exponential and quadratic functions. **(HSF.IF.4)**	**Future Learning Connections** • Connect to extending knowledge to include additional types of functions such as trigonometric, rational, and polynomial. **(HSA.CED.1-4)** • Connect to communicating relevant domain and range for all types of functions. **(HSF.IF.4)**

Clarification Statement

- HSA.CED.A.1: The repertoire of **functions** that is acquired during high school allows students to create more **complex equations**, including equations arising from **linear** and **quadratic expressions**, and **simple rational** and **exponential expressions**.
- HSA.CED.A.2: [Students use complex equations—including equations arising from linear and quadratic expressions, and simple rational and exponential expressions—to model] relationships between **quantities** with **equations in two variables**.
- HSA.CED.A.3: All the standards in the Creating Equations group carry a modeling star, denoting their connection with the Modeling category in high school. This connotes not only an increase in the complexity of the equations studied, but an upgrade of the student's ability in every part of the modeling cycle.



- HSA.CED.A.4: There are situations where an equation is used to describe the **relationship** between a number of different quantities. For example, Ohm's Law V = IR relates the voltage, current, and resistance of an electrical circuit. An equation used in this way is sometimes called a **formula**. It is perhaps best to avoid using the terms "**variable**", "**parameter**", or "**constant**" when working with this formula, because there are six different ways it can be viewed as defining one quantity as a **function** of the other with a third held constant.

Common Misconceptions

- Choosing the correct form of the equation can often be difficult when first introducing exponential and quadratic equations to students. They will often try to make everything linear.
- Students tend to struggle without the benefit of having numbers involved. They will often forget how algebra works while working through problems with this standard.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that previews new contexts for tasks within the unit (e.g., cell phone plans) when studying creating equations that describe numbers or relationships because doing so allows students to better understand that writing expressions, equations, or inequalities to represent data is a highly useful tool that can be used in a variety of different scenarios and when wielded by them, will allow for a broader application of the concepts they are learning.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.EE.A.2: This standard provides a foundation for work with creating equations that describe numbers or relationships because this is the first time that students are being asked to read, write, and evaluate expressions in which letters stand for numbers and many students have trouble making this transition. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with creating equations that describe numbers or relationships benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features such as creating equations, graphs, tables, and verbal models because students comprehend information in a variety of ways that can be illustrated with the various ways of displaying the information to highlight different aspects of the model.

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with creating equations that describe numbers or relationships benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because equations that model the students' real lives and have relevance to them will encourage them to engage more deeply with the content since they will be able to connect it to a future career path or current situation.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with creating equations that describe numbers or relationships benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams because the language of mathematics is more than just one way of representing the information and all of the different ways need to be included and explained in order for students to fully understand the content.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with creating equations that describe numbers or relationships benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) because students who are able to be successful in creating their own equations that represent their real world and can do so in a way that there is a safety net to help them are more likely to want to interact with the information in a variety of ways.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with creating equations that describe numbers or relationships benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) because numbers and relationships are a foundational topic in algebra and providing the connections and context for how using the relationships that you can create in order to solve problems given different parameters will allow students to have a more flexible way of approaching problems in the future.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on creating equations that describe numbers or relationships by clarifying mathematical ideas and/or concepts through a short mini-lesson because creating equations has such a broad level of application, from linear and proportional to exponential, quadratic, logarithmic, and trigonometric meaning that this has the opportunity to be studied from many different perspectives and the better that is understood about one type of problem, the better it will be understood for the others.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit creating equations that describe numbers or relationships by addressing conceptual understanding because creating equations is best done in the context of a real-world problem and understanding the underlying relationships of why a particular equation is preferred over another will allow students to more readily choose the appropriate type of equation in the future. ...

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying creating equations that describe numbers or relationships because it would allow them to build the context of why they are building equations and the purposes of what using the equations would allow them to do. For example, they could explore the link between how building an equation to model the cost of a project based on the material costs and size constraints can help when calculating costs in manufacturing and construction.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying creating equations that describe numbers or relationships the use of mathematical representations within the classroom is critical because creating equations is a skill that focuses on transforming the world around us into numbers and symbols and in doing so, care needs to be taken to emphasize that this stripping down of the world is not a discarding of the cultural aspects of the situation. Making the connections between the math that is being used and the thing it is being used to analyze is imperative in this context.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 1054727

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra	■■□	Heart of Algebra	Linear functions	4. Make connections between verbal, tabular, algebraic, and graphical representations of a linear function, by a. deriving one representation from the other;	No Calculator

1054727

The graph in the xy-plane of the linear function f contains the point $(3,4)$. For every increase of 5 units in x, $f(x)$ increases by 3 units. Which of the following equations defines the function?

A. $f(x) = -\frac{5}{3}x + 9$

B. $f(x) = -\frac{3}{5}x + \frac{29}{5}$

C. $f(x) = \frac{3}{5}x + \frac{11}{5}$

D. $f(x) = \frac{5}{3}x - 1$

Cash box
http://tasks.illustrativemathematics.org/content-standards/HSA/CED/A/tasks/462

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on creating equations that describe numbers or relationships, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, this could be looking at the utility bills for someone in the neighborhood and seeing how the rates are calculated using the unit rate of energy or water.	Economics: Linear programming with a system of inequalities is often used to model the constraint of resources for production. Consider providing a connection where students are starting their own business and must maximize profit or production with the possible solutions of the system. Science: There are many formulas in science such as Ohm's Law and the Doppler formulas that may require isolating and solving for a specific variable given certain conditions. Consider providing a connection where students must rearrange the same formulas in multiple ways to highlight different quantities of interest.

NEW MEXICO
Public Education Department

HS: ALGEBRA- REASONING WITH EQUATIONS AND INEQUALITIES

Cluster Statement: A: Understand solving equations as a process of reasoning and explain the reasoning.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.REI.A.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. *Note: Algebra 1 focuses on mastering linear; learn as general principle*	SMP 3: Students can construct viable arguments and critique the reasoning of others by comparing and justifying the set of steps used to solve equations and obtain solutions. SMP 5: Students can use tools by using algebra tiles, tape diagrams, multi-colored chips and more to develop the formal application of algebra as an abstract tool for solving mathematical problems.	• Explain why an equation is equivalent when performing operations to isolate a variable. • Construct arguments for equality using visual representations. • Justify reasoning for elimination of coefficients and/or constants and other steps using multiple types of operations, including multiplication of fractions. **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy:** Understand, Apply, Evaluate

<u>**Previous Learning Connections**</u>	<u>**Current Learning Connections**</u>	<u>**Future Learning Connections**</u>
• Connect to applying the associative, commutative, distributive, and identity properties. **(3.OA.5)** • Connect to learning math properties and their names.**(7.NS.1-2)** • Connect to using variables to write expressions and equations. **(6.EE.2)** • Connecting to solving linear equations. **(7.EE.4, 8.EE.7)**	• Connect to creating and solving equations and inequalities in one variable. **(HSA.CED.1, HSA.REI.3)**	• Connect to justifying steps in solving rational and radical equations. **(HSA.REI.2)** • Connect to justifying steps in writing proofs for geometry. **(HSG.CO.9-11, HSG.SRT.4-5)**

Clarification Statement
HSA.REI.A.1: A written sequence of steps to **solve** an **equation** is code for a narrative line of **reasoning** using words like "**if**," "**then**," "**for all**," " and "**there exists**." In the process of learning to solve equations, students learn certain standard **"if-then" moves**, for example "if x = y then x + 2 = y + 2." The danger in learning **algebra** is that students emerge with nothing but the moves, which may make it difficult to detect incorrect or made-up moves later. Thus, the first requirement in the standards in this domain is that students understand that solving equations is a process of reasoning. This does not necessarily mean that they always write out the

full text; part of the advantage of **algebraic notation** is its compactness. Once students know what the code stands for, they can start writing in code.

Fragments of reasoning

$$x^2 = 4$$
$$x^2 - 4 = 0$$
$$(x-2)(x+2) = 0$$
$$x = 2, -2$$

This sequence of equations is short-hand for a line of reasoning:

> If x is a number whose square is 4, then $x^2 - 4 = 0$. Since $x^2 - 4 = (x-2)(x+2)$ for all numbers x, it follows that $(x-2)(x+2) = 0$. So either $x - 2 = 0$, in which case $x = 2$, or $x + 2 = 0$, in which case $x = -2$.

More might be said: a justification of the last step, for example, or a check that 2 and -2 actually do satisfy the equation, which has not been proved by this line of reasoning.

Common Misconceptions

- Students do not recognize equality is a relationship between two quantities or, more generally two mathematical expressions, asserting that the quantities have the same value, or that the expressions represent the same mathematical object.
- Students may perform inappropriate operations on polynomials. Students may subtract from coefficients and constants when subtracting on both sides of an equation or multiply only coefficients when multiplying both sides of an equation.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying solving equations and explaining each step because students may need to justify the inverse operation used in each step with viable arguments. Students may practice expressing their mathematical thinking verbally and symbolically.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.EE.B.5: This standard provides a foundation for work with reasoning and solving one-variable equations because students need to understand each step of solving one-variable equations and explain the reason for each step. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with understanding and explaining the reasoning of solving equations benefit when learning experiences include ways to recruit interest such as providing choices in their strategies of solving equations and in their reasoning because students make connections of their prior knowledge of solving equations in different problems. By showing a different order of applying the inverse operations to the equations, students gain new skills and knowledge of solving complex equations and deeper understanding of solving equations.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with explaining reasoning of each step of solving equations benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities because students engage in meaningful discourse to construct viable arguments with the support of the cooperative learning group. Students justify and make connections with reasoning of other strategies used by other learners in the cooperative learning groups.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with explaining the reasoning of each step of solving equations and constructing viable argument benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because students build their reasoning and viable argument using their prior knowledge of solving one-step or two-step equations. Students connect their understanding of inverse operation to justify each step of solving equations.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with explaining the reasoning of solving equations benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because students justify solving the equations in multiple ways and communicate their mathematical thinking verbally and symbolically. By presenting their mathematical thinking in multiple ways, students make connections of conceptual knowledge and gain fluency in procedural knowledge.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with explaining the reasoning of solving equations with viable arguments benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations because students apply the knowledge of solving one-variable equations to solving literal equations. Students identify the patterns of solving equations and make generalization of solving and rearranging equations.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on explaining the reason of each step of solving equations by critiquing student approaches/solutions to make connections through a short mini-lesson because students need to understand why the specific inverse operation is used and develop the viable argument using properties of equality.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit explaining the steps of solving equations by offering opportunities to understand and explore different strategies because students need to understand why some steps are interchangeable when solving the equations. Students need to explain the order of applying the inverse operations and how that relates to the order of operation of the equations.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying solving complex equations and explaining the steps because students may deepen their understanding of inverse operation, such as logarithm as the inverse operation of exponent. Students explore strategies of solving equations with complex operations and justify their reason in cooperative learning groups.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying understanding solving equations as a process of reasoning and explaining the reasoning facilitating meaningful mathematical discourse is critical because students practice expressing their mathematical thinking using the content language. Students compare and evaluate different entry points of solving equations. Students defend their strategies by constructing viable arguments and build confidence in math.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: https://satsuitequestionbank.collegeboard.org/

Question ID 1053197

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math	■■□	Passport to Advanced Mathematics	Nonlinear equations in one variable and systems of equations in two variables	1. Make strategic use of algebraic structure, the properties of operations, and reasoning about equality to b. solve simple rational and radical equations in one variable;	No Calculator

1053197

If $\frac{8}{x} = 160$, what is the value of x ?

A. 1,280

B. 80

C. 20

D. 0.05

Rationale

Choice D is correct. Multiplying both sides of the given equation by x yields $160x = 8$. Dividing both sides of the equation $160x = 8$ by 160 results in $x = \frac{8}{160}$. Reducing $\frac{8}{160}$ to its simplest form gives $x = \frac{1}{20}$, or its decimal

Zero Product Property 1
http://tasks.illustrativemathematics.org/content-standards/HSA/REI/A/1/tasks/2141

Relevance to families and communities:

During a unit focused on solving equations as a process of reasoning consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different ways of relating the steps of solving equations in real-life application of using equations. Students might work backward to solve for the unknown quantity is the same as students use inverse operations to solve the equation.

Cross-Curricular Connections:

Language Arts: Justifying reasoning is a form of persuasive writing, as students are trying to get others to agree that their solving process is appropriate and accurate. Consider providing a connection for students to write out the full text (as referenced above) in more of an essay format.

Science: When students write up a lab report they often must detail how they tested their hypothesis and clarify why they performed their experiment in a specific way. Consider providing a connection where students must make some "prediction" or hypothesis about an equation prior to solving and then write up their solving method in a format like a lab report.

NEW MEXICO
Public Education Department

HS: ALGEBRA- REASONING WITH EQUATIONS AND INEQUALITIES

Cluster Statement: B: Solve equations and inequalities in one variable.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.REI.B.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. *Note: Algebra 1 focuses on linear inequalities; literal that are linear in the variables being solved for; quadratics with real solutions*	SMP 2: Students can reason abstractly and quantitatively by solving equations with and without models. SMP 8: Students look for and express regularity in repeated reasoning by connecting the steps to solve an equation or inequality with variable coefficients to an equation or inequality with integer coefficients.	• Solve linear equations, including ones that require using the distributive property, combining like terms, variables on both sides and rational coefficients. • Solve literal equations to isolate a specific variable (e.g., rewriting point slope form to solve for *m*). • Solve linear inequalities, including ones with negative coefficients. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSA.REI.B.4: Solve quadratic equations in one variable. • **HSA.REI.B.4.A: Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.** • **HSA.REI.B.4.B: Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the**	SMP 5: Students can use tools by using pictures, algebra tiles and/or symbols to explain the concept underlying completing the square. SMP 7: Students can look for and make use of structure by choosing the most efficient method to solve a quadratic equation.	• Derive the quadratic formula from the general form of a quadratic equation. • Solve quadratic equations in one variable with real solutions by inspection, taking square roots, completing the square, using the quadratic formula and factoring. • Identify the number and types of solutions of a quadratic equation using the discriminant. **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy:** Understand, Apply, Analyze, Evaluate

equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. *Note: Algebra 1 focuses on linear inequalities; literal that are linear in the variables being solved for; quadratics with real solutions*		
<u>Previous Learning Connections</u> • Connect to solving equations and inequalities in one variable. **(7.EE.4, 8.EE.7)** • Connect to solving equations involving squares and square roots. **(8.EE.2)**	**<u>Current Learning Connections</u>** • Connect to solving quadratic equations and relating solutions to the graph of the function. **(HSF.IF.7)** • Connect to use completing the square and factoring to rewrite quadratic functions in vertex and intercept form to identify key features of the graph. **(HSS.SSE.3)**	**<u>Future Learning Connections</u>** • Connect to solving additional types of nonlinear equations. **(HSA.REI.2)** • Connect to relating knowledge of solving quadratic equations to complex numbers, solving rational equations, trigonometric equations, and trigonometric form. **(HSN.CN.7, HSA.REI.2, HSF.TF.5, 7)** • Connect to understanding the need for a variety of methods (factoring, completing the square, and using quadratic formula) when solving other types of equations, such as parabolas, hyperbolas, and ellipses. **(HSG.GPE.A)**
Clarification Statement • HSE.REI.B.3: With an understanding of **solving equations** as a reasoning process, students can organize the various methods for solving different types of equations into a coherent picture. For example, solving **linear equations** involves only steps that are reversible (adding a **constant** to both sides, multiplying both sides by a **non-zero** constant, t**ransforming an expression** on one side into an **equivalent expression**). Therefore, solving linear equations does not produce **extraneous solutions**. • HSE.REI.B.4a: The key step in **completing the square** involves at its heart **factoring**. And the **quadratic formula** is nothing more than an encapsulation of the method of completing the square, expressing the actions repeated in solving a collection of quadratic equations with **numerical coefficients** with a single formula. (MP.8) • HSE.REI.B.4b: It is traditional for students to spend a lot of time on various techniques of solving **quadratic equations**, which are often presented as if they are completely unrelated (factoring, completing the square, the quadratic formula). Students with an understanding of the underlying reasoning behind all these methods are opportunistic in their application, choosing the method that best suits the situation at hand.		
Common Misconceptions • Since the steps for solving addition and subtraction equations and inequalities are similar, students often forget to change the direction of the inequality sign when multiplying or dividing by a negative coefficient. • Students will often gravitate toward one solution method or another and try to use it in every possible situation given rather than paying attention to the structure of the equation and choosing the method that is most appropriate to use based on its structure.		

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying solving equations and inequalities in one variable because knowing this will help prevent errors when solving this type of problem. Students will know what to look for and be aware of when approaching the problems.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.EE.B.5Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.: This standard provides a foundation for work with solving the equations and inequalities because students will learn that solving is a process of reasoning to find the numbers which make an equation true, which can include checking if a given number is a solution. Although the process of reasoning will eventually lead to standard methods for solving equations, students should study examples where looking for structure pays off. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners solving equations and inequalities in one variable benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features using different background colors, or changing the size of the text used to present the equations or inequalities because the contrast in font size or background color will help students more accurately depict where their focus should be. It also makes it easier for students to read taking at least one potential barrier out of their way.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with solving equations and inequalities in one variable benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics representations and scaffolds because this cluster is a continuation of standards from earlier grades. Students are deepening and broadening their skills with solving equations in Algebra 1. The more representations, examples, and problem types they see, the better they will become at solving equations and inequalities. Additionally, students need to be given multiple entry points and different strategies based on their learning style when approaching problems within this cluster. This will help them to internalize their learning as well as take ownership of it.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with solving equations and inequalities in one variable benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because pre-teaching or reteaching what students have learned in prior grades will give you an idea of what they remember about solving equations and inequalities and what might need to be retaught. Some students are experts while others have no clue where to begin. In pre-teaching with simpler equations students have a lower entry point into the lesson and you can build on the skills they have from prior courses.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with solving equations and inequalities in one variable benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches, strategies, skills, etc.) because the more ideas and solution methods students are exposed to the more likely they are to find a few that they can use successfully. Students need multiple ways to address problems and when introduced at this early stage it shows students that there is more than one way to approach the problem and that different doesn't always mean wrong.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with solving equations and inequalities in one variable benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because students need repetition in this foundational skill in order to internalize it. Solving equations and inequalities is a process that can take on many different forms. The more that students can practice the more strategies they will learn and make sense of. With additional practice students are also able to better decide which strategy is more suited to which problem type.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on solving equations and inequalities in one variable by providing specific feedback to students on their work through a short mini-lesson because < completing a task that compares equations and inequalities side by side and using the previous learned steps in solving both problems allows them to practice the skills that they have learned previously and reinforce them.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit solving equations and inequalities in one variable by offering opportunities to understand and explore different strategies> because students need opportunities to explore different methods and find which one works best for them. ...

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying solving equations and inequalities in one variable because making connections help students appreciate learning the concept more and gives them opportunities to see where it may be going.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying Solving equations and inequalities in one variable the pattern of questions within the classroom is critical because promoting student learning in It should connect students' lived experiences and interests (their only resources for learning something new) to disciplinary problems in the world. For example, how are verbal and algebraic models and formulas used to represent real life situations? This allows students to come up with their own ideas and make it personable.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 4788931

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra	■□□	Heart of Algebra	Linear equations in two variables	3. For a linear equation in two variables that represents a context b. given a value of one quantity in the relationship, find a value of the other, if it exists.	Calculator

The equation $y=0.1x$ models the relationship between the number of different pieces of music a certain pianist practices, y, during an x-minute practice session. How many pieces did the pianist practice if the session lasted 30 minutes?

A. 1

B. 3

C. 10

D. 30

Rationale

Choice B is correct. It's given that the equation $y=0.1x$ models the relationship between the number of different pieces of music a certain pianist practices, y, and the number of minutes in a practice session, x. Since it's given that the session lasted 30 minutes, the number of pieces the pianist practiced can be found by substituting 30 for x in the given equation, which yields $y=0.1(30)$, or $y=3$.

Choices A and C are incorrect and may result from misinterpreting the values in the equation. Choice D is incorrect. This is the given value of x, not the value of y.

Reasoning with linear inequalities: http://tasks.illustrativemathematics.org/content-standards/HSA/REI/B/3/tasks/807

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on solving equations and inequalities in one variable, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, allowing students the autonomy to choose and create problems relevant to their home culture provides students a connection to the world of mathematics.	Science: Projectile motion is modeled by quadratic functions. Consider providing a connection for students to experiment with projectile motion by tossing objects themselves, possibly using technology, and then exchanging equations with another classmate or group to solve. Language Arts: Explaining a process is a form of expository writing, as students are trying to give facts and information. Consider providing a connection for students to write out the derivation of the quadratic formula from standard form to help them see and explain how the two forms are related.

HS: ALGEBRA- REASONING WITH EQUATIONS AND INEQUALITIES

Cluster Statement: C: Solve systems of equations.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.REI.C.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. *Note: Algebra 1 focuses on Linear-linear and linear-quadratic.*	SMP3: Students can construct viable arguments to verify why replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. SMP 8: Students look for and express regularity in repeated reasoning by working with different sets of equations to come to understand why the elimination method works.	• Transform a given system of two equations in two variables into an equivalent system that has the same solutions as the original system. • Prove that both systems have the same solution. **Webb's Depth of Knowledge:** 2-3 **Bloom's Taxonomy:** Apply, Analyze, Evaluate
Standard Text **HSA.REI.C.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.** *Note: Algebra 1 focuses on Linear-linear and linear-quadratic.*	**Standard for Mathematical Practices** SMP 3: Students can construct viable arguments by explaining why the graphing, elimination, or substitution method is the best method to solve a system of equations. SMP 7: Students can look for and make use of structure by recognizing systems of equations that have no solution or infinite solutions.	**Students who demonstrate understanding can:** • Solve a system of linear equations using substitution. • Solve a system of linear equations using elimination. • Solve a system of linear equations by graphing by hand. • Solve a system of linear equations using graphing technology (or Desmos) to estimate more complicated solutions (non-terminating rational solutions). • Differentiate among situations where one solution, no solutions or infinite solutions occur. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze

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<table>
<tr>
<td rowspan="3">Standard Text

HSA.REI.C.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = -3x and the circle $x^2 + y^2 = 3$.

Note: Algebra 1 focuses on Linear-linear and linear-quadratic.</td>
<td rowspan="3">Standard for Mathematical Practices

SMP 5: Students can use tools by graphing by hand or using technology and using algebraic methods or with CAS.

SMP 7: Students can look for and make use of structure by recognizing systems consisting of one linear and one quadratic equation that have no solution, one solution or two solutions.</td>
<td>Students who demonstrate understanding can:
• Solve a simple system of a linear equation and a quadratic equation algebraically.
• Solve a simple system of a linear equation and a quadratic equation by graphing by hand.
• Differentiate among situations where one solution, no solutions or two solutions occur.</td>
</tr>
<tr>
<td>Webb's Depth of Knowledge: 1-2</td>
</tr>
<tr>
<td>Bloom's Taxonomy:
Understand, Apply, Analyze</td>
</tr>
<tr>
<td><u>Previous Learning Connections</u>
• Connect to solving systems of linear equations with a focus on graphing and substitution. (8.EE.8)</td>
<td><u>Current Learning Connections</u>
• Connect to creating a system of linear equations or inequalities in a real- world context. (HSA.CED.3)</td>
<td><u>Future Learning Connections</u>
• Connect to using matrices to solve systems of linear equations. (HSA.REI.8-9)</td>
</tr>
<tr>
<td colspan="3">Clarification Statement
• HSA.REI.C.5: Student work with solving systems of equations starts the same way as work with solving equations in one variable; with an understanding behind the various techniques. An important step is realizing that a solution to a system of equations must be a solution [to] all the equations in the system simultaneously. Then the process of adding one equation to another is understood as "if the two sides of one equation are equal, and the two sides of another equation are equal, then the sum of the left sides of the two equations is equal to the sum of the right sides." Since this reasoning applies equally to subtraction, the process of adding one equation to another is reversible, and therefore leads to an equivalent system of equations.
• HSA.REI.C.6: [Systems of two linear equations with two variables] also have the advantage that a good graphical visualization is available; a pair (x,y) satisfies two equations in two variables if it is on both their graphs, and therefore an intersection point of the graphs.
• HSA.REI.C.7: Another important method of solving systems is the method of substitution. Again, this can be understood in terms of simultaneity; if (x, y) satisfies two equations simultaneously, then the expression for y in terms of x obtained from the first equation should form a true statement when substituted into the second equation. Since a linear equation can always be solved for one of the variables in it, this is a good method when just one of the equations in a system is linear.</td>
</tr>
<tr>
<td colspan="3">Common Misconceptions
• Students may not realize that a solution to a system of equations must be a solution of all the equations in the system simultaneously and find a pair (x, y) that only fits one of the equations.
• Students may incorrectly apply basic integer operations when substituting one equation into another one.
• Students may believe that a line and a circle have nothing in common.</td>
</tr>
</table>

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying solving systems of equations because understanding common errors will help clarify understanding and avoid making the same mistakes.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8. EE.C.8 Analyze and solve pairs of simultaneous linear equations. This standard provides a foundation for work with solving equations simultaneously graphically, algebraically, or with a matrix because understanding that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with solving systems of equations benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because students struggle with the solving systems of equations and then resort to wondering why they have to learn it. When we make the problems relevant, they can see the applicable nature of this cluster and the necessity of learning how to solve symptoms of equations in many ways.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with solving systems of equations benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that is substantive and informative rather than comparative or competitive because students need to not only know what mistakes they made but they need to learn from them. When we give substantive and informative feedback it is feedback FOR learning rather than just an arbitrary number (or letter) for a grade.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with solving systems of equations benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because whether you teach solution methods in isolation or in some other way you can build on what students already know and understand. You can take the methods and structures they are more familiar with in solving linear equations and inequalities in one variable and apply them to systems of linear equations and other types of systems of equations. In this way students can build on what they already know about solving and realize that they are using similar ideas, with similar results on a more complicated problem type.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with solving systems of equations benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because the standards within this cluster indicate that students should use all three methods for solving systems of equations (graphically, with substitution and elimination). However, they tend to gravitate to the solution method that comes easiest to them or the one that they understand the most. Because of this we need to accept their first attempt and encourage them to try another method with the same problem. In this way they again can see that different approaches will produce the same results. In turn they will build their confidence with using different solution methods and recognize that just because they approach a problem in a different way from their peers does not mean that they've done the problem incorrectly especially if they come up with the same solution.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with solving systems of equations benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing interactive representations that guide exploration and new understandings because students will be able to differentiate that there is more than one way to solve a problem in the real world and know how to pick the best solution method.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on solving system of equations by revisiting student thinking through a short mini lesson because sometimes students need a refresher in prior knowledge to help them continue in the task.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit system of equations by confronting student misconceptions because learning from other students' mistakes can help develop their own understanding and help them to not continue to make the same mistakes. ...

Extension

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What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying solving systems of equations because they can relate the concept to a real world problem and see how this will benefit in real life. Making connections with them and applying solving a system to a real-life situation will allow them to make connections to other concepts as well.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying solving system of equations the pattern of questions within the classroom is critical because promoting student learning in It should connect students' lived experiences and interests (their only resources for learning something new) to disciplinary problems in the world, systems can be used when trying to determine if you'll make more money at one job or another, taking multiple variables into account, such as salary, benefits and commissions. For example, how would you describe in writing the graphic and algebraic solutions to systems of linear equations using key, technical vocabulary in expanded and some complex sentence? This allows students to really see if they understand the concept. How can you create your own real-world problem?

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 5346050

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra	■■■	Heart of Algebra	Systems of two linear equations in two variables	6. Fluently solve a system of linear equations in two variables.	No Calculator

5346050

$$-3x+5y=1$$
$$2x-3y=2$$

If (x,y) is the solution to the given system of equations, what is the value of x ?

Rationale
The correct answer is 13. The solution to a system of two linear equations is the point (x,y) that satisfies both equations. Rewriting the equations so the coefficients of y are additive inverses, then adding the equations can eliminate y. Multiplying both sides of the first equation by 3 and multiplying both sides of the second equation by 5 yield an equivalent system of equations: $-9x+15y=3$ and $10x-15y=10$. Adding these equations to eliminate y yields $x=13$.

Products and Reciprocals: http://tasks.illustrativemathematics.org/content-standards/HSA/REI/C/tasks/911	
Relevance to families and communities: During a unit focused on solving system of equations, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, connecting systems to their future will allow students the opportunity to understand there are many variables that influence their future goals and some variables are dependent on other variables. For example, whether they attend or where they attend college is dependent on money, grades, etc. By connecting systems of equations to their future goals, students learn how variables are connected and influence each other.	**Cross-Curricular Connections:** Science: Projectile motion is modeled by quadratic functions and height is modeled by linear functions. Consider providing a connection for students to experiment with projectile motion by tossing objects themselves, possibly using technology, and then exchanging their system equations with another classmate or group to solve. Social Studies: In high school the New Mexico Social Studies Standards state students should “use quantitative data to analyze economic information”. Consider providing a connection for students to work with system of equations involving economic data.

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HS: ALGEBRA- REASONING WITH EQUATIONS AND INEQUALITIES

Cluster Statement: D: Represent and solve equations and inequalities graphically.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSA.REI.D.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). *Note: Algebra 1 focuses on linear and exponential; learn as general principle*	SMP 2: Students can reason abstractly and quantitatively by relating graphical representations to contextualized situations. SMP 4: Students can model with mathematics by using graphical approaches to represent solutions to a two-variable equation.	• Explain and verify that every point (x, y) on the graph of a linear or exponential equation represents all values for x and y that make the equation true. • Identify points that are solutions to an equation given a graph of a linear or exponential equation. **Webb's Depth of Knowledge:** 1 **Bloom's Taxonomy:** Remember, Understand
Standard Text **HSA.REI.D.11: Explain why the x-coordinates of the points where the graphs of the equations *y = f(x)* and *y = g(x)* intersect are the solutions of the equation *f(x) = g(x)*; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/or *g(x)* are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*** *Note: Algebra 1 focuses on linear and exponential; learn as general principle*	**Standard for Mathematical Practices** SMP 3: Students can construct viable arguments by explaining how the x-coordinate of a solution to the system $y = f(x)$ and $y = g(x)$ solves $f(x) = g(x)$. SMP 5: Students can use tools by finding solution(s) of system of equations from graph or tables. SMP 7: Students look for and make use of structure by explaining in their own words how and when a solution is given as a point (x, y) versus a value (x = a).	**Students who demonstrate understanding can:** • Recognize what the solution $y = f(x)$ and $y = g(x)$ means on a graph. • Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. • Find approximate solutions for the system $y = f(x)$ and $y = g(x)$ using graphs and tables. • Find successive approximations and use them to solve the system $y = f(x)$ and $y = g(x)$. **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy:** Understand, Apply, Analyze, Evaluate

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.REI.D.12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. *Note: Algebra 1 focuses on linear and exponential; learn as general principle*	SMP 2: Students can reason abstractly and quantitatively by creating graphs and solving systems of inequalities with their models. SMP 5: Students can use tools by using graph paper and/or technology to graph linear inequalities and systems of linear inequalities.	• Determine whether the boundary line of a linear inequality is inclusive (solid) or is exclusive (broken) of the solution. • Determine which half-plane is the solution to a linear inequality • Graph the solution set to a system of linear inequalities in two variables as the intersection. of the corresponding half-planes. • Identify points that are a solution or non-solution to a linear inequality or system of linear inequalities. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
Previous Learning Connections • Connect to using variables to write expressions, equations, and inequalities. **(6.EE.2)** • Connect to graphing one variable inequalities on a number line. **(7.EE.4)** • Connect to graphing linear equations. **(8.EE.5)** • Connect to graphing systems of linear equations. **(8.EE.8)**	**Current Learning Connections** • Connect to interpreting statements, key features, and solutions of linear, quadratic, and exponential functions in terms of context. **(HSA.CED.1,3)** • Connect to graphing linear, quadratic, and exponential functions. **(HSA.CED.2)** • Connect to creating linear, quadratic, and exponential functions. **(HSA.CED.1-2)** • Connect to using graphs of linear, quadratic, and exponential functions to solve real-world contexts. **(HSA.CED.2)**	**Future Learning Connections** • Connect to apply these principles to different types of functions. (HSA.REI.11)

Clarification Statement

HSA.REI.D.11: Just as the **algebraic** work with **equations** can be reduced to a series of algebraic moves unsupported by reasoning, so can the **graphical visualization of solutions**. The simple idea that an equation f(x) = g(x) can be **solved (approximately)** by graphing y = f(x) and y = g(x) and finding the **intersection points** involves a number of pieces of conceptual understanding. [This method] seeks to convert an equation in one variable, f(x) = g(x), to a s**ystem of equations** in two **variables**, y = f(x) and y = g(x), by introducing a second variable y and **setting it equal** to each side of the equation. If x is a solution to the original equation, then f(x) and g(x) are equal, and thus **(x, y)** is a solution to the new system.

Common Misconceptions

- Students often interpret the solutions to an equation or graphical representation of an equation as only integer values.

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- Students may believe an estimate of a value between two integer points is sufficient, but the standard states that students should find successive approximations to approximate the solution.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying representing and solving equations and inequalities graphically because they will be taking the graphing of single points to graphing lines and equations as a set.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.EE.B.5: This standard provides a foundation for work to represent and solve equations and inequalities graphically because substituting numerical values into an equation to determine if the equation is true, the student will comprehend that the answer is a solution. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because students understand how equations and inequalities can be used to model real-life problems. Students relate and interpret the solution(s) of the equations or inequalities in the context of the problems.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because students persist on finding the solutions of equations and inequalities using the graphs constructed. Students check and interpret the solutions in the context of the problem to make sense of their mathematical thinking.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where appropriate because students understand the multiple representations of the solutions and make connections to the graphs that represents the equations and inequalities. Students connect the multiple representations to make sense of the meaning of the solution in the context of the problems.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because students use multiple ways, including graphing calculators and graph paper, to construct the graphic representations of the equations and inequalities. Students compare and verify the solutions from different representations to defend the solutions.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations because students apply their interpretation and knowledge of the solutions to new problems in the context of the situation. Students also extend their knowledge of solving equations and inequalities graphically to solving equations and inequalities algebraically.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on representing and solving equations and inequalities graphically by clarifying mathematical ideas and/or concepts through a short mini-lesson because helping students to understand what the different parts of the graph are telling them will help them to make better understanding of the graphs themselves.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit representing and solving equations and inequalities graphically by confronting student misconceptions because graphs can be misleading if read incorrectly and lead to quite a number of misconceptions, especially when it comes to how accurate the answers you are getting from them are. ...

Extension

	What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM? • For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying representing and solving equations and inequalities graphically because some students will pick up on the nuances of graphing quite quickly by comparison and could investigate further along points of inquiry such as how changing windows, scaling, or other aspects of the graph effects the readability and usefulness of it as a tool.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying representing and solving equations and inequalities graphically the use of mathematical representations within the classroom is critical because students are given a situation in two variables and they must find the value of one variable given the value of the other, create an equation to represent the situation, use technology to create a graph, and interpret each representation. Understanding how lines and tables represent solution sets of linear relationships will help students make sense of graphs of and solutions to linear inequalities, and later, to make sense of solutions to systems of linear equations in their Algebra 1 class.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 422195

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math	■■□	Passport to Advanced Mathematics	Nonlinear equations in one variable and systems of equations in two variables	1. Make strategic use of algebraic structure, the properties of operations, and reasoning about equality to f. solve systems of linear and nonlinear equations in two variables, including relating the solutions to the graphs of the equations in the system.	No Calculator

422195

$y = x^2$
$2y + 6 = 2(x + 3)$

If (x, y) is a solution of the system of equations above and x > 0, what is the value of xy ?

A. 1

B. 2

C. 3

D. 9

Rationale

Choice A is correct. Substituting x^2 for y in the second equation gives $2(x^2) + 6 = 2(x + 3)$. This equation can be solved as follows:

$2x^2 + 6 = 2x + 6$	Apply the distributive property.
$2x^2 + 6 - 2x - 6 = 0$	Subtract 2x and 6 from both sides of the equation.
$2x^2 - 2x = 0$	Combine like terms.
$2x(x - 1) = 0$	Factor both terms on the left side of the equation by 2x.

Thus, $x = 0$ and $x = 1$ are the solutions to the system. Since $x > 0$, only $x = 1$ needs to be considered. The value of y when $x = 1$ is $y = x^2 = 1^2 = 1$. Therefore, the value of xy is $(1)(1) = 1$.

Choices B, C, and D are incorrect and likely result from a computational or conceptual error when solving this system of equations.

Ideal Gas Law: http://tasks.illustrativemathematics.org/content-standards/HSA/REI/D/11/tasks/1925

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on representing and solving equations and inequalities graphically, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, bringing in the different languages spoken in the home and connecting it to the tools available to translate different languages, <i.e. Google translate, closed captions on televisions,	Computer Science: Computer programs use functions to define the points used to graph the animation on a computer. Consider providing a connection where students can write from scratch or compile premade selections to create code that will result in their own animations. Social Studies: In high school the New Mexico Social Studies Standards state students should "use quantitative data to analyze economic information". Consider

etc.> make connections that show that in the culture of mathematics, tools are used to translate mathematics and help us make sense of what we are seeing.	providing a connection for students to work with a context that compares two situations that each include a standard base fee and additional charges per unit of some quantity.

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HS: FUNCTIONS- INTERPRETING FUNCTIONS

Cluster Statement: A: Understand the concept of a function and use function notation.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f(x)*. *Note: Algebra 1 focuses on learning as general principle; focus on linear and exponential and on arithmetic and geometric sequences*	SMP 2: Students can reason abstractly and quantitatively by interpreting the relation of domain and range of functions related to a problem given algebraically or graphically within a context. SMP 4: Students can model with mathematics by creating examples of what is and what is not a function using different representations, including graphs, tables, symbols and contexts.	• Distinguish between functions and nonfunctions from a graph. • Distinguish between functions and nonfunctions from a table. • Distinguish between functions and nonfunctions from an equation • State the domain and range given a graph. • Understand when an equation is a function y is replaced with f(x) **Webb's Depth of Knowledge:** 1 **Bloom's Taxonomy:** Remember and Understand
Standard Text **HSF.IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.** *Note: Algebra 1 focuses on learning as general principle; focus on linear and exponential and on arithmetic and geometric sequences*	**Standard for Mathematical Practices** SMP 2: Students can reason abstractly and quantitatively by using and interpret function notation to identify relationships between domain and range. SMP 6: Students can attend to precision by accurately and appropriately using vocabulary and symbols to write functions in function notation and describe different characteristics of graphs.	**Students who demonstrate understanding can:** • Evaluate a function given in function notation for an input. • Find the input for a given output when given in function notation. • Identify the domain and range for any given function presented in function notation or given as a verbal description in terms of a context. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand and Apply

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.IF.A.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.* *Note: Algebra 1 focuses on learning as general principle; focus on linear and exponential and on arithmetic and geometric sequences*	SMP 4: Students can model with mathematics by interpreting parts of sequences in relation to a context. SMP 7: Students can look for and make use of structure by determining that a sequence with which they are working is a function by analyzing the data they are given, whether it is in list form or a graph.	• Observe a sequence as a function whose domain consists of integers. • Observe a sequence that is defined recursively whose domain consists of only integers. **Webb's Depth of Knowledge:** 1 **Bloom's Taxonomy:** Remember and Understand
<u>Previous Learning Connections</u> • Connect to analyzing proportional relationships and solving real-world math problems using numerical and algebraic expressions and equations. **(7.RPA.2-3)** • Connect to describing the functional relationship between two quantities qualitatively by analyzing a graph. **(8.F.5)** • Connect to constructing a function to model a linear relationship. **(8.F.4)**	**<u>Current Learning Connections</u>** • Connect to writing recursive and explicit formulas for arithmetic and geometric sequences. **(HSF.BF.2)** • Connect to writing functions for linear, quadratic, and exponential relationships. **(HSF.BF.1- 2)** •	**<u>Future Learning Connections</u>** • Connect to use function notation with all types of functions. **(HSF.IF.2)** • Connect to deriving the formula for a geometric series. **(HSA.SSE.4)**

Clarification Statement

- HSF.IF.A.1: Students need to understand that a **function** from one **set** (called the **domain**) to another set (called the **range**) **assigns** to each **element** of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $\boldsymbol{f(x)}$ denotes the **output** of f **corresponding** to the **input** x. The **graph** of f is the graph of the **equation** $y = f(x)$. 8.F.A are foundational standards; however, this is students' first opportunity to work with function notation as it is explicitly left out of the Grade 8 standards.
- HSF.IF.A.2: Students should be able to use function notation in a flexible way such as knowing how to **plug in a value** and get the corresponding output. They should also be able to understand and use x and F(x) interchangeably with x and y when explaining the context of a problem. Students should know that all they must do is **isolate an equation** for y and then replace it with f(x) (read as "**f of x**").
- HSF.IF.A.3: Students should recognize that **sequences** are functions, sometimes defined **recursively**, whose domain is a **subset** of the **integers**. Students should see **patterns** emerge when comparing the **x and y values** to each other. Students should know that these patterns are not coincidences and, students should know that these patterns can be thought of as sequences, or a **list** of numbers. Sequences can be either **arithmetic** (where the same number is added or subtracted) or **geometric** (where the same number is multiplied or divided).

Common Misconceptions

- Students do not recognize f(x) = is the same as y =. They also will often confuse f(x) with the product of f and x and not recognize that it is a form of notation.

- Students often show a lack of understanding for what 'n' represents. Students often struggle understanding the notation of recurrence sequences. Using different values of n for a given term.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying the concept of a function and function notation because the foundation for this cluster is developed in 8th grade. Students are introduced to functions as relationships having a unique output for input. Building from this idea is a crucial connection for students developing a deeper understanding of functions and function notation.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.F.A.1: This standard provides a foundation for work with the concept of a function and function notation because it is the foundational concept of the function. Understanding the definition of a function is crucial to making sense of the more complicated functions that are seen in Algebra 1. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with the concepts of a function and function notation benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because it is often difficult to relate sequences to an Algebra 1 student's daily life. By using tasks that involve things like video game scores to work with sequences as functions students can engage in mathematics related to something that many of them work with or use/play daily.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with the concepts of functions and function notation benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success because such feedback is invaluable to developing a depth of understanding with this cluster of conceptual standards.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with the concepts of a function and function notation benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because the foundation for this standard is laid in 8th grade. Because it is conceptual in nature and includes symbolic representations that will be used throughout the students mathematical career, making the connections to prior understandings is vital to guiding students onward through the more complicated functions introduced in Algebra 1 and later in Algebra 2.

Expression and Communication: H*ow will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with the concepts of a function and function notation benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using concept mapping tools because students can link their understanding of a function from 8th grade examples to function notation. Since this cluster of standards is conceptual in nature any "linking" that we can do to broaden and deepen their understanding is the key to this cluster.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with the concepts of a function and function notation benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing templates, graphic organizers, concept maps to support note-taking because connecting ideas to what students already know and moving their thinking forward (from what they say in 8th grade to Algebra 1 and on through Algebra 2) with functions and function notation will support a deeper and greater understanding of those ideas.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on the concept of a function and function notation by clarifying mathematical ideas and/or concepts through a short mini lesson because the cluster is conceptual in nature. Making sense of the concepts is key to analyzing them and interpreting in the next two clusters.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit the concept of a function and function notation by addressing conceptual understanding because this cluster is conceptual in nature. Anything that we do to deepen students understanding of the concept of function and function notation will be a key to extend their understanding of functions and function notation in additional standards within this domain.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as addressing conceptual understanding when studying the concept of a function and function notation because students gain a deeper understanding of functions once, they see applications in real life disciplines other than mathematics. In making cross curricular links students will not only deepen their understanding of the widely applicable nature of functions but also prepare themselves for the next levels of analyzing and interpreting functions.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying the concept of a function and function notation the types of mathematical tasks are critical because this cluster is conceptual in nature. The types of vocabulary introduced/continued within this cluster are vital to success in future mathematics especially those within the domain of interpreting functions. Students who are unfamiliar with the idea of a function or the concept of function notation will struggle with these foundational ideas if explicit instruction is neglected.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 5441675

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra	■□□	Heart of Algebra	Linear functions	3. For a linear function that represents a context a. interpret the meaning of an input/output pair, constant, variable, factor, term, or graph based on the context, including situations where seeing structure provides an advantage;	Calculator

5441675

Questions 11 and 12 refer to the following information.

t	C(t)
1	8.5
2	11
3	13.5
4	16

The length C(t), in inches, of a channel catfish in an Iowa river *t* years after the first year of life can be approximated by the linear function *C*. Some values of C(t) are given in the table above.

$F(t) = 3t + 4$

The length F(t), in inches, of a flathead catfish in the same Iowa river *t* years after the first year of life can be approximated by the linear function *F*, defined by the equation above.

According to the model, which of the following is closest to the expected age, to the nearest whole year, of a flathead catfish that is 31 inches long?

A. 10 years old

B. 13 years old

C. 98 years old

D. 106 years old

Rationale

Choice A is correct. It is given that the length $F(t)$, in inches, of a flathead catfish in the river t years after the first year of life can be approximated using the function $F(t) = 3t + 4$. The question asks for the expected age when a catfish is 31 inches long, which can be represented by substituting 31 for $F(t)$, which gives $31 = 3t + 4$. Subtracting 4 from both sides of this equation gives $27 = 3t$, and then dividing both sides by 3 gives $9 = t$. This means that approximately 9 years after the first year of life, or at the age $1 + 9 = 10$ years old, a flathead catfish is expected to be 31 inches long.

Choice B is incorrect and may result from substituting 31 for $F(t)$ in the linear function F, but solving for t by adding 4 to both sides of the equation, rather than subtracting 4, before dividing both sides by 3 and adding 1 to the result. Choice C is incorrect and may result from substituting 31 for t, rather than for $F(t)$, in the linear function F, then solving for $F(t)$ and adding 1 to the result. Choice D is incorrect and may result from substituting 31 for t, rather than for $F(t)$, in the linear function F, then rewriting the right-hand side of the function as $3(31 + 4)$ and adding 1 to the result.

Interpret the graph: http://tasks.illustrativemathematics.org/content-standards/HSF/IF/A/tasks/636
The Customers: http://tasks.illustrativemathematics.org/content-standards/HSF/IF/A/1/tasks/624
The Parking Lot: http://s3.amazonaws.com/illustrativemathematics/attachments/000/008/832/original/public_task_588.pdf?1462392946

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on the concept of a function and function notation, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, allowing students to look at home budgets, utility bills (the cost as a function of usage etc.) or even bringing in examples of functions from various careers represented at home can help students make connections between the abstract idea of functions and how/where they exist in real life.	Science: Radioactive decay is a function that is a sequence. Consider providing a connection where students know the half-life and starting amount of a substance and use that to define a function and determine the amount left after a certain amount of time.

HS: FUNCTIONS- INTERPRETING FUNCTIONS

Cluster Statement: B: Interpret functions that arise in applications in terms of the context.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ** *Note: Algebra 1 focuses on linear, exponential, and quadratic*	SMP 1: Students can make sense of problems and persevere in solving them by thinking through the meaning of the key features in graphs as relative to a given context. SMP 4: Students can model with mathematics by creating an approximate graph that could model a given context.	• Identify intercepts of a function. • identify intervals where the function is increasing. • Identify intervals where the function is decreasing. • Identify intervals where the function is positive. • Identify intervals where the function is negative. • Identify relative maximums of a function. • Identify relative minimums of a function. • Identify symmetries in the functions. • Identify end behavior of the functions. • Sketch graphs given a list of key features or a verbal model. • Sketch functions that model key feature behavior. • Label intercepts and intervals of a graph. • Interpret where the function is increasing, decreasing, positive, or negative. • Interpret relative maximums and minimums. • Interpret various symmetries, end behaviors, and periodicity.
		Webb's Depth of Knowledge: 1-2
		Bloom's Taxonomy: Understand, Apply, Analyze

Standard Text **HSF.IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.** *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.** *Note: Algebra 1 focuses on linear, exponential, and quadratic*	**Standard for Mathematical Practices** SMP 3: Students can construct viable arguments by explaining why the domain of a given context is discrete or continuous. SMP 4: Students will model with mathematics by connecting a function to the context it represents using quantities.	**Students who demonstrate understanding can:** • Make connections between a graph of a function and its domain. • Make connections between the graph of a function and the context it describes. • Identify when the domain of a given context is discrete or continuous and explain why. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
Standard Text **HSF.IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *** *Note: Algebra 1 focuses on linear, exponential, and quadratic*	**Standard for Mathematical Practices** SMP 4: Students can model with mathematics by interpreting the average rate of change within the context of a problem. SMP 5: Students can use tools by using tables and graphs to determine the average rate of change over a specified interval.	**Students who demonstrate understanding can:** • Calculate the average rate of change of a function over a specified interval presented symbolically. • Calculate the average rate of change of a function over a specified interval presented in a table. • Interpret the average rate of change of a function over a specified interval presented symbolically for a given context. • Interpret the average rate of change of a function over a specified interval presented in a table for a given context. • Estimate the rate of change of a function from a graph. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
Previous Learning Connections • Connect to interpreting the equation y =mx + b as a linear function and using the equation to solve problems in context. **(8.F.3)** • Connect to interpreting key features of linear equations in	**Current Learning Connections** • Connect to discovering features of families of functions. **(HSF.IF.7)** • Connect to distinguishing between situations modeled by linear and exponential functions. **(HSF.LE.1)**	**Future Learning Connections** • Connect to finding key features of the entire family of functions. **(HSF.IF.4)**

relation to a contextual situation. **(8.F.4)**		

Clarification Statement

- HSF.IF.B.4: Students interpret the **key features** of the different **functions** listed in the standard. When given a **table** or **graph** of a function that models a **real-life situation**, explain the meaning of the characteristics of the table or graph in the **context** of the problem. Key features of a **linear function** are **slope** and **intercepts**, of a **quadratic function** are **intervals of increase/decrease**, **positive/negative**, **maximum/minimum**, **symmetry**, and **intercepts**, of an **exponential function** include **y-intercept** and **increasing/decreasing intervals** and of an **absolute value** include **y-intercept**, **minimum or maximum**, **increasing or decreasing intervals**, and **symmetry**.
- HSF.IF.B.5: Students should focus their attention on possible **input and output values**, framing them as the **domain and range of a function**. When given a description of a function that represents a situation, the students should determine reasonable domain and range. Students relate the domain of a function to its graph and, where applicable, to the **quantitative relationship** it describes. Students need to explain the reasonableness of a domain for a given context. Students should understand that the domain of a function is the set of all possible inputs and the range is the set of all possible outputs. Also looking at if a function is **continuous** (time, amount of liquid filling a container) or discrete (number of people or things) and connecting back to number classifications
- HSF.IF.B.6: Students will calculate and interpret the **average rate of change** of a linear, quadratic, **piecewise linea**r (to include absolute value), and exponential function (presented **symbolically** or as a table) over a specified **interval.** Students will **estimate** the rate of change from a graph. In addition to finding average rates of change from functions given symbolically, graphically, or in a table, students may collect **data** from **experiments** or **simulations** (ex. falling ball, velocity of a car, etc.) and find average rates of change over various intervals.

Common Misconceptions

- Students may confuse independent and dependent variables.
- Students may believe that the domain for all functions is all real numbers.
- Students may struggle with the concepts of rate of change and slope.
- Students may focus on the y values of the graph instead of the x values of the interval, when identifying key features of a graph.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that previews new contexts for tasks within the unit (e.g., cell phone plans) when studying the interpretations of functions that arise in applications in terms of a context because understanding the key aspects of a context is the key to unlocking a problem for students.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.F.B.5: This standard provides a foundation for work with interpreting functions that arise in applications in terms of a context because the given standard is the foundational piece of interpreting linear functions. Once students can efficiently and accurately interpret linear functions, they can apply that knowledge to the more complex quadratic and exponential functions of Algebra 1. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with interpreting functions that arise in applications in terms of a context benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features such as the size of text, images, graphs, tables, or other visual content; contrast between background and text or image; color used for information or emphasis because the important part of this cluster is first making sense of the application problem and then interpreting the functions related to the problem. These accommodations can take away visual barriers for students with visual acuity issues.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with interpreting functions that arise in applications in terms of a context benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because students often struggle with making sense of application problems. When they are adequately supported and encouraged to persevere it builds their confidence in applying mathematics (which is what this cluster is all about - application problems involving functions).

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with interpreting functions that arise in applications in terms of a context benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf because often seeing/reading/hearing the application problem in a student's first language allows them to make more sense of the given problem and then break it down in order make the necessary interpretations. In this case we are taking away ONE of the two translations they need to make.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with interpreting functions that arise in applications in terms of a context benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of novel solutions to authentic problems because when students are able to make sense of a mathematics problem (especially with functions and applications) they are more likely to retain the skills that they learn. In allowing them to use multiple procedures or multiple modalities of explaining their thinking or displaying their thinking in approaching a problem we are giving them ownership of learning.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with interpreting functions that arise in applications in terms of a context benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because students struggle with interpreting functions in context. When we build in supports, design and ask appropriate guiding questions, and support them through a productive struggle (wrestling mathematics) with the mathematics their success will build up their confidence and understanding of how mathematics in the real-world works.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on interpreting functions that arise is applications in terms of the context by providing specific feedback to students on their work through a short mini-lesson because in interpreting functions within a context providing feedback and allowing students to revise their work can be a powerful tool in deepening their understanding.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit interpreting functions that arise in applications in terms of the context by confronting student misconceptions because as in the section on re-teach targeted in this cluster students need feedback for learning. They need to see what they don't understand, celebrate their successes and revise their work to deepen their understanding of functions and interpreting functions in context.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying the interpretation of functions that arise in applications in terms of the contest because there are so many opportunities to make connections between the abstract and isolated nature of functions in mathematics and applications in science, history, psychology, sociology and other topics that may be of greater interest to students. If they can research functions in other disciplines they will have more "buy in" to the importance of interpreting functions.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Task: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)." For example, when interpreting functions that arise in applications in the terms of a context the types of mathematical tasks are critical because student engagement in this area leads to greater understanding of the key features of functions and how they relate to the context. When students are beginning to make sense of the parts of a function (or its various representations), they need it to be related to an idea they already understand. In doing this we aren't trying to teach the concept in the application and the mathematics because the students already understand the context and can focus on the mathematics of the task.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 1054722

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra	■■□	Heart of Algebra	Linear functions	3. For a linear function that represents a context a. interpret the meaning of an input/output pair, constant, variable, factor, term, or graph based on the context, including situations where seeing structure provides an advantage;	No Calculator

$$A = 1{,}321 + 0.3433m$$

The equation above can be used to estimate the body surface area A, in square centimeters, of a child with mass m, in grams, where $3{,}000 \le m \le 30{,}000$. Which of the following statements is consistent with the equation?

A. For each increase of 1 gram in mass, A increases by approximately 0.3433 square centimeter.

B. For each increase of 0.3433 gram in mass, A increases by approximately 1 square centimeter.

C. For each increase of 1 gram in mass, A increases by approximately 1,321 square centimeters.

D. For each increase of 1,321 grams in mass, A increases by approximately 1 square centimeter.

Rationale

Choice A is correct. The equation can be represented by a linear graph, where the slope represents the ratio of the change in the dependent variable for every 1-unit change in the independent variable. In this context, the slope is the change in A, the body surface area in square centimeters, for every increase by 1 in m, the mass in grams. In the equation, the slope is represented by the coefficient m of the independent variable, which is 0.3433. Thus, for each increase of 1 gram in mass, A increases by approximately 0.3433 square centimeter.

Choice B is incorrect and may result from incorrectly interpreting the slope in this context. Choices C and D are incorrect. These choices compare a mass, in grams, to an area, in square centimeters. However, the value 1,321 refers only to the initial estimated body surface area A, in square centimeters.

Interpret the graph: http://tasks.illustrativemathematics.org/content-standards/HSF/IF/A/tasks/636
The Customers: http://tasks.illustrativemathematics.org/content-standards/HSF/IF/A/1/tasks/624
The Parking Lot:
http://s3.amazonaws.com/illustrativemathematics/attachments/000/008/832/original/public_task_588.pdf?1462392946

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on Interpreting functions that arise in applications in terms of a context, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, allowing students to look at home budgets, utility bills (the cost as a function of usage etc.) or even bringing in examples of functions from various careers represented at home can help students make connections between the abstract idea of functions and how/where they exist in real life.	Science: Average rate of change can be modeled in contexts involving temperature, speed or height. Consider providing a connection where students collect bivariate data and that make a contextualized explanation of an average rate of change for a model they have created.

HS: FUNCTIONS- INTERPRETING FUNCTIONS

Cluster Statement: C: Analyze functions using different representations.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. * • HSF.IF.C.7.A: Graph linear and quadratic functions and show intercepts, maxima, and minima. • HSF.IF.C.7.B: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. • HSF.IF.C.7.E: Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. *Note: Algebra 1 focuses on linear, exponential, quadratic, absolute value, step, and piecewise-defined.*	SMP 4: Students can model with mathematics by graphing linear and quadratic functions using by-hand and technology methods. SMP 7: Students can look for and make use of structure by recognizing linear and quadratic families from their symbolic and graphical forms.	• Graph functions expressed symbolically showing key features of the graph by hand in simple cases and with technology for more complicated cases. • Graph linear functions showing intercepts. • Graph quadratic functions showing intercepts, maxima and minima. • Graph piecewise defined functions (step functions and absolute value functions) showing intercepts, maxima, and minima. • Compare and contrast linear, quadratic and exponential functions. • Explain issues of domain, range and usefulness when examining piecewise-defined functions. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.IF.C.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. • HSF.IF.C.8.A: Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. • HSF.IF.C.8.B: Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay. *Note: Algebra 1 focuses on linear, exponential, quadratic, absolute value, step, and piecewise-defined.*	SMP 4: Students can model with mathematics by interpreting zeros, intervals where the function is increasing or decreasing, extrema and symmetry within a context. SMP 7: Students can look for and make use of structure by using rearranging functions to highlight key features.	• Rewrite a function to find and highlight key features. • Factor a quadratic expression to find zeros, extrema and symmetry • Interpret the meaning of zeros, extrema and symmetry within the context of a problem. • Complete the square for a quadratic function to reveal its key features. • Interpret the key features of a quadratic expression in terms of the context it represents. • Use properties of exponents to relate parts of an exponential function to its context (e.g., describe the initial value, growth/decay rate or factor and the growth period). • Identify how key features of an exponential function relate to characteristics in a real-world context. • Classify real-world problems as an exponential growth or decay. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSF.IF.C.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* *Note: Algebra 1 focuses on linear, exponential, quadratic, absolute value, step, and piecewise defined.*	SMP 5: Students can use tools by working flexibly with multiple representations. SMP 7: Students can look for and make use of structure by comparing the similarities and differences of linear, quadratic, and exponential functions.	• Make comparisons between functions in different forms using their knowledge of key features. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply,Analyze

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• Connect to graphing linear functions. **(8.F.5)** • Connect to comparing properties of linear functions represented in different ways. **(8.F.2)** • Connect to identifying and using key features of linear functions. **(8.F.4)** • Connect to writing linear equations. **(8.F.4)**	• Connect to writing linear, quadratic, and exponential functions to describe relationships between quantities. **(HSA.CED.1-3)** • Connect to analyzing transformations of parent functions for linear, quadratic, and exponential functions. **(HSF.BF.3)**	• Connect to graphing all parent functions by hand and using technology and identifying their key features. **(HSF.IF.7)** • Connect to factoring to complete the square with quadratic functions with complex zeros. **(HSN.CN.7)**

Clarification Statement

- HSF.IF.C.7: Students should be able to describe the significant features of different **functions graphically** and **algebraically**. Students should be able to use the **significant features** to **sketch** the graph of the function. Students should graph **linear** and **quadratic functions** and show **intercepts**, **maxima**, and **minima.** Students should know the **slope-intercept form** of linear functions, y = mx + b, and how to extract enough information from the equation to be able to draw it. When graphing **roots**, remember that for $\sqrt[n]{x}$, if n is **even,** the domain includes all **positive integers**. Otherwise, **negative** values are included as well. When graphing roots of the for $y = a\sqrt{x} + b$, remember the **y-intercept** is b. Students should remember that roots are **fractional exponents.** Students should know to look at the **highest degree of the polynomial** and its **coefficient,** ax^n. If n is even, the function will extend either **up** or **down** on both ends (as x goes to **positive** or **negative infinity**). If n is **odd**, they'll go in **opposite directions**. If a is positive, the even powered functions will go up and the odd powered functions will start down and go up. If a is negative, the even powered functions will go down, and the odd powered functions will start up and go down.
- HSF.IF.C.8: Students should be able to rewrite quadratic and **exponential functions** in different ways to find key features of the expression and interpret those key features in terms of the context they represent. Students should be able to find the **x-intercepts** of a quadratic function using both **factoring** and **completing the square**.
- HSF.IF.C.9: Students should be able to compare two given functions (linear, exponential, quadratic) whether that be as a function or **equation**, in a **table,** in a **graph,** or by **verbal description**. Students should start by knowing the difference between linear, quadratic and exponential functions, and be able to identify them by equation and by graph. Students should be able to compare two functions even when they're both represented differently. To do this successfully, they must be able to **translate** between an equation, a graph, words, and a table of values, and understand how certain aspects of one **representation** impact the rest.

Common Misconceptions

- Students may have difficulty identifying the key features needed to sketch the graphs or identifying those features algebraically.
- Students may have difficulty with contextualizing and decontextualizing expressions.
- Students will often confuse functions given in a table as a representation of a finite set of numbers rather than a subset of the entire function. They also may have difficulty with the abstractness of determining what is happening with a function over intervals of the domain that they cannot see.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when analyzing of functions using different representations because it is in this cluster that students begging to broaden the scope of the functions they are working with. It is in this section that students are specifically introduced to the ideas of quadratic and exponential functions as well as piecewise defined an absolute value function. In allowing them time to struggle and grapple with the mathematics we are allowing them to make sense of the functions and internalize the understanding of the functions key features when given them in various ways.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.EE.A.3: This standard provides a foundation for work analyzing functions using different representations because this standard lays the foundation for order of operations and understanding the idea of equivalent expressions. The ideas presented in this standard allow students to start slow with expressions that are linear in nature leading up to the use of the distributive property as well as associative and commutative properties that are the precursors for factoring and rearranging higher order functions. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with analyzing functions using different representations benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as because this provides multiple entry points for students. It also supports the cluster in that the standards within it require multiple representations of functions. If students start with the one they understand best and then work their way through the others we build ownership and then deeper engagement and understanding of the multiple ways in which we represent functions (symbolically, graphically, or in a table).

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with analyzing functions using different representations benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as using prompts or scaffolds for visualizing desired outcomes because while students may find one method of representing a function or another more approachable they have to make sense of them all. As we guide them through their work, we can help them build confidence and perseverance with analyzing functions.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with analyzing functions using different representations benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because when students understand the expectations of representations they will be asked to create they are more likely to engage in the work and complete it with more understanding.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with analyzing functions using different representations benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as composing in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video because the primary goal of this cluster is to take a function within its context and make sense of it. Students can use multiple algebraic methods of describing the key features of a function - but they can use the ideas listed here to help tell the story of the function.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with analyzing functions using different representations benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because students need to engage in this cluster in many ways. They first see it with linear functions in 8th grade. In Algebra 1 they will see it with linear, quadratic, and exponential functions. They will work with this cluster further in Algebra 2. Students will need to practice, practice, practice. Making sense of functions and their key features is a life skill and the more problems they see and work with the more widely applicable they will see that standard is.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on analyzing functions using different representations by critiquing student approaches/solutions to make connections through a short mini-lesson because so much of this cluster can be learned through student choice of solution method. When we allow students to share their thinking and make connections between their work and that of others, they are encouraged to try a solution method that they hadn't tried before and might be more efficient. They can also see their errors and make revisions. Jo Boaler (youcubed.com) tells us that brain research suggests that we learn more from when we make mistakes than we do when we get things right all the time. Therefore, constructive criticism and feedback that is more meaningful than just a percentage, and vital for our students' success in learning mathematics.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit analyzing functions using different representations by offering opportunities to understand and explore different strategies because as stated above students will approach the problems with the method that makes the most sense to them at first even if it isn't the most efficient strategy. Looking at ideas from other students allows kids to engage in math practice 5 and perhaps make more meaning of different more efficient strategies. We know that we can pick the best strategy from the outset of the problem - but it's because we have a lot of practice and often, we can't necessarily explain why we chose a specific method. It is helpful for our students to think about why one strategy might be better than another and learning when to use specific strategies based on the problem type, they are given when analyzing functions given in different ways.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when analyzing functions given in different ways because this cluster is widely applicable to other disciplines such as science and statistics. If students can explore something of interest to them related to this cluster, they may think of ways to analyze functions that makes more sense to them and their peers. Allowing them to explore the widely applicable nature of functions given in multiple representations will also allow them to become more informed citizens of our society at large.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when analyzing functions using different representations the use of mathematical representations within the classroom is critical because it is the focus of this cluster. Students must be able to connect a table to the algebraic written function, and its graph (in any order). All three representations are vital to making sense in mathematics applications. Students often come to us with strengths using one or more of those representations and we can build on those strengths and extend them to the other representations. In connecting what they already know to what they need to add to their "toolbox", students build strength in mathematical representations.

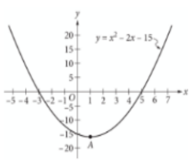
NEW MEXICO
Public Education Department

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 18494

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math	■■■	Passport to Advanced Mathematics	Nonlinear functions	2. For a quadratic or exponential function, d. determine the most suitable form of the expression representing the output of the function to display key features of the context, including i. selecting the form of a quadratic that displays the initial value, the zeros, or the extreme value;	Calculator

18494



Which of the following is an equivalent form of the equation of the graph shown in the xy-plane above, from which the coordinates of vertex A can be identified as constants in the equation?

A. $y=(x+3)(x-5)$

B. $y=(x-3)(x+5)$

C. $y=x(x-2)-15$

D. $y=(x-1)^2-16$

Rationale

Choice D is correct. Any quadratic function q can be written in the form $q(x)=a(x-h)^2+k$, where a, h, and k are constants and (h,k) is the vertex of the parabola when q is graphed in the coordinate plane. This form can be reached by completing the square in the expression that defines q. The equation of the graph is $y=x^2-2x-15$. Since the coefficient of x is -2, this equation can be written in terms of $(x-1)^2=x^2-2x+1$ as follows: $y=x^2-2x-15=(x^2-2x+1)-16=(x-1)^2-16$. From this form of the equation, the coordinates of the vertex can be read as $(1,-16)$.

Choices A and C are incorrect because the coordinates of the vertex A do not appear as constants in these equations. Choice B is incorrect because it is not equivalent to the given equation.

Interpret the graph: http://tasks.illustrativemathematics.org/content-standards/HSF/IF/A/tasks/636
The Customers: http://tasks.illustrativemathematics.org/content-standards/HSF/IF/A/1/tasks/624
The Parking Lot:
http://s3.amazonaws.com/illustrativemathematics/attachments/000/008/832/original/public_task_588.pdf?1462392946

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on analyzing functions using different representations, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, allowing students to look at home budgets, utility bills (the cost as a function of usage etc.) or even bringing in examples of functions from various careers represented at home can help students make connections between the abstract idea of functions and how/where they exist in real life. You can then extend these functions by having students make tables, graphs, and write functions related to what they find at home.	Science: In high school the NGSS build on K–8 experiences and progresses to using, synthesizing, and developing models to predict and show relationships among variables between systems and their components in the natural and designed worlds. Consider providing a connection for students to use a model based on evidence to illustrate the relationships between systems or between components of a system.

NEW MEXICO
Public Education Department

HS: FUNCTIONS- BUILDING FUNCTIONS

Cluster Statement: A: Build a function that models a relationship between two quantities.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.BF.A.1: Write a function that describes a relationship between two quantities.* • HSF.BF.A.1.A Determine an explicit expression, a recursive process, or steps for calculation from a context. • HSF.BF.A.1.B Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* *Note: Algebra 1 focuses on linear, exponential, and quadratic.*	SMP 4: Students can model with mathematics by discovering patterns in each contextual problem and creating verbal, symbolic or explicit symbolic rules to describe them. SMP 7: Students can look for and make use of structure by using the operations of math to combine functions.	• Write an explicit expression to model linear, exponential and quadratic relationships. • Write a recursive expression to model linear, exponential and quadratic relationships. • Add, subtract multiply and divide functions related to a given context. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSF.BF.A.2: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. * *Note: Algebra 1 focuses on linear, exponential, and quadratic.*	SMP 4: Students can model with mathematics by writing using recursive steps to model a context by writing the formula. SMP 8: Students can look for and express regularity in repeated reasoning by using the recursive steps to recognize relationships between the	• Write arithmetic sequences recursively and explicitly. • Write geometric sequences recursively and explicitly. • Translate between recursive and explicit formulas. • Model situations using the formulas. • Relate arithmetic sequences to linear function and geometric sequences to exponential functions.

	pattern and the symbolic representation of the pattern.	**Webb's Depth of Knowledge:** 1-2
		Bloom's Taxonomy: Understand, Apply, Analyze
Previous Learning Connections • Connect to interpreting the equation y = mx + b as defining a linear function. (8.F.3) • Connect to comparing properties of two functions, each represented in a different way. (8.F.2)	**Current Learning Connections** • Connect to identifying patterns in the function's rate of change, specifying intervals of increase and decrease, and graphing to model functions. **(HSF.IF.4,6)** • Connect to discussing the relative strength sand weaknesses of each representation and which are most efficient to be able to assist them in making symbolic functions. **(HSF.IF.9)** • Connect to recognizing situations that grow by a constant rate or percent. **(HSF.LE.1)**	**Future Learning Connections** • Connect to continuing to write arithmetic and geometric sequences. **(HSF.LE.2)** • Connect to using geometric series to find the sum. **(HSA.SSE.4)** • Connect to performing operations with all parent functions and composing functions. **(HSF.BF.1)**

Clarification Statement

- HSF.BF.A.1: Students should write **functions** for given **relationships** between **quantities**. Students can use functions to **model real-life situations** and make **predictions**. Students should be able to use functions describe relationships between two quantities, usually x and f(x), where f(x) is some **output** value that depends on the **input** value x. Within a context, students should be able to express a given relationship as a function.
- HSF.BF.A.2: Students should write **formulas** for **arithmetic** and **geometric sequences** with both **explicit** and **recursive formulas**. They should be able to relate these to a context they represent and be able to **transition** from one form to the other. Students should know that they can write explicit functions recursively, too. For instance, with every year that passes, your age increases by 1. It can be interpreted as constantly adding 1 to the age you were before. In other words, write your age as f(x) = f(x – 1) + 1 starting with f(1) = 1. Students should know how to recognize that arithmetic functions that take the explicit form A(n) = A(1) + (n – 1)d have the recursive form A(n) = A(n – 1) + d and geometric functions with the form G(n) = G(1) × r^{n-1} have the recursive form G(n) = G_{n-1}. × r.

Common Misconceptions

- Students may want to try to use a linear function, specifically the slope-intercept form for every situation.
- Students may tend to focus on the symbolic form of a function and may need additional support in working with other forms.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that previews new contexts for tasks within the unit (e.g., cell phone plans) when studying building a function that models a relationship between two quantities because to discuss possible strategies and viable solutions.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.F.A.3 This standard provides a foundation for work with building a function that models a relationship between two quantities because students identify the type of relationship the two quantities have (linear, non-linear, exponential). If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with building a function that models a relationship between two quantities benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features given an example connected to this standard such as the size of text, images, graphs, tables, or other visual content; contrast between background and text or image; color used for information or emphasis; volume or rate of speech or sound; speed or timing of video, animation, sound, simulations, etc.; layout of visual or other elements; font used for print materials because students will be able to recognize various situations and can create representations that will allow them to understand and be able to share their understanding with others. When a student recognizes a situation, they are then able to create a table, equation, and graph to explain and show their thinking using various forms (paper, technology).

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with building a function that models a relationship between two quantities benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that is substantive and informative rather than comparative or competitive because students will think about what was their thinking process solving the problem instead of asking "is this correct?" students will talk and discuss with other students and demonstrate how they came about to the conclusion that they are displaying. If students use (MP 3) Construct viable arguments and critique the reasoning of others. The students themselves will determine if their conclusion is the correct one for the given problem.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with building a function that models a relationship between two quantities benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) because students can go back and use the support that will aid them in clarifying and understanding what the problem wants them to do. If students do not understand what they are solving for and how they need to represent it is usually the first cause of the students getting confused and frustrated.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with building a function that models a relationship between two quantities benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because when students are given appropriate tools the students are then able to demonstrate what they are thinking without having to worry about if they are drawing the graph correctly. They can focus on the mathematics and show their representation of their solution. In the real-world technology is used most of the time when graphing is needed.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with building a function that models a relationship between two quantities benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because students need tools that will allow them to be able to determine if their thinking, planning and process is helping them understand the problem and how to solve it.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on building a function that models a relationship between two quantities by revisiting student thinking through a short mini lesson because some students have trouble writing their thinking and they just need more time to explain what they are thinking.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on building a function that models a relationship between two quantities by helping students move from specific answers to generalizations for certain types of problems because students need to understand that the content used in this unit is not only useful for one relationship between two quantities. It is a concept that they will continue to use every time that they have a relationship between two quantities.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when studying building a function that models a relationship between two quantities because students will be able to explore relationships between

	two quantities in different perspectives. With the problem being open-ended it allows the students to explain and describe their thinking without feeling pressured to one specific answer.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying building a function that models a relationship between two quantities the pattern of questions within the classroom is critical because by posing purposeful questions you will be able to scaffold the activity to provide multiple entry points meeting students where they are at.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 1474177

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra	■■■	Heart of Algebra	Linear functions	5. Write the rule for a linear function given two input/output pairs or one input/output pair and the rate of change.	Calculator

Population of Greenleaf, Idaho

Year	Population
2000	862
2010	846

The table above shows the population of Greenleaf, Idaho, for the years 2000 and 2010. If the relationship between population and year is linear, which of the following functions P models the population of Greenleaf t years after 2000?

A. $P(t) = 862 - 1.6t$

B. $P(t) = 862 - 16t$

C. $P(t) = 862 + 16(t - 2{,}000)$

D. $P(t) = 862 - 1.6(t - 2{,}000)$

Rationale

Choice A is correct. It is given that the relationship between population and year is linear; therefore, the function that models the population t years after 2000 is of the form $P(t) = mt + b$, where m is the slope and b is the population when $t = 0$. In the year 2000, $t = 0$. Therefore, $b = 862$. The slope is given by $m = \frac{P(10) - P(0)}{10 - 0} = \frac{846 - 862}{10 - 0} = \frac{-16}{10} = -1.6$. Therefore, $P(t) = -1.6t + 862$, which is equivalent to the equation in choice A.

Choice B is incorrect and may be the result of incorrectly calculating the slope as just the change in the value of P. Choice C is incorrect and may be the result of the same error as in choice B, in addition to incorrectly using t to represent the year, instead of the number of years after 2000. Choice D is incorrect and may be the result of incorrectly using t to represent the year instead of the number of years after 2000.

https://docs.google.com/a/hobbsschools.net/viewer?a=v&pid=sites&srcid=c2pjaXNkLm9yZ3xjb21tb24tY29yZS1tYXRoZW1hdGljcy1hc3Nlc3NtZW50LWZpZWxkLXRlc3Rpbmctc2l0ZXxneDoyNjA3Mzc0N2YwZDBkZDMz

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on building a function that models a relationship between two quantities, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, finding something that you do in your family and create a function model to show someone can create a strong connection between your school tasks and your life tasks.	Science: In high school the NGSS students should apply concepts of statistics and probability to explain the variation and distribution of expressed traits in a population. Consider providing a connection for students to examine scientific data and predict the effect of a change in one variable on another. https://www.nextgenscience.org/topic-arrangement/hsinheritance-and-variation-traits

NEW MEXICO Public Education Department

HS: FUNCTIONS- BUILDING FUNCTIONS

Cluster Statement: B: Build new functions from existing functions.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.BF.B.3: Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. *Note: Algebra 1 focuses on linear, exponential, quadratic, and absolute value*	SMP 5: Students can use tools by using graphing calculators or technology to experiment with parent functions and the results when different transformations are applied. SMP 8: Students look for and express regularity in repeated reasoning by exploring different expressions for transformations of f(x) and generalizing the effects.	• Identify vertical transformations from a function or a graph. • Identify horizontal transformations from a function or a graph. • Identify a shrink or a stretch from a function or a graph. • Write the results from such transformations. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
Standard Text HSF.BF.B.4: Find inverse functions. • HSF.BF.B.4.A: Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x^3 or f(x) = (x+1)/(x-1) for x ≠ 1.* *Note: Algebra 1 focuses on linear only*	**Standard for Mathematical Practices** SMP.6 Students can attend to precision by understanding that some functions do not have an inverse unless there is some sort of restriction on the domain. SMP 7: Students can look for and make use of structure by recognizing that the ordered pair (x, y) is reversed for a function's inverse.	**Students who demonstrate understanding can:** • Write the inverse of a simple function. • Relate using an inverse as an operation that undoes another operation. • Determine restrictions on the domain to allow for an inverse to exist. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
Previous Learning Connections • Connect to recognizing and understanding that all linear functions can be written in the form y = mx + b. **(8.F.3)** • Connect to graphing linear relationships. **(8.F.5)**	**Current Learning Connections** • Connect to graphing linear, quadratic, and exponential relationships. **(HSF.IF.4)**	**Future Learning Connections** • Connect to extending transformation patterns to all functions. **(HSF.BF.3)** • Connect to graph transformations and compositions of transformations on a coordinate plane. **(HSF.BF.1)**

Clarification Statement

- HSF.BF.B.3: Students should describe the effect of **stretches, shrinkages**, **vertical and horizontal transformations** of **linear, quadratic** and **exponential functions**. They should be able to find the value of the **transformation** when given a **graph** and be able to explain effects of transformations using technology. Students should know that adding a **constant** k to a function will change the graph of the function depending not only on the value of the constant, but on where it is inserted as well. If $y = f(x)$ is changed to $y = f(x) + k$, the curve will **shift** vertically (up for $k > 0$, down if $k < 0$). Adding k to x such that $y = f(x + k)$ will shift the curve horizontally (left for $k > 0$, right for $k < 0$). Multiplying f(x) by a constant k stretches ($k > 1$) or **squishes** ($0 < k < 1$) the graph vertically. If $k < 0$, the graph is also **flipped** over the **x-axis**. Multiplying x by k stretches ($k > 0$) or squishes ($k < 0$) the graph horizontally.
- HSF.BF.B.4: Students should be able to find the **inverse** of **simple linear functions** and recognize that other functions may not have an inverse unless there are **restrictions** placed on the **domain**. If $f(x) = y$ is a function, the inverse function can be found by switching the place of x and y ($f(y) = x$), and then solving for y so that $f-1(x) = y$. For instance, if the function f(x) is $y = 2x^3$, then the inverse function $f^{-1}(x)$ consists of switching the places of x and y ($x = 2y^3$) and then solving for y.

Common Misconceptions

- Students often have difficulty determining the direction of the horizontal shifts.
- Students often confuse the notation for the inverse and negative numbers.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying building new functions from existing functions because students will need to make connections to the previous standard. If they still have misconceptions it is better to address before the new standard is introduced to reduce the amount of future confusion.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.F.A.3: This standard provides a foundation for work with building new functions from existing functions because students identify the type of relationship the two quantities have (linear, non-linear, exponential) and they can create new functions. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user.*

- For example, learners engaging with building new functions from existing functions benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features give an example connected to this standard such as the size of text, images, graphs, tables, or other visual content; contrast between background and text or image; color used for information or emphasis; volume or rate of speech or sound; speed or timing of video, animation, sound, simulations, etc.; layout of visual or other elements; font used for print materials because students can compare the functions using various displays.

Build
Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence.*

- For example, learners engaging with building new functions from existing functions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as such as providing feedback that is substantive and informative rather than comparative or competitive because students will begin to see the relationship between parent functions and how changes to the parent function move the graph. When students look for and express regularity in repeated reasoning by exploring different expressions for transformations of f(x) and generalizing the effects (MP.8) they begin to understand how changing a function rule moves the graph of the function.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with building new functions from existing functions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as using technology (graphing calculators, desmos.com) because students can explore function types, how changes to the function rule moves the graph of the function and begin to see the relationship between the function rule and the graph.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with building new functions from existing functions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as allowing access to graphing calculators, graph paper, colored pencils, desmos.com provides students with the opportunity to choose appropriate tools (MP.4) They can focus on the mathematics and show their representation of their solution. In the real-world technology is used most of the time when graphing is needed.

Internalize
Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with building new functions from existing functions benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because students must have the opportunity to choose the appropriate tool to determine if their thinking, planning and process is helping them understand the problem and how to solve it.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on Build new functions from existing functions by critiquing student approaches/solutions to make connections through a short mini-lesson because by having students critiquing their work or others they are able to make connections which they can use to help them build new functions.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after the unit building new functions from existing functions by addressing conceptual understanding because this will inform the teacher what the student understands and why it is important to understand why building new functions from existing functions is useful.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students. when studying building new functions from existing functions because some students can do the assignments but sometimes do not fully understand the concept. This will allow them to focus on the concept and not just on finishing the problems.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying building new functions from existing functions, the use of mathematical representations within the classroom is critical because students will need different representations when creating new functions from existing functions. Students will need to make connections to their previous "mathematical and cultural "knowledge.

NEW MEXICO
Public Education Department

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 4788993

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math	■■□	Passport to Advanced Mathematics	Nonlinear functions	2. For a quadratic or exponential function, e. make connections between tabular, algebraic, and graphical representations of the function, by iii. determining how a graph is affected by a change to its equation, including a vertical shift or scaling of the graph.	Calculator

4788993

In the xy-plane, which of the following changes to the graph of the equation $y=x^2+3$ will result in the graph of the equation $y=(x^2+3)-6$?

A. A shift 6 units to the left

B. A shift 6 units to the right

C. A shift 6 units upward

D. A shift 6 units downward

Rationale

Choice D is correct. The graph of the equation $y=(x^2+3)-6$ is similar to the graph of $y=x^2+3$ in the xy-plane. The constant -6 in the equation $y=(x^2+3)-6$ indicates a vertical shift of 6 units downward from the graph of $y=x^2+3$. Therefore, a shift of 6 units downward of the graph of $y=x^2+3$ will result in the graph of $y=(x^2+3)-6$.
Choice A is incorrect. A shift of 6 units to the left is equivalent to the graph of $y=(x+6)^2+3$. Choice B is incorrect. A shift of 6 units to the right is equivalent to the graph of $y=(x-6)^2+3$. Choice C is incorrect. A shift 6 units upward is equivalent to the graph of $y=(x^2+3)+6$.

https://docs.google.com/a/hobbsschools.net/viewer?a=v&pid=sites&srcid=c2pjaXNkLm9yZ3xjb21tb24tY29yZS1tYXRoZW1hdGljcy1hc3Nlc3NtZW50LWZpZWxkLXRlc3Rpbmctc2l0ZXxneDo1ODhmMGI5NGNlNGU3MDYy

Relevance to families and communities:

During a unit focused on building new functions from existing functions., consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example compare functions that represent your community that you can find on line. This can create a strong connection between your school tasks and your community.

Cross-Curricular Connections:

Science: The equation for velocity, M(v) = $6v^2$, is one where the variable, v, has directions. Therefore, an inverse function of M(v) cannot give back both a positive and negative velocity. Consider providing a connection for students to consider how they will handle this situation.

<table>
<tr><th colspan="3">HS: FUNCTIONS – LINEAR, QUADRATIC, & EXPONENTIAL MODELS*</th></tr>
<tr><td colspan="3">Cluster Statement: A: Construct and compare linear, quadratic, and exponential models and solve problems.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.</td></tr>
<tr><td rowspan="3">Standard Text

HSF.LE.A.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.
• HSF.LE.A.1.A: Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
• HSF.LE.A.1.B: Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
• HSF.LE.A.1.C: Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</td><td rowspan="3">Standard for Mathematical Practices

SMP 3: Students can construct viable arguments and critique the reasoning of others when comparing linear and exponential functions, proving that linear functions grow by equal differences over equal intervals and exponential functions grow by equal factors over equal intervals.

SMP 8: Students look for and express regularity in repeated reasoning by showing that the rate of change over any given interval of a linear function is the same.</td><td>Students who demonstrate understanding can:
• Compare linear and exponential functions in various ways.
• Show that linear functions have a common difference and that exponential functions have a common ratio.
• Determine when a relationship is growing by a constant difference.
• Determine when a relationship grows by a common ratio.</td></tr>
<tr><td>Webb's Depth of Knowledge: 1-3</td></tr>
<tr><td>Bloom's Taxonomy:
Understand, Apply, Analyze, Evaluate</td></tr>
<tr><td>Standard Text

HSF.LE.A.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</td><td>Standard for Mathematical Practices

SMP 3: Students can construct viable arguments by convincing classmates when a set of data represents linear or exponential relationships.

SMP 5: Students can use tools strategically by determining whether a given description of a numerical relationship is linear or exponential using a table, graph or verbal description.</td><td>Students who demonstrate understanding can:
• Write linear and exponential functions (including arithmetic and geometric sequences) based on a graph.
• Write linear and exponential functions (including arithmetic and geometric sequences) based on a description of a relationship.
• Write linear and exponential functions (including arithmetic and geometric sequences) based</td></tr>
</table>

		on two ordered pairs (including from a table). • Decide whether a relationship is linear, or exponential given a table, graph or verbal description.
		Webb's Depth of Knowledge: 1-2
		Bloom's Taxonomy: Understand, Apply, Analyze
Standard Text HSF.LE.A.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	**Standard for Mathematical Practices** SMP 5: Students can use tools strategically by using technology to determine relation rates of change and make conclusions. SMP 8: Students look for and express regularity in repeated reasoning by comparing exponential, quadratic and linear functions (either with graphs or tables) to realize that a quantity increasing exponentially will exceed a quantity growing linearly or quadratically.	**Students who demonstrate understanding can:** • Explore rates of change of different functions using graphs or tables. • Generalize that an exponential growth function will exceed a linear or quadratic function eventually. • identify situations where this phenomenon is occurring.
		Webb's Depth of Knowledge: 1-2
		Bloom's Taxonomy: Understand, Apply, Analyze
<u>Previous Learning Connections</u> • Connect to determining the growth of a linear expression by taking the ratio of rise over run for any two distinct points on the same line. **(8.EE.6)** • Connect to relating the information gathered by the ratio of rise over run to the linear equation in terms of input and output. **(8.F.4)**	**<u>Current Learning Connections</u>** • Connect to examining contextual information and distinguishing if the solution can be modeled with linear or exponential functions. **(HSA.CED.3)** • Connect to writing arithmetic and geometric sequences both recursively and with an explicit formula to model situations. **(HSF.BF.1)** • Connect to relating the knowledge of linear functions to exponential and polynomial functions and comparing their behaviors. **(HSF.If.9)**	**<u>Future Learning Connections</u>** • Connect to extending their knowledge of linear, quadratic and exponential situations to different types of functions and making comparisons. **(HSF.IF.7-9)**

Clarification Statement

- HSE.LE.A.1: Students should be able to differentiate between **exponential** and **linear functions** by determining whether given relationships have a **common difference** or a **common ratio**. Students have to know the differences between linear functions and exponential functions. In simplest terms, a linear function one that takes the **form y = mx + b** and an exponential function is one in which **y = ax**.

- HSE.LE.A.2: When given a variety of descriptions (whether words, **graphs**, or **tables**), student should write linear and exponential functions. Students should determine and explain (orally and in writing) whether **relationships**—in descriptions, tables, **equations**, or graphs—are functions. In a table, students should recall that when the **difference** in **interval** is **constant**, we can presume that our equation is most likely linear. In this case it is simply a matter of f(x) = x + 1. Students should understand that when graphs are involved, students should **plot** points. That way, students can assemble a list of **input and output values** from the graph. As for descriptions, words to watch out for are "exponential," "linear," "**multiple**," "**constant**," and "**factor.**"
- HSE.LE.A.3: Students should understand that a function **growing** exponentially will eventually overtake or grow faster that either a linear or quadratic function. Students should be able to compare linear and exponential relationships by performing calculations and by interpreting graphs that show two **growth patterns**. Students should be able to **prove** that eventually, as long as the functions are headed in the same direction, a quantity increasing exponentially will "beat" linear, quadratic, and polynomial functions.

Common Misconceptions

- Students may not realize when a table or set of points increases by an interval other than 1 and not take the effect of this into account when finding the common difference or ration.
- Students may find it difficult to attend to direction and rates of change, making it hard to then compare the graphs.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that <rehearses prior learning when studying Construct And Compare Linear, Quadratic, And Exponential Models And Solve Problems because this allows students to go over what they previously learned and think about the process and skills needed to construct models of functions.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.F.A.2 This standard provides a foundation for work with Construct And Compare Linear, Quadratic, And Exponential Models And Solve Problems because students compared properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). This sets up the concept they will need for this cluster. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Physical Action: *How will the learning for students provide a variety of methods for navigation to support access?*

- For example, learners engaging with constructing and comparing linear, quadratic, and exponential models and solve problems benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as varying methods for response and navigation by providing alternatives to requirements for rate, timing, speed, and range of motor action with instructional materials, physical manipulatives, and technologies; physically responding or indicating selections; physically interacting with materials by hand, voice, single switch, joystick, keyboard, or adapted keyboard because when students are given different options of constructing function models and they can identify the model they created then students are able to see the mathematics behind the problems.

Build
Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with constructing and comparing linear, quadratic, and exponential models and solve problems. benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing prompts that guide learners in when and how to ask peers and/or teachers for help because when students are given the opportunity to question and explore more deeply their work is that they can be better understood and more thoughtful opinions and decisions are made.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with constructing and comparing linear, quadratic, and exponential models and solve problems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because the students will be able to solve the problem and not be confused about what the problem is requiring them to do since they know the vocabulary and the symbols needed. Students will be more prepared and therefore more confident as they work thru the problems.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with constructing and comparing linear, quadratic, and exponential models and solve problems. benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing different approaches to motivate, guide, feedback or inform students of progress towards fluency because students like instant feedback and teachers been able to have different forms of giving feedback will only keep the students engaged, because they like to hear different form of been told that they are doing a good job instead of "yes, you are correct or good job."

Internalize
Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with constructing and comparing linear, quadratic, and exponential models and solving problems benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as addressing subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") because when students are constructing models and they can explain them to others then the students know if they are on the right path of solving the problem. If there are misconceptions, they can find them as they explain their model and they can make changes as needed.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on constructing and comparing linear, quadratic, and exponential models and solving problems by critiquing student approaches/solutions to make connections through a short mini lesson because students will be given an opportunity to hear vocabulary and revisit concepts and skills needed to construct various models.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on constructing and comparing linear, quadratic, and exponential models and solving problems offering opportunities to understand and explore different strategies because when students are able to have various opportunities to understand and explore different strategies then they are able to think of the different models that they have learned about and use that connection to solve the problem by choosing the strategy that they understand the best.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying constructing and comparing linear, quadratic, and exponential models and solving problems because students can design their own learning path and select the resources, guides and information they will need to discover new information ad think critically about it.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying constructing and comparing linear, quadratic, and exponential models and solve problems the types of mathematical tasks are critical because the tasks need to be engaging and allow students to use multiple solution strategies which will give the students opportunities to make comparisons.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: https://satsuitequestionbank.collegeboard.org/

Question ID 5440666

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Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra	■■■	Heart of Algebra	Linear functions	2. Create a linear function to model a relationship between two quantities.	Calculator

5440666

The dwarf planet Makemake completes one orbit around the Sun every 310 years. Which of the following functions r models the number of orbits of Makemake in t years?

A. $r(t) = 310 + t$

B. $r(t) = 310t$

C. $r(t) = \frac{t}{310}$

D. $r(t) = \frac{310}{t}$

Rationale

Choice C is correct. It's given that Makemake completes one orbit around the Sun every 310 years. This can be represented by the ratio $\frac{1 \text{ orbit}}{310 \text{ years}}$. The number of orbits r in t years can be determined by setting up an equivalent ratio and solving the proportion for r: $\frac{1 \text{ orbit}}{310 \text{ years}} = \frac{r \text{ orbits}}{t \text{ years}}$. Cross-multiplying yields $310r = 1t$, or $r = \frac{t}{310}$. Therefore, the function $r(t) = \frac{t}{310}$ models the number of orbits of Makemake in t years.

Choice A is incorrect and results from adding the time needed for one orbit and the number of years t. Choice B is incorrect and results from multiplying the time needed for one orbit by the number of years t. Choice D is incorrect and results from dividing the time needed for one orbit by the number of years t.

https://www.map.mathshell.org/tasks.php?unit=HN08&collection=9&redir=1

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on constructing and comparing linear, quadratic, and exponential models and solve problems, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example while looking at the current events that are occurring in your community or in the world create a model and determine the type of model it represents. This will create a strong connection on math tasks and current events that affect your life.	Science: Exponential functions can model population growth. However, they will ultimately be limited by resource availability. Consider providing a connection where students track and/or predict when and why this will happen for a given population.

<table>
<tr><th colspan="3">HS: FUNCTIONS – LINEAR, QUADRATIC, & EXPONENTIAL MODELS*</th></tr>
<tr><td colspan="3">Cluster Statement: B: Interpret expressions for functions in terms of the situation they model.</td></tr>
<tr>
<td rowspan="3">Standard Text

HSF.LE.B.5: Interpret the parameters in a linear or exponential function in terms of a context.

Note: Algebra 1 focuses on linear and exponential of form $f(x)=b^x+k$.</td>
<td rowspan="3">Standard for Mathematical Practices

SMP 2: Students can reason abstractly and quantitatively by associating parts of a function or graph of a function to its meaning within a given context.

SMP 7: Students can look for and make use of structure by understanding and analyzing linear and exponential functions in order to describe their key features in terms of a context.</td>
<td>Students who demonstrate understanding can:
• Explain the meanings of inputs and outputs of both exponential ($y=b^x+k$) and linear functions in terms of a given context.
• Explain the meaning of parts of functions in terms of context (e.g., if x is ice cream cones 5x means 5 times the number of ice cream cones).
• Identify the parameters of a linear or exponential equation and know the parameters may be different based on the context (parameters include initial values, rate of change or growth factor/rate, etc.).</td>
</tr>
<tr><td>Webb's Depth of Knowledge: 1-2</td></tr>
<tr><td>Bloom's Taxonomy:
Understand, Apply, Analyze</td></tr>
<tr>
<td><u>Previous Learning Connections</u>
• Connect to understanding slope is a rate of change expressed as the ratio of rise over run for any two distinct points on the same line. (8.EE.6)
• Connect to relating the information gathered by the ratio of rise over run to the linear equation and understand that a change in slope will cause the steepness of the line to change. (8.F.2)
• Connect to simplifying exponential expressions using the Rules of Exponents. (8.EE.1)</td>
<td><u>Current Learning Connections</u>
• Connect to using paper and pencil, graphing calculators, graphing programs, spreadsheets, or other graphing technologies to model and interpret parameters in linear, quadratic, or exponential functions. Parameters may include slope, y-intercept, base value, and vertical shifts. (HSF.IF.7, HSF.BF.3)
• Connect to studying functions to develop contextual understanding on parameter changes in linear and exponential function situations. (HSF.LE.1-3)</td>
<td><u>Future Learning Connections</u>
• Connect to extending analysis to different types of functions and interpreting the key features in modeling situations. (HSF.IF.4-6, 7)</td>
</tr>
</table>

Clarification Statement
HSF.LE.B.4: Student should be able to describe parts of **linear** and **exponential functions** in terms of a context. They should be able to describe **slope** and **y-intercepts** or A and B (when in **standard form**) for a linear function and describe and/or differentiate between the **initial value** and **growth factor** for an exponential function. In more complex problems such as exponentials and **polynomials**, it may be useful to break down the problem so that it's clearly understood what is changing by how much for every what. **Translating** the equation into words or vice versa may help understand the equation in terms of the overall **context**. (For instance, every additional packet of gum sold, denoted by x, increases the **revenue** y by 0.95 dollars. That's what the equation y = 0.95x ultimately means.)

Common Misconceptions
- Students often confuse decay factor with the rate of decay.
- Students may be able to identify the slope and y-intercept but not understand their meaning.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying Interpreting expressions for functions in terms of the situation they model because this allows students to go over what they previously learned and think about the misconceptions they had about the process and skills needed to construct models of functions. This will benefit students when interpreting expressions.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.F.B.4: This standard provides a foundation for work with interpreting expressions for functions in terms of the situation they model because this standard asks students to construct a function to model a linear relationship between two quantities. Students learned to read values from a table or from a graph and interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access
Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with Interpret expressions for functions in terms of the situation they model benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because when students use personal experiences and then model them thru mathematics the students are more interested in doing the task and finding out the outcome. The students become curious about how their personal experience would look thru mathematics.

Build
Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with interpreting expressions for functions in terms of the situation they model benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as prompting or requiring learners to explicitly formulate or restate learning goals because when students understand the learning goal they will stay on task and work on meeting the goal. Students need a goal so that they can reach it.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with <Interpret expressions for functions in terms of the situation they model benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because students will need their previous knowledge of functions to be able to interpret the model in terms of the situation.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with interpreting expressions for functions in terms of the situation they model benefit when learning experiences attend to the multiple ways, students can express knowledge, ideas, and concepts such as using social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) because this will support the interest of the students by allowing them to use different media to represent their model and interpretation of the functions.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with interpreting expressions for functions in terms of the situation they model benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) because when students are able to connect a model of a function relation to any situation then they have created an understanding of how mathematics can be connected to real life.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on interpreting expressions for functions in terms of the situation they model. By providing specific feedback to students on their work through a short mini-lesson, misconceptions can be addressed immediately.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit interpreting expressions for functions in terms of the situation they model by confronting student misconceptions because students will be aware of them and avoid them next time they are exposed to the same task.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying interpreting expressions for functions in terms of the situation they model because this allows students to make connections not only with the mathematics but notice how this unit is related to other topics.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time." For example, when studying interpreting expressions for functions in terms of the situation they model, eliciting and using student thinking is critical because students need to feel comfortable that their peers will validate their thinking so they can share what they did. Sharing is an opportunity to learn from our mistakes and from others.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 19789

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra	■■□	Heart of Algebra	Linear functions	3. For a linear function that represents a context a. interpret the meaning of an input/output pair, constant, variable, factor, term, or graph based on the context, including situations where seeing structure provides an advantage;	Calculator

19789

The average number of students per classroom at Central High School from 2000 to 2010 can be modeled by the equation $y = 0.56x + 27.2$, where x represents the number of years since 2000, and y represents the average number of students per classroom. Which of the following best describes the meaning of the number 0.56 in the equation?

A. The total number of students at the school in 2000

B. The average number of students per classroom in 2000

C. The estimated increase in the average number of students per classroom each year

D. The estimated difference between the average number of students per classroom in 2010 and in 2000

Rationale

Choice C is correct. In the equation $y = 0.56x + 27.2$, the value of x increases by 1 for each year that passes. Each time x increases by 1, y increases by 0.56 since 0.56 is the slope of the graph of this equation. Since y represents the average number of students per classroom in the year represented by x, it follows that, according to the model, the estimated increase each year in the average number of students per classroom at Central High School is 0.56. Choice A is incorrect because the total number of students in the school in 2000 is the product of the average number of students per classroom and the total number of classrooms, which would appropriately be approximated by the y-intercept (27.2) times the total number of classrooms, which is not given. Choice B is incorrect because the average number of students per classroom in 2000 is given by the y-intercept of the graph of the equation, but the question is asking for the meaning of the number 0.56, which is the slope. Choice D is incorrect because 0.56 represents the estimated yearly change in the average number of students per classroom. The estimated difference between the average number of students per classroom in 2010 and 2000 is 0.56 times the number of years that have passed between 2000 and 2010, that is, $0.56 \times 10 = 5.6$.

https://www.map.mathshell.org/tasks.php?unit=HN08&collection=9&redir=1
https://docs.google.com/a/hobbsschools.net/viewer?a=v&pid=sites&srcid=c2pjaXNkLm9yZ3xjb21tb24tY29yZS1tYXRoZW1hdGljcy1hc3Nlc3NtZW50LWZpZWxkLXRlc3Rpbmctc2l0ZXxneDo3NTkzOTczZTJhN2M1MjJk

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on interpreting expressions for functions in terms of the situation they model, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, think of a special event or activity that you have done. How would you represent it as a function and how would it model the	Science: Colony Collapse Disorder refers to the drastic loss of honeybees and honeybee colonies, such as what has been observed around the world in recent decades. Consider providing a connection where students construct models based on the data and then use those models to describe factors affecting the bee colony populations.

event or activity? Using the mathematics, you will create a stronger connection between your personal life and mathematics.	

NEW MEXICO
Public Education Department

HS: STATISTICS & PROBABILITY — INTERPRETING CATEGORICAL & QUANTITATIVE DATA

Cluster Statement: A: Summarize, represent, and interpret data on a single count or measurement variable

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSS.ID. A.1: Represent data with plots on the real number line (dot plots, histograms, and box plots).	SMP 1: Students can make sense of problems and persevere in solving them by analyzing the data they are given and determining the best method for representing the data (dot plots, histograms, box plots). SMP 3: Students can construct viable arguments by defending their reasoning for their chosen display.	• Summarize data using a dot plot. • Summarize data using a histogram. • Summarize data using a box plot. • Know when each of these is appropriate to be used **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSS.ID.A.2: Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. *Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.*	SMP 2: Students can reason abstractly and quantitatively by determining the appropriate measure of center or spread to use in describing a data set based on its shape. SMP 3: Students can construct viable arguments by justifying appropriate measures of center based on distribution shape and unusual features.	• Calculate the median, mean, interquartile range, and standard deviation of a set of data. • Identify and describe differences in two sets of data base on these calculations. • Identify and describe the shape of a set of data (skewness, symmetric, bimodal, normal). **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze

<table>
<tr>
<td rowspan="3">Standard Text

HSS.ID.A.3: Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).</td>
<td rowspan="3">Standard for Mathematical Practices

SMP 2: Students can reason abstractly and quantitatively by using the center shape and spread of data sets in order to compare them and interpret the meaning of those measures of center and spread within their given contexts.

SMP 6: Students can attend to precision by recognizing and naming different shapes and their characteristics of center, shape and spread.</td>
<td>Students who demonstrate understanding can:
• Describe how the presence or removal of an outlier changes the shape, center and spread of a data set.
• Explain why some data sets will tend towards skewness (e.g., tests scores with an upper limit that students do well on tend to be left skewed while heights tend to be more normally distributed).
• Recognize that the shape of the data is usually connected to the relative positions of the mean and median (e.g., left skewed data has a mean that is lower than the median).</td>
</tr>
<tr>
<td>Webb's Depth of Knowledge: 1-2</td>
</tr>
<tr>
<td>Bloom's Taxonomy:
Understand, Apply, Analyze</td>
</tr>
<tr>
<td><u>Previous Learning Connections</u>
• Connect to plotting points on a coordinate grid. (5.G.1-2)
• Connect to plotting data on dot plots and boxplot. (6.SP.4)
• Connect to describe center and spread in a data distribution. (6.SP.5)</td>
<td><u>Current Learning Connections</u>
• Connect to how outliers can affect data and skew data.
• Connect to classroom test scores or heights of students (something relevant to them)</td>
<td><u>Future Learning Connections</u>
• Connect to using a standard deviation to make conclusions about a set of data. (HSS.IS.4)
• Investigate normal distributions within a context. (HSS.IS.4)
• Calculate confidence intervals based on a normal curve, mean and standard deviation. (HSS.IC.4)</td>
</tr>
<tr>
<td colspan="3">Clarification Statement
• HSS.ID.A.1: Students should not only be able to construct each of these plot types but be able to do so in a way that shows the data in a meaningful way considering things like spread and center. For example, making bins of appropriate width in when making a histogram or appropriate spacing on a number line for a dot plot or a box plot. Students should know that a dot plot is a diagram that represents a data set using dots over the number line. A histogram is a diagram that shows a data set as a series of rectangles that shows how often data occur within a given interval. A box plot, also called a box and whisker plot, is a diagram that shows a data set as a distribution along the number line, divided into four equal parts using the median (the middle data value) and the upper and lower quartiles (median of upper and lower half of data, respectively).
• HSS.ID.A.2: Students should use statistics appropriately to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Students should know that the center of data can give us a good sense of the data set overall. The center of the data is exactly what it sounds like: a representation of the middle of the data, or a typical value. It gives us a good first guess as to where on the number line the data will fall. Students should know the two types of centers of data: mean and median. The mean, or average, is the sum of all the data points divided by the number of data points, while the median is the value that splits the data into two intervals.</td>
</tr>
</table>

- HSS.ID.A.3: Students understand and use the context of the data to explain why its distribution takes on a shape (e.g., Is the data **skewed**? Are there **outliers**?). Students understand that the higher the value of a measure of **variability**, the more spread out the data set is. Measures of variability are **range** (100% of data), standard deviation (68-95-99.7% of data), and interquartile range (50% of data). Students explain the effect of any outliers on the shape, center, and spread of the data sets.

Common Misconceptions

- Students may forget to arrange the data in numerical order before finding key numbers needed for creating a box plot.
- When doing normal distribution calculations, students often report the area to the left of a boundary when they are asked about the area to the right of the boundary.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying the center, spread and overall shape of the data sets because students need to understand the conceptual knowledge of center and spread in the context of the problems. Students need to interpret the information numerically and graphically.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.SP.A.2 and 6.SP.B.5: This standard provides a foundation for work with interpreting the center and spread of the data sets because students use their prior knowledge to compare the center and spread of different data sets and make implication in the context of the problems. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with representing and interpreting the data set of one variable benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because students represent and interpret data sets of real-life situations. Students understand the application and implication of using math and make connections to their prior experience.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with representing and interpreting the data sets of one variable benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports because students use their background knowledge to support their understanding and interpretation of the data set. Students contribute their knowledge and understanding to support their peers to understand the meaning of the shape, center and spread of the data sets through meaningful discourse.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with interpreting and comparing data sets of one variable benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams because students interpret the center and spread of the data sets graphically and numerically. Students connect their understanding of the center and spread of data sets in the context of the problems using graphs, charts and math models.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with interpreting the shape, the center and spread of the data sets in the context of the problems benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem because students interpret the center and spread of the data plots graphically and numerically. Students make connections of the information graphically, numerically and verbally in the context of the data.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with interpreting and comparing the center and spread of the data sets of one variable benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) because students use the understanding of center and spread to compare and interpret different data sets to describe the implication of the solutions in the context of the problems.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on interpretation of the center and the spread of the data sets by clarifying mathematical ideas and/or concepts through a short mini-lesson because students use different measures of center and spread to explain the meaning of center and spread in the context of the data set. Students use the interpretation of center and spread to compare two different data sets.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit interpreting the center and spread of the data sets graphically and numerically by addressing conceptual understanding because students need to interpret the center and spread of the data sets graphically and numerically. Students interpret and compare the center and spread of 2 data sets graphically. ...

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying interpretation of shape, center and spread of data sets because students may predict the impact on shape, center and spread of data sets when the sample changes. Students may justify possible bias of the sampling method by comparing the shape, center and spread of the data.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

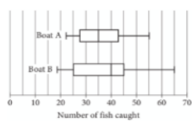
Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying data on a single variable the pattern of questions within the classroom is critical because teachers use open-ended questions to scaffold the information that students can summarize and interpret from the data set. Students explore different perspectives of interpreting the data and its implication.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 4170248

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	**Math**	**Problem Solving and Data Analysis**	■■□	**Problem Solving and Data Analysis**	**One variable data: Distributions and measures of center and spread**	**1. Choose an appropriate graphical representation for a given data set.**	**Calculator**

4170248



The box plots above summarize the distribution of the number of fish caught each day on two commercial fishing boats for a season. By how many fish does the median number of fish caught each day on Boat B exceed the median number on Boat A?

Rationale

The correct answer is 5. According to the Boat A box plot, the median number of fish caught on Boat A is 35. According to the Boat B box plot, the median number of fish caught on Boat B is 40. The number of fish by which the median number of fish caught each day on Boat B exceeds the median number on Boat A is $40 - 35$, or 5.

https://www.insidemathematics.org/sites/default/files/materials/archery.pd

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on summarizing, representing and interpreting data in a single variable, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, using data that is relevant to the community and the summary of the data provides useful information to students regarding to their families, culture and community.	Social Studies: In high school the New Mexico Social Studies Standards state students should "explain how to use technological tools to research data, verify facts and information, and communicate findings." Consider providing a connection for students to write a report describing and analyzing a specific set of data.

NEW MEXICO
Public Education Department

<table>
<tr><th colspan="3">HS: STATISTICS & PROBABILITY — INTERPRETING CATEGORICAL & QUANTITATIVE DATA</th></tr>
<tr><td colspan="3">Cluster Statement: B: Summarize, represent, and interpret data on two categorical and quantitative variables</td></tr>
<tr><td rowspan="3">Standard Text

HSS.ID.B.5: Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Note: Algebra 1 focuses on Linear, discuss general principle</td>
<td rowspan="3">Standard for Mathematical Practices

SMP 2: Students can reason abstractly and quantitatively by completing two-way tables and interpreting frequencies found in them based on the context of the data.

SMP 6: Students can attend to precision by identifying and finding the relative frequency from a two-way table based on what's asked.</td>
<td>Students who demonstrate understanding can:
Summarize data by creating a two-way frequency table for two categories of data.
Calculate relative frequencies in the context of data.
Identify and describe correlations in relative frequencies that could signify possible causation.</td></tr>
<tr><td>Webb's Depth of Knowledge: 1-2</td></tr>
<tr><td>Bloom's Taxonomy:
Understand, Apply, Analyze</td></tr>
<tr><td rowspan="3">Standard Text

HSS.ID.B.6: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
HSS.ID.B.6.A: Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
HSS.ID.B.6.B: Informally assess the fit of a function by plotting and analyzing residuals.
HSS.ID. B.6.C: Fit a linear function for a</td>
<td rowspan="3">Standard for Mathematical Practices

SMP 4: Students can model with mathematics by writing functions that represent data in various contexts.

SMP 5: Students use tools strategically by using technology to create scatter-plots, calculate a best-fit function, and establish its reasonableness based on a residual plot.</td>
<td>Students who demonstrate understanding can:
Summarize data by creating a scatter plot for two quantitative variables.
Identify and describe how these two variables are related (e.g., positive, negative or no correlation).
Explain how strongly or negatively correlated two variables are.
Determine what type of curve is most appropriate to represent a given set of data.
Create a residual graph.
Determine if a curve is an appropriate model based on the residual graph.
Estimate a line of best fit for a scatterplot of data that is linearly related.</td></tr>
<tr><td>Webb's Depth of Knowledge: 1-2</td></tr>
<tr><td>Bloom's Taxonomy:
Understand, Apply, Analyze</td></tr>
</table>

scatter plot that suggests a linear association. *Note: Algebra 1 focuses on Linear, discuss general principle*		
<u>Previous Learning Connections</u> • Connect to constructing and interpreting two-way tables using frequencies and relative frequencies. **(8.SP.4)** • Connect to using relative frequencies to describe a possible association between two variables. **(8.SP.4)** • Connect to constructing and interpreting scatterplots. **(8.SP.1)** • Connect to constructing an equation or a function to model a linear relationship and determine/interpret the slope and y-intercept. **(8.F.4)**	**<u>Current Learning Connections</u>** • Connect to writing functions for linear, quadratic, and exponential representations. **(HSA.CED.1-3)** • Connect to **i**dentifying representations as linear, quadratic and exponential models. **(HSF.B.4)** • Connect to constructing and comparing linear and exponential functions to solve problems. **(HSF.LE.1)**	**<u>Future Learning Connections</u>** • Connect to constructing and interpreting two-way frequency tables to determine independence. **(HSS.CP.4)** • Connect to constructing and interpreting two-way frequency tables to calculate conditional probabilities. **(HSS.CP.4)**

Clarification Statement

- HSS.ID.B.5: Students will develop an understanding and analyze the vocabulary of **two-way frequency tables**. Students will learn the table entries are the **joint frequencies**, **row** and **column totals** constitute the **marginal frequencies**, and dividing joint or marginal frequencies by the total number of subjects define **relative frequencies**, respectively. Students will also know **conditional relative frequencies** are determined by focusing on a specific row or column of the table and are particularly useful in determining any associations between the two **variables**. Students are flexible in identifying and interpreting the information from a two-way frequency table. They complete **calculations** to determine frequencies and use those frequencies to describe and compare variables.
- HSS.ID.B.6: Students represent **data** on two **quantitative variables** on a **scatter plot**, and describe how the variables are related. Students will **fit a function to the data**, use functions fitted to data to solve problems in the context of the data, use given functions or choose a **function** suggested by the context. (Emphasis is on **linear and quadratic models**.) Students will also informally assess the **fit of a function** by **plotting** and **analyzin**g r**esiduals** and fit a linear function for a scatter plot that suggests a **linear association**.

Common Misconceptions

- Students may not consider outliers when analyzing data and determining the best fit of a function.
- Students often have difficult separating causation and association with contextual data sets.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when describing possible associations between bi-variate data. Students may benefit from min-lesson on the types of outcomes and examples of data (hot chocolate sales and temperature). Students may also benefit from a mini lesson on how to construct and analyze a two-way frequency table and how the frequency table can help determine possible associations.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.SP.4 provides a foundation for work with understanding patterns of association of bivariate data by displaying frequencies and relative frequencies in a two-way table. If students have unfinished learning with this standard providing opportunities for students to analyze and discuss possible associations will provide them with the opportunity for on grade level learning. Also discussing the difference between causation and correlation will benefit students before attempting on grade level work for this standard.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with summarizing, representing and interpreting data of 2 categorical or quantitative variables benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because students make meaningful connections to real-life data of 2 quantities. By using real-life examples, students understand and interpret the different data sets in the context of the problems based on their prior experience.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with informally assessing the fit of function by plotting and analyzing residuals benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as engaging learners in assessment discussions of what constitutes excellence because students discuss the features of the functions and how the function best match the data sets. Students justify the best fit function to the given data set with viable arguments and evidence.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with representing the data sets with mathematical model using functions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to the structure because students represent the 2 quantities in the data sets using the variables and model relationship of the quantities using variables and functions. Students interpret and describe the mathematical model used in the context of the data verbally and symbolically.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with using functions fitted to data to solve problems in the context of the data benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing different approaches to motivate, guide, feedback or inform students of progress towards fluency because students receive frequent feedback to guide their mathematical thinking of justifying which function fit the data sets. Students will test the data set and analyze the tread to check for the best fit to make sense of their mathematical thinking.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with representing and interpreting the quantities of the data sets using function benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations because students interpret and predict the correlation of the 2 quantities in the data sets using function. Students may make generalization and prediction of the quantities in the context of the problems with evidence and viable arguments.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on modeling the relation of the 2 variables with the appropriate mathematical model by revisiting student thinking through a short mini-lesson because students need to explain the features of the data sets or the graphs/plots. Students connect those features to the features of different functions.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit choosing the appropriate functions to model the 2 quantitative variables of the data by confronting student misconceptions because students need to use the functions model to make prediction and implication of the data. Students need to justify if the prediction and the implication make sense in the context of the data.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying representing the data with a function model because students explain the interval(s) when the function model fits the scatter plot. Students describe the possible situation when the function model does not fit the scatter plot and possible explanation.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students. For example, when studying data on two variables supporting productive struggle is critical because students may explore multiple ways of representing and interpreting two variables data sets. Students explore different functions to model the data set and defend their choice. When the function is not the best fit for the domain, students need to develop flexible solutions and define underlying assumptions about the math model used.

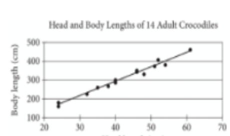
Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

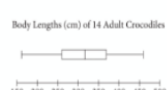
Question ID 4170271

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Problem Solving and Data Analysis	■□□	Problem Solving and Data Analysis	Two-variable data: Models and scatterplots	1. Using a model that fits the data in a scatterplot, compare values predicted by the model to values given in the data set.	Calculator

4170271

Questions 3-5 refer to the following information.





The scatterplot above represents the head lengths, in centimeters (cm), and body lengths, in cm, of 14 adult crocodiles. The line of best fit for the data is also shown. The box plot above summarizes the body lengths of the 14 crocodiles.

For an adult crocodile with a head length of 30 cm, which of the following is closest to the body length, in cm, predicted by the line of best fit?

For an adult crocodile with a head length of 30 cm, which of the following is closest to the body length, in cm, predicted by the line of best fit?

A. 180

B. 215

C. 250

D. 275

Rationale

Choice B is correct. It's given that the adult crocodile has a head length of 30 cm. Based on the line of best fit shown in the scatterplot, an adult crocodile that has a head length of 30 cm is predicted to have a body length that falls between 200 cm and 250 cm. Only choice B gives a body length that is between 200 cm and 250 cm.
Choice A is incorrect. This is approximately the minimum body length predicted by the line of best fit. Choices C and D are incorrect and may result from misinterpreting the scatterplot.

https://tasks.illustrativemathematics.org/content-standards/tasks/1887

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on summarizing, representing and interpreting data on two variables, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, using data that is relevant to the community and the summary of the data provides useful information to students regarding to their families, culture and community.	Social Studies: In high school the New Mexico Social Studies Standards state students should "explain how to use technological tools to research data, verify facts and information, and communicate findings." Consider providing a connection for students to determine the best fit of a function for a set of data and explain their choice.

HS: STATISTICS & PROBABILITY — INTERPRETING CATEGORICAL & QUANTITATIVE DATA

Cluster Statement: C: Interpret linear models

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSS.ID.C.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. *Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.*	SMP 2: Students can reason abstractly and quantitatively by describing the meaning of a slope and y-intercept related to linear (or nearly linear) bivariate set of data. SMP 7: Students can use the structure of the equation for a line of best fit or a graph of the data to describe its rate of change and intercept in context of the data.	• Explain the slope and intercept of a linear model in the context of data from a visual model. • Explain the slope and intercept of a linear model in the context of data from written notation. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSS.ID.C.8: Compute (using technology) and interpret the correlation coefficient of a linear fit.	SMP 2: Students reason abstractly and quantitatively by making sense of correlation coefficients and their relationship to different situations. SMP 5: Students can use e tools strategically by using technology to calculate correlation coefficients to determine the strength of a model.	• Find the correlation coefficient using technology. • Describe the meaning of the correlation coefficient of a given set of data in the context of the problem **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSS.ID.C.9: Distinguish between correlation and causation.	SMP 2: Students can reason abstractly and quantitatively by determining when a situation has correlation without causation, correlation with causation or correlation with possible causation. SMP 3: Students can construct viable arguments by verbally justifying whether two variables show causation.	• Explain the difference between correlation and causation. • Give examples of variables that are correlated, but have no logical causal connection (e.g., number of bee stings and ice cream sales both go up in the summer but there isn't a causal link between the two). • Give examples of variables that have both correlation and a high likelihood of causation (e.g., the amount of time spent studying for an exam and the score on the exam). • Give examples of variables that are neither correlated nor have a causal link (e.g., the number of shoes you own and how many students are in your third period class).
		Webb's Depth of Knowledge: 1-2
		Bloom's Taxonomy: Understand, Apply, Analyze
<u>Previous Learning Connections</u> • Connect to plotting points in a coordinate grid and constructing an equation or a function to model the linear relationship. **(5.G.1-2)** • Connect to constructing and interpreting scatterplots. **(8.SP.1)** • Connect to constructing an equation or a function to model a linear relationship and determining/interpreting the slope and y-intercept. **(8.F.4)**	**<u>Current Learning Connections</u>** • Connect to explaining the slope and y-intercept as they relate to the context of the original problem. **(HSF.IF.4)** • Connect to creating linear functions and using them to solve problems. **(HSA.CED.1-3)** • Connect to interpreting key features of graphs and functions. **(HSF.IF.4)**	**<u>Future Learning Connections</u>** • Connect to determining which function fits the data. (linear, exponential, quadratic).
Clarification Statement • HSS.ID.C.7: Students interpret the **slope (rate of change)** and the **y-intercept (constant term)** of a **linear model** in the context of the **data**. Students may use **graphing calculators or software** to create **representations of data sets**, create linear models, and to assist student them in interpreting the data. Students should know that all linear models take the form **y = mx + b** where m is the slope and b is the y-intercept.		

- HSS.ID.C.8: Students should compute (using technology) and interpret the **correlation coefficient** of a **linear fi**t and use it to understand the strength of a **linear relationship**. Students should match the correlation coefficient to its appropriate **scatter plot** and linear model.

  Students should use the correlation coefficient to determine the **goodness of fit** for a linear model. Students should know that it has the symbol r and that it ranges from -1 to 1. A coefficient equal to 1.0 suggests a **positive correlation** between the data. This means that as the **independent variable** (x) **increases** so does the **dependent variable** (y).

  A correlation coefficient equal to -1.0 suggests a **negative correlation** between the data, or as the independent variable (x) increases, the dependent variable **decreases**.

  If the coefficient equals 0, there is no **linear correlation**. However, just because the linear correlation coefficient equals 0 doesn't mean there is not another type of correlation between the data. Students should also know that in addition to being positive or negative, the correlation coefficient can be **weak** or **strong**. The closer the correlation is to -1 or 1, the stronger the correlation.

- HSS.ID.C.9: Students should be able to do more than just give the definition of correlation and **causation**. This should be developed as a skill of critical thinking where students are expected to first look at every set of data to determine if it is appropriate to be making comparisons between them. Students need to remember that correlation does not imply causation. For example, let's say we find that there's a strong positive linear correlation between the age of a tree and how many apples it produces. In fact, this correlation is so strong that r = 0.99. Does that mean the age of the tree causes more apples to grow? Can there be other factors? (e.g., What about rainfall? Did the farmer use fertilizer? Did he prune the trees? What were the summer and winter temperatures? Any one of these factors may have influenced the number of apples.)

Common Misconceptions

- Students do not always know that slope, rate of change, and steepness are interchangeable.
- Students may try to determine the appropriateness of a line of best fit based only on the value of r.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying representation of linear function because students need to understand the features of the linear model and the meaning of the features. Students need to rehearse different ways of writing the linear model.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.F.8.4: This standard provides a foundation for work with constructing a function to model a linear relationship between 2 quantities because students need to know the information needed to model the linear function. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with interpreting the linear model in the context of the data benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because students make meaningful connections to real-life data sets. By using real-life examples, students understand and interpret the linear model in the context of the problems based on their prior experience and make implication of the data sets.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with interpreting the slope (rate of change) and the intercept (the constant term) of a linear model in the context of the data benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because students use examples relevant to their interest to interpret the meaning of slope and intercept in the context of the data. Students expand their understanding of slope and intercept to describe the implication of the linear model in the context of the data.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with interpreting the meaning of slope and intercept of a linear model in the context of the data benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because students make connection of their prior knowledge of slope and intercept of linear model in the context of the data. Students interpret the features of the linear model in the context of the problems verbally and symbolically.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with computing (using technology and interpreting the correlation coefficient of a linear fit benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because students use graphing calculators and graph paper to calculate and validate the correlation coefficient of a linear fit. Students use the correlation coefficient to explain the relationship of the two quantities of the data set.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with interpreting the linear model in the context of the data and distinguishing between correlation and causation benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations because students use their understanding of the rate of change and constant of the linear model to describe the correlation coefficient of the data set. Students interpret the trend of the data using the correlation coefficient. Students distinguish between correlation and causation by plotting analyzing the data.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on modeling the scatter plot with a linear function by clarifying mathematical ideas and/or concepts through a short mini-lesson because students need to use the appropriate information to model the scatter plot with linear function. Students understand the connection of rate of change and intercept of the linear function in context of the data.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit representing the scatter plot with linear function by addressing conceptual understanding because students explain the implication of rate of change and intercepts of the linear function in the context of the data. Students use the rate of change and intercepts to predict the trend of the behavior and describe the correlation.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying modeling with linear function and calculating the correlation because students explain the similarities and differences of correlation and causation. Students describe and compare real-life data that has a correlation and real-life data that has causation.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

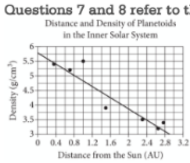
Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others.” For example, when studying interpreting linear models for data facilitating meaningful mathematical discourse is critical because students might use different linear models to represent the data sets. Students interpret and summarize the data differently using their linear models. Students compare the conclusion and defend the solution by constructing viable arguments.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 423228

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	**Problem Solving and Data Analysis**	■□□	**Problem Solving and Data Analysis**	**Two-variable data: Models and scatterplots**	**1. Using a model that fits the data in a scatterplot, compare values predicted by the model to values given in the data set.**	**Calculator**

Questions 7 and 8 refer to the following information.



The scatterplot above shows the densities of 7 planetoids, in grams per cubic centimeter, with respect to their average distances from the Sun in astronomical units (AU). The line of best fit is also shown.

An astronomer has discovered a new planetoid about 1.2 AU from the Sun. According to the line of best fit, which of the following best approximates the density of the planetoid, in grams per cubic centimeter?

A. 3.6

B. 4.1

C. 4.6

D. 5.5

Rationale

Choice C is correct. According to the line of best fit, a planetoid with a distance from the Sun of 1.2 AU has a predicted density between $4.5\ g/cm^3$ and $4.75\ g/cm^3$. The only choice in this range is 4.6.

Choices A, B, and D are incorrect and may result from misreading the information in the scatterplot.

https://tasks.illustrativemathematics.org/content-standards/tasks/1888

Relevance to families and communities:

During a unit focused on interpreting linear models of the data, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, relating the mathematical models used to real-life data to interpret and predict the trend of data in order to provide useful information for decision-making in the community.

Cross-Curricular Connections:

Science: In high school the NNSS state students should “use mathematical and/or computational representations to support explanations of factors that affect carrying capacity of ecosystems at different scales.” Consider providing a connection based where students must analyze data and consider the relationship among various factors including boundaries, resources, climate, and competition.

https://www.nextgenscience.org/topic-arrangement/hsinterdependent-relationships-ecosystems

Section 3: Resources, References, and Glossary

Resources

Evidence-Based Resources	English Learner Resources	MLSS Resources	Mathematics Standard Resources
What Works Clearinghouse Best Evidence Encyclopedia Evidence for Every Students Succeeds Act Evidence in Education Lab	World-Class Instructional Design and Assessment (WIDA) Standards USCALE Language Routines for Mathematics English Language Development Standards Spanish Language Development Standards	NM Multi-Layered System of Supports (MLSS) Universal Design for Learning Guidelines Achieve the Core: Instructional Routines for Mathematics Project Zero Thinking Routines	Focus by Grade Level and Widely Applicable Prerequisites High school Coherence Map College-and Career Ready Math Shifts Fostering Math Practices: Routines for the Mathematical Practices

Planning Guidance for Multi-Layered Systems of Support: Core Instruction[10]

Core Instructional Planning must reflect and leverage scientific insights into how humans learn in order to ensure all students are ready for success, thus the following guidance for optimizing teaching and learning is grounded in the **Universal Design Learning (UDL) Framework**

Key design questions, planning actions, and potential strategies are provided below, with respect to guidance for minimizing barriers to learning and optimizing (1) universal ACCESS to learning experiences, (2) opportunities for students to BUILD their understanding of the **Learning Goal**, and (3) INTERNALIZATION of the Learning Goal.

Optimizing Universal ACCESS to Learning Experiences	
ENGAGEMENT ⍰ How will you provide multiple options for recruiting interest?	**Recruiting Student Interest**: ⍰ What do you anticipate in the range of student interest for this lesson? ▯ Plan for options for recruiting student interest: ❒ provide choice (e.g. sequence or timing of task completion) ❒ set personal academic goals ❒ provide contextualized examples connected to their lives ❒ support culturally relevant connections (i.e home culture) ❒ create socially relevant tasks ❒ provide novel & relevant problems to make sense of complex ideas in creative ways

[10] Adapted from: CAST (2018). *Universal Design for Learning Guidelines version 2.2*. Retrieved from http://udlguidelines.cast.org

	❒ provide time for self-reflection about content & activities ❒ create accepting and supportive classroom climate ❒ utilize **instructional routines** to involve all students
REPRESENTATION ⍰ How will you reduce barriers to perceiving the information presented in this lesson?	**Perception**: ⍰ What do you anticipate about the range in how students will perceive information presented in this lesson? ▯ Plan for different modalities and formats to reduce barriers to learning: ❒ display information in a flexible format to vary perceptual features ❒ offer alternatives for auditory information ❒ offer alternatives for visual information
ACTION & EXPRESSION ⍰ How will the learning for students provide a variety of methods for navigation to support access?	**Physical Action**: ⍰ What do you anticipate about the range in how students will physically navigate and respond to the learning experience? ▯ Plan a variety of methods for response and navigation of learning experiences by offering alternatives to: ❒ requirements for rate, timing, speed, and range of motor action with instructional materials, manipulatives, and technologies ❒ physically indicating selections ❒ interacting with materials by hand, voice, keyboard, etc.

Opportunities for Students to BUILD their Understanding	
ENGAGEMENT ⍰ How will the learning for students provide options for sustaining effort and persistence?	**Sustaining Effort & Persistence**: ⍰ What do you anticipate about the range in student effort? ▯ Plan multiple methods for attending to student attention and affect by: ❒ prompting learners to explicitly formulate or restate learning goals ❒ displaying the learning goals in multiple ways ❒ using prompts or scaffolds for visualizing desired outcomes ❒ engaging assessment discussions of what constitutes excellence ❒ generating relevant examples with students that connect to their cultural background and interests ❒ providing alternatives in the math representations and scaffolds ❒ creating cooperative groups with clear goals, roles, responsibilities ❒ providing prompts to guide when and how to ask for help ❒ supporting opportunities for peer interactions and supports (e.g. peer tutors) ❒ constructing communities of learners engaged in common interests ❒ creating expectations for group work (e.g., rubrics, norms, etc.) ❒ providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge ❒ providing feedback that: ❒ emphasizes effort, improvement, and achieving a standard rather than on relative performance ❒ is frequent, timely, and specific ❒ is informative rather than comparative or competitive

	❒ models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success
REPRESENTATION ? How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners?	**Language & Symbols**: ? What do you anticipate about the range of student background experience and vocabulary? ▯ Plan multiple methods for attending to linguistic and nonlinguistic representations of mathematics to ensure universal clarity by: ❒ pre-teaching vocabulary and symbols in ways that promote connection to the learners' experience and prior knowledge ❒ graphic symbols with alternative text descriptions ❒ highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to structure ❒ embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) ❒ embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) ❒ highlighting structural relations or make them more explicit ❒ making connections to previously learned structures ❒ making relationships between elements explicit (e.g., highlighting the transition words in an argument, links between ideas, etc.) ❒ allowing the use of text-to-speech and automatic voicing with digital mathematical notation (math ml) ❒ allowing flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) ❒ clarification of notation through lists of key terms ❒ making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf ❒ linking key vocabulary words to definitions and pronunciations in both dominant and heritage languages ❒ defining domain-specific vocabulary (e.g., "map key" in social studies) using both domain-specific and common terms ❒ electronic translation tools or links to multilingual web glossaries ❒ embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc) ❒ presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) ❒ making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams
ACTION & EXPRESSION ? How will the learning provide multiple	**Expression & Communication**: ? What do you anticipate about the range in how students will express their thinking in the learning environment? ▯ Plan multiple methods for attending to the various ways in which students can express knowledge, ideas, and concepts by providing:

modalities for students to easily express knowledge, ideas, and concepts in the learning environment?	❒ options to compose in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video ❒ use of social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) ❒ flexibility in using a variety of problem solving strategies ❒ spell or grammar checkers, word prediction software ❒ text-to-speech software, human dictation, recording ❒ calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper ❒ sentence starters or sentence strips ❒ concept mapping tools ❒ Computer-Aided-Design (CAD) or mathematical notation software ❒ virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) ❒ multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches) ❒ multiple examples of novel solutions to authentic problems ❒ different approaches to motivate, guide, feedback or inform students of progress towards fluency ❒ scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) ❒ differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners)

Optimizing INTERNALIZATION of the Learning Goal	
ENGAGEMENT ? How will the design of the learning strategically support students to effectively cope and engage with the environment?	**Self-Regulation**: ? What do you anticipate about barriers to student engagement? ▯ Plan to address barriers to engagement by promoting healthy responses and interactions, and ownership of learning goals: ❒ metacognitive approaches to frustration when doing mathematics ❒ increase length of on-task orientation through distractions ❒ frequent self-reflection and self-reinforcements ❒ address subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") ❒ offer devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the math practices for the purpose of monitoring and improving ❒ use activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely
REPRESENTATION ? How will the learning support transforming accessible information into usable knowledge	**Comprehension**: ? What do you anticipate about barriers to student comprehension? ▯ Plan to address barriers to comprehension by intentionally building connections to prior understandings and experiences, relating meaningful information to learning goals,

that is accessible for future learning and decision-making?	providing a process for meaning making of new learning, and applying learning to new contexts: ❒ incorporate explicit opportunities for review and practice ❒ note-taking templates, graphic organizers, concept maps ❒ scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) ❒ explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) ❒ opportunities over time to revisit key ideas and connections ❒ make explicit cross-curricular connections ❒ highlight key elements in tasks, graphics, diagrams, formulas ❒ outlines, graphic organizers, unit organizer routines, concept organizer routines, and concept mastery routines to emphasize key ideas and relationships ❒ multiple examples & non-examples ❒ cues and prompts to draw attention to critical features ❒ highlight previously learned skills that can be used to solve unfamiliar problems ❒ options for organizing and possible approaches (tables and representations for processing mathematical operations) ❒ interactive representations that guide exploration and new understandings ❒ introduce graduated scaffolds that support information processing strategies ❒ tasks with multiple entry points and optional pathways ❒ "Chunk" information into smaller elements ❒ remove unnecessary distractions unless essential to learning goal ❒ anchor instruction by linking to and activating relevant prior knowledge (e.g., using visual imagery, concept anchoring, or concept mastery routines) ❒ pre-teach critical prerequisite concepts via demonstration or representations ❒ embed new ideas in familiar ideas and contexts (e.g., use of analogy, metaphor, drama, music, film, etc.) ❒ advanced organizers (e.g., KWL methods, concept maps) ❒ bridge concepts with relevant analogies and metaphors
ACCESS ACTION & EXPRESSION ⍰ How will the learning for students support the development of executive functions to allow them to take advantage of their environment?	**Executive Functions**: ⍰ What do you anticipate about barriers to students demonstrating what they know? ▯ Plan to address barriers to demonstrating understanding by providing opportunities for students to set goals, formulate plans, use tools and processes to support organization and memory, and analyze their growth in learning and how to build from it: ❒ prompts and scaffolds to estimate effort, resources, difficulty ❒ models and examples of process and product of goal-setting ❒ guides and checklists for scaffolding goal-setting ❒ post goals, objectives, and schedules in an obvious place ❒ embed prompts to "show and explain your work" ❒ checklists and project plan templates for understanding the problem, prioritization, sequences, and schedules of steps ❒ embed coaches/mentors to demonstrate think-alouds of process ❒ guides to break long-term goals into short-term objectives ❒ graphic organizers/templates for organizing information & data ❒ embed prompts for categorizing and systematizing ❒ checklists and guides for note-taking ❒ asking questions to guide self-monitoring and reflection ❒ showing representations of progress (e.g., before and after photos, graphs/charts showing progress, process portfolios)

	❒ prompt learners to identify type of feedback or advice they seek ❒ templates to guide self-reflection on quality & completeness ❒ differentiated models of self-assessment strategies (e.g., role-playing, video reviews, peer feedback) ❒ assessment checklists, scoring rubrics, and multiple examples of annotated student work/performance examples

Planning Guidance for Culturally and Linguistically Responsive Instruction[11]

In order to ensure our students from marginalized cultures and languages view themselves as confident and competent learners and doers of mathematics within and outside of the classroom, educators must intentionally plan ways to counteract the negative or missing images and representations that exist in our curricular resources. The guiding questions below support the design of lessons that validate, affirm, build, and bridge home and school culture for learners of mathematics:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

In addition, Aguirre and her colleagues[12] define **mathematical identities** as the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives. Many students see themselves as "not good at math" and approach math with fear and lack of confidence. Their identity, developed through earlier years of schooling, has the potential to affect their school and career choices.

Five Equity-Based Mathematics Teaching Practices[13]

Go deep with mathematics. Develop students' conceptual understanding, procedural fluency, and problem solving and reasoning.

Leverage multiple mathematical competencies. Use students' different mathematical strengths as a resource for learning.

Affirm mathematics learners' identities. Promote student participation and value different ways of contributing.

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[11] This resource relied heavily on the work of: Hollie, S. (2011). Culturally and linguistically responsive teaching and learning. Teacher Created Materials. (see also, https://www.culturallyresponsive.org/vabb)

[12] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices. Reston, VA: National Council of Teachers of Mathematics (p. 14).

[13] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc. (p.6). (adapted from Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013) (p. 43).

Challenge spaces of marginality**. Embrace student competencies, value multiple mathematical contributions, and position students as sources of expertise.

Draw on multiple resources of knowledge (mathematics, language, culture, family). Tap students' knowledge and experiences as resources for mathematics learning.

The following lesson design strategies support Culturally and Linguistically Responsive Instruction, specific examples for each cluster of standards can be found in part 2 of the document. These were adapted from the Promoting Equity section of the Taking Action series published by NCTM.[14]

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.

Mathematical Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher.

Modifying Mathematical Tasks: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)."

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics.

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages.

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we

[14] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians.

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others."

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time."

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students.

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Glossary[15]

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (- 3/4) = (- 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[16]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. See: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances
from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten.
For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

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[15] Glossary and tables taken from: Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/

[16] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[17] See also: median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.
Integer. A number expressible in the form a or -a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line.
Also known as a dot plot.[18]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[19] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 Ã· 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 Ã— 4/3 = 4/3 Ã— 3/4 = 1.

[17] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

[18] Adapted from Wisconsin Department of Public Instruction, op. cit.

[19] To be more precise, this defines the arithmetic mean.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin,
selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also*: uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form *a/b* or – *a/b* for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also*: terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[20]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar
model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

[20] Adapted from Wisconsin Department of Public Instruction, op. cit.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also*: median, first quartile, interquartile range.

Table 1: Common addition and subtraction.[1]

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 =5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now?5-2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?? -2 = 3
	TOTAL UNKNOWN	ADDEND UNKNOWN	BOTH ADDENDS UNKNOWN[2]
PUT TOGETHER / TAKE APART[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5-3 = ?	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? 5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1, 5 = 2 + 3, 5 = 3 + 2
COMPARE	DIFFERENCE UNKNOWN	BIGGER UNKNOWN	SMALLER UNKNOWN
	(“How many more?” version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?(“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? 2 + ? = 5, 5 – 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with “more”):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2: Common multiplication and division situations.[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	3 x ? = 18, and 18 ÷ 3 = ?	? x 6 = 18, and 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	a x b = ?	a x ? = p and p ÷ a = ?	? x b = p, and p ÷ b = ?

[1]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3: The properties of operations.

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	(a +b) + c = a + (b+c)
Commutative property of addition	a + b = b + a

Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists -*a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) x c = a x (b x c)
Commutative property of multiplication	a x b = b x a
Multiplicative identity property 1	a x 1 = 1 x a = a
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that a x $1/a$ = $1/a$ x a = 1
Distributive property of multiplication over additions	a x (b + c) = a x b + a x c

Table 4: The properties of equality.

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a.
Symmetric property of equality	If a = b, then b = a.
Transitive property of equality	If a = b and b = c , then a = c.
Addition property of equality	If a = b, then a +c = b + c.
Subtraction property of equality	If a = b then a – c = b – c.
Multiplication property of equality	If a = b, then a x c = b x c.
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c.
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality.

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b.
If a > b and b > c then a > c.
If a > b, b < a.
If a > b, then -a <-b.
If a > b, then a ± c > b ± c.
If a > b and c > 0, then a x c > b x c.
If a > b and c < 0, then a x c < b x c.
If a > b and c > 0, then a ÷ c > b ÷ c.
If a > b and c < 0, then a ÷ c < b ÷ c.