NEW MEXICO
Public Education Department

HS: GEOMETRY-CONGRUENCE

Cluster Statement: C: Prove geometric theorems

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSG.CO.C.9 **Prove theorems about lines and angles.** *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	SMP 2 Students reason abstractly and quantitatively by using Theorems and postulates to prove relationships between angles formed by intersecting lines and systems of intersecting lines SMP 5 Students use appropriate tools strategically by using dynamic geometry tools to solve problems. These tools might include pencil and paper, concrete models, rulers, protractors, compasses, software, apps, and calculators SMP 7 Students look for and make use of structure by learning the language of geometric theorems and postulates to frame problem solving SMP 8 Students look for and express regularity in repeated reasoning by generalizing rules, theorems, and postulates to similar problems and situations	• Prove the vertical angle theorem and alternate interior angle theorem. • Prove corresponding angles are congruent. • Prove the converse of the alternate interior angle theorem and the corresponding angle theorem and use it to show that two lines are parallel. • Use perpendicular bisectors to locate the circumcenter of a triangle and to find the center of a circle given three points on the circle. • Express proofs both in writing and orally by using precise mathematical language • Examine and critique proofs produced by other students as well as their own **Webb's Depth of Knowledge:** 2-3 **Bloom's Taxonomy:** apply, analyze

Standard Text **HSG.CO.C.10** **Prove theorems about triangles.** *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	**Standard for Mathematical Practices** SMP 2 Students reason abstractly and quantitatively by using Theorems and postulates to prove relationships between angles formed by intersecting lines and systems of intersecting lines SMP 5 Students use appropriate tools strategically by using dynamic geometry tools to solve problems. These tools might include pencil and paper, concrete models, rulers, protractors, compasses, software, apps, and calculators SMP 7 Students look for and make use of structure by learning the language of geometric theorems and postulates to frame problem solving SMP 8 Students look for and express regularity in repeated reasoning by generalizing rules, theorems, and postulates to similar problems and situations	**Students who demonstrate understanding can:** • Prove and apply that the sum of the interior angles of a triangle is 180°. • Prove and apply that the base angles of an isosceles triangle are congruent. • Prove and apply the midsegment (midline) of triangle theorem. • Prove that the medians of a triangle meet at a point, a point of concurrency. • Prove and apply that the exterior angle theorem. • Determine the conditions for forming a triangle, when given three lengths. **Webb's Depth of Knowledge: 1-2** **Bloom's Taxonomy:** understand, apply
Standard Text **HSG.CO.C.11** **Prove theorems about parallelograms.** *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	**Standard for Mathematical Practices** SMP 2 Students reason abstractly and quantitatively by using Theorems and postulates to prove relationships between angles formed by intersecting lines and systems of intersecting lines SMP 5 Students use appropriate tools strategically by using dynamic geometry tools to solve problems. These tools might include pencil and paper, concrete models, rulers,	**Students who demonstrate understanding can:** • Prove properties of parallelograms and then apply them. • Prove the properties of rectangles and then apply them. • Prove the properties of rhombi and then apply them. • Prove the properties of squares and then apply them. • Classify a quadrilateral by its properties. • Identify the conditions necessary to prove that a quadrilateral is a parallelogram.

	protractors, compasses, software, apps, and calculators SMP 7 Students look for and make use of structure by learning the language of geometric theorems and postulates to frame problem solving SMP 8 Students look for and express regularity in repeated reasoning by generalizing rules, theorems, and postulates to similar problems and situations	**Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** understand, apply
Previous Learning Connections In 7th grade, students use facts about supplementary, complementary, vertical, and adjacent angles. In 8th grade, learners use informal arguments to establish facts about the angle sum and exterior angle of triangles, and about the angles created when parallel lines are cut by a transversal. These angle facts connect this cluster as students apply them when creating proofs.	**Current Learning Connections** The formalized theorems within this cluster will be used to build theorems and proofs for concepts in future clusters within the Geometry course.	**Future Learning Connections** Understanding the logical flow of developing a proof will be used in future courses such as when proving trigonometric identities.

Clarification Statement
Students focus on formalizing geometric proof structure and language. They write formal proofs focusing on angle relationships, triangle segment and angle relationships, and parallelogram properties.

Common Misconceptions
Students may have a hard time generalizing and spend unnecessary time looking for multiple counterexamples to prove or disprove a proof, or, they may assume a conjecture is always true because it worked in all examples that were explored. Additionally, they may assume the converse of a statement is always true.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., structured proofs) when studying proving geometric theorems because students may be unfamiliar with traditionally structured mathematical proofs.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- G.CO.B6: This standard provides a foundation for work with proving geometric theorems because this is where students have formalized an understanding of congruence in terms of rigid motion, which is required for every aspect of this cluster. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with proving geometric theorems benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for auditory information (e.g. connecting an oral argument for a theorem to a written proof to an image/series of images that display the argument, and in some cases, a written proof in native language) because students may be able to articulate their thoughts using one of these methods, but may not see how to produce a formal proof.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with proving geometric theorems benefit when learning experiences attend to student's attention and affect to support sustained effort and concentration such as using prompts or scaffolds for visualizing desired outcomes (e.g. two-column proof outline) because students may fail to recognize what is being asked of them when constructing a proof, or how to properly get from point A to point B.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with proving geometric theorems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) because students may be able to explain their mathematical thinking using one particular modality as opposed to always creating formal proofs, etc.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with proving geometric theorems benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches, strategies, skills, etc.) because students see arguments/proofs in different ways (visual, oral, physical proof, written, etc.). Eliciting a variety of examples can help students to see math from a different perspective.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with proving geometric theorems benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because formal mathematical proofs can feel overwhelming to students. Modeling for students that choosing a starting point and following it through, whether it arrives at the desired outcome or not, can still be helpful and meaningful can encourage students to pick up a proof from wherever they are comfortable and try it when they may have otherwise not attempted it to begin with.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on proving geometric theorems by <critiquing student approaches/solutions to make connections through a short mini-lesson because students will often see different methods of progressing through a proof which can benefit all students to see. Further, students may make specific claims or statements that are unsupported or incorrect, and critiquing this reasoning will strengthen all students' understanding of the content.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit about proving geometric theorems by offering opportunities to understand and explore different strategies because students may struggle with writing their own proofs, particularly in the beginning. It may be helpful to provide many examples and ask students to analyze each statement as to whether it can be mathematically supported or not. This could also be an opportunity to allow students to show their thinking with pictures rather than a formal proof structure.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying proving geometric theorems because giving students the opportunity to select certain theorems to prove can be both challenging and engaging for those with a firm grasp of the process.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

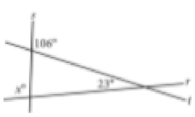
Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion, we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics, we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying Proving Geometric Theorems facilitating meaningful mathematical discourse is critical because proofs of theorems can frequently be seen from multiple perspectives. Purposefully sequencing, discussing and validating these different perspectives can help students internalize their worth in the classroom. It can also show students that knowing a piece of the larger puzzle often "unlocks" the door to the next step in a proof. The key is in selecting and sequencing a variety of perspectives to share out and focus on how it helps the process, rather than whether it is complete and correct.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
SAT Item #: 422005 The linked assessment question addresses G-CO.C., specifically the question requires students to know and apply theorems about exterior and interior angles.

CollegeBoard — **Question ID 422005**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	**Math**	**Additional Topics in Math**	**Hard**	**Additional Topics in Math**	**Lines, angles, and triangles**	**4. Know and directly apply relevant theorems such as b. triangle similarity and congruence criteria;**	**No Calculator**

Intersecting lines r, s, and t are shown below.



What is the value of x ?

Question Difficulty: Hard

The correct answer is 97. The intersecting lines form a triangle, and the angle with measure of x° is an exterior angle of this triangle. The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles of the triangle. One of these angles has measure of 23° and the other, which is supplementary to the angle with measure 106°, has measure of $180^\circ - 106^\circ = 74^\circ$. Therefore, the value of x is $23 + 74 = 97$.

Additional Assessment:

Midpoints of the Sides of a Parallelogram: https://tasks.illustrativemathematics.org/content-standards/HSG/CO/C/11/tasks/35
The linked assessment question addresses G-CO.C, specifically the question requires students to prove segments of a parallelogram are congruent. Students use knowledge of corresponding parts of congruent triangles are congruent to form arguments. This assessment should be given to students after they've worked with corresponding parts of triangles. Students will engage in SMP 2 and SMP 7.

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on proving geometric theorems, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, the history of geometry spans across cultures. This history can be incorporated as students learn how to build and create formal proofs from informal diagrams, experimentation and/or oral arguments.	Links to history can be made by exploring how geometric proofs developed over time in different cultures. Further, links to computer science can be made by discussing and displaying how coding reflects a variety of steps to get from one point to another, as is mirrored in proofs.