New Mexico Mathematics Instructional Scope for Algebra 2

June 2020

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Overview

This mathematics instructional scope was created by a cohort of New Mexico educators and the New Mexico Public Education Department. This document is organized into three sections. Section 1 describes how to use this document to support equitable and excellent mathematics instruction. Section 2 contains planning support for each cluster of mathematics standards within the grade level or course. Section 3 provides additional resources, references, and glossary.

The intention of this document is to act as companion during the planning process alongside High Quality Instructional Materials (HQIM). A sample template is presented to show a quick snapshot of planning supports provided within each cluster of standards in section 2.

During the creation of this document, we leveraged the work of other states, organizations, and educators from across country and the world. This work would not have been possible without all that came before it and we wish to express our sincerest gratitude for everyone that contributed to the resources listed within our references. This document is a work in progress and in some circumstances, our team of New Mexico educators may have embedded content from resources that have yet to be cited, as these elements are discovered in the use of this tool the references in section 3 will be updated.

Section 1: New Mexico Instructional Scope for Supporting Equitable and Excellent Mathematics Instruction

To better understand the planning supports provided in section 2, for each cluster of standards, this section provides a brief description of each planning support including: *what* support is provided; *why* the planning support is critical for equitable and excellent mathematics instruction; and, *how* to use the planning support with HQIM.

Cluster Statement

What: The New Mexico Mathematics Standards are grouped by Domains with somewhere between 4 to 10 domains per grade level. Within each domain the standards are arranged around clusters. Cluster statements summarize groups of related standards. The cluster statement planning support also indicates if the clusters is major, supporting, or additional work of the grade.

Why: The New Mexico Mathematics Standards require a stronger *focus*[1] on the way time and energy are spent in the mathematics classroom. Students should spend the large majority of their time (65-85%) on the major clusters of the grade/course. Supporting clusters and, where appropriate, additional clusters should be connected to and engage students in the major work of the grade.

How: When planning with your HQIM consider the time being devoted to major versus additional or supporting clusters. Major Work of each grade should be designed to provide students with strong foundations for future mathematical work which will require more time than additional or supporting clusters. Consider also the ways the HQIM makes explicit for students the connections between additional and supporting clusters and the major work of the grade.

Standard Text

What: Each cluster level support document contains the text of each standard within the cluster.

Why: The cluster statement and standards are meant to be read together to understand the structure of the standards. By grouping the standards within the cluster the connectedness of the standards is reinforced.

How: The text of the standards should always ground all planning with HQIM. Reading the standards within a cluster intentionally focuses on the connections within and among the standards.

Standards for Mathematical Practice

What: The Standards for Mathematical Practice describe the varieties of expertise and habits of mind that mathematics educators at all levels should seek to develop in their students.

Why: Equitable and excellent mathematics instruction supports students in becoming confident and competent mathematicians. By engaging with the standards for mathematical practice students are engaging in the practice of doing mathematics and development of mathematical habits of mind—the ability to think mathematically, analyze situations, understand relationships, and adapt what they know to solve a wide range of problems, including problems they may not look like any they have encountered before.[2]

How: When planning with HQIM it is critical to consider the connections between the content standards and the standards for mathematical practice. The planning supports highlight a few practices in which students could engage when learning the content of the standard. Note it is not necessary or even appropriate to engage in all of the practices every day, rather choosing a few and spending time intentionally supporting students in learning both the what (content standards) and the how (standards for mathematical practice) will create a stronger foundation for ongoing learning.

Students Who Demonstrate Understanding Can (Webb's Depth of Knowledge and Bloom's Taxonomy)

What: The New Mexico Mathematics Standards include each aspect of mathematical rigor: conceptual understanding, procedural skill and fluency, and application to the real world.[3] This planning support considers which aspect(s) of rigor are within each standard and then identifies academics skills students need to demonstrate comprehension of

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[1] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

[2] Seeley, C. L. (2016). Math is Supposed to Make Sense. In *Making sense of math: How to help every student become a mathematical thinker and problem solver*. Alexandria, VA, USA: ASCD. (P. 13)

[3]Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

the standard and associated mathematical practices. The statements also highlight both the receptive (listening and reading) and expressive (speaking and writing) parts of language by considering the types of mathematical representations (verbal, visual, symbolic, contextual, physical) within the standard and what students need to do with them. The planning supports also provide information about two common classifications on cognitive complexity, Webb's Depth of Knowledge and Bloom's Taxonomy.

Why: Analyzing standards alongside the standards for mathematical practice provide a fuller picture of the mathematical competencies demanded in the standard.

How: When planning for a cluster of standards with your HQIM a critical first step is to analyze the content and language demands of the standards and standards for mathematical practice. The analysis can be used to inform formative assessment, or it can be used to plan/design appropriate formative assessment.[4] The planning supports provide a possible break-down of the standard that can serve as the basis for this sort analysis.

Connections

What: The New Mexico Mathematics Standards are designed around coherent progressions of learning. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but an extension of previous learning.[5] The connections to previous, current and future learning make this coherence visible.

Why: Students build stronger foundations for learning when they see mathematics as an inter-connected discipline of relationships rather than discrete skills and knowledge. The intentional inclusion of connections to previous, current, and future learning can support a more inter-connected understanding of mathematics.

How: When planning with HQIM use the connection planning supports to find ways to support students in making explicit connections within their study of mathematics.

Clarification Statement

What: The clarification statement provides greater clarity for teachers in understanding the purpose of the standards within a cluster.

Why: The New Mexico Mathematics Standards illustrate how progressions support student learning within each major domain of mathematics. The clarification statement provides additional context about the ways each cluster of standards supports student learning of the larger learning progression.

How: When planning with HQIM use the clarification statement to support an understanding of how the materials use specific types of representations or change the learning sequence from instructional approaches not grounded in progressions of learning.

Common Misconceptions

What: This planning support identifies some of the common misconceptions students develop about a mathematical topic.

Why: Students create misconceptions based on an over generalization of patterns they notice or an over reliance on rules rather than underlying mathematics. Rules in mathematics expire[6] over time (e.g., you can't subtract 1-3) as students expand their knowledge of mathematics (e.g., from whole numbers to rational numbers). It is critical to understand some of the common misconceptions students can develop so we can address them directly with students and continue to build a strong foundation for their mathematical learning.

[4] English Learners Success Forum. (2020). ELSF | Resource: Analyzing Content and Language Demands. Retrieved from https://www.elsuccessforum.org/resources/math-analyzing-content-and-language-demands
[5] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics
[6] Cardone, T. (n.d.). Nix the Tricks. Retrieved from https://nixthetricks.com/

How: When planning with your HQIM look for ways to directly address with students some common misconceptions. The planning supports in this document provide some possible misconceptions and your HQIM might include additional ones. The goal is not to avoid misconceptions, they are a natural part of the learning process, but we want to support students in exploring the misconception and modifying incorrect or partial understandings.

Multi-Layered System of Supports/Suggested Instructional Strategies

What: The section on Multi-Layered Systems of Supports(MLSS)/Suggested Instructional Strategies is designed to support teachers in planning for the needs of all students. Each section includes options for pre-teaching, reteaching, extensions and core instructional supports for students. Targeted pre-teaching and reteaching support student's acquisition of the knowledge and skills identified in the New Mexico Mathematics Standards to support student success with high-quality differentiated instruction. Intensive supports may be provided for a longer duration, more frequently, smaller groups, or otherwise be more intensive than targeted supports. Progress monitoring should occur to assess students' responses to additional supports, see Standards Aligned Instructionally Embedded Formative Assessment Resources.

Why: MLSS is a holistic framework that guides educators, those closest to the student, to intervene quickly when students need additional supports. The framework moves away from the "wait to fail" model and empowers teachers to use their professional judgement to make data-informed decisions regarding the students in their classrooms to ensure academic success with the grade level expectations of the New Mexico Mathematics Standards.

How: When planning with your HQIM use the suggestions for pre-teaching as a starting point to determine if some or all of the students in your classroom may need targeted or intensive pre-teaching at the start of unit to ensure they can access the grade level material with the unit. The core-instruction and reteach sections work together to support planning within a unit, look for the ways the materials are supporting greater access for all students and providing options to revisit materials based on formative assessments. The planning supports for each cluster are grounded in the Universal Design Learning (UDL) Framework, additional planning supports based on this framework can be found in Section 3 of this document in the part titled, Planning Guidance for Multi-Layered Systems of Support: Core Instruction.

Culturally and Linguistically Responsive Instruction

What: Culturally and Linguistically Responsive Instruction (CLRI), or the practice of situational appropriateness, requires educators to contribute to a positive school climate by validating and affirming students' home languages and cultures. Validation is making the home culture and language legitimate, while affirmation is affirming or making clear that the home culture and language are positive assets. It is also the intentional effort to reverse negative stereotypes of non-dominant cultures and languages and must be intentional and purposeful, consistent and authentic, and proactive and reactive. Building and bridging is the extension of validation and affirmation. By building and bridging students learning to toggle between home culture and linguistic behaviors and expectations and the school culture and linguistic behaviors and expectations. The building component focuses on creating connections between the home culture and language and the expectations of school culture and language for success in school. The bridging component focuses on creating opportunities to practice situational appropriateness or utilizing appropriate cultural and linguistic behaviors.[7]

Why: The mathematical identities of students are shaped by the messages they receive about their ability to do mathematics and the power of mathematics in their lives outside of school.[8] Mathematics educators must intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages. In addition, create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to supports students in creating mathematical identities as capable mathematicians within school and society.

[7] Hollie, S. (2011). *Culturally and linguistically responsive teaching and learning*. Teacher Created Materials.
[8] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics. (P. 14)

How: When planning instruction is critical to consider ways to validate/affirm and build/bridge from your students cultural and linguistic assets. The planning supports for each cluster provide an example of how to support equity-based teaching practices. Look for additional ways within your HQIM to ensure all students develop strong mathematical identities.

Standards Aligned Instructionally Embedded Formative Assessment Resources

What: Formative Assessment is the planned, ongoing process used by all students and teachers during learning and teaching to elicit and use evidence of student learning to improve student understanding of the outcomes and support students to become directed learners. All New Mexico educators have access to standards aligned instructionally embedded formative assessments: iStation at K-2; Cognia at 3-8, and the SAT Suite Question Bank at 9-12. These are intended to be used during instruction for each at each grade alongside assessments within your HQIM.

Why: When student thinking is made visible the teacher can examine the progression of learning towards the goals of the standards and adjust instruction as necessary. By including students in the assessment and analysis process students become strategic and goal-directed with their learning.

How: The planning supports at each cluster provide an example of a task that addresses one more aspect of the cluster of standards. This example can be used to discuss possible responses by students and next steps for instruction. A similar process can then be used to identify additional items from one of the formative assessment resources provided by NM PED and your HQIM.

Relevance to Families and Communities

What: Relevance to families and communities requires finding the relevance of mathematics outside of the classroom by connecting to families and communities and learning about varied and often unexpected ways they use mathematics.

Why: When school mathematics is connected to the mathematics outside of school students can build a bridge between their ways of thinking about quantities outside and inside school created a bridge between home and school.

How: When planning at the year and unit level with you HQIM find ways to intentionally learn from your families and communities the cultural and linguistic ways they use mathematics outside of school.

Cross-Curricular Connections

What: New Mexico defines cross-curricular connections as connections between two or more areas of study made by teachers or students within the structure of a subject.

Why: The purpose of planning cross-curricular connections in an instructional sequence is to ensure that students build connections and recognize the relevance of mathematics beyond the mathematics classroom.

How: When planning with HQIM look for opportunities to make explicit connections to other content areas such as the examples provided for each cluster.

Template of the New Mexico Cluster Level Planning Support for the New Mexico Mathematics Standards

<table>
<tr><td colspan="4"><GRADE/COURSE/DOMAIN ABBREVIATION: DOMAIN NAME></td></tr>
<tr><td colspan="4">Cluster Statement: Statement from New Mexico Mathematics Standards summarize a group of related standards.

Major/Additional/Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)
Identifies if the cluster is major, additional or supporting work of the grade.</td></tr>
<tr><td rowspan="3">Standard Text
Full text of the standard</td><td rowspan="3" colspan="2">Standard for Mathematical Practices
The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.</td><td>Students who demonstrate understanding can: The cognitive skills students perform to demonstrate to comprehension of a standard.</td></tr>
<tr><td>Depth Of Knowledge: Correlation of standard to Webb’s Depth of Knowledge</td></tr>
<tr><td>Bloom’s Taxonomy: Correlation of standard to Bloom’s Taxonomy</td></tr>
<tr><td>Connections to Previous Learning:
Supports student connections to learning from previous grade levels.</td><td colspan="2">Connections to Current Learning
Supports student connections to learning within the grade level.</td><td>Connections to Future Learning
Supports student connections to learning in a future grade.</td></tr>
<tr><td colspan="4">Clarification Statement: Clarifies the language of the standard.</td></tr>
<tr><td colspan="4">Common Misconceptions: Guidance on where a student misconception or misunderstanding could potentially occur.</td></tr>
<tr><td colspan="4">Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies
Pre-Teach
Pre-teach (targeted): Guidance for how to activate students’ knowledge to support their learning.
Pre-teach (intensive): Guidance for how to use earlier grade standards to build a strong foundational understanding upon which to build grade level concepts.

Core Instruction
Access: Guidance for optimizing universal access to learning experiences.
Build: Guidance for supporting students build their understanding of the cluster.
Internalize: Guidance for ensuring student internalization of the learning goal.

Re-teach
Re-teach (targeted): Guidance for adjusting instruction during a unit by using formative assessment data.
Re-teach (intensive): Guidance for analyzing assessment data to identify content that would benefit from more intensive reteaching.
Extension Ideas: Suggestions that offer additional challenges to ‘broaden’ students’ knowledge of the mathematics within the cluster.</td></tr>
<tr><td colspan="4">Culturally and Linguistically Responsive Instruction: Provides equity based instructional suggestions aligned to the cluster of standards</td></tr>
<tr><td colspan="4">Standards Aligned Instructionally Embedded Formative Assessment Resources: Includes reference to high-quality formative assessment resources, including examples from New Mexico’s formative assessment banks.</td></tr>
<tr><td colspan="2">Relevance to Families and Communities:
Connecting with families and communities to create relevant connections between mathematics inside and outside of school.</td><td colspan="2">Cross Curricular Connections: Includes examples of how the cluster provides opportunities to connect to other disciplines such as literacy, science, social studies, and the arts.</td></tr>
</table>

Section 2: Cluster Level Planning Support for the New Mexico Mathematics Standards

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[9] Appendix A of the Common Core State Standards was used to determine the standards within each high school course, (+) were not included in this version of the instructional scope.

Linear, Quadratic, & Exponential Models

HSF.LE.A

Strand: Statistics & Probability

Interpreting Categorical and Quantitative Data

HSS.ID.A

Making Inferences & Justifying Conclusions

HSS.IC.A

HSS.IC.B

HS: NUMBER AND QUANTITY- THE COMPLEX NUMBER SYSTEM		
Cluster Statement: A: Perform arithmetic operations with complex numbers.		
Standard Text HSN.CN.A.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	**Standard for Mathematical Practices** SMP.1 Students can make sense of problems and persevere in solving them. Students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. SMP.2 Students can reason abstractly and quantitatively. Students make sense of quantities and their relationships in problem situations. SMP.6 Students attend to precision by communicating precisely to others. They use clear and precise definitions in discussion with others and in their own reasoning.	**Students who demonstrate understanding can:** • Identify the real number and the imaginary number of a complex number • Define an imaginary number (i.e. $i^2 = -1$). • Define complex numbers. • Find the complex conjugate. • Describe complex numbers in terms of their real and imaginary parts. **Webb's Depth Of Knowledge:** 1-2 **Bloom's Taxonomy:** understand
Standard Text HSN.CN.A.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	**Standard for Mathematical Practices** SMP.1 Students can make sense of problems and persevere in solving them. Students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. SMP.2 Students can reason abstractly and quantitatively.	**Students who demonstrate understanding can:** • Recognize that $i^2 = -1$ • Use the properties of operations to add and subtract complex numbers • Use the distributive property and the relation $i^2 = -1$ to multiply complex numbers. • Apply the commutative, associative, and distributive properties to complex numbers in order to add, subtract, and multiply. **Webb's Depth of Knowledge:** 1-2

	Students make sense of quantities and their relationships in problem situations. SMP.6 Students attend to precision by communicating precisely to others. They use clear and precise definitions in discussion with others and in their own reasoning.	**Bloom's Taxonomy:** understand, apply
Previous Learning Connections In Algebra 1, students solved quadratic equations using a variety of methods. Their solutions however were limited to real solutions.	**Current Learning Connections** Students will learn to solve quadratic and higher-order polynomial equations that have complex answers as those found within this cluster.	**Future Learning Connections** Students will relate this knowledge of complex numbers to solving rational equations, trigonometric equations and trigonometric form in subsequent math courses (Pre-Calculus, AP Calculus, College Algebra, etc).

Clarification Statement
Complex numbers expand the number system to include square roots of negative numbers and allows applications of complex numbers to electronics. Students use the properties of operations as it applies to complex numbers to simplify expressions and to build foundations to solve quadratic equations having complex solutions.

Common Misconceptions
Since most variables are letters and symbols, students may confuse i as a variable.

Students may try to simplify a complex number by combining the real part and the imaginary part.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., number lines) when studying to perform arithmetic operations with complex numbers because students no longer will be using real numbers on both axes in their graphs. The y-axis will be used for the imaginary numbers.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.NS.A.1: This standard provides a foundation for work with perform arithmetic operations with complex numbers because all numbers are classified as rational or irrational and the real number part of the complex numbers is still either. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?

- For example, learners engaging with <perform operations with complex numbers> benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features such as graphs, tables, videos, and symbols using different color markers/pencils to emphasis the real and imaginary numbers because students will be introduced to imaginary numbers and need the variety of representation. Students will recognize and make the connection that the operations properties on the real and imaginary numbers will be similar.

Build

Effort and Perception: How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?

- For example, learners engaging with <perform operations with complex numbers> benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features such as graphs, tables, videos, and symbols using different color markers/pencils to emphasis the real and imaginary numbers because students will be introduced to imaginary numbers and need the variety of representation. Students will recognize and make the connection that the operations properties on the real and imaginary numbers will be similar.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with <performs arithmetic operations with complex numbers> benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as <highlighting structural relations or make them more explicit> because <the imaginary number is introduced and students identify the real and imaginary numbers>.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with <perform arithmetic operations on complex numbers> benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as <providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use

differing approaches, strategies, skills, etc.)> because <students get a choice on the strategies that they can make a connection to and have the same outcome>.

Internalize

Comprehension: How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?

- For example, learners engaging with <perform arithmetic operations with complex numbers> benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing options for organizing and possible approaches (tables and representations for processing mathematical operations) because < vertically alignment of the real and complex numbers helps organize the student’s work and keep the numbers separated. Boxes help the students multiply binomials and trinomials. Multiple examples give student choices on which method to use.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on performing arithmetic operations with complex numbers by revisiting student thinking through a short mini-lesson because one of the students’ misconception is that the i is another variable. Check for misconceptions using aggressive monitoring.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit perform arithmetic operations with complex numbers by offering opportunities to understand and explore different strategies because students may make the connection between the properties of equations and the procedures within the complex number operations.

Extension

What type of extension will offer additional challenges to ‘broaden’ your student’s knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying to perform arithmetic operations with complex numbers because students explore how the operations will be used in later lessons by watching a short video.

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time." For example, when studying, performing arithmetic operations with complex numbers eliciting and using student thinking is critical because making mistakes and finding the error, students make adjustments and begin asking questions without any repercussions. They are comfortable and know mistakes are allowed and corrections can be made. Mistakes allow students to try instead of leaving questions blank. Challenging questions can also lead to critical thinking and when a task is complete, whether it's right or wrong, students feel the ownership of learning.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

CollegeBoard | Question ID 5344950

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Additional Topics in Math	Hard	Additional Topics in Math	Complex numbers	1. Apply knowledge and understanding of the complex number system to add, subtract, multiply, and divide with complex numbers and solve problems.	No Calculator

$i^2+(-i)^2$

In the complex number system, what is the value of the given expression? (Note: $i=\sqrt{-1}$
)

Question Difficulty: Hard

A. -2

B. 0

C. 2

D. $2i$

Choice A is correct. The power of a product property states that $(xy)^a=x^ay^a$. Using this property, the second term of the given expression can be rewritten as $(-1\times i)^2=(-1)^2i^2$, or i^2. Substituting i^2 in place of $(-i)^2$ in the given expression yields i^2+i^2, or $2i^2$. Since $i=\sqrt{-1}$, $i^2=-1$ and $2i^2=2(-1)$, or -2.

Choice B is incorrect and may result from rewriting $(-i)^2$ as $-i^2$ instead of i^2. Choice C is incorrect and may result from rewriting i^2 as 1 instead of -1. Choice D is incorrect and may result from rewriting i^2 as i instead of -1.

This type of assessment question requires students to simplify an expression using powers of the imaginary unit. Students will engage in SMP 6 as they must attend to the sign of their answers carefully.

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on performing arithmetic operations with complex numbers, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, learning about the history of the complex number systems and how the complex numbers originated and used in other countries. Different families can contribute small history pieces and will eventually turn into a big presentation to the class by the student.	**Science -** Science and Electrical Engineering use complex numbers, especially when dealing with light and radio wave. http://faculty.wcas.northwestern.edu/~infocom/Ideas/electric.html **History -** The ancient Greeks once believed that all numbers were rational numbers; that is, that every number could be expressed as the ratio of two integers, and they were very disturbed when it was demonstrated that the measure of the hypotenuse of an isosceles right triangle, having arms of unit measure, was not a rational number. http://mathforum.org/library/drmath/view/55747.html

HS: NUMBER AND QUANTITY- THE COMPLEX NUMBER SYSTEM

Cluster Statement: C: Use complex numbers in polynomial identities and equations.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSN.CN.C.7 Solve quadratic equations with real coefficients that have complex solutions.	SMP 2 Students reason abstractly and quantitatively by explaining their solutions to quadratics and their representations relative to roots of the quadratic. SMP 3 Students construct viable arguments and critique the reasoning of others by justifying solutions and appropriate techniques for solving different quadratic equations.	• Determine the number and nature of quadratic solutions; • Solve a quadratic equation using various methods (e.g., factoring, completing the square, quadratic formula) **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** understand, apply
Previous Learning Connections In Algebra 1, students solved quadratic equations using a variety of methods. Their solutions were limited however to real solutions.	**Current Learning Connections** Students learn to solve polynomial equations that have complex answers.	**Future Learning Connections** Students will connect this knowledge of complex numbers to solving rational equations, trigonometric equations and trigonometric form in subsequent math courses (Pre-Calculus, AP Calculus, College Algebra, etc).

Clarification Statement
Students will be able to use multiple methods to solve quadratic equations with complex solutions.

Common Misconceptions
Students may confuse non-real, imaginary and irrational numbers.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying using complex numbers in polynomial identities and equations because students will have to recall prior knowledge from previous grade levels.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.EE.A.1: This standard provides a foundation for work using complex numbers in polynomial identity and equations because students should be able to apply properties of operation strategies. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: How will the learning for students provide multiple options for recruiting student interest?

- For example, learners engaging with using complex numbers in polynomial identities and equations benefit when learning experiences include ways to recruit interest such as setting personal academic goals because students who are involved in setting their own goals are vested in their learning. Students tend to take more pride in learning the material and become more connected to the learning. Students should be involved in setting goals for the current content as well as for looking forward to future content connections. Students need to be offered choices in how to obtain their goals (such as how to practice their skills solving polynomials and using complex numbers).

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with using complex numbers in polynomial identities and equations benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because supporting students to build individual skills in self-determination and not giving up can improve their learning potential. When students are motivated and encouraged to be successful, they generally are in the face of adversity and difficulty.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with using complex numbers in polynomial identities and equations benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) because <students are able to find resources quickly to aid in

any misconceptions of the content when referencing what a complex number is for example or how to distinguish between polynomial identities and polynomial equations.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with using complex numbers in polynomial identities and equations benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because students are presented with a variety of strategies may find one strategy easier to comprehend than another which will provide students with a better understanding of the content of polynomials and complex numbers.

Internalize:

Executive Functions: How will the learning for students support the development of executive functions to allow them to take advantage of their environment?

- For example, learners engaging with using complex numbers in polynomial identities and equations benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as posting goals, objectives, and schedules in an obvious place because students will be able to focus on the objectives that have been presented and set goals for achieving the learning associated with those objectives and posting them in an obvious place makes it easy for students to refer back to them throughout the learning.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on using complex numbers in polynomial identities and equations by providing specific feedback to students on their work through a short mini-lesson because students who are having difficulty or who may be struggling will be able to get immediate feedback which will help them to better understand possible misconceptions..

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit using complex numbers in polynomial identities and equations by offering opportunities to understand and explore different strategies because students will be able to visual different perspectives with the different strategies and may get a better understanding of the content being presented..

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying using complex numbers in polynomial

	identities and equations because students will be able to direct their studying to the specific areas that they need further clarification in.

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?
Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying using complex numbers in polynomial identities and equations, goal setting is critical because students are able to make connections to their learning and prior knowledge can be accessed when goals are clearly identified.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Illustrative mathematics
http://tasks.illustrativemathematics.org/content-standards/HSN/CN/C/7/tasks/1690
The goal of this task is to solve quadratic equations with complex roots by completing the square. Students could of course directly use the quadratic formula, but going through the process of completing the square helps reinforce the mathematics behind the quadratic formula. The teacher may wish to have students graph the solutions so that they can see that the imaginary solutions are reflections of one another about the real axis. In the case of the equation x2+x+1=0 these two solutions also lie on the unit circle and they are third roots of unity, that is, the roots of this equation equal 1 when they are raised to the third power.
Additional Assessment:
https://algebra-equation.com/solving-quadratic-equation.html
https://member.mathhelp.com/courses/middle_and_high_school/14/chapter/11/lesson/4085?tab=test&tabItem=1

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on using complex numbers in polynomial identities and equations, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different complex numbers and making reference to how they could be used at home or in the community can be a great way to connect the tasks to their own personal tasks.	For centuries there were quadratic equations that were deemed not to have solutions. Equations like x^2 = -1 and x^2 -2x + 2 = 0 have no solutions among the positive and negative numbers. The problem in seeking solutions to equations like these two is that the squares of positive and negative numbers are both positive. Solutions for equations like these can be found, however, if we decide to invent a completely new number whose square is -1; of course, it is not a number that we have seen before. We name this number "i". The square of -i is also -1. http://mathforum.org/library/drmath/view/55747.html Reactant particles sometimes collide with one other and yet remain unchanged by the collision. Other times, the collision leads to the formation of products. The state of

	the particles that is in between the reactants and products is called the activated complex. An activated complex is an unstable arrangement of atoms that exists momentarily at the peak of the activation energy barrier. Because of its high energy, the activated complex exists for an extremely short period of time (about 10−13 s). There is equal likelihood that the activated complex either reforms the original reactants or goes on to form products. **Activated Complex**

NEW MEXICO
Public Education Department

HS: ALGEBRA- SEEING STRUCTURE IN EXPRESSIONS

Cluster Statement: A: Interpret the structure of expressions.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.
Note, the A-SSE domain is especially important in the high school content standards overall as a widely applicable prerequisite.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.SSE.A.1: Interpret expressions that represent a quantity in terms of its context. • **HSA.SSE.A.1.A: Interpret parts of an expression, such as terms, factors, and coefficients.** • **HSA.SSE.A.1.B: Interpret complicated expressions by viewing one or more of their parts as a single entity.** ***For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*** *Note: Algebra 2 focuses polynomial and rational.*	SMP 4: Students can model with mathematics by identifying the meaning of the terms, factors, and coefficients of polynomial and rational expressions in context. SMP 7: Students can look for and make use of structure in expressions by seeing how the structure of an algebraic expression reveals properties of the function it defines.	• Identify how parts of an expression relate to a real-world situation. • Interpret how parts of an expression relate to a real-world situation. • Interpret algebraic expressions that describe real-world scenarios, including parts within an expression and using grouping strategies to interpret expressions. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Remember, Understand, Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSA.SSE.A.2: Use the structure of an expression to identify ways to rewrite it. ***For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*** *Note: Algebra 2 focuses polynomial and rational.*	SMP 3: Students can construct viable arguments by explaining whether expressions are equivalent using mathematical justifications. SMP 8: Students look for and express regularity in repeated reasoning by classifying expressions.	• Identify patterns of factoring. • Factor a polynomial or rational expression. • Classify expressions by method of factoring. • Apply different algebraic properties to an expression to produce an equivalent form. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Remember, Understand, Apply

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• Connect to the work with linear, quadratic, and exponential expressions in Algebra 1 (HAS.SSE.A) • Connect to rewriting quadratic functions to find specific key features. (HSA.SSE.B.3)	• Connect to rewriting formulas to highlight quantities of interest. (HSA.CED.4)	• Connect to work with expressions of all function types.

Clarification Statement

- HSA.SEE.A.1: Algebra 1 emphasized linear, exponential and quadratic expressions. The work of Algebra 2 is to generalize that work to polynomial and rational expressions by examining real-world situations that can be modeled by algebraic expressions and explaining how parts of the expression describe different aspects of the situation.
- HSA.SEE.A.2: Seeing structure in expressions entails a dynamic view of an algebraic expression, in which potential rearrangements and manipulations are ever present. An important skill for college readiness is the ability to try possible manipulations mentally without having to carry them out, and to see which ones might be fruitful and which not. Emphasize that there are many algebraic properties that can be used to write equivalent forms of an expression. Complex, linear and non-linear equations need to be addressed.

Common Misconceptions

- Students may confuse equations with expressions. The focus in this cluster is on analyzing expressions.
- Students may confuse the order of operations when they simplify an expression.
- Students may not have a conceptual basis for patterns and therefore struggle to recognize and apply them to new situations.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying to interpret the structure of expressions because with the new vocabulary terms, it can get confusing. Students need to differentiate between expression and equation.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 5.OA.A.2: This standard provides a foundation for work with interpreting the structure of expressions because students write out the numerical expression without the calculation. Students become comfortable with using the vocabulary words: difference, greater than, multiple, etc. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
- 6.EE.A.4: This standard provides a foundation for work with interpreting the structure of expressions because being able to tell if two expressions are equivalent is the building blocks for being able to construct and deconstruct expressions to use their structure. Being able to tell if what you have done to an expression essentially changes it or not leads to the understanding of how to use these changes to manipulate the expressions and equations to better understand their structure. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access
Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features such as the size of text, images, graphs, tables, or other visual content, contrast between background and text or image; color used for information or emphasis; volume or rate of speech or sound; speed or timing of video, animation, sound, simulations, etc.; layout of visual or other elements; font used for print materials because in properly prepared digital materials, the display of the same information is very malleable and customizable. Such malleability provides options for increasing the perceptual clarity and salience of information for a wide range of learners and adjustments for preferences of others. While these customizations are difficult with print materials, they are commonly available automatically in digital materials, though it cannot be assumed that because it is digital it is accessible as many digital materials are equally inaccessible.

Build
Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because assessment is most productive for sustaining engagement when the feedback is relevant, constructive, accessible, consequential, and timely. But the type of feedback is also critical in helping learners to sustain the motivation and effort essential to learning. Mastery-oriented feedback is the type of feedback that guides learners toward mastery rather than a fixed notion of performance or compliance. It also emphasizes the role of effort and practice rather than "intelligence" or inherent "ability" as an important factor in guiding learners toward successful long-term habits and learning practices. These distinctions may be particularly important for learners whose disabilities have been interpreted, by either themselves or their caregivers, as permanently constraining and fixed.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams because providing alternatives—especially illustrations, simulations, images or interactive graphics—can make the information in text more comprehensible for any learner and accessible for some who would find it completely inaccessible in text.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) because fluency is also built through many opportunities for performance, be it in the form of an essay or a dramatic production. Performance helps learners because it allows them to synthesize their learning in personally relevant ways.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as addressing subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") because reminders, models, checklists, and so forth can assist learners in choosing and trying an adaptive strategy for managing and directing their emotional responses to external events (e.g., strategies for coping with anxiety-producing social settings or for reducing task-irrelevant distracters) or internal events (e.g., strategies for decreasing rumination on depressive or anxiety-producing ideation).

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on interpreting the structure of expressions by examining tasks from a different perspective through a short mini-lesson because students learn differently. Auditory learners may need an explanation and some one-on-one explanations. Students may also learn from one another. Videos and group collaborations are also a great way to help students understand a lesson from a different perspective.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit interprets the structure of expressions by confronting student misconceptions because one-on-one explanations of mistakes made will help the students make the connections to their mistakes. Students may also have their ah-ha moment by recognizing their own mistake.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying interpreting the structure of expressions because structuring of an expression is the foundation for creating expressions and equations in word problems. Students need to analyze the word problems and pick out the important phrases and create an expression/equation.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion, we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the way students talk about who is and isn't capable of mathematics, we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying, interpreting the structure of expressions facilitating meaningful mathematical discourse is critical because teachers need to lead to some sort of discourse that will ensure that all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration of their own cultures and languages. Interpretation can be critical because students may vary depending on how they learned the previous concepts. Mathematics discourse must lead to digging deeper understanding on why, how and what without deviating from their cultures and beliefs. Answers from the questions may depend on how they perceived and interpreted base from the given activities.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: SAT

If $\frac{\sqrt{x^5}}{\sqrt[3]{x^4}} = x^{\frac{a}{b}}$ for all positive values of x, what is the value of $\frac{a}{b}$?

Relevance to families and communities:

During a unit focused on interpreting the structure of expressions , consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, During a unit focused on counting, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different

Cross-Curricular Connections:

Science:
Earth's Place in the Universe
Use mathematical or computational representations to predict the motion of orbiting objects in the solar system. This a connection because each of the terms in the equations model an aspect in the motion of orbiting objects.

Let's go to Mars!

structures for the number names across the languages in your classroom can lead to a more robust understanding of number for all students by making connections to the different structures of number-names in other languages.	This activity is designed for students familiar with advanced algebra concepts. In this lesson, students will: • Use algebraic computations to determine the relative positions of Earth and Mars during which an optimal (low-energy) transfer of a spacecraft can occur. • Combine this information with planetary-position data to determine the next launch opportunity to Mars.

HS: ALGEBRA- SEEING STRUCTURE IN EXPRESSIONS

Cluster Statement: B: Write expressions in equivalent forms to solve problems.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.SSE.B.4: Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. ***For example, calculate mortgage payments.***	SMP 4: Students model with mathematics modeling real-life situations mathematically.	• Find the sums of finite geometric series; find the common ratio. • Use an infinite series as a model; apply a given formula for the sum of a finite geometric series by solving for the isolated variable. • Apply a given formula for the sum of a finite geometric series to solve for a coefficient. • Apply a given formula for the sum of a finite geometric series to justify real world scenarios.
		Webb's Depth of Knowledge: 1-2
		Bloom's Taxonomy: apply
<u>Previous Learning Connections</u> • In Algebra I students have studied exponential growth and decay, so can identify first terms and common ratios. Students have written arithmetic and geometric sequences both recursively and explicitly. Students have also used arithmetic and geometric sequences to model situations.	**<u>Current Learning Connections</u>** • Students will transfer previous learning to geometric series.	**<u>Future Learning Connections</u>** • This is an important concept for Calculus when learning about Riemann sums, series, and sequences.

Clarification Statement
Introduce **geometric sequences**. Students need to identify the **common ratio**, nth **term**, and previous term. Students calculate the nth term substituting the common ratio and the first term. Students apply the formula for the sum of the finite **geometric series** by solving for an isolated variable or for a coefficient. Students model real-world applications and should explain in contextual situations.

Common Misconceptions

Geometric series are obtained through a series of additions or subtraction. When applying the sum formula for geometric series, students may subtract the values in the numerator before applying the exponent. Remind them that exponents are evaluated before addition and subtraction in the order of operation.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying about writing expressions in equivalent forms to solve problems because students will see how the prior knowledge and the new lesson will be connected. The formula for the finite geometric series will make more sense when the connection is made by reviewing the binomial expansion or multiplying two binomials.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.EE.A.1: This standard provides a foundation for work with writing expressions in equivalent forms because performing operations on binomials is critical. Students need to learn how to add, multiple, and subtract to derive the formula for finite geometric series. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because to recruit all learners equally, it is critical to provide options that optimize what is relevant, valuable, and meaningful to the learner.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that is substantive and informative rather than comparative or competitive because there is not one best right way to write and equation to solve a problem and the nuance need to be highlighted so that students can improve their work over time.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or* *puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) because students are often asked to interpret a scenario with information given in one form to answer a question and if they don't understand the context of what is happening, it makes it nearly impossible to complete the task.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because the concept is to be able to look at the different forms, not the incidental calculations needed to do so.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because the path forward is not always clear when trying to find the best way to solve a problem, with the most obvious path not always being the most efficient path to the solution.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on writing expressions in equivalent forms by critiquing student approaches/solutions to make connections through a short mini-lesson because making connections using different strategies, students are able to communicate using mathematical terms and the more practice, they'll use the terms easily.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit writing expressions in equivalent forms by helping students move from specific answers to generalizations for certain types of problems because seeing the bigger picture to a detailed problem will address the conceptual understanding and students can analyze the formula to the context of the word problems.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying writing expressions in equivalent forms because explorations give opportunities for collaboration and thinking outside the box to make connections. Exploration allows the students to interact and learn from each other.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying to write expressions in equivalent forms to solve problems the types of mathematical tasks are critical because teachers need to be aware of the real-world problems that need to be used as tasks for students to solve problems. If teachers use tax and sales real -world solving problems different communities apply tax rates differently. Students' solutions would vary. Instruction should be culturally and linguistically appropriate and relevant to allow students engagement and gain their interest.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 19944

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math	■■■	Passport to Advanced Mathematics	Nonlinear functions	2. For a quadratic or exponential function, e. make connections between tabular, algebraic, and graphical representations of the function, by ii. identifying features of one representation given another representation, including maximum and minimum values of the function;	No Calculator

In the quadratic equation above, a is a nonzero constant. The graph of the equation in the xy-plane is a parabola with vertex (c, d). Which of the following is equal to d ?

A. $-9a$

B. $-8a$

C. $-5a$

D. $-2a$

Rationale Choice A is correct. The parabola with equation $y=a(x-2)(x+4)$ crosses the x-axis at the points $(-4,0)$ and $(2,0)$. By symmetry, the x-coordinate of the vertex of the parabola is halfway between the x-coordinates of $(-4,0)$ and $(2,0)$. Thus, the x-coordinate of the vertex is $\frac{-4+2}{2}=-1$. This is the value of c. To find the y-coordinate of the vertex, substitute -1 for x in $y=a(x-2)(x+4)$: $y=a(x-2)(x+4)=a(-1-2)(-1+4)=a(-3)(3)=-9a$ Therefore, the value of d is $-9a$. Choice B is incorrect because the value of the constant term in the equation is not the y-coordinate of the vertex, unless there were no linear terms in the quadratic. Choice C is incorrect and may be the result of a sign error in finding the x-coordinate of the vertex. Choice D is incorrect because the negative of the coefficient of the linear term in the quadratic equation is not the y-coordinate of the vertex.	
Relevance to families and communities: During a unit focused on writing expressions in equivalent forms to solve problems, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, learning about communities have the different tax rate is used in the home and community can a be a great way to connect schools' tasks with home tasks.	**Cross-Curricular Connections:** Art: In the realm of digital art, so many wonderful and playful genres exist that stimulate the imagination, but so few do it with the intricate style of fractal art. Fractal art is achieved through the mathematical calculations of fractal objects being visually displayed, with the use of self-similar transforms that are generated and manipulated with different assigned geometric properties to produce multiple variations of the shape in continually reducing patterns. Sounds extremely technical and not that artistic, true, but these equations create some of the most mesmerizing and inspiring artwork to emerge from the digital art arena. https://fractalfoundation.org/resources/what-are-fractals/ 35 Phenomenal Fractal Art Pictures Engineering: The Invention of Fractal Antennas Dr. Cohen built the first bona fide fractal element antenna in 1988. He is now one of the world's most innovative antenna designers, now with 26 years of professional experience, and 53 years of practical experience, stemming from his 'ham' antenna work over many years. Fractal Antennas website: Invention

NEW MEXICO Public Education Department

<table>
<tr><td colspan="3">HS: ALGEBRA- ARITHMETIC WITH POLYNOMIALS & RATIONAL EXPRESSIONS</td></tr>
<tr><td colspan="3">Cluster Statement: A: Perform arithmetic operations on polynomials.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers</td></tr>
<tr><td rowspan="3">Standard Text

HSA.APR.A.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Note: Algebra 1 focused on linear and quadratic, Algebra 2 focuses on polynomials beyond quadratic.</td><td rowspan="3">Standard for Mathematical Practices

SMP 2: Students can reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations.</td><td>Students who demonstrate understanding can:
• Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
• describe the similarities between the set of integers and the system of polynomials.
• add, subtract, and multiply polynomials.
• determine whether a set or system is closed under a given operation</td></tr>
<tr><td>Webb's Depth of Knowledge: 1</td></tr>
<tr><td>Bloom's Taxonomy:
Remember, Understand</td></tr>
<tr><td><u>Previous Learning Connections</u>
• Connect to applying the properties of <u>integer</u> exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. (8.EE.A.1)</td><td><u>Current Learning Connections</u>
• Connect to using the properties of operations to write expressions in different but equivalent forms. (HSA.SSE.A.2)</td><td><u>Future Learning Connections</u>
• Connect to performing operations with rational expressions (HSA.APR.7)
• Connect to deriving the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems. For example, calculate mortgage payments. (HS.A.SSE.B.4)</td></tr>
<tr><td colspan="3">Clarification Statement
HSA.APR.A.1: The development of polynomials and rational expressions in high school parallels the development of numbers in elementary and middle grades. In elementary school, students might initially see expressions for the same numbers 8 + 3 and 11, or 3/4 and 0.75, as referring to different entities: 8 + 3 might be seen as describing a calculation and 11 is its answer; 3/4 is a fraction and 0.75 is a decimal. They come to understand that these different expressions are different names for the same numbers, that properties of operations allow numbers to be written in different but equivalent forms, and that all of these numbers can be represented as points on the number line. In middle grades, they come to see numbers as forming a unified system, the number system, still represented by points on the number line. The whole numbers expand to the</td></tr>
</table>

integers—with extensions of addition, subtraction, multiplication, and division, and their properties. Fractions expand to the rational numbers—and the four operations and their properties are extended. A similar evolution takes place in algebra. At first algebraic expressions are simply numbers in which one or more letters are used to stand for a number which is either unspecified or unknown. Students learn to use the properties of operations to write expressions in different but equivalent forms. At some point they see equivalent expressions, particularly polynomial and rational expressions, as naming some underlying thing. As they see polynomial expressions as quantities rather than operations to be performed, they can perform operations such as adding, subtracting and multiplying two polynomials and identify that these operations will yield another polynomial, thus making the system of polynomials closed.

Common Misconceptions

- Students might think polynomials are only monomial, binomial, or trinomial.
- Students may not confuse the impact of adding and subtracting polynomials on the degree of the variable.
- Students may not fully distribute the multiplication of polynomials and only multiply like terms.
- When adding and multiplying like terms students may initially confuse $x + x$ as x^2 instead of $2x$.
- Students may not think $x^2 \cdot x = x^3$ is not an example of closure for polynomial multiplication since the result has a different exponent than the factors.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying arithmetic operations on polynomials because the structure of the four basic operations hold true for arithmetic operations on polynomials.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.EE.A.1: This standard provides a foundation for work with arithmetic operations on polynomials because the student must know and apply the properties of integer exponents. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with performing arithmetic operations on polynomials benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because the student can bridge conceptual ideas within a familiar context to procedural mechanics of the cluster.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with performing arithmetic operations on polynomials benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing prompts that guide learners in when and how to ask peers and/or teachers for help because as students perform arithmetic operations on polynomials they can have scripted questions to focus the intent of support provided.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with performing arithmetic operations on polynomials benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because the structure of the four basic operations holds true for polynomials as it does for integers.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with performing arithmetic operations on polynomials benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing scaffolds that can be gradually released with increasing independence and skills because integer arithmetic and polynomial arithmetic are related and the student becomes fluent in the latter as (s)he is gradually released.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with performing arithmetic operations on polynomials benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as highlighting previously learned skills that can be used to solve unfamiliar problems because students use connections with arithmetic of polynomials and functions to explore equivalent expressions and create functions that meet specific conditions.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on arithmetic operations on polynomials by providing specific feedback to students on their work through a short mini-lesson because looking at integer rules for arithmetic operations apply directly to arithmetic operations with polynomials.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on arithmetic operations on polynomials by confronting student misconceptions because integer rules concerning positives and negatives are common errors that lead to misconceptions when performing arithmetic operations on polynomials.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying arithmetic operations on polynomials because addition and subtraction are inverse operations as are multiplication and division.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying performing arithmetic operations on polynomials, the types of mathematical tasks are critical because they build on prior knowledge of arithmetic operations. Time spent on conceptual understanding of the four basic operations (addition, subtraction, multiplication, division) using integers can bridge to the conceptual understanding of those operations of polynomials. From here time can be spent on procedural fluency of the mechanics of the operations with polynomials. As students will be expected to demonstrate proficiency on End of Course exams and the SAT in English, an opportunity presents itself to bridge home language to the language of these exams.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 5094624

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math	■■□	Passport to Advanced Mathematics	Equivalent expressions	2. Fluently add, subtract, and multiply polynomials.	No Calculator

Which of the following is equivalent to the sum of $3x^4+2x^3$ and $4x^4+7x^3$?

A. $16x^{14}$

B. $7x^8+9x^6$

C. $12x^4+14x^3$

D. $7x^4+9x^3$

Rationale

Choice D is correct. Adding the two expressions yields $3x^4+2x^3+4x^4+7x^3$. Because the pair of terms $3x^4$ and $4x^4$ and the pair of terms $2x^3$ and $7x^3$ each contain the same variable raised to the same power, they are like terms and can be combined as $7x^4$ and $9x^3$, respectively. The sum of the given expressions therefore simplifies to $7x^4+9x^3$.
Choice A is incorrect and may result from adding the coefficients and the exponents in the given expressions. Choice B is incorrect and may result from adding the exponents as well as the coefficients of the like terms in the given expressions. Choice C is incorrect and may result from multiplying, rather than adding, the coefficients of the like terms in the given expressions.

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on performing arithmetic operations on polynomials, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, in becoming a critical thinker and problem solver. Students take a skill familiar to them (arithmetic operations with integers) and apply it to something new, arithmetic operations on polynomials. This unit practices learning something new from existing knowledge.	History: The history of exponents dates back many centuries and Euclid are credited with the first known usage of exponents. He used the term 'power' to represent what we know today, how many times a number is multiplying by itself. The ancient Greek mathematicians used, and many other mathematicians added onto the use of exponents as they learned more about their use. Archimedes generalized the same idea of powers and later mathematicians in the Islamic golden age utilized powers of two and three in their work in algebra. In our project, you will see many other mathematicians and their contributions to the development of exponents from the 14th century up to the use of exponents today. https://www.sutori.com/story/history-of-exponents--wNbwYExXdzFNYPh1zFUYhbDc

NEW MEXICO
Public Education Department

HS: ALGEBRA- ARITHMETIC WITH POLYNOMIALS & RATIONAL EXPRESSIONS		
Cluster Statement: B: Understand the relationship between zeros and factors of polynomials. Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers		
Standard Text HSA.APR.B.2: Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	**Standard for Mathematical Practices** SMP 3: Students can construct viable arguments and critique the reasoning of others by explaining in their own words what it means to factor an expression, what a zero of an equation represents and how it relates to its graph. Students will be able to explain how the quotient and remainder of a polynomial division problem are related.	**Students who demonstrate understanding can:** • Define the Remainder Theorem. • Use the Remainder Theorem to show the relationship between a factor and a zero. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply
Standard Text HSA.APR.B.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	**Standard for Mathematical Practices** SMP 3: Students can construct viable arguments and critique the reasoning of others by explaining in their own words what it means to factor an expression, what a zero of an equation represents and how it relates to its graph. Students will be able to explain how the quotient and remainder of a polynomial division problem are related.	**Students who demonstrate understanding can:** • Use polynomial identities to prove numerical relationships. • Determine the degree of a polynomial and the number of possible zeros of that polynomial. • Simplify polynomials into factored forms. • Identify the zeros of the polynomial using the factors. • Plot the zeros of the polynomial on a graph. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Apply, Analyze
Previous Learning Connections • Connect to factoring and completing the square and using the Remainder Theorem (standards A-APR.B.2, F-IF.C.8.a)	**Current Learning Connections** • Connect to calculating the zero in the Remainder Theorem or by factoring to graph the zeros of a polynomial function (standards A-APR.B.2, A-APR.B.3)	**Future Learning Connections** • Connect to graphing key features of polynomial functions to identifying zeros and sketching a graph (standards F-IF.C.7)

Clarification Statement
The zeros of a polynomial are turned into linear factors and can be used to factor polynomials of any power. The degree of a polynomial will indicate the maximum number of zeros of the polynomial.

Common Misconceptions
- Division problems never have a remainder; it is okay to write R-value.
- Students often forget to distribute the −1 which is equivalent to subtraction, to terms inside the parenthesis.
- Students might make errors in signs when doing synthetic division and synthetic substitution because values are added rather than subtracted as in long division. Remind them that terms are always added for synthetic substitution and synthetic division. When listing the coefficients, there may be missing degrees and students will forget to write a zero.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*
- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying understanding the relationship between zeros and factors of polynomials because when using the Remainder Theorem students must use the opposite sign of the factor in the dividend. Students must also know how to factor polynomials when the leading coefficient is equal to 1 and not equal to 1 which can lead to common misconceptions.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*
- A.SSE.B.3.A and A.APR.B.6: These standards provide a foundation for work with understanding the relationship between zeros and factors of polynomials because students will be producing equivalent forms of polynomials to reveal properties of the expression (in this case the factored form will reveal zeros; the Remainder Theorem can be used instead of long division to check if a factor is a zero of the expression). . If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*
- For example, learners engaging with APR.B benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because finding the zeros of a polynomial can be bridged in context to a socially or culturally relevant prompt.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*
- For example, learners engaging with understanding the relationship between zeros and factors of polynomials benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the

face of challenge because the learner can use specific supports (eg. graph to check zeros, substitution to check zeros) to encourage self-awareness.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with understanding the relationship between zeros and factors of polynomials benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) because finding and checking zeros of polynomials can be found by factoring/ using Remainder Theorem and checked graphically.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding the relationship between zeros and factors of polynomials benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because polynomials can be factored using different strategies depending on the structure of the polynomial.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with understanding the relationship between zeros and factors of polynomials benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as highlighting previously learned skills that can be used to solve unfamiliar problems because the learner can link skills learned with factoring quadratics to unfamiliar polynomials of higher degree.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understanding the relationship between zeros and factors of polynomials by critiquing student approaches/solutions to make connections through a short mini-lesson because zeros of polynomials must match the graph of the polynomial. By critiquing other students work, the learner can immediately make connections to the correctness of the work by observing a graph.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on understanding the relationship between zeros and factors of polynomials by offering opportunities to understand and explore different strategies because investigating graphs to identify zeros and using area models to factor polynomials can offer structure to the student as a starting point.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students. when studying understanding the relationship between zeros and factors of polynomials because some learners may be ready to factor more difficult polynomials.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

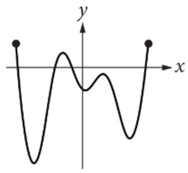
Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying understanding the relationship between zeros and factors of polynomials the use of mathematical representations within the classroom is critical because graphing technology and area models can be used to factor polynomials and check the zeros of those factors. Mathematics can be designed in a context to connect home culture or interests in a way that a polynomial function could represent a quantity where its solution(s) could represent a critical value within the context. For example, a cubic function could represent the profit of a fundraiser given the cost of a ticket for the fundraiser. The zero(s) of the function would represent the break-even point. The context of the fundraiser could be framed around a cultural interest. A graph could be used to show a representation of the function to support the learner in bridging different representations.

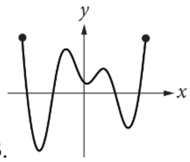
Standards Aligned Instructionally Embedded Formative Assessment Resources:

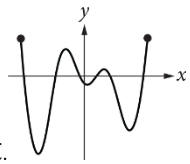
Source: SAT https://satsuitequestionbank.collegeboard.org/results

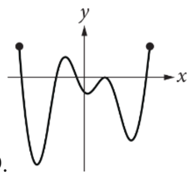
20079	■■□	Passport to Advanced Math	Passport to Advanced Mathematics	Nonlinear functions	3. For a factorable or factored polynomial or simple rational function, b. understand and use the fact that for the graph of y = f(x), the solutions to f(x) = 0 correspond to x-intercepts of the graph and f(0) corresponds to the y-intercept of the graph; interpret these key features in terms of a context;	Calculator

If the function f has five distinct zeros, which of the following could represent the complete graph of f in the xy-plane?

- A. 

- B. 

- C. 

- D. 

Rationale

- Choice D is correct. A zero of a function corresponds to an x-intercept of the graph of the function in the xy-plane. Therefore, the complete graph of the function f, which has five distinct zeros, must have five x-intercepts. Only the graph in choice D has five x-intercepts, and therefore, this is the only one of the given graphs that could be the complete graph of *f* in the xy-plane.
- Choices A, B, and C are incorrect. The number of x-intercepts of each of these graphs is not equal to five; therefore, none of these graphs could be the complete graph of f, which has five distinct zeros.

Relevance to families and communities:
During a unit focused on understanding the relationship between zeros and factors of polynomials, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, students practice using mathematical tools to solve new problems. Students learn to identify what tools are at their disposal when connecting the zeros of a polynomial to its factors. This practices them for life outside the math classroom by providing skills that are lifelong.

Cross-Curricular Connections:
Science:
This approach is extended to a spherical body rolling on a curved path. Assuming that a curved path can be approximated by a sequence of many very short inclines, the problem is approached as a body rolling on this sequence of inclines, solving each with the work-energy theorem. Defining the curved path as a differentiable function, the slope of each incline is obtained through the function
Teach Engineering Roller Coaster - Spherical Body rolling

Social Studies:
Not much is really known about the Pythagoreans or their rather mysterious founder, Pythagoras. Several different accounts of the Pythagoreans have come down to us from antiquity. Plato and Aristotle both reference the Pythagoreans throughout their philosophical writings. Even still, the true nature of the "cult of Pythagoras" is often shrouded in mystery. Pythagorious and his followers were a mystical society that placed great importance on the mathematical relations of the universe. There is no denying that they contributed greatly to the

	area of mathematics and philosophy. One needs only to reflect on the Pythagorean theorem, a mathematical principle said to have been discovered by Pythagoras himself, to appreciate the profound impact they had on the development of scientific thought. https://classicalwisdom.com/philosophy/cult-of-pythagoras/

NEW MEXICO
Public Education Department

HS: ALGEBRA- ARITHMETIC WITH POLYNOMIALS & RATIONAL EXPRESSIONS

Cluster Statement: C: Use polynomial identities to solve problems.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.APR.C.4: Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	SMP 7: Students look for and make use of structure when given a polynomial function in factored form. Students will be able to find the zeros, plot the zeros and then make a sketch of the graph of that is reflective of the function (and its other key features). SMP 8: Students look for and express regularity in repeated reasoning when connecting the representations of functions on the coordinate plane with their zeros and factored forms.	• Understand that polynomial identities include but are not limited to the product of the sum and difference of two terms, the difference of two squares, the sum and difference of two cubes, the square of a binomial, etc. • Prove polynomial identities by showing steps and providing reasons. • Illustrate how polynomial identities are used to determine numerical relationships. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply
<u>Previous Learning Connections</u> • Students are building on their knowledge of zeros and factors of quadratics learned in Algebra 1.	**<u>Current Learning Connections</u>** • Students are learning about factoring with polynomials of degrees higher than 2 (perfect cubes, quadratics, factor by grouping, etc). Students are also understanding that not all polynomials are factorable, but still can by divided by another polynomial. Students continue to build their understanding of how factored form relates to zeros on a graph. Later in the year, these skills are used in simplifying rational expressions.	**<u>Future Learning Connections</u>** • In 4th year math (Pre-Calculus, Calculus, and college level math) students will build on their factoring skills (with rational expressions and trigonometric expressions). Students will also determine zeros of trigonometric functions in subsequent math courses.

Clarification Statement
Students make systematic lists of all arrangements and count the number of unique subgroups. Students use prior knowledge of counting techniques to calculate the number of combinations.

Common Misconceptions
- There are no y-axis zeros.
- Easily get lost with the different coefficients and degrees.
- Multiplying the degrees.

- Students might incorrectly expand binomial expressions by choosing the wrong row of coefficients in Pascal's triangle. Remind students that for an exponent of n, choose row n of Pascal's triangle. Row n will be the row with the value n as the second entry.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., Pascal's Triangle, the Binomial Theorem) when studying the use of polynomial identities to solve problems because there are structures that exist to make expanding binomials more efficient and effective.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- A.SSE.A.2: This standard provides a foundation for work with the use of polynomial identities to solve problems because students have looked at the structure of an expression to identify ways to rewrite it. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with using polynomial identities to solve problems benefit when learning experiences include ways to recruit interest such as providing time for self-reflection about the content and activities because the learner can reflect on how the Binomial Theorem connects to expanding polynomials of higher degrees.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with using polynomial identities to solve problems benefit when learning experiences attend to student's attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics representations and scaffolds because the learner can use area models as an alternative to the expansion of binomials.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with using polynomial identities to solve problems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because area models, although not efficient for higher degree binomials, can still be used to expand binomials of higher degrees.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with using polynomial identities to solve problems benefit when learning experiences attend to the multiple ways' students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because the student can use the Binomial Theorem or area models but expand binomials.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with using polynomial identities to solve problems benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because although the power of the binomial is a positive integer, the coefficients of the binomial terms may be positive or negative.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on the use of polynomial identities to solve problems by revisiting student thinking through a short mini-lesson because looking at other students' work can support all learners in understanding the structure of the Binomial Theorem. Learners listening to their peers explain their thinking can benefit all as student thinking is delivered in student friendly language.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on the use of polynomial identities to solve problems by offering opportunities to understand and explore different strategies because some students will insist on using area models or the FOIL method for expanding a binomial regardless of the power of the binomial. As the Binomial Theorem will be more effective and efficient to expand certain binomials, pockets of students are afforded the opportunity to explore and apply previous strategies.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various strategies when studying the use of polynomial identities to solve problems because some learners can investigate and explain when it would be more appropriate to use an area model, the FOIL method, or the Binomial Theorem when expanding binomials.

Culturally and Linguistically Responsive Instruction: **Validate/Affirm**: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? **Build/Bridge**: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society? Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying the use of polynomial identities to solve problems the types of mathematical tasks are critical because, for example, students' familiarity of structure can support the expansion of binomials. When squaring a binomial, students have a working knowledge of area models and the FOIL method. As the power of a binomial grows, these methods break down and become messy. The student can then decide when it would be more efficient to use the Binomial Theorem and Pascal's Triangle to expand a binomial. Conceptual understanding of expanding binomials will lead to procedural fluency as the student decides what method works best for their learning style.
Standards Aligned Instructionally Embedded Formative Assessment Resources: *Source:* Illustrative Mathematics Felicia notices what appears to be an interesting pattern between powers of 11 and powers of x+1: $11^0 = 1 \qquad (x+1)^0 = 1$ $11^1 = 11 \qquad (x+1)1 = x+1$ $11^2 = 121 \qquad (x+1)^2 = x^2+2x+1$ • The digits of the number 11n are the same as the coefficients of the polynomial $(x+1)^n$. Is this always true? • Does this pattern continue for n=3 and n=4? • What is the answer to Felicia's question? IM Commentary This task has students combine polynomial arithmetic with pattern-matching. Students can expand powers of x+1 using either repeated multiplication (A-APR.1) or by the binomial theorem (A-APR.5), and then are asked to analyze the question of whether the similarity of coefficients with the digits of powers of 11 is a coincidence. Identifying patterns, as Felicia has done, is an important part of mathematics. In this case, there is a deep relationship between the numbers and polynomials that Felicia is investigating; on the other hand, further consideration shows that the pattern does not continue. It is important for students not only to identify patterns but also to look more deeply to understand whether or not the patterns are "generalizable" or true because of some essential mathematical structure.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on the use of polynomial identities to solve problems, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, the student makes	In this activity, students relate the graph of a rational function to the graphs of the polynomial functions of its numerator and denominator. Students graph these polynomials one at a time and identify their y-intercepts and zeros. [Asymtotes and Zeros of Rational Functions: Algebra 2](#)

use of structure and decides on a method to expand binomials that is effective and efficient and makes sense to them. The Standards for Mathematical Practice come alive as they use polynomial identities to solve problems as the bridge to perseverance and making use of structure and repeated reasoning.	

HS: ALGEBRA- ARITHMETIC WITH POLYNOMIALS & RATIONAL EXPRESSIONS

Cluster Statement: D: Rewrite rational expressions.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSA.APR.D.6: Rewrite simple rational expressions in different forms; write ${}^{a(x)}/_{b(x)}$ in the form $q(x) + {}^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	SMP 3: Students construct viable arguments and critique the reasoning of others by explaining the steps in creating equivalent forms of rational expressions, including identifying the quotient and the remainder as a fraction with the divisor as the denominator. SMP 5: Students use appropriate tools strategically when recognizing which method is appropriate to use in a variety of situations. SMP 7: Students look for and make use of structure when gaining procedural fluency and conceptual understanding of how and why to rewrite rational expressions as quotients and remainders.	• Divide polynomials using long division. • Divide polynomials using synthetic division. • Relate the algorithm of dividing multi-digit integers with polynomial long division. • Perform partial fraction decomposition. • Determine the quotient and remainder of rational expressions using inspection, long division, and/or a computer algebra system. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply
Previous Learning Connections	**Current Learning Connections**	**Future Learning Connections**
• Students are building on their knowledge of factors of quadratics learned in Algebra 1.	• Students will use the skills learned to factor and divide polynomials to simplify rational expressions.	• Students will be able to perform all operations with rational expressions.

Clarification Statement
Rational expressions can be rewritten using properties of fractions and elementary numerical algorithms.

Common Misconceptions

- Students may forget to write the polynomial in descending order.
- Students may not recognize a missing term in the divisor or dividend and forget to insert a zero for the missing term.
- Students might make errors in signs when doing synthetic division and synthetic substitution because values are added rather than subtracted as in long division. Remind them that terms are always added for synthetic substitution and synthetic division.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying rewriting rational expressions because arithmetic operation with polynomials and factoring will be used when rewriting rational expressions.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- A.SSE.A.2: This standard provides a foundation for work with rewriting rational expressions because students use the structure of an expression to identify ways to rewrite it. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with rewriting rational expressions benefit when learning experiences include ways to recruit interest such as creating accepting and supportive classroom climate because peers might take different paths in rewriting rational expressions, all of which are acceptable methods.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with rewriting rational expressions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities because rewriting rational expressions may have multiple entry points depending on how the rational expression is structured.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with rewriting rational expressions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because tools such as factoring, synthetic division, and operations with fractions might lend themselves to be efficient and effective in rewriting rational expressions.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with rewriting rational expressions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing different approaches to motivate, guide, feedback or inform students of progress towards fluency because the learner might not know when a rational expression has been fully rewritten as a simplified form.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with rewriting rational expressions benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because rewriting rational expressions involves being fluent in fraction operations which has been a traditional frustration with students.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on rewriting rational expressions by providing specific feedback to students on their work through a short mini lesson because an expression might not be fully simplified. Students might not have applied a full set of mathematical properties to rewrite a rational expression and may benefit from focused feedback on where to go next in their work.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit rewriting rational expressions by confronting student misconceptions because the student might rewrite a rational expression incorrectly and simplify a polynomial incorrectly (e.g., (x+ y)2 = x2+ y2) or might have factored a polynomial incorrectly.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying rewriting rational expressions because some students will be ready for more complex rational expressions with more complex terms than others. Pockets of students can be paired homogeneously by ability to work on more complex rational expressions to explore in greater depth.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying rewriting rational expressions, the use of mathematical representations within the classroom is critical because students are in fact rewriting simple rational expressions as equivalent representations. Students are asked to draw on their mathematical competence by simplifying rational expressions previously learned within this standard domain. For example, a student might have to factor a numerator and/or denominator to simplify a rational expression. Or a student might have to perform synthetic division to rewrite a rational expression as an equivalent representation. These skills could build a bridge for students to position them as competent and capable mathematicians and leverage further study of mathematics.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: SAT

Which of the following is equivalent to $\frac{4x^2+6x}{4x+2}$?

- A. x
- B. $x+4$
- C. $x-\frac{2}{4x+2}$
- D. $x+1-\frac{2}{4x+2}$

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on rewriting rational expressions, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, learners look for and make use of structure and make sense of problems and persevere in solving them. As students rewrite rational expressions in equivalent forms, they are building confidence in taking what they know to apply to problem solving scenarios. These Mathematical Practices skills exist outside of school as the student builds their critical thinking skills through rewriting rational expressions.	Medicine and Analytical Chemistry: MRI and NMR Spectroscopy involves Fast Fourier Transformation that allows the creation of images from the "ringing" after the atoms are subjected to radio waves in strong magnetic fields. The Fourier series consists of terms of increasing orders. (An Algorithm for the Machine Calculation of Complex Fourier Series by James W. Cooley and John W. Tukey)

HS: ALGEBRA- CREATING EQUATIONS

Cluster Statement: A: Create equations that describe numbers or relationships.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

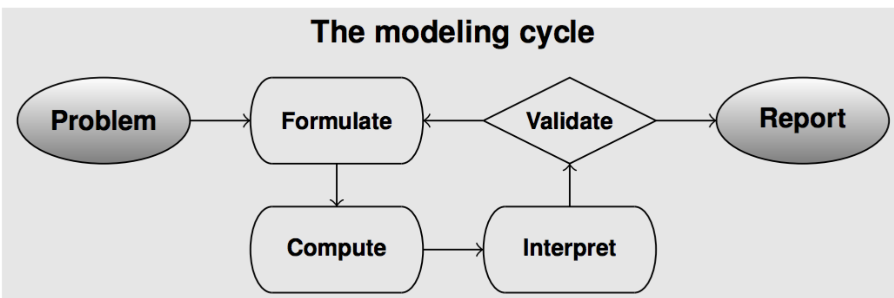
Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.CED.A.1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* *Note: Algebra 1 focuses on linear, quadratic, and exponential (integer inputs only), Algebra 2 focuses on equations using all available types of expressions, including simple root functions.*	SMP 1: Students can make sense of problems and persevere in solving them by appropriately choosing a function type when creating an equation or inequalities in one variable to solve a problem. SMP 4: Students can model with mathematics by creating equations or inequalities with one variable when given a problem with context.	• Create equations and inequalities in one variable and use them to solve problems. • Write equations in one variable and use them to solve problems. • Write inequalities in one variable and use them to solve problems **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSA.CED.A.2: Create equations in two or more variables to represent relationships between quantities, graph equations on coordinate axes with labels and scales. *Note: Algebra 1 focuses on linear, quadratic, and exponential (integer inputs only), Algebra 2 focuses on equations using all available types of expressions, including simple root functions.*	SMP 1: Students can make sense of problems and persevere in solving them by appropriately choosing from among linear, exponential and quadratic functions when creating a system of equations or inequalities in two or more variables to solve a problem. SMP 4: Students can model with mathematics by creating a system equations or inequalities in two or more variable when given a problem with context.	• Create equations in two or more variables based on a given context. • Write equations in two or more variables based on a given context. • Graph equations on coordinate axes with scales clearly labeling the axes, defining what the values on the axes represent and the unit of measure. • Select intervals for the scale that are appropriate for the context and display adequate information about the relationship. • Analyze points on and off a graph and interpret them in context. **Webb's Depth of Knowledge:** 1-2

		Bloom's Taxonomy: Understand, Apply,Analyze
Standard Text **HSA.CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.** ***For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*** *Note: Algebra 1 focuses on linear, quadratic, and exponential (integer inputs only), Algebra 2 focuses on equations using all available types of expressions, including simple root functions.*	**Standard for Mathematical Practices** SMP 2: Students can reason abstractly and quantitatively by contextually, analytically and graphically checking a solution set of inequalities to determine the viability of each solution. SMP 4: Students can model with mathematics by representing constraints using equations or inequalities and systems of equations and/or inequalities when given a problem with context.	**Students who demonstrate understanding can:** • Identify constraints of equations, inequalities, and systems of equations and inequalities given a context. • Interpret solutions of equations, inequalities, and systems of equations and inequalities as viable or non-viable given a context. • Interpret solutions analytically and graphically to answer questions about the quantities in context. **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy:** Understand, Apply, Analyze, Evaluate
Standard Text **HSA.CED.A.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.** ***For example, rearrange Ohm's law V = IR to highlight resistance R.*** *Note: Algebra 1 focuses on linear, quadratic, and exponential (integer inputs only), Algebra 2 focuses on equations using all available types of expressions, including simple root functions.*	**Standard for Mathematical Practices** SMP 4: Students can model with mathematics by applying literal when given a problem in context. SMP 7: Students can reflect and recognize the various structures in mathematic formulas and use them when solving problems requiring those formulas.	**Students who demonstrate understanding can:** • Solve for a specified variable in a literal equation. • Solve formulas for a specified variable of interest. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply
Previous Learning Connections • Connect to the work of Algebra 1 around linear, quadratic, and exponential (integer inputs only) with this cluster. **(HSA.CED.A)** • Connect to graphing systems of equations and inequalities.**(HSA.REI.7)**	**Current Learning Connections** • Connect to communicating relevant domain and range for linear, exponential and quadratic functions.**(HSF.IF.4)** • Connect to graphing equations and inequalities.**(HSF.IF.7)**	**Future Learning Connections** • Connect to extending knowledge to include additional types of functions such as trigonometric, rational, and polynomial. **(HSA.CED.1-4)** • Connect to communicating relevant domain and range

• Connect to solving equations in one variable including those equations with coefficients represented by variables.**(HSA.REI.3-4)**		for all types of functions. **(HSF.IF.4)**

Clarification Statement

- Equations and inequalities can be created to represent and solve real world and mathematical problems.
- Students check their solutions to real-world problems which can be found by modeling them with equations and graphs.
- Constraints are necessary to balance a mathematical model with real-world context. Variable quantities may be able to take on only certain values and expressing these restrictions, or constraints, algebraically in an important part of modeling with mathematics.
- Formulas are equations with specific meaning that show the relationship between two or more quantities and are written in the same way literal equations are solved for a given variable, by isolating the desired variable on one side of the equation.
- All the standards in the Creating Equations group carry a modeling star, denoting their connection with the Modeling category in high school. This connotes not only an increase in the complexity of the equations studied, but an upgrade of the student's ability in every part of the modeling cycle.



Common Misconceptions

- Student may believe only linear and quadratic expressions can be used within inequalities.
- Students may believe that ellipses and hyperbolas are the same, but are reversed on the axis
- Students may believe absolute value cannot be inverted and struggle when there is more than one term inside absolute value
- Students may believe that mid-term and distance are the same thing and confuse the formulas.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying creating equations that describe numbers or relationships because this cluster requires students to create equations they have already studied from relationships and contexts. A recap of the key features of the families of functions studied can help students more easily apply their prior learnings to these problems.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.F.B.4: This standard provides a foundation for work with creating equations that describe numbers or relationships because this standard called on students to specifically write linear equations from a given relationship and explain the parts of the equation in context of a scenario. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with creating equations that describe numbers or relationships benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because this selecting relevant and/or recognizable topics can ease the stress of students in discussing the mathematics and engaging with the material.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with creating equations that describe numbers or relationships benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports (e.g., peer-tutors) because students may each pick out different important features in a given problem and, through sharing their findings, can help each other move to the next step in a problem solving process. Consider structures such as think-write-pair share where all students have time to consider the problem, write what they recognize as important or write a question they have about the problem and then share with a partner or group. Structures like this give all students support in engaging with the content and in discussing the mathematics.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with creating equations that describe numbers or relationships benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making relationships between elements explicit (constant change is linear, half-life/doubling is exponential, “at least” and “no more than” create inequalities, etc.) because identifying these key terms in a word problem are the key to understanding which equation to create. Providing support in translating from context to equation can help students guide themselves through a problem.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*
For example, learners engaging with creating equations that describe numbers or relationships benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches,

strategies, skills, etc.) because students may be able to reason using a table, graph or by analyzing keywords in descriptions. Each of these techniques can illuminate new pathways to solutions and can show students a variety of ways to approach a problem.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with creating equations that describe numbers or relationships benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because this cluster requires students to consider a context, extract the important information that reveals features of a specific family of functions and then model the scenario with an equation. There are many steps where students may feel lost or stuck. Providing students with prompting questions they can ask themselves like "what information is given?" or "is the change constant?" may help them to progress through the problems.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on creating equations that describe numbers or relationships by clarifying mathematical ideas and/or concepts through a short mini-lesson because students may see problems as having one specific solution when infinitely many solutions are appropriate. Students may benefit from revisiting contexts with inequalities and discussing many potential solutions and why they each make sense in context of the problem. Further, students may benefit from discussing why a solution can be found mathematically but why it may not make sense in context of a problem.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit creating equations that describe numbers or relationships by addressing conceptual understanding because students must have a firm grasp of the features of equations and inequalities before they can model scenarios with them. Students may require support in conceptualizing the different families of functions and/or the difference between an equation and an inequality.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when studying creating equations that describe numbers or relationships because once students are fluent in applying equations to contexts, they can be challenged by selecting their own problems relating to specific careers or interests.

NEW MEXICO
Public Education Department

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying HS.CED.A: Create equations that describe numbers or relationships cluster the types of mathematical tasks are critical because fluency in Algebra is akin to becoming fluent in a spoken or written language. Fluency is essential to obtaining a deep understanding of the function and meaning of any language. Algebra is no different.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 19489

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra	■□□	Heart of Algebra	Linear inequalities in one or two variables	1. Create and use linear inequalities in one or two variables to solve problems in a variety of contexts.	Calculator

19489

Wyatt can husk at least 12 dozen ears of corn per hour and at most 18 dozen ears of corn per hour. Based on this information, what is a possible amount of time, in hours, that it could take Wyatt to husk 72 dozen ears of corn?

Rationale
The correct answer is any number between 4 and 6, inclusive. Since Wyatt can husk at least 12 dozen ears of corn per hour, it will take him no more than $\frac{72}{12} = 6$ hours to husk 72 dozen ears of corn. On the other hand, since Wyatt can husk at most 18 dozen ears of corn per hour,
it will take him at least $\frac{72}{18} = 4$ hours to husk 72 dozen ears of corn.
Therefore, the possible times it could take Wyatt to husk 72 dozen ears of corn are 4 hours to 6 hours, inclusive. Any number between 4 and 6, inclusive, can be gridded as the correct answer.

Relevance to families and communities:
During a unit focused on HS.CED.A: Create equations that describe numbers or relationships cluster, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, how statistics are used to describe how the risk of different cultural and ethnic groups for developing breast cancer and how this might affect medical breast cancer screening frequency recommendations.
Example 1: During a unit focused on creating equations in two variables, consider options for

Cross-Curricular Connections:

Economics: Linear programming with a system of inequalities is often used to model the constraint of resources for production. Consider providing a connection where students are starting their own business and must maximize profit or production with the possible solutions of the system.

Science: There are many formulas in science such as Ohm's Law and the Doppler formulas that may require isolating and solving for a specific variable given certain conditions. Consider providing a connection where

learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, exploring how changing the structure for an equation is similar to how a sentence can be re-structured to convey different meanings depending on the structure of the words.	students must rearrange the same formulas in multiple ways to highlight different quantities of interest.

<table>
<tr><td colspan="3">HS: ALGEBRA- REASONING WITH EQUATIONS & INEQUALITIES</td></tr>
<tr><td colspan="3">Cluster Statement: A: Understand solving equations as a process of reasoning and explain the reasoning.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers</td></tr>
<tr><td rowspan="3">Standard Text

REI.A.2: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.</td><td rowspan="3">Standard for Mathematical Practices

SMP 6: Students attend to precision when determining true solutions (not extraneous).</td><td>Students who demonstrate understanding can:
• Determine the domain of a rational function.
• Determine the domain of a radical function.
• Solve radical equations in one variable.
• Solve rational equations in one variable.
• Explain and give examples how extraneous solutions may arise when solving rational and radical equations.</td></tr>
<tr><td>Webb's Depth of Knowledge: 1-3</td></tr>
<tr><td>Bloom's Taxonomy:
Apply, Analyze</td></tr>
<tr><td><u>Previous Learning Connections</u>
• In Algebra 1, students solved linear and quadratic equations.</td><td><u>Current Learning Connections</u>
• In this course, students will extend their solving skills learned in Algebra 1 to rational and radical equations. Students will also relate the solving of other nonlinear equations learned in this course to solving rational and radical equations.</td><td><u>Future Learning Connections</u>
• In future math classes students will solve more challenging nonlinear equations, including trigonometric equations.</td></tr>
<tr><td colspan="3">Clarification Statement
• This cluster builds on the framework of solving equations and extends it to rational and radical equations (and the knowledge of extraneous solutions).
• Equations are solved as a process of reasoning using properties of operations and equality, which can justify each step of the process. Students solve simple rational and radical equations using a variety of methods and explain why and where in the solution process the extraneous solution arose.</td></tr>
<tr><td colspan="3">Common Misconceptions
• Students may struggle identifying when there is an extraneous solution.
• Struggle with expression versus equation.
• When students multiply or divide both sides of an inequality by a negative value, they forget to reverse the inequality symbol.
• Students may sometimes forget to consider the cases when the LCD is positive and negative when solving a rational inequality algebraically,</td></tr>
</table>

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying solving equations and explaining each step because students may need to justify the inverse operation used in each step with viable arguments. Students may practice expressing their mathematical thinking verbally and symbolically.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.EE.B.5: This standard provides a foundation for work with reasoning and solving one-variable equations because students need to understand each step of solving one-variable equations and explain the reason for each step. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with understanding and explaining the reasoning of solving equations benefit when learning experiences include ways to recruit interest such as providing choices in their strategies of solving equations and in their reasoning because students make connections of their prior knowledge of solving equations in different problems. By showing a different order of applying the inverse operations to the equations, students gain new skills and knowledge of solving complex equations and deeper understanding of solving equations.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with explaining reasoning of each step of solving equations benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities because students engage in meaningful discourse to construct viable arguments with the support of the cooperative learning group. Students justify and make connections with reasoning of other strategies used by other learners in the cooperative learning groups.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with explaining the reasoning of each step of solving equations and constructing viable argument benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because students build their reasoning and viable argument using their prior knowledge of solving one-step or two-step equations. Students connect their understanding of inverse operation to justify each step of solving equations.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with explaining the reasoning of solving equations benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because students justify solving the equations in multiple ways and communicate their mathematical thinking verbally and symbolically. By presenting their mathematical thinking in multiple ways, students make connections of conceptual knowledge and gain fluency in procedural knowledge.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with explaining the reasoning of solving equations with viable arguments benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations because students apply the knowledge of solving one-variable equations to solving literal equations. Students identify the patterns of solving equations and make generalization of solving and rearranging equations.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on explaining the reason of each step of solving equations by critiquing student approaches/solutions to make connections through a short mini-lesson because students need to understand why the specific inverse operation is used and develop the viable argument using properties of equality.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit explaining the steps of solving equations by offering opportunities to understand and explore different strategies because students need to understand why some steps are interchangeable when solving the equations. Students need to explain the order of applying the inverse operations and how that relates to the order of operation of the equations.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying solving complex equations and explaining the steps because students may deepen their understanding of inverse operation, such as logarithm as the inverse operation of exponent. Students explore strategies of solving equations with complex operations and justify their reason in cooperative learning groups.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying understanding solving equations as a process of reasoning and explaining the reasoning the use of mathematical representations within the classroom is critical because linguistically and culturally speaking, experiences of students provide different and varied types of representations for solving mathematical problems varies on different types of exposures they have in their home and communities. Students' abilities in explaining mathematical reasoning varies on their language preferences and representations.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: An Extraneous Solution Illustrative Mathematics

Megan is working solving the equation

$$\frac{2}{x^2-1} - \frac{1}{x-1} = \frac{1}{x+1}.$$

She says

If I clear the denominators I find that the only solution is $x = 1$ but when I substitute in $x = 1$ the equation does not make any sense.

a. Is Megan's work correct?

b. Why does Megan's method produce an x value that does not solve the equation?

IM Commentary
The goal of this task is to examine how extraneous solutions can arise when solving rational equations. The task presents an operation, "clearing denominators," which appears to lead to a contradiction. To resolve the contradiction, we examine more carefully what is happening when we clear denominators (MP6). One way to describe the process is that we find a common denominator for both sides and set the numerators equal to each other. This gives solutions to the original equation provided the solutions are in the domain of the rational functions on both sides, that is, provided they are not zeros of one or more of the denominators. In this case, the solution x=1 makes the numerators equal to one another but also makes the denominators of two of the expressions zero, and so x=1 is an extraneous solution.

Another way to think about the process is to connect it to the familiar process students have learned from solving linear equations of multiplying both sides by a non-zero constant to get an equivalent equation. In this case, clearing denominators amounts to multiplying both sides by $x2-1$. The problem is that when $x=1$ or $x=-1$ this is multiplying both sides by zero. So, the operation only produces an equivalent equation if you stay away from those two values, and consider them separately at the end of the process. Students may also experiment with graphs of the functions on the left hand and right-hand side of the equation to see that they are never equal. This confirms Megan's work which shows that if the two expressions are equal to one another then $x=1$, but this is not possible.	
Relevance to families and communities: During a unit focused on understanding solving equations as a process of reasoning and explaining the reasoning, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, some students learned differently when it comes to expressing their ideas linguistically speaking they learned different languages. Learning about the different structures for the process and explaining it across the languages in the classroom can lead to a more robust understanding of solving equations for all students by making connections to the different structures of understanding solving equations in other languages.	**Cross-Curricular Connections:** This lesson uses right triangle trigonometry and a rational function to explore the percent of your visual field that is occupied by the area of a television. Sofa Away From Me: A Lesson by Mathalicious

HS: ALGEBRA- REASONING WITH EQUATIONS & INEQUALITIES

Cluster Statement: D: Represent and solve equations and inequalities graphically.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.REI.D.11: Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* *Note: Algebra 1 focuses on linear and exponential. In Algebra 2 the focus is on combining polynomial, rational, radical, absolute value, and exponential functions.*	SMP 3: Students can construct viable arguments by explaining how the x-coordinate of a solution to the system *y = f(x)* and *y = g(x)* solves *f(x) = g(x).* SMP 5: Students can use tools by finding solution(s) of system of equations from graph or tables. SMP 7: Students look for and make use of structure by explaining in their own words how and when a solution is given as a point (x, y) versus a value (x = a).	• Recognize what the solution *y = f(x)* and *y = g(x)* means on a graph. • Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. • Find approximate solutions for the system $y = f(x)$ and $y = g(x)$ using graphs and tables. • Find successive approximations and use them to solve the system $y = f(x)$ and $y = g(x)$. • Use paper/pencil or technology to produce a table of values. • Explain what x-coordinate of a common ordered pair represents in the context of the problem. **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy:** Understand, Apply, Analyze, Evaluate
<u>Previous Learning Connections</u> • Connect to the work of Algebra 1 in this cluster around linear and exponential. **(HSA.REI.D)**	**<u>Current Learning Connections</u>** • Connect this cluster across all of Algebra 2, particularly when each new function is presented.	**<u>Future Learning Connections</u>** • Connect this cluster to future work around determining specific solutions to new functions (i.e. zeros). Also, in Calculus, when students discuss the area between two curves and volume with rotation.

Clarification Statement

HSA.REI.D.11: Just as the **algebraic** work with **equations** can be reduced to a series of algebraic moves unsupported by reasoning, so can the **graphical visualization of solutions**. The simple idea that an equation f(x) = g(x) can be **solved (approximately)** by graphing y = f(x) and y = g(x) and finding the **intersection points** involves a number of pieces of conceptual understanding. [This method] seeks to convert an equation in one variable, f(x) = g(x), to a s**ystem of equations** in two **variables**, y = f(x) and y = g(x), by introducing a second variable y and **setting it equal** to each side of the equation. If x is a solution to the original equation, then f(x) and g(x) are equal, and thus **(x, y)** is a solution to the new system.

Common Misconceptions

- Students often interpret the solutions to an equation or graphical representation of an equation as only integer values.
- Students may believe an estimate of a value between two integer points is sufficient, but the standard states that students should find successive approximations to approximate the solution.
- Students believe the graph of a function is simply a line or curve "connecting the dots," without recognizing that the graph represents all solutions to the equation.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., number lines) when studying represent and solve equations and inequalities graphically because different representations within the same problem, students make the connection between the graph, table, word problem, and equations. When two expressions are equal to each other, the variable equal to a numerical value is the solution algebraically and graphically.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.EE.B.5: This standard provides a foundation for work to represent and solve equations and inequalities graphically because substituting numerical values into an equation to determine if the equation is true, the student will comprehend that the answer is a solution. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features give an example connected to this standard such as the size of text, images, graphs, tables, or other visual content; contrast between background and text or image; color used for information or emphasis; volume or rate of speech or sound; speed or timing of video, animation, sound, simulations, etc.; layout of visual or other elements; font used for print material because teachers and learners should work together to attain the best match of features to learning needs.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because students persist on finding the solutions of equations and inequalities using the graphs constructed. Students check and interpret the solutions in the context of the problem to make sense of their mathematical thinking.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where appropriate because students understand the multiple representations of the solutions and make connections to the graphs that represents the equations and inequalities. Students connect the multiple representations to make sense of the meaning of the solution in the context of the problems.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because students use multiple ways, including graphing calculators and graph paper, to construct the graphic representations of the equations and inequalities. Students compare and verify the solutions from different representations to defend the solutions.

Internalize

Comprehension: *How will the learning for students' support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations because students apply their interpretation and knowledge of the solutions to new problems in the context of the situation. Students also extend their knowledge of solving equations and inequalities graphically to solving equations and inequalities algebraically.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on represent and solve equations and inequalities graphically by critiquing student approaches/solutions to make connections through a short mini-lesson because connections between solution (algebraically) and intersection (graphically) are equivalent. When students compare answers graphically and algebraically, intersections are the solution.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit to represent and solve equations and inequalities graphically by offering opportunities to understand and explore different strategies because interpreting solution using the different representations allows the students to visualize the answer that was only written as a system of equations or two expressions equal to each other. Students can check their work graphically to confirm their answer.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying represent and solve equations and inequalities graphically because different types of equations (logarithmic, exponential, trigonometric, etc) can use the graphing method to find solutions to word problems or algebraically.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying represent and solve equations and inequalities graphically the use of mathematical representations within the classroom is critical because students given a situation in two variables and they must find the value of one variable given the value of the other, create an equation to represent the situation, use technology to create a graph, and interpret each representation. Understanding how lines and tables represent solution sets of linear relationships will help students make sense of graphs of and solutions to linear inequalities, and later, to make sense of solutions to systems of linear equations in their Algebra 1 class.

Standards Aligned Instructionally Embedded Formative Assessment Resources: *Source:* SAT $x+1=\frac{2}{x+1}$ *Question 1474935 Answers* In the equation above, which of the following is a possible value of $x+1$? • A. $1-\sqrt{2}$ • B. $\sqrt{2}$ • C. 2 • D. 4 How Many Solutions?	
Relevance to families and communities: During a unit focused on representing and solving equations and inequalities graphically, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, bringing in the different languages spoken in the home and connecting it to the tools available to translate different languages, i.e. Google translate, closed captions on televisions, etc. make connections that show that in the culture of mathematics, tools are used to translate mathematics and help us make sense of what we are seeing.	**Cross-Curricular Connections:** Students will model projectile motion in both function and parametric graphing. This was designed as an in-class modeling activity to be used prior to actually launching air-powered projectile rockets. A set of data is given in a spreadsheet and students create model functions using a variety of methods: vertex form (then using grab and move to fit the curve to the data points), standard form (using matrices), and quadratic regression. Rocket Simulation Activity

HS: FUNCTIONS- INTERPRETING FUNCTIONS

Cluster Statement: B: Interpret functions that arise in applications in terms of the context.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.IF.B.4 **For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship**. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. *Note: Algebra 1 focuses on linear, exponential, and quadratic. In Algebra 2 emphasize selection of appropriate models.*	MP1: Students can make sense of problems and persevere in solving them by thinking through the meaning of the key features in graphs as relative to a given context. MP.4: Students can model with mathematics by creating an approximate graph that could model a given context.	• Identify intercepts of a function. • identify intervals where the function is increasing. • Identify intervals where the function is decreasing. • Identify intervals where the function is positive. • Identify intervals where the function is negative. • Identify relative maximums of a function. • Identify relative minimums of a function. • Identify symmetries in the functions. • Identify end behavior of the functions. • Sketch graphs given a list of key features or a verbal model. • Sketch functions that model key feature behavior. • Label intercepts and intervals of a graph. • Interpret where the function is increasing, decreasing, positive, or negative. • Interpret relative maximums and minimums. • Interpret various symmetries, end behaviors, and periodicity.
		Webb's Depth Of Knowledge: 1-2
		Bloom's Taxonomy: Understand, Apply and Analyze

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<table>
<tr>
<td rowspan="3">Standard Text

HSF.IF.B.5
Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Note: Algebra 1 focuses on linear, exponential, and quadratic. In Algebra 2 emphasize selection of appropriate models.</td>
<td rowspan="3">Standard for Mathematical Practices

SMP 3: Students can construct viable arguments by explaining why the domain of a given context is discrete or continuous.

SMP 4: Students will model with mathematics by connecting a function to the context it represents using quantities.</td>
<td>Students who demonstrate understanding can:
• Make connections between a graph of a function and its domain.
• Make connections between the graph of a function and the context it describes.
• Identify when the domain of a given context is discrete or continuous and explain why.</td>
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<tr><td>Webb's Depth of Knowledge: 1-2</td></tr>
<tr><td>Bloom's Taxonomy: Understand, Apply and Analyze</td></tr>
<tr>
<td rowspan="3">Standard Text

HSF.IF.B.6
Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Note: Algebra 1 focuses on linear, exponential, and quadratic. In Algebra 2 emphasize selection of appropriate models.</td>
<td rowspan="3">Standard for Mathematical Practices

SMP 4: Students can model with mathematics by interpreting the average rate of change within the context of a problem.

SMP 5: Students can use tools by using tables and graphs to determine the average rate of change over a specified interval.</td>
<td>Students who demonstrate understanding can:
• Calculate the average rate of change of a function over a specified interval presented symbolically.
• Calculate the average rate of change of a function over a specified interval presented in a table.
• Interpret the average rate of change of a function over a specified interval presented symbolically for a given context.
• Interpret the average rate of change of a function over a specified interval presented in a table for a given context.
• Estimate the rate of change of a function from a graph.</td>
</tr>
<tr><td>Webb's Depth of Knowledge: 1-2</td></tr>
<tr><td>Bloom's Taxonomy: Understand, Apply and Analyze</td></tr>
</table>

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• Connect to the work of Algebra 1 within this cluster around linear, quadratic, and exponential. **(HSF.IF.B)**	• Connect to discovering features of families of functions. **(HSF.IF.7)** • Connect to finding key features of the entire family of functions. **(HSF.IF.4)**	• Connect to the work with trigonometric functions. **(HST.TF.B)**

Clarification Statement

HSF.IF.B.4: Students interpret the **key features** of the different **functions** listed in the standard. When given a **table** or **graph** of a function that models a **real-life situation**, explain the meaning of the characteristics of the table or graph in the **context** of the problem.

Key features of a **linear function** are **slope** and **intercepts**, of a **quadratic function** are **intervals of increase/decrease**, **positive/negative**, **maximum/minimum**, **symmetry**, and **intercepts**, of an **exponential function** include **y-intercept** and **increasing/decreasing intervals** and of an **absolute value** include **y-intercept**, **minimum or maximum**, **increasing or decreasing intervals**, and **symmetry**.

HSF.IF.B.5: Students should focus their attention on possible **input and output values**, framing them as the **domain and range of a function**. When given a description of a function that represents a situation, the students should determine reasonable domain and range. Students relate the domain of a function to its graph and, where applicable, to the **quantitative relationship** it describes. Students need to explain the reasonableness of a domain for a given context.

Students should understand that the domain of a function is the set of all possible inputs and the range is the set of all possible outputs. Also looking at if a function is **continuous** (time, amount of liquid filling a container) or discrete (number of people or things) and connecting back to number classifications

HSF.IF.B.6: Students will calculate and interpret the **average rate of change** of a linear, quadratic, **piecewise linea**r (to include absolute value), and exponential function (presented **symbolically** or as a table) over a specified **interval.** Students will **estimate** the rate of change from a graph. In addition to finding average rates of change from functions given symbolically, graphically, or in a table, students may collect **data** from **experiments** or **simulations** (ex. falling ball, velocity of a car, etc.) and find average rates of change over various intervals.

Common Misconceptions

Students may confuse scatter plots and correlations.

Students may focus on the y values of the graph instead of the x values of the interval, when identifying key features of a graph.

Students may have difficulty understanding domain.

Students may confuse independent and dependent variables.

Students may confuse shift with rate of change

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying interpret functions that arise in applications in terms of the context because quantities in graphs and tables should be interpreted in context to the problem and domains should be within the context.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.F.A.1: This standard provides a foundation for work with interpret functions that arise in applications in terms of the context because students need to interpret the ordered pairs on the graph for analyzing or making predictions. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: How will the learning for students provide multiple options for recruiting student interest?

- For example, learners engaging with interpret functions that arise in applications in terms of the context benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because when given a function, students can explain the x and y variables. When given in an application context, students need to make a connection with their lives and explain the variables in the function.

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with interpret functions that arise in applications in terms of the context benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because collaboration within a group of student can generate some effective communication skill and this creates peer mentoring for group members that need help.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with interpret functions that arise in applications in terms of the context benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to the structure because interpreting an equation with the independent variable and dependent variables contextual to the problem will help the students interpret the graph and the average rate of change and other important characteristics of the graph.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with interpret functions that arise in applications in terms of the context benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as composing in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video using physical manipulatives (e.g., blocks, 3D models, base-ten blocks) because learners from different cultures and backgrounds need exposure using multiple tools to construct graphs and tables. Students make connections and interpret the functions when they are presented with different tools.

Internalize
Comprehension: How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?

- For example, learners engaging with interpret functions that arise in applications in terms of the context benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing templates, graphic organizers, concept maps to support note-taking because <students need to internalize and interpret the information from the sketch of a graph, table, verbal description or equation. Students explore multiple representation, and, in the end, all interpretation should have the same results.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- This standard 8.F.A.1 provides a foundation for work with interpret functions that arise in applications in terms of the context because students need to interpret the ordered pairs on the graph for analyzing or making predictions. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit interpret functions that arise in applications in terms of the context by addressing conceptual understanding because students need to make connections about the numbers they choose for their domain and range in context with the problem. They will need to interpret the characteristics of a graph.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying interpret functions that arise in applications in terms of the context because students can interpret their own graphs and explore how graphs can be integrated according their interest.

Culturally and Linguistically Responsive Instruction:

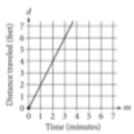
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying interpret functions that arise in applications in terms of the context the types of mathematical tasks are critical because clearly defined tasks set the routine for interaction and support for students. Interpreting and sketching key characteristics of graphs and tables, students make the connections graphically, verbally, table, and symbolically. Allowing students to explain, think out loud, making conjectures, and communicate with peers to come up with mathematical ideas.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

CollegeBoard	Question ID 1474415						
Assessment **SAT**	Test **Math**	Cross-Test and Subscore **Heart of Algebra**	Difficulty **Easy**	Primary Dimension **Heart of Algebra**	Secondary Dimension **Linear functions**	Tertiary Dimension **4. Make connections between verbal, tabular, algebraic, and graphical representations of a linear function, by c. determining how a graph is affected by a change to its equation.**	Calculator **No Calculator**



The graph above shows the distance traveled d, in feet, by a product on a conveyor belt m minutes after the product is placed on the belt. Which of the following equations correctly relates d and m ?

Question Difficulty: Easy

A. $d = 2m$

B. $d = \frac{1}{2}m$

C. $d = m + 2$

D. $d = 2m + 2$

Choice A is correct. The line passes through the origin. Therefore, this is a relationship of the form $d = km$, where k is a constant representing the slope of the graph. To find the value of k, choose a point (m, d) on the graph of the line other than the origin and substitute the values of m and d into the equation. For example, if the point $(2, 4)$ is chosen, then $4 = k(2)$, and $k = 2$. Therefore, the equation of the line is $d = 2m$.

Choice B is incorrect and may result from calculating the slope of the line as the change in time over the change in distance traveled instead of the change in distance traveled over the change in time. Choices C and D are incorrect because each of these equations represents a line with a d-intercept of 2. However, the graph shows a line with a d-intercept of 0.

SAT Item

This type of assessment question requires students to analyze a graph in context and create a linear equation to fit. Students will engage with SMP 7 as they use the structure of the line in the graph to write an equation.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on interpret functions that arise in applications in terms of the context, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, families create their own story and make the connection with a table and graph. Key features should be included. Every family will have a different story.	In this lesson, students use exponential decay and rational functions to understand why addicted patients seek more and stronger opioids to alleviate their pain. Students discuss the role that various parties played in creating the crisis and ways they can help to solve it. House of Pain: A Lesson by Mathalicious

<table>
<tr><th colspan="3">HS: FUNCTIONS- INTERPRETING FUNCTIONS</th></tr>
<tr><td colspan="3">Cluster Statement: C: Analyze functions using different representations.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers</td></tr>
<tr><td rowspan="3">Standard Text

HSF.IF.C.7
Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

• HSF.IF.C.7.B: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
• HSF.IF.C.7.C: Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
• HSF.IF.C.7.E: Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Note: Algebra 1 focuses on linear, exponential, quadratic, absolute value, step, and piecewise defined. Algebra 2 focuses on using key features to guide selection of appropriate type of model function.</td>
<td rowspan="3">Standard for Mathematical Practices

SMP 4: Students can model with mathematics by focusing on using key features to guide the selection of an appropriate type of function to model a context.

SMP 7: Students can look for and make use of structure by recognizing key symbolic and graphical features to identify the type of a function.</td>
<td>Students who demonstrate understanding can:
• Graph exponential, logarithmic, and trigonometric functions.
• Describe key features of exponential, logarithmic, and trigonometric functions.
• Graph functions expressed symbolically showing key features of the graph by hand in simple cases and with technology for more complicated cases.
• Compare and contrast functions.</td></tr>
<tr><td>Webb's Depth Of Knowledge: 1-2</td></tr>
<tr><td>Bloom's Taxonomy: Understand, Apply and Analyze</td></tr>
</table>

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. • HSF.IF.C.8.A: Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. • HSF.IF.C.8.B: Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay. *Note: Algebra 1 focuses on linear, exponential, quadratic, absolute value, step, and piecewise defined. Algebra 2 focuses on using key features to guide selection of appropriate type of model function.*	SMP 4: Students can model with mathematics by interpreting zeros, intervals where the function is increasing or decreasing, extrema and symmetry within a context. SMP 7: Students can look for and make use of structure by using rearranging functions to highlight key features.	• Rewrite a function to find and highlight key features. • Factor a quadratic expression to find zeros, extrema and symmetry • Interpret the meaning of zeros, extrema and symmetry within the context of a problem. • Complete the square for a quadratic function to reveal its key features. • Interpret the key features of a quadratic expression in terms of the context it represents. • Use properties of exponents to relate parts of an exponential function to its context (e.g., describe the initial value, growth/decay rate or factor and the growth period). • Identify how key features of an exponential function relate to characteristics in a real-world context. • Classify real-world problems as an exponential growth or decay. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply and Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSF.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* *Note: Algebra 1 focuses on linear, exponential, quadratic, absolute value, step, and piecewise defined. Algebra 2 focuses on using key*	SMP 5: Students can use tools by working flexibly with multiple representations. SMP 7: Students can look for and make use of structure by comparing the similarities and differences of functions.	• Make comparisons between functions in different forms using their knowledge of key features.

features to guide selection of appropriate type of model function.		**Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply and Analyze
Previous Learning Connections • Connect to the work in Algebra 1 with this cluster which focused on linear, exponential, quadratic, absolute value, step, and piecewise defined by supporting Algebra 2 students to focus on using key features to guide selection of an appropriate type of model function.	**Current Learning Connections** • Connect to writing linear, quadratic, and exponential functions to describe relationships between quantities. **(HSA.CED.1-3)** • Connect to analyzing transformations of parent functions for linear, quadratic, and exponential functions. **(HSF.BF.3)**	**Future Learning Connections** • Connect to graphing all parent functions by hand and using technology and identifying their key features. **(HSF.IF.7)** • Connect to factoring to complete the square with quadratic functions with complex zeros. **(HSN.CN.7)**

Clarification Statement

HSF.IF.C.7: Students should be able to describe the significant features of different **functions graphically** and **algebraically**. Students should be able to use the **significant features** to **sketch** the graph of the function. Students should graph **linear** and **quadratic functions** and show **intercepts**, **maxima**, and **minima.** Students should know the **slope-intercept form** of linear functions, y = mx + b, and how to extract enough information from the equation to be able to draw it. When graphing **roots**, remember that for $\sqrt[n]{x}$, if n is **even,** the domain includes all **positive integers**. Otherwise, **negative** values are included as well. When graphing roots of the for $y = a\sqrt{x} + b$, remember the **y-intercept** is b. Students should remember that roots are **fractional exponents.** Students should know to look at the **highest degree of the polynomial** and its **coefficient,** ax^n. If n is even, the function will extend either **up** or **down** on both ends (as x goes to **positive** or **negative infinity**). If n is **odd**, they'll go in **opposite directions**. If a is positive, the even powered functions will go up and the odd powered functions will start down and go up. If a is negative, the even powered functions will go down, and the odd powered functions will start up and go down.

HSF.IF.C.8: Students should be able to rewrite quadratic and **exponential functions** in different ways to find key features of the expression and interpret those key features in terms of the context they represent. Students should be able to find the **x-intercepts** of a quadratic function using both **factoring** and **completing the square**.

HSF.IF.C.9: Students should be able to compare two given functions (linear, exponential, quadratic) whether that be as a function or **equation**, in a **table,** in a **graph,** or by **verbal description**. Students should start by knowing the difference between linear, quadratic and exponential functions, and be able to identify them by equation and by graph. Students should be able to compare two functions even when they're both represented differently. To do this successfully, they must be able to **translate** between an equation, a graph, words, and a table of values, and understand how certain aspects of one **representation** impact the rest.

Common Misconceptions

Students may have difficulty identifying the key features needed to sketch the graphs or identifying those features algebraically.

Students may have difficulty with contextualizing and decontextualizing expressions.

Students will often confuse functions given in a table as a representation of a finite set of numbers rather than a subset of the entire function. They also may have difficulty with the abstractness of determining what is happening with a function over intervals of the domain that they cannot see.

Students may not distinguish between the different type of logarithms, i.e., natural logs, when using calculator

Students may struggle with applying translations, stretches, compressions, and reflections to a parent function.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., number lines) when studying analyze functions using different representations because connections between the different representations, students need exposure to the different family functions (linear, quadratic, polynomial, etc.)

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- This standard 8.F.A.1 provides a foundation for work with analyzing functions using different representations because order pairs of numbers have a relationship and the point is represented on the graph contextually. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Physical Action: How will the learning for students provide a variety of methods for navigation to support access?

- For example, learners engaging with analyze functions using different representations benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as varying methods for response and navigation by providing alternatives to <requirements for rate, timing, speed, and range of motor action with instructional materials, physical manipulatives, and technologies; physically responding or indicating selections; physically interacting with materials by hand, voice, single switch, joystick, keyboard, or adapted keyboard> because real-world hands-on activities require communication and collaboration. Integrate technology to collect data and graph different functions. Technology can be used to graph, calculate regressions and analyze different features of the graph.

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with analyzing functions using different representations benefit when learning experiences attend to students' attention and affect to support sustained effort and concentration such as constructing communities of learners engaged in common interests or activities because different ideas from multiple team members can create collaboration and peer tutoring. Immediate feedback can also open discussions/ideas to further the project.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with analyze functions using different representations benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) because making the connection verbally, graphically, numerically and algebraically, students explain the context of the graphs paying attention to the key features. Highlight key words in word problems. interpret the graphs in context using the rate of change, making predictions, intercepts, maximum and minimum values and decreasing/increasing.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with analyzing functions using different representations benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because visually displayed on graph paper and using the graphing calculator to check their work, students become more confident and appreciate the different tools available to construct the graphs.

Internalize

Comprehension: How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?

- For example, learners engaging with analyze functions using different representations benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as highlighting previously learned skills that can be used to solve unfamiliar problems because background knowledge using different hands-on strategies like paper folding, graphic organizers, technology, students make the connection with new skills.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on analyze functions using different representations by providing specific feedback to students on their work through a short mini lesson because immediate feedback provides support for learning. There are several family functions with different key features and interpretation.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on analyzing functions using different representations by offering opportunities to understand and explore different strategies because explaining the context of the problem verbally, graphically and writing, students comprehend the different family functions/equations.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying analyze functions using different representations because making a real-world connection with a choice to select what the learner is interested in will make a deeper connection to the mathematical concept and skill.

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying analyzing functions using different representations the pattern of questions within the classroom is critical because asking open ended questions allows the students to think, answer and have a reason for their answer. Ask probing questions that allow students to elaborate and clarify their different graphs and key representations. The explanation of the different family functions from linear to trigonometric functions include all types of learners from the low to the high so everyone feels included.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
http://tasks.illustrativemathematics.org/content-standards/HSF/IF/C/9/tasks/1279
This type of assessment question requires students to analyze function and a graph and compare the key features in context of a scenario. Students will engage with SMP 7 as they use the structure of both the equation and the graph to answer questions in context.
Additional assessment:
Analyzing Graphs
https://www.map.mathshell.org/lessons.php?unit=9245&collection=8

https://www.map.mathshell.org/lessons.php?unit=9240&collection=8

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on analyzing functions using different representations, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, use community data and technology to graph, calculate and analyze the regression between the variables. Data and graphs can be used to make comparisons between the communities.	Many of the Navajo rug designs you will discover by following the project will be good examples of symmetrical balance. Symmetrical balance is a type of visual balance where the overall composition is arranged to look like it is the same on both sides of the center of the design. In other words, it is a design which could be folded in half, and as the design folds, each part of the design would match up with its symmetrical counterpart on the opposite side of the center. The rug design on the right is symmetrical left-to-right. If a line was drawn vertically down the center of the rug, the arrangement of shapes and colors would appear to be exactly the opposite of each other on both sides of that line. Design a Navajo Rug

NEW MEXICO
Public Education Department

HS: FUNCTIONS- BUILDING FUNCTIONS

Cluster Statement: A: Build a function that models a relationship between two quantities.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

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Standard Text

HSF.BF.A.1: Write a function that describes a relationship between two quantities.*

- HSF.BF.A.1.B Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

Note: Algebra 1 focuses on linear, exponential, and quadratic. Algebra 2 includes all types of functions studied.

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Standard for Mathematical Practices

SMP 2: Students can reason abstractly and quantitatively to make sense of quantities and their relationships in problem situations.

SMP 4: Students can model with mathematics by discovering patterns in each contextual problem and creating verbal, symbolic or explicit symbolic rules to describe them.

SMP 7: Students can look for and make use of structure by using the operations of math to combine functions.

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Students who demonstrate understanding can:

- Build a function using different functions and arithmetic operations in context.

Webb's Depth of Knowledge: 1-2

Bloom's Taxonomy: Understand, Apply, Analyze

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<u>Previous Learning Connections</u>

- Connect to Algebra 1 work focusing on linear, exponential, and quadratic within this cluster.
- Connect to recognizing situations that grow by a constant rate or percent. **(HSF.LE.1)**

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<u>Current Learning Connections</u>

- Connect to identifying patterns in the function's rate of change, specifying intervals of increase and decrease, and graphing to model functions. **(HSF.IF.4,6)**
- Connect to discussing the relative strength sand weaknesses of each representation and which are most efficient to be able to assist them in making symbolic functions. **(HSF.IF.9)**

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<u>Future Learning Connections</u>

- Connect to arithmetic and geometric sequences and using them to model situations. **(HSF.BF.A.2)**

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Clarification Statement

HSF.BF.A.1: Students should write **functions** for given **relationships** between **quantities**. Students can use functions to **model real-life situations** and make **predictions**. Students should be able to use functions describe relationships between two quantities, usually x and f(x), where f(x) is some **output** value that depends on the **input** value x. Within a context, students should be able to express a given relationship as a function.

Common Misconceptions

- Students may want to try to use a linear function, specifically the slope-intercept form for every situation.
- Students may tend to focus on the symbolic form of a function and may need additional support in working with other forms.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that previews new contexts for tasks within the unit (e.g., cell phone plans) when studying building a function that models a relationship between two quantities because the new contexts will show an alignment to new material that will be covered.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.F.B.4: This standard provides a foundation for work with building a function that makes a relationship between two quantities because prior learning on constructing a function modeling a linear relationship between two quantities is learned and will be expanded on in the current unit. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with building a function that models a relationship between two quantities benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because <building functions will provide practice for student and keep them engaged in the actual solving and working with the function.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with building a function that models a relationship between two quantities benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as prompting or requiring learners to explicitly formulate or restate learning goals because students can continue working towards goals when they are prompted to formulate and restate learning goals which keeps them focused.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with building a function that models a relationship between two quantities benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because students understanding the vocabulary and symbols before the instruction allows for them to connect with the content when it is being explained and they will have a better understanding of what the content is explaining as it is being taught.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with building a function that models a relationship between two quantities benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks, 3D models, base-ten blocks) because students can make a visual connection to the material being learned. With multiple types of learners, visuals provide a different perspective of the content and can aid in building the functions.

Internalize

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with <insert the mathematics examined in the cluster> benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as posting goals, objectives, and schedules in an obvious place because students will be able to quickly refer to the posted goals and objectives for the lesson. Posting these into an obvious place allows for less time with students not on task and allows for them to remain focused on the outcomes for the lesson and reflecting on building functions.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on building a function that models a relationship between two quantities by revisiting student thinking through a short mini-lesson because students should be able to recall prior knowledge in the content previously learned and can use that prior knowledge to build on current content.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit building a function that models a relationship between two quantities by helping students move from specific answers to generalizations for certain types of problems because recalling prior knowledge will aid the student with current understanding and show the alignment to prior knowledge and will engage the student in the current content.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying building a function that models a relationship between two quantities because activating prior knowledge will allow for students to take the understanding of the current content to a greater level and will allow for better understanding of the content.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Task: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)." For example, when studying building a function that models a relationship between two quantities the types of mathematical tasks are critical because when students are able to make connections, it is easier for them to learn and store information, like making a connection to background knowledge or prior learning to create an optimal environment for learning, as they bring this knowledge with them to class each day.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/HSF/BF/A/2/tasks/1695
This type of assessment question requires students to analyze a number pattern described in context and fit both a recursive function to the pattern and use it to answer questions. Students will engage with SMP 1 and SMP 4 as they persevere to express the pattern mathematically and model the scenario with an equation.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on building a function that models a relationship between two quantities, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the various ways that functions relate to quantities can be a great way to connect school and home with making references to those quantities that can be encountered at home and how they relate to the tasks being created in the classroom. Making that connection allows for students to become more comfortable with learning the content and provides evidence of prior knowledge that the student can bring into the lessons.	Science: In high school the NGSS students should apply concepts of statistics and probability to explain the variation and distribution of expressed traits in a population. Consider providing a connection for students to examine scientific data and predict the effect of a change in one variable on another. https://www.nextgenscience.org/topic-arrangement/hsinheritance-and-variation-traits

NEW MEXICO
Public Education Department

HS: FUNCTIONS- BUILDING FUNCTIONS

Cluster Statement: B: Build new functions from existing functions.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.BF.B.3 **Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.** *Note: Algebra 1 focuses on linear, exponential, quadratic, and absolute value. Algebra 2 includes simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types.*	SMP 5: Students can use tools by using graphing calculators or technology to experiment with parent functions and the results when different transformations are applied. SMP 8: Students look for and express regularity in repeated reasoning by exploring different expressions for transformations of f(x) and generalizing the effects.	• Identify vertical transformations from a function or a graph. • Identify horizontal transformations from a function or a graph. • Identify a shrink or a stretch from a function or a graph. • Write the results from such transformations. • Recognize odd and even functions. • Identify transformations of a function on a graph. • Describe the effects of transformations on parent functions. **Webb's Depth Of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply and Analyze
Standard Text HSF.BF.B.4.A Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.* *Note: Algebra 1 focuses on linear, exponential, quadratic, and absolute value. Algebra 2 includes simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types.*	**Standard for Mathematical Practices** SMP.6 Students can attend to precision by understanding that some functions do not have an inverse unless there is some sort of restriction on the domain. SMP 7: Students can look for and make use of structure by recognizing that the ordered pair (x, y) is reversed for a function's inverse.	**Students who demonstrate understanding can:** • Write the inverse of a function. • Determine restrictions on the domain to allow for an inverse to exist. • Relate using an inverse as an operation that undoes another operation. • Solve an equation of the form f(x)=c for a function f that has an inverse and write an expression for the inverse. **Webb's Depth of Knowledge:** 1-2

		Bloom's Taxonomy: Understand, Apply and Analyze
Previous Learning Connections • Connect to the work in Algebra 1 with linear, exponential, quadratic, and absolute value functions for this cluster. **(HSF.BF.B)**	**Current Learning Connections** • Connect to graphing functional relationships. **(HSF.IF.4)** • Connect to trigonometric functions. **(HS.F-TF.B)**	**Future Learning Connections** • Connecting to understanding the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents. **(HS.F-BF.B.5)**

Clarification Statement

HSF.BF.B.3: Students should describe the effect of **stretches, shrinkages**, **vertical and horizontal transformations** on **functions**. They should be able to find the value of the **transformation** when given a **graph** and be able to explain effects of transformations using technology. Students should know that adding a **constant** k to a function will change the graph of the function depending not only on the value of the constant, but on where it is inserted as well. If y = f(x) is changed to y = f(x) + k, the curve will **shift** vertically (up for k > 0, down if k < 0). Adding k to x such that y = f(x + k) will shift the curve horizontally (left for k > 0, right for k < 0). Multiplying f(x) by a constant k stretches (k > 1) or **squishes** (0 < k < 1) the graph vertically. If k < 0, the graph is also **flipped** over the **x-axis**. Multiplying x by k stretches (k > 0) or squishes (k < 0) the graph horizontally.

HSF.BF.B.4: Students should be able to find the **inverse** of **functions** and recognize that other functions may not have an inverse unless there are **restrictions** placed on the **domain**. If f(x) = y is a function, the inverse function can be found by switching the place of x and y (f(y) = x), and then solving for y so that f-1(x) = y. For instance, if the function f(x) is $y = 2x^3$, then the inverse function $f^{-1}(x)$ consists of switching the places of x and y ($x = 2y^3$) and then solving for y.

Common Misconceptions

Students often have difficulty determining the direction of the horizontal shifts.

Students often confuse the notation for the inverse and negative numbers.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying building new functions from existing functions because in prior lessons and grade levels, students have been introduced to many aspects and content of functions.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- HS.F-IF.A.1: This standard provides a foundation for work with building new functions from existing functions because this prerequisite has students understand functions based on domain and range. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?

- For example, learners engaging with building new functions from existing functions benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information for all images, graphics, video, or animations; touch equivalents (tactile graphics or objects of reference) for key visuals that represent concepts; objects and spatial models to convey perspective or interaction; auditory cues for key concepts and transitions in visual information because offering alternatives provides more flexibility for students to obtain the necessary information and skills needed to successfully complete the tasks.

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with building new functions from existing functions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as displaying the learning goals in multiple ways because this supports students to build individual skills in self-determination and pushes them to not give up on the content and will improve their potential for learning the content.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with building new functions from existing functions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) because embedding the supports provides accessibility for all which can help in achieving to link or associate alternate representations of meaning for students.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with building new functions from existing functions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because when students are presented with various strategies they are able to explore the variety of methods used to solve problems and come up with solutions.

Internalize

Self-Regulation: How will the design of the learning strategically support students to effectively cope and engage with the environment?

- For example, learners engaging with building new functions from existing functions benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because allowing students to become aware and understand their own thought processes allows for students to overcome those frustrations that they are experiencing in mathematics and for them to move beyond them to overcome them.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on building new functions from existing functions by revisiting student thinking through a short mini-lesson because students will be able to activate prior learning on functions and make the connection between the content.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit building new functions from existing functions by offering opportunities to understand and explore different strategies because providing students with various strategies allows for further depth in understanding and further delving into the depth of the content.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying building new functions from existing functions because when students are able to draw connections between various topics, the learning potential is increased and prior knowledge activation is improved upon and built upon.

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of

low expectations and low achievement. For example, when studying building new functions from existing functions goal setting is critical because it allows students to take ownership of the content and what the expectations for learning are as they are clearly identified while making a meaningful connection between the learning and daily lives.	
Standards Aligned Instructionally Embedded Formative Assessment Resources: http://tasks.illustrativemathematics.org/content-standards/HSF/BF/B/3/tasks/742 This type of assessment question requires students to apply vertical and horizontal translations as well as a reflection to a given graph. Further, students are asked to identify the location of specific coordinates on the new graphs. This will engage students with SMP 7 as they use the structure of the graph, the expression of the transformation and/or a table of values to create new graphs and identify the imaged points. https://www.engageny.org/resource/algebra-i-module-3-topic-c-lesson-17	
Relevance to families and communities: During a unit focused on building new functions from existing functions, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the characteristics of building functions from existing functions allows for making connections to building in any concept which can be done at home such as working on a home project that requires building on to something that already exists. Students can make connections to prior learning (something that already exists) and build onto that knowledge.	**Cross-Curricular Connections:** Science: The equation for velocity, $M(v) = 6v^2$, is one where the variable, v, has directions. Therefore, an inverse function of M(v) cannot give back both a positive and negative velocity. Consider providing a connection for students to consider how they will handle this situation.

<table>
<tr><th colspan="3">HS: FUNCTIONS- TRIGONOMETRIC FUNCTIONS</th></tr>
<tr><td colspan="3">Cluster Statement: A: Extend the domain of trigonometric functions using the unit circle.</td></tr>
<tr><td rowspan="3">Standard Text

HSF.TF.A.1
Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.</td><td rowspan="3">Standard for Mathematical Practices

SMP.2:
Students reason abstractly and quantitatively by understanding the relationship between angle measures in radians and the quadrant that the terminal side of the reference angle lies.

SMP.5:
Students use appropriate tools strategically by using calculators with both degrees and radians and setting the calculator for the correct unit of measure.</td><td>Students who demonstrate understanding can:
• Find the measures of an angle in standard position and the reference angle.
• Find arc length using radian measure on the unit circle.
• Convert between degrees and radians.</td></tr>
<tr><td>Webb's Depth of Knowledge: 1-2</td></tr>
<tr><td>Bloom's Taxonomy: understand, apply</td></tr>
<tr><td rowspan="2">Standard Text

HSF.TF.A.2
Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.</td><td rowspan="2">Standard for Mathematical Practices

SMP 6.
Students attend to precision by understanding when to give a rounded answer (typically in degrees) vs. an actual answer (typically in radians)
SMP 8
Students look for and express regularity in repeated reasoning by using their knowledge of special right triangles and relate it to the unit circle.</td><td>Students who demonstrate understanding can:
• Identify the relationship between the unit circle and the coordinate plane.</td></tr>
<tr><td>Webb's Depth of Knowledge: 1-2</td></tr>
</table>

		Bloom's Taxonomy: understand
Previous Learning Connections In Geometry, students learned the relationship of trigonometric ratios and special right triangles (which leads to evaluating on the unit circle). Students also learned about radian measure and how to convert to degree measure.	**Current Learning Connections** Students will use this knowledge of trigonometric functions (sine and cosine) and apply transformations and identify key features of these trigonometric functions. They will also be able to evaluate trig ratios on a coordinate plane (not on the unit circle).	**Future Learning Connections** In Precalculus and Calculus courses, students will connect this learning cluster to other trigonometric ratios and functions (tangent, cosecant, secant and cotangent) and inverse trigonometric functions.

Clarification Statement

Students will be able to extend their knowledge of circle and trigonometric ratios (sine and cosine) to arc length, evaluating using a unit circle, and graphing trigonometric functions (sine and cosine).

Common Misconceptions

Students may confuse the direction of positive and negative angle measures, writing the wrong sign with the measure.

Students may mix-up radian and degree measures

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time)when studying extending the domain of trigonometric functions using the unit circle because this is the first time many students will be working with a unit circle so providing that visual at the very beginning and explaining its purpose can be helpful in later parts of the lessons.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- F.IF.A1: This standard provides a foundation for work with extending the domain of trigonometric functions using the unit circle because this is the standard where students gain conceptual understanding of domain and range as sets of inputs and outputs for a given function. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

- For example, learners engaging with extending the domain of trigonometric functions using the unit circle benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for auditory information because some students may have a hard time understanding the cyclic nature of these functions when described only in words or with numbers. Consider displaying a rotating wheel or other circular object to show why values repeat over time. This tangible object may help solidify the concept of periodicity.

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with extending the domain of trigonometric functions using the unit circle benefit when learning experiences attend to student's attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities

because students tend to struggle in the details (reducing a fraction, applying the correct sign in each quadrant, misidentified reference angle, etc). By setting up a cooperative group where each student has a role, the group knows who to look to for support at each step and provides structure to how groups work so that no group is sitting "stuck."

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with extending the domain of trigonometric functions using the unit circle benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because understand the concept of equivalence from work with prior functions, we can extend that knowledge to this family of functions. Showing students that trigonometric functions evaluate the same whether in radian or degree can help students to check their work and can take some of the mystery out of these functions once students see they evaluate numerically and create graphs just like any other family of functions we have worked with.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with extending the domain of trigonometric functions using the unit circle benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing virtual or concrete mathematics (e.g. a unit circle with reference angles marked and/or degree and radian measures shown) because it is easy for students to get stuck in the details and not see the larger concept. Providing a visual that allows students to quickly identify equivalent angle measurements or reference angles in a given quadrant can free their minds to focus on how the domain of these functions is not limited to a specified set of values.

Internalize

- For example, learners engaging with extending the domain of trigonometric functions using the unit circle benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) because students will have previously worked with converting units and therefore relating radian measurements as just another unit will ease potential fear of this new concept. Further, students have familiarity with right triangle calculations which can be useful in calculating sin, cos and tan within the unit circle. Showing that the values of the trigonometric functions are not arbitrary but arise from concepts they are familiar with will help build connections between prior knowledge and new learning.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on extending the domain of trigonometric functions using the unit circle by providing specific feedback to students on their work through a short mini-lesson because students may have a fear of trigonometry that may affect their willingness to interact with the content. Providing student specific feedback, identifying things students are doing correct and/or effort students are putting forth can encourage students to continue engaging with the material. There are many opportunities for students to make minor errors and if this is the focus, students may shut down. Focused and encouraging feedback can help to counteract any feelings that arise as errors are made.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit extending the domain of trigonometric functions using the unit circle by addressing conceptual understanding because the goal of this cluster is understanding the basis of periodic functions, rather than procedural computations. Students may lack conceptual understanding of domain/range, radian measures of angles and/or the periodicity of functions. Each of these concepts should be explored so students can adequately express their understanding.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying extending the domain of trigonometric functions using the

	unit circle because once students understand the behavior of periodic functions they can extend the pattern on their own with little guidance from the teacher.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying extending the domain of trigonometric functions using the unit circle facilitating meaningful mathematical discourse is critical because some students will naturally see and understand the patterns of periodic functions quicker than others. Eliciting peer-to-peer explanations can build a cooperative environment in the classroom. Further, by providing sentence frames for students, all students can engage in mathematical conversations, even if they are not sure what the solution is. When students speak about mathematics, the engage meaningfully in the content and deepen their understanding.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
SAT Item

CollegeBoard | Question ID 423225

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	**Math**	**Additional Topics in Math**	**Hard**	**Additional Topics in Math**	**Circles**	**6. Convert between angle measures in degrees and radians.**	**No Calculator**

The number of radians in a 720-degree angle can be written as $a\pi$, where a is a constant. What is the value of a ?

Question Difficulty: Hard

The correct answer is 4. There are π radians in a $180°$ angle. An angle measure of $720°$ is 4 times greater than an angle measure of $180°$. Therefore, the number of radians in a $720°$ angle is 4π.

This type of assessment question requires students to translate an angle measurement from degrees to radians. Students may use a conversion factor to do so or may use the fact that pi radians is equivalent to 180 degrees,

and the given angle is 4 times 180 degrees. Students will engage with SMP 7 in either case as they use the structure of the measurement to convert to different unit of measurement.	
Relevance to families and communities: During a unit focused on extending the domain of trigonometric functions using the unit circle, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, periodicity refers to a pattern that repeats over time. Consider providing examples of numeric and non-numeric patterns that exist (springs bouncing, pendulums swinging, temperature fluctuations throughout the year, hunger growing and decreasing as you approach mealtime) and then ask students to provide some examples of their own.	**Cross-Curricular Connections:** Many of the Navajo rug designs you will discover by following the project will be good examples of symmetrical balance. Symmetrical balance is a type of visual balance where the overall composition is arranged to look like it is the same on both sides of the center of the design. In other words, it is a design which could be folded in half, and as the design folds, each part of the design would match up with its symmetrical counterpart on the opposite side of the center. The rug design on the right is symmetrical left-to-right. If a line was drawn vertically down the center of the rug, the arrangement of shapes and colors would appear to be exactly the opposite of each other on both sides of that line. Design a Navajo Rug

HS: FUNCTIONS- TRIGONOMETRIC FUNCTIONS

Cluster Statement: B: Model periodic phenomena with trigonometric functions.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	SMP 4 Students model with mathematics. By demonstrating how trigonometric functions can be used to model real-life periodic phenomena. SMP 8 Students look for and express regularity in repeated reasoning when examining trigonometric functions as periodic in nature. Students will also utilize patterns in problem solving.	• Graph sine and cosine functions using radian and degrees. • Identify properties of the sine function. • Model a real-world situation using trigonometric functions. Students can then use inverse trigonometric functions to find solutions. **Webb's Depth Of Knowledge:** 1-2 **Bloom's Taxonomy:** understand, apply
Previous Learning Connections In Geometry and Algebra II, students have defined trigonometric ratios using the acute angles of right triangles	**Current Learning Connections** In Algebra II, students define inverse functions an	**Future Learning Connections** Inverse trigonometric functions play a major role in Calculus, when using operations such as differentiation and integration.

Clarification Statement
Students apply the concept of inverse functions to trigonometric functions and use that concept to solve problems.

Common Misconceptions
Students may mix-up sine and cosine.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying modelling periodic phenomena with trigonometric functions because there is new vocabulary introduced in this cluster that relates to the graphs of trigonometric functions. These terms will then be applied to contextual scenarios. By rehearsing how to precisely use these terms to describe graphs can support student's later work in using them to describe scenarios.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- F-IF.C.7: This standard provides a foundation for work with modelling periodic phenomena with trigonometric functions because this standard is where students focused on describing key features of other function families. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: How will the learning for students provide multiple options for recruiting student interest?

- For example, learners engaging with modelling periodic phenomena with trigonometric functions benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because many relevant concepts are roughly modeled by periodic functions. This is a perfect opportunity to incorporate student interests and/or social issues into mathematics (e.g. is climate change real or do temperatures just fluctuate naturally?). Students can, therefore, see the power of mathematics and discuss features of periodic functions grounded in context.

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with modelling periodic phenomena with trigonometric functions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics representations and scaffolds because students can reason solutions to real-world problems using tables, graphs, logical reasoning and/or calculations with functions. Allowing students to engage in a variety of representations and looking for connections between them can strengthen their understanding of these functions and their features.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with modelling periodic phenomena with trigonometric functions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because the new vocabulary/key features of these graphs are unique to the family of periodic functions. Showing these terms with definitions and images at the beginning can help students see what the end-goal of the lesson will be.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with modelling periodic phenomena with trigonometric functions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because trigonometric functions are frequently graphed with a domain in radians and students may not be efficient yet in setting up a graph by hand in this unit of measurement. Establishing the appropriate domain on a calculator or graphing software and/or providing the set-up of a graph on graph paper can allow students to focus on the key features of the graphs rather than the detail of setting up the problem.

Internalize
Self-Regulation: How will the design of the learning strategically support students to effectively cope and engage with the environment?

- For example, learners engaging with modelling periodic phenomena with trigonometric functions benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because the concept of periodic functions will be new to many students and, therefore, students may be tempted to stop working when they feel stuck. Prompting students to look for the key features (midline as middle point, amplitude as how high does it go, etc.) can allow students to begin to “unstick” themselves. If we set up cooperative learning groups, this can further allow students to learn from each other and share/critique their reasoning.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on modelling periodic phenomena with trigonometric functions by critiquing student approaches/solutions to make connections through a short mini-lesson because students may formulate solutions from multiple perspectives (table, graph, calculation, logical reasoning). Presenting multiple solution methods allows students to think from a new perspective and analyze the features of these functions in a new way. This may illuminate errors in their own work or in the work of others.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit modelling periodic phenomena with trigonometric functions by offering opportunities to understand and explore different strategies because students may, at times, have an easier time solving a problem using a table rather than a graph or equation and vice versa. Allowing students, the opportunity to explore these different strategies and to discuss their usefulness will help students deepen their understanding of the concepts as well as build their skills of using different representations to solve problems. ...

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as <open ended tasks linking multiple disciplines when studying modelling periodic phenomena with trigonometric functions because once a student has a conceptual understanding of periodic functions, the applications are easy to see in science and career specific scenarios. Linking these can allow students to explore their interests beyond the mathematics.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Task: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)." For example, when studying modelling periodic phenomena with trigonometric functions the types of mathematical tasks are critical because we often experience patterns in the real-world, whether it be related to science or societal issues. This is an opportunity for students to explore claims like "climate change is not real" or "the violent crime rate always rises in the warmer months" mathematically by attempting to apply features of periodic functions to describe them, or showing that these features do not exist within the data.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
http://tasks.illustrativemathematics.org/content-standards/HSF/TF/B/5/tasks/816

This type of assessment question requires students to model the population of two species using trigonometric functions given a table of data. Since students may use positive or negative sine and cosine functions to model the data, consider having students work in cooperate groups and comparing solutions between groups. Students will engage with SMP 1, SMP 4 and if comparing work with other groups, SMP 3.

Relevance to families and communities:

During a unit focused on modelling periodic phenomena with trigonometric functions, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, have students provide examples of music they enjoy and discuss how music creates sound waves that are modeled by trigonometric functions. Using free online tools, you can create sound wave images (as a class or on an individual basis) and discuss the features of these images as they relate to periodic functions.

Cross-Curricular Connections:

Many of the Navajo rug designs you will discover by following the project will be good examples of symmetrical balance. Symmetrical balance is a type of visual balance where the overall composition is arranged to look like it is the same on both sides of the center of the design. In other words, it is a design which could be folded in half, and as the design folds, each part of the design would match up with its symmetrical counterpart on the opposite side of the center. The rug design on the right is symmetrical left-to-right. If a line was drawn vertically down the center of the rug, the arrangement of shapes and colors would appear to be exactly the opposite of each other on both sides of that line.
Design a Navajo Rug

HS: FUNCTIONS- TRIGONOMETRIC FUNCTIONS

Cluster Statement: C: Prove and apply trigonometric identities.

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSF.TF.C.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	SMP 3 Students can construct viable arguments and critique the reasoning of others when proving the Pythagorean Identity. SMP 5. Students use appropriate tools strategically by selecting an appropriate method to calculate trig ratios in all quadrants. SMP 6. Students attend to precision when determining whether a trigonometric ratio should be positive or negative based on the information given (such as quadrant or other restrictions).	• Use the concepts from the Pythagorean identity to calculate trigonometric ratios in any quadrant on the coordinate plane **Webb's Depth Of Knowledge:** 1-2 **Bloom's Taxonomy:** understand, apply
Previous Learning Connections In Geometry, students learned the relationship of trigonometric ratios. In 8th grade, Algebra 1, and Geometry, students also learned the Pythagorean Theorem and how to graph on a coordinate plane.	**Current Learning Connections** Students will use their knowledge of the unit circle and relate that to determine trigonometric ratios not on the unit circle.	**Future Learning Connections** In Precalculus and Calculus courses, students will connect this learning cluster to other trigonometric ratios (tangent, cosecant, secant and cotangent).

Clarification Statement
Students will make connections between their knowledge of the Pythagorean Theorem, trigonometric ratios, the unit circle and coordinate plane.

Common Misconceptions
Students may struggle to explain how the identities frame responses.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying proving and applying trigonometric identities because algebraic proofs can be very challenging for students and we want to confront that fact by providing extra supports and time for students to engage with the material, whether their work is exactly correct or not. The extra time experiencing the material will build deeper understanding.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.GB.7: This standard provides a foundation for work with proving and applying trigonometric identities because students used the visual structure of right triangles to apply the Pythagorean theorem. This process can be thought of as a concrete version of the algebraic process this cluster calls for. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?

- For example, learners engaging with proving and applying trigonometric identities benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features because trigonometric identities may appear in a variety of rearranged formats. Highlighting these different formats can help students to see the identities in proofs, even when they are not shown in one specific way.

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with proving and applying trigonometric identities benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that emphasizes effort, improvement, and achieving a standard rather than on relative performance because students may not be successful and completing every single proof and our focus should be on engaging with the proof. Focusing feedback on the step's students did will encourage students to continue trying rather than focus on completing every single proof perfectly.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with proving and applying trigonometric identities benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as highlighting structural relations or make them more explicit because some students may struggle to see the identities in rearranged formats. Provide students with, or have students create, a sheet that shows common trigonometric identities re-written in a variety of ways, so they have a quick reference as they move through proofs.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with proving and applying trigonometric identities benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing different approaches to motivate, guide, feedback or inform students of progress towards fluency because students may start these proofs in a variety of ways. Showing different approaches, even those that are incorrect or incomplete, serve as opportunities for students to discuss the mathematics behind the approaches which may spark ideas and/or deepen conceptual understanding of the process.

Internalize
Executive Functions: How will the learning for students support the development of executive functions to allow them to take advantage of their environment?

- For example, learners engaging with proving and applying trigonometric identities benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as asking questions to guide self-monitoring and reflection because students frequently feel stuck either starting or through the process of a proof using trigonometric identities. Providing prompting questions like "is everything in terms of sin and cos?", "do you see any version of the Pythagorean identity?", etc. can help students find ways to progress through a proof when they feel stuck.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on proving and applying trigonometric identities by critiquing student approaches/solutions to make connections through a short mini-lesson because students frequently are able to take certain steps in a proof but may find themselves feeling stuck. Whether this is because they have made an error or just cannot see the next step, showing these approaches to their peers can help students make connections between the work they did and that of others, as well as critique steps that are different.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit proving and applying trigonometric identities by confronting student misconceptions because students may expect that each proof will progress in the same format, or that identities will always appear in the same way. Every proof is different and showing students that identities can be applied in a variety of ways may help them feel freed from looking for specific instances of the identities. ...

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students. when studying proving and applying trigonometric identities because when students see the patterns and process clearly, we should allow them to challenge themselves at their own pace to try increasingly more challenging proofs.

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying proving and applying trigonometric identities goal setting is critical because these algebraic proofs can be very challenging for students and if they do not have a productive mind frame as a starting point, they will struggle to make any progress. Encouraging students to set a goal to simply start these proofs by choosing to work from left to right or right to left or a goal to accurately rewrite an expression in terms of sin/cos can give students the support they need in engaging with difficult material. The importance is not that they can complete every proof, but rather that they have a goal they can achieve, and they work mathematically toward that goal.

Standards Aligned Instructionally Embedded Formative Assessment Resources: http://tasks.illustrativemathematics.org/content-standards/HSF/TF/C/8/tasks/1835 This type of assessment question requires students to take a given ratio for a trigonometric function and use it to exactly state the value of two other trigonometric functions. This will require knowledge of the trigonometric functions as ratios of side lengths and possibly trigonometric identities. Students will engage with SMP 7 as they use the structure of the ratio to determine the remaining trigonometric values. Additional Assessment: https://www.map.mathshell.org/lessons.php?unit=9255&collection=8&redir=1	
Relevance to families and communities: During a unit focused on proving and applying trigonometric identities, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, connect the process of proofs with the concept of replacing equivalent items can allow students to relate to the process. Students may have experienced this in recipes (replacing butter with oil), in making purchases (choosing one brand over another), etc. Use this as an opportunity to talk about equivalence. In some instances, the outcome may be changed by replacement, but in this mathematical process, equivalence is preserved.	**Cross-Curricular Connections:** Economics – Substitution and utility when making purchases

HS: FUNCTIONS- LINEAR, QUADRATIC, & EXPONENTIAL MODELS

Cluster Statement: A: Construct and compare linear, quadratic, and exponential models and solve problems.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.LE.A.4 **For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.**	SMP 4 Students model with mathematics.by applying concepts of logarithms to problems in context. SMP 7. Students look for and make use of structure by writing a logarithmic model given an exponential model and vice versa.	• Use the properties of logs. • Describe the key features of logs. • Use logarithmic form to solve exponential models. **Webb's Depth Of Knowledge:** 1-2 **Bloom's Taxonomy:** understand, apply
Previous Learning Connections In 8th grade and Algebra 1, students learned about exponential models.	**Current Learning Connections** Students will use this knowledge to solve more complex logarithmic problems that include the use of logarithmic properties.	**Future Learning Connections** Students will build on this knowledge of exponents and logarithms in future math courses.

Clarification Statement
Students will be able to go back and forth between an exponential model and logarithmic model and know when one model may be more useful than another one to solve problems in context.

Common Misconceptions
Students may mix-up direction of term with value of exponents.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying constructing and comparing linear, quadratic and exponential models and using them so solve problem because these are all function families that should have been previously studied. Students, therefore, may be able to spark each other's memory about the shapes of graphs, patterns in numbers and/or forms of equations.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.F.B4/F.BF.A2: This standard provides a foundation for work with constructing and comparing linear, quadratic and exponential models and using them so solve problem because the 8th grade standard is when students modeled linear data with linear functions and the high school standard is when students build a function to model arithmetic and geometric sequences. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: How will the learning for students provide multiple options for recruiting student interest?

- For example, learners engaging with constructing and comparing linear, quadratic and exponential models and using them to solve problems benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because a variety of relevant concepts can be modeled with these functions, allowing a teacher to tailor this learning to fit the social, cultural and/or career interests of their students.

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with constructing and comparing linear, quadratic and exponential models and using them so solve problems benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics representations and scaffolds because students may see these models better through a table, graph or equation. Allowing multiple representations gives opportunity to draw connections between the representations and reasoning of others.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with constructing and comparing linear, quadratic and exponential models and using them so solve problems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because these functions are not always discussed at the same time. Linear functions are a prior grade skill while quadratics and exponentials are explored in depth, often independently. Reviewing the key characteristics of these structures allows students to wrestle with which function type best fits a given scenario.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with constructing and comparing linear, quadratic and exponential models and using them so solve problems benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because student reasoning may stem from a table, graph and/or work with a function rule. All strategies may lead to correct solutions and provide opportunity to build connections across representations and critique reasoning.

Internalize

Executive Functions: How will the learning for students support the development of executive functions to allow them to take advantage of their environment?

- For example, learners engaging with constructing and comparing linear, quadratic and exponential models and using them so solve problems benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as embedding prompts to "show and explain your work" (e.g., portfolio review, art critiques) because understanding student thinking provides insight into their level of understanding as well as their ability to use a variety of representations to support their solutions. This allows for specific feedback, clarification and opportunities to push students beyond their comfort zone.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging constructing and comparing linear, quadratic and exponential models and using them so solve problem by critiquing student approaches/solutions to make connections through a short mini lesson because explanations may arise using tables, graphs and function algebra. Building connections between these approaches can illuminate errors as well as push students beyond estimations and toward exact answers.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit constructing and comparing linear, quadratic and exponential models and using them to solve problem by addressing conceptual understanding because the basis of this cluster is in strategically selecting a model based on characteristics provided. If students do not have a firm understanding of the characteristics of linear, quadratic and exponential functions, they will not be able to select or therefore use an appropriate model to solve problems.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying constructing and comparing linear,

	quadratic and exponential models and using them to solve problems because challenging students to select a data set to model requires them to reason with the different types of data sets available, strategically explore models and interpret their findings in context. This can also serve to reach the interest of these students on a deeper level.

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Task: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)." For example, when studying constructing and comparing linear, quadratic and exponential models and using them so solve problems, the types of mathematical tasks are critical because this is a golden opportunity to show the relevancy and usefulness of mathematics, whether it relates to sports, careers, social data, etc. Every student can find a use for this mathematics given the proper context.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
http://tasks.illustrativemathematics.org/content-standards/HSF/LE/A/tasks/213

This type of assessment question requires students to analyze and compare two exponential functions. Students will engage with SMP 7 as they use the structure of the exponential function to analyze a scenario and, if allowed use of technology, SMP 5 as they use graphs to reason their solutions.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on constructing and comparing linear, quadratic and exponential models and using them so solve problems, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example collecting/researching a socially relevant data set to fit with a mathematical model. This context allows students to reason with and discuss the mathematics but also provides a purpose and drives engagement.	Many of the Navajo rug designs you will discover by following the project will be good examples of symmetrical balance. Symmetrical balance is a type of visual balance where the overall composition is arranged to look like it is the same on both sides of the center of the design. In other words, it is a design which could be folded in half, and as the design folds, each part of the design would match up with its symmetrical counterpart on the opposite side of the center. The rug design on the right is symmetrical left-to-right. If a line was drawn vertically down the center of the rug, the arrangement of shapes and colors would appear to be exactly the opposite of each other on both sides of that line. Design a Navajo Rug

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<table>
<tr><th colspan="3">HS: STATISTICS & PROBABILITY- INTERPRETING CATEGORICAL & QUANTITATIVE DATA</th></tr>
<tr><td colspan="3">Cluster Statement: A: Summarize, represent, and interpret data on a single count or measurement variable</td></tr>
<tr>
<td rowspan="3">Standard Text

HSS.ID.A.4
Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.</td>
<td rowspan="3">Standard for Mathematical Practices

SMP2: Students can reason abstractly and quantitatively by expressing relationships between different measures by finding the measures of center and dispersion using quantitative reasoning.

SMP3: Students can construct viable arguments and critique the reasoning of others by justifying their reason for using a specific measure of center based on the data dispersion.

SMP 7: Students can look for and make use of structure by learning to see visually the relationship between probability of a population and a normal curve.

SMP 8: Students can look for and express regularity in repeated reasoning by determinin the effects of skewed data sets and extreme values on their measures of center.</td>
<td>Students who demonstrate understanding can:
• Explain data distributions: when the data is normal, mean and standard deviation are used to represent the data; when data is skewed, median and Interquartile Range (IQR) are used to represent the data.
• Explain why normal distribution can be used to estimate population percentages.
• Estimate areas under a normal curve.
• Calculate mean, median (Q2), standard deviation, IQR (Q1 & Q3).
• Calculate z- score based on mean and standard deviation.
• Use the empirical rule to understand area under the distribution
• Apply this knowledge to estimate population percentages.
• Use real world data to determine distributions.</td>
</tr>
<tr><td>Webb's Depth Of Knowledge: 1-4</td></tr>
<tr><td>Bloom's Taxonomy: remember, understand, apply, analyze, evaluate</td></tr>
<tr>
<td><u>Previous Learning Connections</u>

• Connect to work in previous math courses: when students learned to determine mean, median, mode, range, IQR, minimum, maximum. Students also learned how to graph data distributions (e.g., histograms, box plots).</td>
<td><u>Current Learning Connections</u>

• Connect students using this information to make inferences and justify conclusions from sample survey, experiments and observational studies.</td>
<td><u>Future Learning Connections</u>

• Connect to future work students may do in subsequent statistics course (AP or college level).</td>
</tr>
</table>

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Clarification Statement
At this level, students are not expected to fit **normal curves** to data. (In fact, it is rather complicated to rescale data plots to be **density plots** and then find the best fitting curve.) Instead, the aim is to look for broad **approximations**, with application of the rather rough "**empirical rule**" (also called the 68%–95% Rule) for distributions that are somewhat **bell-shaped**. The better the bell, the better the approximation. Using such approximations is partial justification for the introduction of the **standard deviation**.

Common Misconceptions
Students may believe all data follows a normal distribution because of prior work with a sampling distribution of means.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying Summarize, Represent, And Interpret Data On A Single Count Or Measurement Variable because there are common misconceptions which learners often encounter. Addressing these misconceptions will often resolve issues that would otherwise have to be addressed in more time consuming one-on-one instruction.
- For example: Learners often misunderstand the meaning of range and standard deviation. Directly targeting this misconception can often allow students to move forward without time-consuming one-on-one instruction.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.SP.B.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.: This standard provides a foundation for work with Interpreting Categorical And Quantitative Data because learners which don't understand the meaning and use of different types of data graphs will have difficulty interpreting them for meaning when solving problems. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Physical Action: *How will the learning for students provide a variety of methods for navigation to support access?*

- For example, learners engaging with Interpreting Data benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as varying how statistical data and analytical results are displayed or presented because insight into a problem is often gained by viewing the data using different perspectives.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with Interpreting Data benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as creating expectations for group work (e.g., rubrics, norms, etc.)s because students are more likely to fully engage in an activity when they are being held accountable within their peers.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with Interpreting Data benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) because Interpreting Data requires a good understanding of a large number of concepts that some students have trouble processing at one time, producing cognitive overload. Reducing the cognitive load enables learners to process at a higher level since less energy and time is spent on remembering concepts.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with Interpreting Data benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing different approaches to motivate, guide, feedback or inform students of progress towards fluency because insightful interpretation of data requires a high level of conceptual understanding and processing that is honed through practice, trial and error and feedback.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with Interpretation of data benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as using activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely because learners often benefit from guiding questions and other feedback which help them overcome conceptual and reasoning errors and help them develop reasoning strategies that are useful in solving similar problems.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on summarize, represent, and interpret data on a single Count or measurement variable by providing specific feedback to students on their work through a short mini-lesson because learners often get hung up on a single misconception or step. Clearing that step will often enable the learner to progress in solving the problem.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?* • For example, some students may benefit from intensive extra time during and after a unit in the Summarize, Represent, And Interpret Data On A Single Count Or Measurement Variable cluster by helping students move from specific answers to generalizations for certain types of problems because learners sometimes have difficulties with recall or have difficulty with the cognitive load of recalling and processing information spread out over one or more days. This is especially true when instruction is spread over weekends or breaks. Intensive Reteaching eases this cognitive load and empowers learners to accomplish the task. **Extension** *What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?* • For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying how to summarize, represent, and interpret data on a single count or measurement variable, such as non-Gaussian distributions because there may be learners that need additional challenges beyond the curriculum.
Culturally and Linguistically Responsive Instruction: **Validate/Affirm**: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? **Build/Bridge**: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society? Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying HS.ID.A: Summarize, represent, and interpret data on a single count or measurement variable cluster, goal setting is critical because statistics are often used to help describe how a specific ethnic or cultural group is doing and what needs they may need. The census is the largest statistical tool the United States uses to help with this objective.
Standards Aligned Instructionally Embedded Formative Assessment Resources: http://tasks.illustrativemathematics.org/content-standards/HSS/ID/A/4/tasks/1020 This type of assessment question requires students to analyze a scenario approximately modeled by a normal distribution with given mean and standard deviation to make and support a statistical claim. Students will engage with SMP 3 by making and supporting statistical claims. If students share their solutions with others, SMP 3 can be further targeted by critiquing the solutions of peers.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on analyzing data, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different data	Social Studies: In high school the New Mexico Social Studies Standards state students should "explain how to use technological tools to research data, verify facts and information, and communicate findings." Consider

analytical techniques helps provide a robust set of results which provide differing points of view. For example, mean and median of a data set can be used to estimate the symmetry of a curve, providing insight into the number of people that might be affected at the extremes of the population.	providing a connection for students to write a report describing and analyzing a specific set of data.

HS: STATISTICS & PROBABILITY- MAKING INFERENCES & JUSTIFYING CONCLUSIONS

Cluster Statement: A: Understand and evaluate random processes underlying statistical experiments

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSS.IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	SMP 2: Students reason abstractly and quantitatively using statistics from random samples to make inferences about populations. SMP 3: Students can construct viable arguments about populations parameters based on statistics from random samples and can critique the inferences of studies about a population by examining the process for creating a random sample. SMP 4: Students can model population parameters based on statistics from random samples.	• Explain the difference between bias and unbiased sampling. • Explain why the statistical measures of a random sample should be roughly the same as the statistical measures of the population. • Make inferences about a population based on a random sample. **Webb's Depth of Knowledge:** 2-3 **Bloom's Taxonomy:** Understand and Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSS.IC.A.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model*?	SMP 2: Students can reason abstractly and quantitatively by proposing sampling techniques and exploring their nature to produce bias and unbiased results. SMP 5: Students can use appropriate tools strategically (e.g., spinner, die, coin, cards, computer simulation) to analyze a model. SMP 8: Students can look for and express regularity in repeated	• Explain the difference between a rare event and an ordinary event. • Use tools to analyze results from data-generating process, i.e., create a simulation. • Make inferences about a model to decide if a model is consistent with the result given. **Webb's Depth of Knowledge:** 3-4

	reasoning to determine if events are consistent or inconsistent through simulation.	**Bloom's Taxonomy:** Analyze and Evaluate
Previous Learning Connections • Connect to the work students have done to determine mean, median, mode, range, IQR, minimum, maximum.	**Current Learning Connections** • Connect to work throughout the Statistics and Probability domain around interpreting and making inferences about populations based upon sample quantitative data.	**Future Learning Connections** • Connect to future work with Statistics and Probability in college level courses and careers.

Clarification Statement

Students move beyond analyzing data to making sound statistical decisions based on probability models. The reasoning process is as follows: develop a statistical question in the form of a hypothesis (supposition) about a population parameter; choose a probability model for collecting data relevant to that parameter; collect data; compare the results seen in the data with what is expected under the hypothesis.

Common Misconceptions

Students may struggle with the difference between rare and impossible.

Students may struggle with recognizing the difference between random and non-repeating events. Humans tend to believe random means an outcome will not repeat but in large data sets of random outcomes it common to have strings of the same outcome (i.e., flipping a coin and 100 time and finding a string of 5 heads in a row).

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

For example, some learners may benefit from targeted pre-teaching that previews new contexts for tasks within the unit when studying the units in understanding and evaluating random processes underlying statistical experiments because a preview often piques learner curiosity and knows that there is a reason for learning this material.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

7.SP.C.7: This standard focuses on developing a probability model and using it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. This standard provides a foundation for work with units in Understand and Evaluate Random Processes Underlying Statistical Experiments cluster because misconceptions with probabilities will impact a learner's ability to evaluate random processes. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with understanding and evaluating random processes underlying statistical experiments benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as different representations of the data, auditory descriptions of data, electronic methods of manipulating the data into various formats because learners effectively access information in different modes. Providing more modes of access empowers more students to be successful in the task.

Build
Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understanding and evaluating random processes underlying statistical experiments benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because this can encourage students to engage in the problem from both a logical and mathematical perspective.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with understanding and evaluating random processes underlying statistical experiments benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf because these problems tend to be in paragraph form with detailed descriptions. We want to be sure we are assessing student ability to show what they know about the mathematical concepts rather than their ability to read detailed information in English.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding and evaluating random processes underlying statistical experiments benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches, strategies, skills, etc.) because students may express their understanding in a variety of ways. Allowing for multiple solution methods to be shown can illuminate errors as well as connections between the different methods.

Internalize
Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with understanding and evaluating random processes underlying statistical experiments benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as using multiple examples and non-examples to emphasize critical features because students may initially struggle to connect logical reasoning with mathematical justification. Providing several examples of statistical claims that can and cannot be mathematically supported will model for students how to look for evidence to support or refute claims.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*
For example, students may benefit from re-engaging with content during a unit on understanding and evaluating random processes underlying statistical experiments by critiquing student approaches/solutions to make connections through a short mini-lesson because misconceptions about the nature of random events is common and may lead to incorrect conclusions. For example, many people believe that luck is real and can affect the outcome of random events.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*
For example, some students may benefit from intensive extra time during and after a unit in the understanding and evaluating random processes underlying statistical experiments by addressing conceptual understanding because small subtle misconceptions about random processes can lead to invalid or inaccurate decisions. For example, may people believe that if they get cancer once, they are less likely to get it again.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when studying evaluating random processes underlying statistical experiments because many learners want or need to know how this cluster will relate to future career or college interests.

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying HS.IC.B: Summarize, represent, and interpret data on two categorical and quantitative variables cluster goal setting is critical because statistics are often used to help describe how a specific ethnic or cultural group is doing and what needs they may need. Knowing how to interpret and present data in meaningful ways helps develop a more robust model of the community. The 10 year Census is the largest statistical tool the United States uses to help with this objective.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

CollegeBoard | **Question ID 4169753**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	**Math**	**Problem Solving and Data Analysis**	**Medium**	**Problem Solving and Data Analysis**	**Evaluating statistical claims: Observational studies and experiments**	**1. With random samples, describe which population the results can be extended to.**	**Calculator**

A sample of 40 fourth-grade students was selected at random from a certain school. The 40 students completed a survey about the morning announcements, and 32 thought the announcements were helpful. Which of the following is the largest population to which the results of the survey can be applied?

Question Difficulty: Medium

A. The 40 students who were surveyed

B. All fourth-grade students at the school

C. All students at the school

D. All fourth-grade students in the county in which the school is located

Choice B is correct. Selecting a sample of a reasonable size at random to use for a survey allows the results from that survey to be applied to the population from which the sample was selected, but not beyond this population. In this case, the population from which the sample was selected is all fourth-grade students at a certain school. Therefore, the results of the survey can be applied to all fourth-grade students at the school.

Choice A is incorrect. The results of the survey can be applied to the 40 students who were surveyed. However, this isn't the largest group to which the results of the survey can be applied. Choices C and D are incorrect. Since the sample was selected at random from among the fourth-grade students at a certain school, the results of the survey can't be applied to other students at the school or to other fourth-grade students who weren't represented in the survey results. Students in other grades in the school or other fourth-grade students in the country may feel differently about announcements than the fourth-grade students at the school.

This type of assessment question requires students to analyze a scenario and determine what a potential population could have been. Students will engage with SMP 6 as they must carefully read the information to pick out a relevant population.

Relevance to families and communities:
During a unit focused on evaluating random processes underlying statistical experiments consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, how statistics are used to describe how the risk of different cultural and ethnic groups for developing breast cancer and how this might affect medical breast cancer screening frequency recommendations.

Cross-Curricular Connections:

Social Studies: In high school the New Mexico Social Studies Standards state students should "explain how to use technological tools to research data, verify facts and information, and communicate findings." Consider providing a connection for students to determine the best fit of a function for a set of data and explain their choice.

NEW MEXICO
Public Education Department

<table>
<tr><th colspan="3">HS: STATISTICS & PROBABILITY- MAKING INFERENCES & JUSTIFYING CONCLUSIONS</th></tr>
<tr><td colspan="3">Cluster Statement: B: Make inferences and justify conclusions from sample surveys, experiments, and observational studies</td></tr>
<tr><td rowspan="3">Standard Text

HSS.IC.B.3
Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.</td><td rowspan="3">Standard for Mathematical Practices

SMP 3: Students can construct viable arguments and critique the reasoning of others by understanding how decisions based on sample data are related to probability, and that this decision process does not guarantee a correct answer to the underlying statistical question</td><td>Students who demonstrate understanding can:
Recognize that randomization is necessary to making accurate statistical inferences.
Compare and contrast the differences between a sample survey, experiment, and observational study and the advantages to their uses.
Explain how to use random sampling techniques and the importance of random sampling.</td></tr>
<tr><td>Webb’s Depth of Knowledge: 3</td></tr>
<tr><td>Bloom’s Taxonomy:
Understand, Apply</td></tr>
<tr><td rowspan="2">Standard Text

HSS.IC.B.4
Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.</td><td rowspan="2">Standard for Mathematical Practices

SMP 3: Students can construct viable arguments and critique the reasoning of others by using simulation and the collection of data to make inferences.

SMP 5: Students can use appropriate tools strategically to decide which kind of sampling technique to use in different situations (experiment, survey, observational study).</td><td>Students who demonstrate understanding can:
Estimate a population mean and calculate margin of error given a simulation.
Use real world data to determine the population mean and margin of error.</td></tr>
<tr><td>Webb’s Depth of Knowledge: 3</td></tr>
</table>

	SMP 8: Students can look for and express regularity in repeated reasoning by examining simulation data and recognizing the proportion of times the simulation rejected the hypothesis.	**Bloom's Taxonomy:** Understand, Apply
Standard Text HSS.IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	**Standard for Mathematical Practices** SMP 3: Students can construct viable arguments and critique the reasoning of others by using simulation and the collection of data to make inferences. SMP 5: Students can use appropriate tools strategically to decide which kind of sampling technique to use in different situations (experiment, survey, observational study). SMP 8: Students can look for and express regularity in repeated reasoning by examining simulation data and recognizing the proportion of times the simulation rejected the hypothesis.	**Students who demonstrate understanding can:** • Identify differences between parameters. • Compare two treatment groups in an experiment and decide if the difference in parameters is significant. **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy:** Analyze, Evaluate
Standard Text HSS.IC.B.6 Evaluate reports based on data.	**Standard for Mathematical Practices** MP.6; MP.7; MP.8	**Students who demonstrate understanding can:** • Explain the parameters of data and their significance • Define the characteristics of experimental design (control, randomization, and replication). • Evaluate the experimental study design, how the data was gathered, what analysis (numerical or graphical) was used. • Draw conclusions based on graphical and numerical summaries. • Evaluate reports based on data. **Webb's Depth of Knowledge:** 3

		Bloom's Taxonomy: Analyze, Evaluate
Previous Learning Connections • Connect to work previous math courses, where students have learned to determine mean, median, mode, range, IQR, minimum, maximum. Students have also learned how to graph data distributions (e.g., histograms, box plots).	**Current Learning Connections** • Connect to students work with evaluating the randomness of a sample and use this to determine if a specified model is consistent with the results. **(HSS.IC.A)**	**Future Learning Connections** • Connect to work in subsequent statistics course (AP or college level).

Clarification Statement

Once students see how **probability** intertwines with data collection and analysis, students use this knowledge to make **statistical inferences** from data collected in **sample surveys** and in **designed experiments**, aided by **simulation** and the technology that affords it

Students should be able to explain the reasoning in a **statistical decision** and the nature of the error that may have been made.

Student will look at quality of a statistical question to be answered, question clarity, quantization of responses and error calculations, as absolute value of difference of the sample values from the mean.

Common Misconceptions

Students may struggle with distinguishing between the difference between estimates for means and proportions.

Students may have difficulty in relating the margin of error from a simulation model to the inference about a population.

Students may have the misconception that when the margin of error increases that the statistic does not contain the true population parameter.

Students may struggle with using statistical language that includes the possibility of error in measurement rather than absolute language such as always, never, guaranteed.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying the Making Inferences And Justifying Conclusions cluster because learners are often more ready to engage the material at a deeper level when they feel frustrated and confused just enough to have questions that need answered during the upcoming units.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.SP.A: This cluster of standards provides a foundation for work with the units in the HS.S-IC.B: Making Inferences And Justifying Conclusions Cluster because the 7th grade standard provides a foundation for valid random sampling techniques, providing validity for conclusions based on the data. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with making inferences and justifying conclusions from sample surveys, experiments, and observational studies benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because students may have collected data and made a simple comparison. When students have data that is geared towards their interest, they analyze the data and can interpret the marginal error, population mean, or graph.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with making inferences and justifying conclusions from sample surveys, experiments, and observational studies benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as prompting or requiring learners to explicitly formulate or restate learning goals because keeping the end in mind, students will keep their goal in mind as to what they are working towards. Too little data may not support their conclusion, graph, or create a difference in the parameters.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with making inferences and justifying conclusions from sample surveys, experiments, and observational studies benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) because data can be organized with an organizational map. Data can be analyzed, graphed, or used to compare data to another set of data.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with making inferences and justifying conclusions from sample surveys, experiments, and observational studies benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) because social media can be used to collect data by setting up a survey and people can randomly decide to participate all over the world. Or the student can set it up to where only a specific group can participate. Online graphic organizers and graphing software can be used to analyze the data.

Internalize

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with making inferences and justifying conclusions from sample surveys, experiments and observational studies benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as posting goals, objectives, and schedules in an obvious place because students may forget that they are drawing and supporting inferences mathematically. By posting explicit goals/objectives in an obvious place, teachers can remind students to focus their attention on the mathematics that can support their justifications rather than getting lost in the logical argument.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on HS.S-IC.B: Making Inferences And Justifying Conclusions cluster by revisiting student thinking through a short mini-lesson because this conversation can serve as a diagnostic tool so the teacher can prescribe the needed review needed to get the learner moving.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit in the HS.S-IC.B: Making Inferences And Justifying Conclusions cluster by helping students move from specific answers to generalizations for certain types of problems because learners often benefit from seeing the work of more experienced problem solvers.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics because advanced or gifted learners often need or want to explore more into how data is used. For example, a learner could look at how Big Data is being used to make life better but not without potential risks.

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying HS.IC: Making Inferences & Justifying Conclusions cluster goal setting is critical because interpreting statistical results are often used to infer how a specific ethnic or cultural group is doing and what needs they may need. The census is the largest statistical tool the United States uses to help with this objective.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

CollegeBoard Question ID 4789744

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Problem Solving and Data Analysis	Medium	Problem Solving and Data Analysis	Inference from sample statistics and margin of error	1. Use sample mean and sample proportion to estimate population mean and population proportion. Utilize, but do not calculate, margin of error.	Calculator

A bag containing 10,000 beads of assorted colors is purchased from a craft store. To estimate the percent of red beads in the bag, a sample of beads is selected at random. The percent of red beads in the bag was estimated to be 15%, with an associated margin of error of 2%. If r is the actual number of red beads in the bag, which of the following is most plausible?

Question Difficulty: Medium

A. $r > 1{,}700$

B. $1{,}300 < r < 1{,}700$

C. $200 < r < 1{,}500$

D. $r < 1{,}300$

Choice B is correct. It was estimated that 15% of the beads in the bag are red. Since the bag contains 10,000 beads, it follows that there are an estimated $10{,}000 \times 0.15 = 1{,}500$ red beads. It's given that the margin of error is 2%, or $10{,}000 \times 0.02 = 200$ beads. If the estimate is too high, there could plausibly be $1{,}500 - 200 = 1{,}300$ red beads. If the estimate is too low, there could plausibly be $1{,}500 + 200 = 1{,}700$ red beads. Therefore, the most plausible statement of the actual number of red beads in the bag is $1{,}300 < r < 1{,}700$.

Choices A and D are incorrect and may result from misinterpreting the margin of error. It's unlikely that more than 1,700 beads or fewer than 1,300 beads in the bag are red. Choice C is incorrect because 200 is the margin of error for the number of red beads, not the lower bound of the range of red beads.

This type of assessment question requires students to select a reasonable answer using an estimated measure of center and given margin of error. Solutions are provided as inequalities. Students will engage with SMP 1 and SMP 4 as they apply knowledge of margin of error to solve the problem and model their solution using an inequality.

Relevance to families and communities:
During a unit focused on HS.IC: Making Inferences & Justifying Conclusions, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, how statistics are used to describe how the risk of different cultural and ethnic groups for developing breast cancer and how this might affect medical

Cross-Curricular Connections:

Social Studies: Connection to the difference between correlation versus causation when reading data.

breast cancer screening frequency recommendations. During a unit focused on making inferences, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about how data might allow us to infer how an infection might be slowed in a large extended family living in one house.	

Section 3: Resources, References, and Glossary

Resources

Evidence-Based Resources	English Learner Resources	MLSS Resources	Mathematics Standard Resources
What Works Clearinghouse Best Evidence Encyclopedia Evidence for Every Students Succeeds Act Evidence in Education Lab	World-Class Instructional Design and Assessment (WIDA) Standards USCALE Language Routines for Mathematics English Language Development Standards Spanish Language Development Standards	NM Multi-Layered System of Supports (MLSS) Universal Design for Learning Guidelines Achieve the Core: Instructional Routines for Mathematics Project Zero Thinking Routines	Focus by Grade Level and Widely Applicable Prerequisites High school Coherence Map College-and Career Ready Math Shifts Fostering Math Practices: Routines for the Mathematical Practices

Planning Guidance for Multi-Layered Systems of Support: Core Instruction[10]

Core Instructional Planning must reflect and leverage scientific insights into how humans learn in order to ensure all students are ready for success, thus the following guidance for optimizing teaching and learning is grounded in the **Universal Design Learning (UDL) Framework**

Key design questions, planning actions, and potential strategies are provided below, with respect to guidance for minimizing barriers to learning and optimizing (1) universal ACCESS to learning experiences, (2) opportunities for students to BUILD their understanding of the **Learning Goal**, and (3) INTERNALIZATION of the Learning Goal.

Optimizing Universal ACCESS to Learning Experiences	
ENGAGEMENT ? How will you provide multiple options for recruiting interest?	**Recruiting Student Interest**: ? What do you anticipate in the range of student interest for this lesson? ▯ Plan for options for recruiting student interest: ❒ provide choice (e.g. sequence or timing of task completion) ❒ set personal academic goals ❒ provide contextualized examples connected to their lives ❒ support culturally relevant connections (i.e home culture) ❒ create socially relevant tasks ❒ provide novel & relevant problems to make sense of complex ideas in creative ways

[10] Adapted from: CAST (2018). *Universal Design for Learning Guidelines version 2.2*. Retrieved from http://udlguidelines.cast.org

	❒ provide time for self-reflection about content & activities ❒ create accepting and supportive classroom climate ❒ utilize **instructional routines** to involve all students
REPRESENTATION ⍰ How will you reduce barriers to perceiving the information presented in this lesson?	**Perception**: ⍰ What do you anticipate about the range in how students will perceive information presented in this lesson? ▯ Plan for different modalities and formats to reduce barriers to learning: ❒ display information in a flexible format to vary perceptual features ❒ offer alternatives for auditory information ❒ offer alternatives for visual information
ACTION & EXPRESSION ⍰ How will the learning for students provide a variety of methods for navigation to support access?	**Physical Action**: ⍰ What do you anticipate about the range in how students will physically navigate and respond to the learning experience? ▯ Plan a variety of methods for response and navigation of learning experiences by offering alternatives to: ❒ requirements for rate, timing, speed, and range of motor action with instructional materials, manipulatives, and technologies ❒ physically indicating selections ❒ interacting with materials by hand, voice, keyboard, etc.

Opportunities for Students to BUILD their Understanding	
ENGAGEMENT ⍰ How will the learning for students provide options for sustaining effort and persistence?	**Sustaining Effort & Persistence**: ⍰ What do you anticipate about the range in student effort? ▯ Plan multiple methods for attending to student attention and affect by: ❒ prompting learners to explicitly formulate or restate learning goals ❒ displaying the learning goals in multiple ways ❒ using prompts or scaffolds for visualizing desired outcomes ❒ engaging assessment discussions of what constitutes excellence ❒ generating relevant examples with students that connect to their cultural background and interests ❒ providing alternatives in the math representations and scaffolds ❒ creating cooperative groups with clear goals, roles, responsibilities ❒ providing prompts to guide when and how to ask for help ❒ supporting opportunities for peer interactions and supports (e.g. peer tutors) ❒ constructing communities of learners engaged in common interests ❒ creating expectations for group work (e.g., rubrics, norms, etc.) ❒ providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge ❒ providing feedback that: ❒ emphasizes effort, improvement, and achieving a standard rather than on relative performance ❒ is frequent, timely, and specific ❒ is informative rather than comparative or competitive

	❒ models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success
REPRESENTATION ⍰ How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners?	**Language & Symbols:** ⍰ What do you anticipate about the range of student background experience and vocabulary? ▯ Plan multiple methods for attending to linguistic and nonlinguistic representations of mathematics to ensure universal clarity by: ❒ pre-teaching vocabulary and symbols in ways that promote connection to the learners' experience and prior knowledge ❒ graphic symbols with alternative text descriptions ❒ highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to structure ❒ embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) ❒ embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) ❒ highlighting structural relations or make them more explicit ❒ making connections to previously learned structures ❒ making relationships between elements explicit (e.g., highlighting the transition words in an argument, links between ideas, etc.) ❒ allowing the use of text-to-speech and automatic voicing with digital mathematical notation (math ml) ❒ allowing flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) ❒ clarification of notation through lists of key terms ❒ making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf ❒ linking key vocabulary words to definitions and pronunciations in both dominant and heritage languages ❒ defining domain-specific vocabulary (e.g., "map key" in social studies) using both domain-specific and common terms ❒ electronic translation tools or links to multilingual web glossaries ❒ embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc) ❒ presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) ❒ making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams
ACTION & EXPRESSION ⍰ How will the learning provide multiple	**Expression & Communication:** ⍰ What do you anticipate about the range in how students will express their thinking in the learning environment? ▯ Plan multiple methods for attending to the various ways in which students can express knowledge, ideas, and concepts by providing:

modalities for students to easily express knowledge, ideas, and concepts in the learning environment?	❒ options to compose in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video ❒ use of social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) ❒ flexibility in using a variety of problem solving strategies ❒ spell or grammar checkers, word prediction software ❒ text-to-speech software, human dictation, recording ❒ calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper ❒ sentence starters or sentence strips ❒ concept mapping tools ❒ Computer-Aided-Design (CAD) or mathematical notation software ❒ virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) ❒ multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches) ❒ multiple examples of novel solutions to authentic problems ❒ different approaches to motivate, guide, feedback or inform students of progress towards fluency ❒ scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) ❒ differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners)

Optimizing INTERNALIZATION of the Learning Goal	
ENGAGEMENT ? How will the design of the learning strategically support students to effectively cope and engage with the environment?	**Self-Regulation**: ? What do you anticipate about barriers to student engagement? ▯ Plan to address barriers to engagement by promoting healthy responses and interactions, and ownership of learning goals: ❒ metacognitive approaches to frustration when doing mathematics ❒ increase length of on-task orientation through distractions ❒ frequent self-reflection and self-reinforcements ❒ address subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") ❒ offer devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the math practices for the purpose of monitoring and improving ❒ use activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely
REPRESENTATION ? How will the learning support transforming accessible information into usable knowledge	**Comprehension**: ? What do you anticipate about barriers to student comprehension? ▯ Plan to address barriers to comprehension by intentionally building connections to prior understandings and experiences, relating meaningful information to learning goals,

that is accessible for future learning and decision-making?	providing a process for meaning making of new learning, and applying learning to new contexts: ❒ incorporate explicit opportunities for review and practice ❒ note-taking templates, graphic organizers, concept maps ❒ scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) ❒ explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) ❒ opportunities over time to revisit key ideas and connections ❒ make explicit cross-curricular connections ❒ highlight key elements in tasks, graphics, diagrams, formulas ❒ outlines, graphic organizers, unit organizer routines, concept organizer routines, and concept mastery routines to emphasize key ideas and relationships ❒ multiple examples & non-examples ❒ cues and prompts to draw attention to critical features ❒ highlight previously learned skills that can be used to solve unfamiliar problems ❒ options for organizing and possible approaches (tables and representations for processing mathematical operations) ❒ interactive representations that guide exploration and new understandings ❒ introduce graduated scaffolds that support information processing strategies ❒ tasks with multiple entry points and optional pathways ❒ "Chunk" information into smaller elements ❒ remove unnecessary distractions unless essential to learning goal ❒ anchor instruction by linking to and activating relevant prior knowledge (e.g., using visual imagery, concept anchoring, or concept mastery routines) ❒ pre-teach critical prerequisite concepts via demonstration or representations ❒ embed new ideas in familiar ideas and contexts (e.g., use of analogy, metaphor, drama, music, film, etc.) ❒ advanced organizers (e.g., KWL methods, concept maps) ❒ bridge concepts with relevant analogies and metaphors
ACCESS ACTION & EXPRESSION ⍰ How will the learning for students support the development of executive functions to allow them to take advantage of their environment?	**Executive Functions**: ⍰ What do you anticipate about barriers to students demonstrating what they know? ▯ Plan to address barriers to demonstrating understanding by providing opportunities for students to set goals, formulate plans, use tools and processes to support organization and memory, and analyze their growth in learning and how to build from it: ❒ prompts and scaffolds to estimate effort, resources, difficulty ❒ models and examples of process and product of goal-setting ❒ guides and checklists for scaffolding goal-setting ❒ post goals, objectives, and schedules in an obvious place ❒ embed prompts to "show and explain your work" ❒ checklists and project plan templates for understanding the problem, prioritization, sequences, and schedules of steps ❒ embed coaches/mentors to demonstrate think-alouds of process ❒ guides to break long-term goals into short-term objectives ❒ graphic organizers/templates for organizing information & data ❒ embed prompts for categorizing and systematizing ❒ checklists and guides for note-taking ❒ asking questions to guide self-monitoring and reflection ❒ showing representations of progress (e.g., before and after photos, graphs/charts showing progress, process portfolios)

	❒ prompt learners to identify type of feedback or advice they seek ❒ templates to guide self-reflection on quality & completeness ❒ differentiated models of self-assessment strategies (e.g., role-playing, video reviews, peer feedback) ❒ assessment checklists, scoring rubrics, and multiple examples of annotated student work/performance examples

Planning Guidance for Culturally and Linguistically Responsive Instruction[11]

In order to ensure our students from marginalized cultures and languages view themselves as confident and competent learners and doers of mathematics within and outside of the classroom, educators must intentionally plan ways to counteract the negative or missing images and representations that exist in our curricular resources. The guiding questions below support the design of lessons that validate, affirm, build, and bridge home and school culture for learners of mathematics:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

In addition, Aguirre and her colleagues[12] define **mathematical identities** as the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives. Many students see themselves as "not good at math" and approach math with fear and lack of confidence. Their identity, developed through earlier years of schooling, has the potential to affect their school and career choices.

Five Equity-Based Mathematics Teaching Practices[13]

Go deep with mathematics. Develop students' conceptual understanding, procedural fluency, and problem solving and reasoning.

Leverage multiple mathematical competencies. Use students' different mathematical strengths as a resource for learning.

Affirm mathematics learners' identities. Promote student participation and value different ways of contributing.

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[11] This resource relied heavily on the work of: Hollie, S. (2011). Culturally and linguistically responsive teaching and learning. Teacher Created Materials. (see also, https://www.culturallyresponsive.org/vabb)

[12] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices. Reston, VA: National Council of Teachers of Mathematics (p. 14).

[13] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc. (p.6). (adapted from Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013) (p. 43).

Challenge spaces of marginality. Embrace student competencies, value multiple mathematical contributions, and position students as sources of expertise.

Draw on multiple resources of knowledge (mathematics, language, culture, family). Tap students' knowledge and experiences as resources for mathematics learning.

The following lesson design strategies support Culturally and Linguistically Responsive Instruction, specific examples for each cluster of standards can be found in part 2 of the document. These were adapted from the Promoting Equity section of the Taking Action series published by NCTM.[14]

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.

Mathematical Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher.

Modifying Mathematical Tasks: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)."

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics.

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages.

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we

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[14] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians.

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others."

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time."

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students.

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Glossary[15]

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (- 3/4) = (- 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[16]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. See: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances
from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten.
For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

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[15] Glossary and tables taken from: Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/
[16] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[17] See also: median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.
Integer. A number expressible in the form a or -a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line.
Also known as a dot plot.[18]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[19] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 Ã· 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 Ã— 4/3 = 4/3 Ã— 3/4 = 1.

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[17] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

[18] Adapted from Wisconsin Department of Public Instruction, op. cit.

[19] To be more precise, this defines the arithmetic mean.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin,
selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also*: uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form *a/b* or – *a/b* for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also*: terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[20]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar
model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

[20] Adapted from Wisconsin Department of Public Instruction, op. cit.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also*: median, first quartile, interquartile range.

Table 1: Common addition and subtraction.[1]

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 =5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now?5-2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?? -2 = 3
	TOTAL UNKNOWN	ADDEND UNKNOWN	BOTH ADDENDS UNKNOWN[2]
PUT TOGETHER / TAKE APART[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5-3 = ?	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? 5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1, 5 = 2 + 3, 5 = 3 + 2
COMPARE	DIFFERENCE UNKNOWN	BIGGER UNKNOWN	SMALLER UNKNOWN
	(“How many more?” version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?(“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? 2 + ? = 5, 5 – 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with “more”):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2: Common multiplication and division situations.[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	3 x ? = 18, and 18 ÷ 3 = ?	? x 6 = 18, and 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	a x b = ?	a x ? = p and p ÷ a = ?	? x b = p, and p ÷ b = ?

[1]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3: The properties of operations.

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	(a +b) + c = a + (b+c)
Commutative property of addition	a + b = b + a

Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists -*a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) x c = a x (b x c)
Commutative property of multiplication	a x b = b x a
Multiplicative identity property 1	a x 1 = 1 x a = a
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that a x $1/a$ = $1/a$ x a = 1
Distributive property of multiplication over additions	a x (b + c) = a x b + a x c

Table 4: The properties of equality.

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a.
Symmetric property of equality	If a = b, then b = a.
Transitive property of equality	If a = b and b = c , then a = c.
Addition property of equality	If a = b, then a +c = b + c.
Subtraction property of equality	If a = b then a – c = b – c.
Multiplication property of equality	If a = b, then a x c = b x c.
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c.
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality.

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b.
If a > b and b > c then a > c.
If a > b, b < a.
If a > b, then -a <-b.
If a > b, then a ± c > b ± c.
If a > b and c > 0, then a x c > b x c.
If a > b and c < 0, then a x c < b x c.
If a > b and c > 0, then a ÷ c > b ÷ c.
If a > b and c < 0, then a ÷ c < b ÷ c.