HS: GEOMETRY- CIRCLES		
Cluster Statement: A: Understand and apply theorems about circles		
Standard Text HSG.C.A.1 Prove that all circles are similar.	**Standard for Mathematical Practices** MP3 Students construct viable arguments and critique reasoning of other by engaging in discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments when discussing circles. MP5 Students use appropriate tools strategically by utilizing tools to prove circle similarity. These tools might include pencil and paper, concrete models, protractors, compasses, calculators, and software or apps.	**Students who demonstrate understanding can:** • Show that all circles are similar by proving that the ratio of a circle's circumference to its diameter for different sized circles is a constant • Calculate the circumference of a circle, given the diameter or radius. • Calculate angles inside and outside of a circle. • Prove that circles are similar. • Compare the ratios of the radius and circumference of multiple circles to determine similarity.
		Webb's Depth of Knowledge: 3-4
		Bloom's Taxonomy: analyze, evaluate
Standard Text **HSG.C.A.2** **Identify and describe relationships among inscribed angles, radii, and chords.** *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	**Standard for Mathematical Practices** MP1 Students make sense of problems and persevere in solving them by interpreting and make meaning of a problem. Students monitor their progress and change their approach to solving if necessary MP3 Students construct viable arguments and critique reasoning of other by engaging in discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments when discussing circles.	**Students who demonstrate understanding can:** • Identify central angles, inscribed angles, circumscribed angles, tangent line and chords on a circle from a drawing. • Construct and explain examples of central angles, inscribed angles, circumscribed angles, tangent line and chords on a circle. • Describe the relationship between central angles, inscribed angles, circumscribed angles, tangent lines, and chords.
		Webb's Depth of Knowledge: 1-2

	MP5 Students use appropriate tools strategically by utilizing tools to prove circle similarity. These tools might include pencil and paper, concrete models, protractors, compasses, calculators, and software or apps.	**Bloom's Taxonomy: understand, apply**
Standard Text HSG.C.A.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	**Standard for Mathematical Practices** MP3 Students construct viable arguments and critique reasoning of other by engaging in discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments when discussing circles. MP5 Students use appropriate tools strategically by utilizing tools to prove circle similarity. These tools might include pencil and paper, concrete models, protractors, compasses, calculators, and software or apps.	**Students who demonstrate understanding can:** • Construct angle bisectors, perpendicular bisectors, inscribed circle of a triangle and circumscribed circle about a triangle. • Prove that opposite angles of an inscribed quadrilateral are supplementary. **Webb's Depth of Knowledge: 1-3** **Bloom's Taxonomy:** apply, analyze
<u>Previous Learning Connections</u> In 8th grade, learners have worked with two-dimensional figures and verified their properties. These skills from 8.G.A. will support students in their ability to work with circle while working within this standard.	**<u>Current Learning Connections</u>** Previous work with similarity will be applied to circles. Construction adds to learning from previous clusters by increasing skills with formal construction, building on angle congruence, perpendicular lines/segments, and properties of polygons. This will lead to work with arcs and areas of sectors, as well as prepare learners for future work in the Geometry course with 3-dimensional geometry and cross-sections.	**<u>Future Learning Connections</u>** Unit circles, their central angles, and reference angles will build on foundational skills learned in this cluster.
Clarification Statement Learners will apply concepts of similarity to circles and their related components, explore inscribed and circumscribed circles and their associated polygons through constructions. This cluster builds many of the basic properties for angles, lines, and segments related to circles. Learners explore those properties and form conjectures. They should then be encouraged to create justifications regarding why their conjectures are correct. Learners refer back to their work with transformations to better understand these relationships.		

Common Misconceptions

Students may try to solve by sketching the circles instead of ensuring precision by using a compass or appropriate tool to construct an accurate circle.

Student might confuse the relationships of central angle, inscribe angles, circumscribe angles, as well as tangent line and chords of a circle.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying understand and apply theorems about circles because new concepts (inscribed, circumscribed and central angles and tangent lines) are connected with prior held concepts (parallel, perpendicular, radii, etc). Students may need extra time to wrestle with the differences between these types of angles/lines and the work they have already mastered.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- SRTA2/CO.C.9: This standard provides a foundation for work with understand and apply theorems about circles because these lay the groundwork for understanding similarity and congruence, both of which are the foundational concepts of this cluster. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with understanding and applying theorems about circles> benefit when learning experiences include ways to recruit interest such as <providing contextualized examples to their lives> because <this will help them identify the relevance of the topics in this cluster to math in the real-world. For this cluster students.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understanding and applying theorems about circles benefit when learning experiences attend to student's attention and affect to support sustained effort and concentration such as using prompts or scaffolds for visualizing desired outcomes because some proofs may not be obvious. Teacher can revisit transformations with students in order to help students make the connections between them and circles.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with <understanding and applying theorems about circles benefit when learning experiences attend to the linguistic and nonlinguistic

representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) because this cluster requires the precise use of academic vocabulary in the area of math. The use of this academic vocabulary will assist students in developing and supporting their rationale of: 1) whether or not proofs are accurate; or 2) identifying the missing step in a proof.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding and applying theorems about circles benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners) because part of this cluster allows students to determine how theorems are related and then justify their reasoning. Students may have varying justifications for their rationale (and several might be right), therefore individual feedback is essential to honor the thinking of all learners in the classroom. In addition, different students may be missing different steps in the proofs so the personalized feedback individualizes the learning per student.

Internalize
Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with understanding and applying theorems about circles benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because this cluster does require the knowledge of material from other content standards. For example, students may find themselves needing to review material on transformations, arcs, central angles, measures of inscribed angles, etc. Providing students with practice to review these skills will lead to greater success in the cluster overall.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understand and apply theorems about circles by critiquing student approaches/solutions to make connections through a short mini-lesson because proofs and constructions can be seen from a variety of perspectives, some correct and others incorrect but will offer insight into common misconceptions all students may have.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit understand and apply theorems about circles by confronting student misconceptions because when students do not have a firm grasp of congruence and similarity, progressing forward and applying those to circles and proofs will be impossible.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying understand and apply theorems about circles because proofs with circles can be challenging and require students to pull facts from a variety of places, some implied and others explicitly stated. This develops and pushes students' abstract thinking skills.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

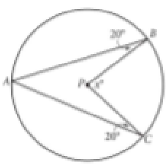
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying <understand and apply theorems about circles> the types of mathematical tasks are critical because they will help students make connections to how the math in this cluster is applicable to real-life context. In some regards, tasks can be designed to highlight various cultures which in turn allows for students to learn about the diversity amongst their peers' cultures. Tasks should be cognizant of culturally responsive academic vocabulary, language, and literacy.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

SAT Item # 422459: The linked assessment question addresses G-C.A., specifically the question requires students to use knowledge of inscribed and central angles.

CollegeBoard	Question ID 422459						
Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	**Math**	**Additional Topics in Math**	**Hard**	**Additional Topics in Math**	**Circles**	**1. Use definitions, properties, and theorems relating to circles and parts of circles, such as radii, diameters, tangents, angles, arcs, arc lengths, and sector areas to solve problems.**	**Calculator**



Point P is the center of the circle in the figure above. What is the value of x ?

Question Difficulty: Hard

> The correct answer is 80. If points A and P are joined, then the triangles that will be formed, APB and APC, are isosceles because $PA = PB = PC$. It follows that the base angles on both triangles each measure $20°$. Angle BAC consists of two base angles; therefore, the measure of angle BAC = $40°$. Since the measure of an angle inscribed in a circle is half the measure of the central angle that intercepts the same arc, it follows that the value of x is $80°$.

Additional Assessment:
http://tasks.illustrativemathematics.org/content-standards/HSG/C/A/1/tasks/1368
The linked assessment question addresses G-C.A, specifically the question requires students to make use of visualizing transformations as well as knowledge of equations for a circle. This assessment should be given to students after they've been introduced to these concepts. Students will engage in SMP 1 and SMP 8.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on understanding and apply theorems about circles, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, providing a task that requires students to calculate the dimensions needed for a new restaurant to build a triangular deck (with one side being the restaurant building) will relate school learning to community/home application.	Art: Consider discussing how inscribed and circumscribed angles may be used in calculating specific designs in landscape, apparel, etc. Designers are often given constraints in which to create an image and may use knowledge of these angles to help design an appropriate image.