NEW MEXICO
Public Education Department

HS: GEOMETRY-CONGRUENCE

Cluster Statement: B: Understand congruence in terms of rigid motions

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSG.CO.B.6 **Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.**	MP3 Students construct viable arguments and critique reasoning of other when proving congruency and describe transformations. MP6 Students attend to Precision by using accurate transformations to map shapes onto other shapes. MP7 Students look for and make use of structure by applying rules of transformations to prove congruence of objects.	• Demonstrate that two figures are congruent if there is a sequence of rigid motions that map one figure to another. • Express in words and in writing that two figures are congruent if and only if they have the same shape and size. • Model composite transformations to map one figure onto another. • Recognize and explain the effects of rigid motion on orientation and location of a figure. • Define congruence as a test to see if two figures are congruent.
		Webb's Depth of Knowledge: 1-2
		Bloom's Taxonomy: understand, apply
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSG.CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	MP3 Students construct viable arguments and critique reasoning of other when proving congruency and describe transformations. MP6 Students attend to Precision by using accurate transformations to map shapes onto other shapes. MP7 Students look for and make use of structure by applying rules of transformations to prove congruence of objects.	• Identify corresponding angles and sides based on congruence statements. • Develop and write congruence statements for two congruent triangles. • Determine if two triangles are congruent based on their corresponding parts. • Compare given figures to determine congruence and indicate whether the figure went through a rigid transformation. • Explain, using rigid motions, why in congruent triangles, corresponding parts must be congruent.

<table>
<tr><td></td><td></td><td>Webb's Depth of Knowledge: 1-2

Bloom's Taxonomy: understand, apply</td></tr>
<tr><td>Standard Text

HSG.CO.B.8
Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.</td><td>Standard for Mathematical Practices

MP3
Students construct viable arguments and critique reasoning of other when proving congruency and describe transformations.

MP6
Students attend to Precision by using accurate transformations to map shapes onto other shapes.

MP7
Students look for and make use of structure by applying rules of transformations to prove congruence of objects.</td><td>Students who demonstrate understanding can:
• Create a method to determine unknown measurements of congruent triangles.
• Explain the approach that was used to determine to congruency of two triangles given limited parts of triangles.
• Explain the approach that was used to determine the congruence of two triangles.
• Apply the criteria of SSS, SAS, ASA to prove triangle congruency.

Webb's Depth of Knowledge:1-2

Bloom's Taxonomy: understand, apply</td></tr>
<tr><td><u>Previous Learning Connections</u>
• In 7th grade, students focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. In 8th grade, students develop understanding of congruence using physical models, transparencies, or geometry software. Students also understand that figures are congruent if the second can be obtained from the first by a sequence of rotations, reflections, and/or translations. These foundational skills are applied within this standard to continue to explore congruence and the impact of rigid motions on geometric shapes.</td><td><u>Current Learning Connections</u>

• Students will use triangle congruence concepts to develop future postulates and theorems. Concepts of triangle congruence serve to build a foundation for work with triangle proofs in future clusters.</td><td><u>Future Learning Connections</u>
• In later courses, students consider triangle congruence and the ambiguous case when working with the Law of Sines and Law of Cosines.</td></tr>
</table>

Clarification Statement:
Students create a definition of triangle congruence in terms of rigid motions. They work to develop a set of criteria for triangle congruence and build a foundation for geometric proofs.

Common Misconceptions
Combinations such as SSA or AAA are also a congruence criterion for triangles. All transformations, including dilations, are rigid motions. Any two figures that have the same area represent a rigid transformation. Students should recognize that the areas remain the same, but preservation of side and angle lengths determine that the transformation is rigid.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying understanding congruence in terms of rigid motion because students may incorrectly interchange congruence and similarity.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.GA.2: This standard provides a foundation for work with understanding congruence in terms of rigid motion because this is where the concept of congruence is solidified in terms of one object being able to be moved directly on top of another and match perfectly. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with experimenting with transformations in the plane benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as descriptions (text or spoken) for transformations, auditory cues and/or vocabulary word wall for key terms because the section is vocabulary heavy and students may not have a satisfactory grasp of prior skills and/or may have a different first language and by providing the visual meaning along with a written description and re-explained orally provides multiple means of access for the students. Providing auditory cues and/or a vocabulary word wall for key terms can reinforce this. Further, using patty paper/trace paper/geometric software to create transformations allows for interaction between the student and the material because it provides opportunities for the student to physically perform transformations, along with seeing a visual, hearing a description, and reading a definition.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with transformations in the plane benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics

representations and scaffolds because students may struggle with visualizing a series of transformations and/or have a hard time abstractly connecting the concept of transformations to angles, circles, parallel and perpendicular lines etc.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with transformations in the plane benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because most, if not all, of these terms have been introduced at prior grades and students will have some incoming concept of some of the terms. Building off their prior knowledge and/or identifying misunderstandings at this point can ensure precise use of language throughout the learning.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with transformations in the plane benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., cut out objects, geometric software, patty paper/trace paper) because students can physically create the transformations and/or series of transformations which may deepen their understanding.

Internalize

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with transformations in the plane benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as embedding prompts to "show and explain your work" because students may see transformations in a variety of ways (eg. one student may see a reflection followed by a rotation where another student sees a series of reflections, etc.).

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understanding congruence in terms of rigid motion by clarifying mathematical ideas and/or concepts through a short mini-lesson because students may incorrectly apply the triangle congruence theorems (ASA, SSS, etc) and could benefit from clarifying these and connecting them to the concept of congruence.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit understanding congruence in terms of rigid motion by helping students move from specific answers to generalizations for certain types of problems because this cluster requires students to prove, generally speaking, if two triangles are congruent

by applying knowledge of congruence rather than using specific triangles with concrete measurements.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying understanding congruence in terms of rigid motion because their established knowledge base of congruence could allow them to generate examples and non-examples of congruent shapes.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

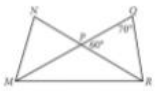
Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying Understanding Congruence in terms of Rigid Motion the types of mathematical tasks are critical because students need the time and experience in connecting prior knowledge of congruence to rigid motions in the plane. Tasks should activate knowledge of both congruence and rigid motions, and build the bridge between them.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

SAT Item #: 422659: The linked assessment question addresses G-CO.B., specifically the question requires students to identify and use congruent angles to find the angle measure in a triangle.

CollegeBoard **Question ID 422659**

SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Lines, angles, and triangles	1. Use concepts and theorems relating to congruence and similarity of triangles to solve problems.	No Calculator



In the figure above, $\overline{MQ}$ and $\overline{NR}$ intersect at point P, $NP = QP$, and $MP = PR$. What is the measure, in degrees, of $\angle QMR$? (Disregard the degree symbol when gridding your answer.)

Question Difficulty: Medium

The correct answer is 30. It is given that the measure of $\angle QPR$ is $60°$. Angle MPR and $\angle QPR$ are collinear and therefore are supplementary angles. This means that the sum of the two angle measures is $180°$, and so the measure of $\angle MPR$ is $120°$. The sum of the angles in a triangle is $180°$. Subtracting the measure of $\angle MPR$ from $180°$ yields the sum of the other angles in the triangle MPR. Since $180 - 120 = 60$, the sum of the measures of $\angle QMR$ and $\angle NRM$ is $60°$. It is given that $MP = PR$, so it follows that triangle MPR is isosceles. Therefore $\angle QMR$ and $\angle NRM$ must be congruent. Since the sum of the measure of these two angles is $60°$, it follows that the measure of each angle is $30°$.

An alternate approach would be to use the exterior angle theorem, noting that the measure of $\angle QPR$ is equal to the sum of the measures of $\angle QMR$ and $\angle NRM$. Since both angles are equal, each of them has a measure of $30°$.

Additional Assessment:
Properties of Congruent Triangles: https://tasks.illustrativemathematics.org/content-standards/HSG/CO/B/7/tasks/1637
The linked assessment question addresses G-CO.B, specifically the question requires students to look at two triangles and connect the concept of congruence of corresponding parts to congruence of shape in terms of rigid motion. Two different approaches are prompted: one assuming the triangles can be mapped on each other and asking students to explain which parts are congruent, and another assuming the triangles are congruent and asking students to create the sequence of transformations to map one onto the other. This assessment should be given to students after they've had time to work with rigid motion and have a firm grasp of naming parts of triangles. Students will engage in SMP 3, SMP 6 and SMP 7.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on Congruence in terms of Rigid Motion, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, teachers can create connections from native language to English by focusing on cognates (Words that sound the same in two different languages). Incorporating the	Computer Science: program to create visual demo of transformations

usage of cognates throughout a unit validates and affirms all languages and can encourage students to explore these terms in language other than their native language.	