NEW MEXICO
Public Education Department

HS: GEOMETRY- CIRCLES

Cluster Statement: B: Find arc lengths and areas of sectors of circles

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSG.C.B.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	SMP 2 Students reason abstractly and quantitatively by calculating the length of an intercepted arc. SMP 4 Students model with mathematics by deriving the area of a sector using similarity. SMP7 Students look for and make use of structure by expecting students to apply rules, look for patterns, and analyze structure.	• Calculate the length of an intercepted arc. • Demonstrate that the constant of proportionality between arc length and the radius of the circle is the radian measure of the central angle. • Derive the formula for the area of a sector using similarity. • Calculate the area of a sector. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** understand, apply
<u>Previous Learning Connections</u> In 7th grade, the formulas for the area and circumference of a circle are learned and then applied to solve problems. They give an informal derivation of the relationship between the circumference and area of a circle.	**<u>Current Learning Connections</u>** Later in the Geometry course when calculating geometric probabilities, students will need to know how to calculate the area of a sector which is taught within this cluster.	**<u>Future Learning Connections</u>** In future courses, students expand on their basic understanding of the radian measure of an angle. They apply radian measures when discovering relationships within the unit circle and while learning trigonometric relationships.

Clarification Statement
This cluster explores the relationship between the length of an arc and the measure of a central angle. Learners develop a definition for the radian measure of an angle and apply radians to find the area of sectors.

Common Misconceptions
Students often struggle with precision while working within this cluster. Small errors in constructions will lead to results that do not work.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying arc Lengths and areas of sectors of circles because understanding here depends on how deeply a student understands earlier concepts such as area of a circle and the terminology involved.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.GB.4 and 6.RP.A2: : This standard provides a foundation for work with arc lengths and areas of sectors of circles because it deals with concept of area of circles and the concept of proportionality. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with finding arc lengths and areas of sectors of circles benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because it will help facilitate the application of importance of the concept by giving the students an opportunity for hands on tactile learning

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with finding arc lengths and areas of sectors of circles benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because allowing students to become self-actualized persistent learners will foster in them a need and a desire for tackling new and challenging areas in their lives. It's more about becoming a life-long learner.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with finding arc lengths and areas of sectors of circles benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) because at this level of geometry this cluster is built upon previously learned concepts and because of that, there are numerous ways to achieve a solution. By encouraging multiple pathways, students can become very well versed in the cluster's concepts.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with finding arc lengths and areas of sectors of circles benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as composing in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video because this cluster focuses on parts of circles and can be very easily adapted into large scale representations. By allowing students to represent using multiple media of their choosing, we are crossing over into other interest categories enjoyed by each student.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with finding arc lengths and areas of sectors of circles benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as addressing subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math")because when students start to feel successful at math, they begin to shed those beliefs of inadequacy. There is a need to create a culture of positive math experiences. Some students enjoy large leaps in improvement, some need recognition for the small, daily victories. It's a matter of knowing your students' needs and abilities.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on Arc Lengths And Areas Of Sectors Of Circles by examining tasks from a different perspective through a short mini-lesson because we are welding concepts together to form a new concept, this process is not automatic and by backing up and looking at the problem from a different point of view

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on arc length and areas of sectors of circles by offering opportunities to understand and explore different strategies because what works for one student may not work for another.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

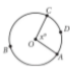
- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying Arc Lengths And Areas Of Sectors Of Circles because linking the concept to something that a student will experience in their own lives will add depth to their experience regarding this concept

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying arc lengths and areas of sectors of circles the types of mathematical tasks are critical because the struggle to be all-inclusive can be an issue. Where students with strong procedural knowledge will easily follow a process, some students will struggle and need adaptation and accommodations. Some ways to address this would be to adapt procedures into the students spoken language, apply terminology and problems from the student's daily life, use hands-on demonstrations, and use bi-lingual grouping of students.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
SAT Item # 5209208: The linked assessment question addresses G-C.B., specifically the question requires students to use knowledge of arc length.

CollegeBoard	Question ID 5209208						
SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Circles	1. Use definitions, properties, and theorems relating to circles and parts of circles, such as radii, diameters, tangents, angles, arcs, arc lengths, and sector areas to solve problems.	No Calculator



The circle above has center O, the length of arc $\overset{\frown}{ADC}$ is 5π, and $x = 100$. What is the length of arc $\overset{\frown}{ABC}$?

Question Difficulty: Medium

A. 9π

B. 13π

C. 18π

D. $\frac{13}{2}\pi$

Choice B is correct. The ratio of the lengths of two arcs of a circle is equal to the ratio of the measures of the central angles that subtend the arcs. It's given that arc $\overset{\frown}{ADC}$ is subtended by a central angle with measure 100°. Since the sum of the measures of the angles about a point is 360°, it follows that arc $\overset{\frown}{ABC}$ is subtended by a central angle with measure $360° - 100° = 260°$. If s is the length of arc $\overset{\frown}{ABC}$, then s must satisfy the ratio $\frac{s}{5\pi} = \frac{260}{100}$. Reducing the fraction $\frac{260}{100}$ to its simplest form gives $\frac{13}{5}$. Therefore, $\frac{s}{5\pi} = \frac{13}{5}$. Multiplying both sides of $\frac{s}{5\pi} = \frac{13}{5}$ by 5π yields $s = 13\pi$.

Choice A is incorrect. This is the length of an arc consisting of exactly half of the circle, but arc $\overset{\frown}{ABC}$ is greater than half of the circle. Choice C is incorrect. This is the total circumference of the circle. Choice D is incorrect. This is half the length of arc $\overset{\frown}{ABC}$, not its full length.

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on arc lengths and areas of sectors of circles , consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, such as using everyday circular items to show how sectors are part of the entire circle.	Economics: Connect to a variety of circular foods, talking about maximizing crust or finding the largest slice.