NEW MEXICO
Public Education Department

HS: GEOMETRY-CONGRUENCE

Cluster Statement: Experiment with transformations in the plane

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
G.CO.A.1 **State and apply precise definitions of angle, circle, perpendicular, parallel, ray, line segment, and distance based on the undefined notions of point, line, and plane.**	SMP1 Students make sense of problems and persevere in solving them by making sense of definable and undefinable terms that exist in geometry. SMP7 Students look for and make use of structure by using a basic understanding of definitions and being able to apply them to generalizations of the rule.	• Demonstrate the knowledge of precise definitions of angles, line, point, plane, circles, perpendicular and parallel lines, and line segments. • Calculate the linear distance and arc length. • Demonstrate the use of proper notation. • Make connections with rigid motions in relation to the definitions of words above. **Webb's Depth of Knowledge:** 2-3 **Bloom's Taxonomy:** Understand, Apply
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
G.CO.A.2 **Represent transformations in the plane. (e.g., using transparencies and/or geometry software) a. Describe transformations as functions that take points in the plane as inputs and give other points as outputs. b. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus dilation).**	SMP 1 Students can make sense of problems and persevere in solving them by representing the transformations in the plane while also describing and comparing them in order to solve problems. SMP 3 Students make arguments and critique the arguments of others when they compare strategies for finding sequences of rigid transformations that take one figure onto another SMP 6 Students can attend to precision by utilizing precise language when describing the transformations with the appropriate mathematics vocabulary.	• Write a function that maps a preimage to its image from an image or a description of a transformation. • Understand which transformations result in figures with congruent sides and angles and which do not. • Represent transformations in the plane. • Compare rigid motions that preserve distance and angle measure (translations, reflections, rotations) to transformations that do not preserve both distance and angle measure (e.g. stretches, dilations). • Understand that rigid motions produce congruent figures while dilations produce similar figures. **Webb's Depth of Knowledge:** 2-3

		Bloom's Taxonomy: Understand, Apply
Standard Text G.CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and/or reflections that map the figure onto itself.	**Standard for Mathematical Practices** SMP1 Students make sense of problems and persevere in solving them by applying transformations to a given shape SMP4 Students model with mathematics by constructing transformations SMP5 Students use appropriate tools strategically by expecting students to consider available tools when solving a mathematical problem. Tools might include pencil and paper, concrete models, rulers, protractors, compasses, calculators, and software or apps. SMP7 Students look for and make use of structure by understanding different types of rotations and/or reflections and be able to generalize these understandings to other objects	**Students who demonstrate understanding can:** • Identify and describe the different symmetries (line symmetry, rotational symmetry, point symmetry) of a figure. • Determine the maximum possible lines of symmetries that exist for a given polygon. • Determine the order and angle of a rotational symmetry. • Determine the symmetries of a parallelogram, rectangle, rhombus, square, trapezoid and regular polygon. • Understand symmetry in terms of transformations" (The Common Core Mathematics Companion). • Explore which shapes are symmetric and what symmetries they will have" (The Common Core Mathematics Companion). • Develop generalizations for the symmetries held by various geometric shapes" (The Common Core Mathematics Companion). • Determine the properties of a shape based on its symmetries" (The Common Core Mathematics Companion).
		Webb's Depth of Knowledge: 2-3
		Bloom's Taxonomy: Understand, Apply

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
G.CO.A.4 **Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.**	SMP3 Students construct viable arguments and critique reasoning of other by justification of method of transformations. SMP7 Students look for and make use of structure by applying the rules and definitions of transformations.	• Describe rotations, reflections and translations. • Determine and apply the properties of the isometric transformations. • Identify which transformation has taken place based on the properties found between the preimage and image. • Identify the orientation relationship between the preimage and image. • Explore properties of transformations using common geometric relationships (e.g., parallel, perpendicular, and congruence)" (The Common Core Mathematics Companion). • Develop definitions of the transformations in terms of their properties" (The Common Core Mathematics Companion). **Webb's Depth of Knowledge:** 2-3 **Bloom's Taxonomy:** understand, apply
Standard Text G.CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure	**Standard for Mathematical Practices** SMP1 Students make sense of problems and persevere in solving them by applying the various types of transformations SMP3 Students construct viable arguments and critique reasoning of other by justification of method of transformations SMP5 Students use appropriate tools strategically by expecting students to consider available tools when	**Students who demonstrate understanding can:** • Model a sequence of transformations including reflections, rotations, and translations with a geometric figure. • Determine the sequence of transformations performed between a given preimage and image. • Describe which single transformation is the result of two reflections over parallel lines. • Describe which single transformation is the result of two reflections over intersecting lines.

	solving a mathematical problem. Tools might include pencil and paper, concrete models, rulers, protractors, compasses, calculators, and software or apps.	• Identify a transformation by its coordinate rule and then apply it to transform the shape. • Demonstrate how some composite transformations are not commutative.
		Depth of Knowledge: 1-2
		Bloom's Taxonomy: Understand, Apply
Previous Learning Connections • This cluster aligns directly to the learning in 8th grade, when students were introduced to congruence and similarity using physical models, transparencies, or geometry software. They have described sequences of rigid motions informally and in terms of coordinates. Learners have verified experimentally the properties of transformations, and describe their effects on two-dimensional figures using coordinates.	**Current Learning Connections** • Transformation definitions will serve as the basis for theorems that will be proven later in the year.	**Future Learning Connections** • In Algebra II, students will connect their knowledge of transformations to functions. They will use transformation language to compare a function to its parent function, identify lines of symmetry, and other characteristics of functions.

Clarification Statement:
Students formalize their transformation language by building precise definitions based on properties. They use formal notation and precise descriptions of transformations and sequences of transformations. Rotational and reflection symmetry are identified with a specific degree of rotation or line(s) of symmetry.

Common Misconceptions
Students may confuse transformation and translation and/or not know how to express the differences between the two terms. Other misconceptions often lie within the connection between the terms (angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.) in the standard G.CO.A.1 and their application in this standard.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying transformations in the plane because students are experimenting with transformations and precise language will be important.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

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- 4.GA.1: This standard provides a foundation for work with transformations in the plane because students should have already built a firm grasp of key vocabulary such as angle, point, line, parallel, perpendicular, etc. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception*: How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with experimenting with transformations in the plane benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as descriptions (text or spoken) for transformations, auditory cues and/or vocabulary word wall for key terms because the section is vocabulary heavy and students may not have a satisfactory grasp of prior skills and/or may have a different first language and by providing the visual meaning along with a written description and re-explained orally provides multiple means of access for the students. Providing auditory cues and/or a vocabulary word wall for key terms can reinforce this. Further, using patty paper/trace paper/geometric software to create transformations allows for interaction between the student and the material because it provides opportunities for the student to physically perform transformations, along with seeing a visual, hearing a description, and reading a definition.

Build

Effort and Persistence*: How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with transformations in the plane benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics representations and scaffolds because students may struggle with visualizing a series of transformations and/or have a hard time abstractly connecting the concept of transformations to angles, circles, parallel and perpendicular lines etc.

Language and Symbols*: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with transformations in the plane benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because most, if not all, of these terms have been introduced at prior grades and students will have some incoming concept of some of the terms. Building off their prior knowledge and/or identifying misunderstandings at this point can ensure precise use of language throughout the learning.

Expression and Communication*: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with transformations in the plane benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., cut out objects, geometric software, patty paper/trace paper) because students can physically create the transformations and/or series of transformations which may deepen their understanding.

Internalize

Executive Functions*: How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with transformations in the plane benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as embedding prompts to "show and explain your work" because students may see transformations in a variety of ways (e.g. one student may see a reflection followed by a rotation where another student sees a series of reflections, etc.).

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on transformations in the plane by clarifying mathematical ideas and/or concepts through a short mini lesson because students may struggle with rotations or reflections about a point/line that is not at the origin.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on transformations in the plane by offering opportunities to understand and explore different strategies because there may be several approaches to map one object onto another.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying transformations in the plane because geometric software can allow students to experiment with combinations of transformations and using such software can allow for abstract application of knowledge of transformations.

Culturally and Linguistically Responsive Instruction:[1]

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

[1] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics.
Boston, M., Dillon, F., & Miller, S. (2017). *TAKING ACTION: IMPLEMENTING EFFECTIVE MATHEMATICS TEACHING PRACTICES IN Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.
Los Angeles, CAUSA. (n.d.). VABB™. Retrieved from https://www.culturallyresponsive.org/vabb

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society? Equity Based Practices (Tasks): The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying Exploring Transformations in the Plane the types of mathematical tasks are critical because the purpose of the standard is for students to explore and experience the transformations for themselves. This requires teachers to select tasks and activities that provide for students to experiment with the transformations in the plane rather than explicitly tell students what steps to take.	
Standards Aligned Instructionally Embedded Formative Assessment Resources: *Source*: http://tasks.illustrativemathematics.org/content-standards/HSG/CO/A/tasks/1468 The linked assessment question addresses G-CO.A, specifically the question requires students to look at lines of symmetry using reflections. Two different arguments are presented using triangle congruence and another which uses rotations and reflections. This assessment should be given to students after they've been introduced to the formal definition of reflections. Students will engage in SMP 1, SMP 7, and potentially SMP 3 depending on if students work in groups to share their solutions.	
Relevance to families and communities: During a unit focused on Exploring Transformations in the Plane, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, viewing artwork from a variety of cultures or having students select and bring in artwork that shows specific transformations can build the bridge between things they may have seen at home to what they are seeing in school. These cultural connections can be made explicitly or can be incorporated into word problems and other abstract application style questions.	**Cross-Curricular Connections:** Art: Support students in making connections in between geometric transformation and art, especially in relation to the idea of repetition and perspective in artwork.[2]

[2] https://uwm.edu/arts/wp-content/uploads/sites/71/2020/01/Peterson_ArtsECOFellowsLesson2019.pdf