NEW MEXICO
Public Education Department

HS: GEOMETRY-GEOMETRIC MEASUREMENT & DIMENSION

Cluster Statement: A: Explain volume formulas and use them to solve problems

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSG.GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	SMP 2 Students reason abstractly and quantitatively by making conjectures about volume of objects with similar dimensions SMP 3 Students construct viable arguments and critique the reasoning of others by constructing and analyzing arguments about volumes SMP 4 Students model with mathematics by constructing multiple representations of given situation in order to make valid arguments and conclusions	• Demonstrate Cavalieri's Principle concretely. • Give an informal argument for circumference and area formulas for circles. • Give an informal argument for volume formulas of cylinders, pyramids, and cones. • Construct viable arguments to validate the circumference of a circle, volume of a cylinder, volume of a pyramid, and volume of a cube by using Cavalieri's Principle. **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy:** understand, apply, analyze
Standard Text **HSG.GMD.A.3** **Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.**	**Standard for Mathematical Practices** SMP 1 Students make sense of problems and persevere in solving them by analyzing the problem and choosing the correct model in which to proceed SMP 4 Students model with mathematics by using derived equations, formulae, and theorems to complete problems SMP 7 Students look for and make use of structure by noting that volume formulae are constructed from the products of the area of the base of	**Students who demonstrate understanding can:** • Identify these geometric shapes: cylinders, pyramids, cones, and spheres. • Calculate volume for cylinders, pyramids, cones and spheres. • Use formulas to solve problems involving three-dimensional figures. • Apply volume to real world problems. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** understand, apply

	an object times its height or some variation on that theme	
Previous Learning Connections In 7th grade, learners worked with area, and circumference which extended to find components needed for surface area and volume. Throughout grades 6, 7, and 8 students calculated the volumes and surface areas of prisms, cones, cylinders, and spheres which will connect to their work figures within this cluster.	**Current Learning Connections** Students will continue to expand their work to include composite figures. They will also justify volume formulas and other constructions.	**Future Learning Connections** In Calculus, students will apply Cavalieri's principle to calculate volumes for solids of rotation.

Clarification Statement

Students move from applying volume formulas to justifying them. Students will be exposed to advanced concepts in an informal setting. Learners will deconstruct complex geometric shapes into basic three-dimensional shapes to calculate their surface areas and volumes.

Common Misconceptions

Students may mix up which formula to use for a given figure.

Students may have difficulty identifying the base of a figure.

When considering units, students may struggle to interpret inches cubed or forget these units in the final answer.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying explaining volume formulas and using them to solve problems because students have to use different formulas that are necessary to support them on solving problems involving volume of three-dimensional figures, such as, cylinder, pyramid, and cone.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. This standard provides a foundation for work with explaining volume formulas and using them to solve problems because students need to master the different formulas and use them to solve real-world problems. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging in explaining volume formulas and use them to solve problems because it will help the student understand the concept at a deeper level than

just plugging in numbers and making calculations. When the student can give an informal argument for what makes up a volume formula, their understanding will be at a greater depth of knowledge.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with volume formulas and use them to solve problems benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because allowing students to become self-directed learners will foster in them a need to explore concepts deeply. to become investigators

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging volume formulas and use them to solve problems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) because when using volume formulas the background knowledge gives so much understanding of how these formulas are derived. If a student understands the language and symbology of volume as a concept, they do not need to even remember the exact formula. The student will be able to construct a volume calculation using the parts of the object.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging volume formulas and use them to solve problems benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as composing in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video because in this cluster the focus on the volume of an object can be very easily adapted into large scale representations such as constructions or by measuring everyday objects. By allowing students to represent using multiple media of their choosing, we are jumping into interest categories enjoyed by each student.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging volume formulas and use them to solve problems benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as addressing subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math")because there is a need to create a culture of positive math experiences. So many students do not have that, they have a history of feeling as if they were failures.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on GMD.A: Explain Volume Formulas and Use Them to Solve Problems by providing specific feedback to students on their work through a short mini-lesson because teachers need to monitor students on how they use different formulas and use them precisely to answer a given problem. Providing students specific feedback will help students to create strong justifications of their answers.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit GMD.A: Explain Volume Formulas and Use Them to Solve Problems by helping students move from specific answers to generalizations for certain types of problems because students need to master how they justify answers mathematically and base from specific concepts to come up with a general, accurate and precise answers.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying explaining volume formulas and using them to solve problems because these topics are interconnected and it will help students to master the whole concepts. Students need to have a strong foundation of different formulas and let them use it to formulate and come up with justifications on volumes.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

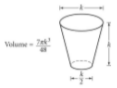
Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students. For example, when studying, explaining volume formulas and use them to solve problems supporting productive struggle is critical because students may not see the connections of other formulas to volume and they need those to solve problems involving volume. However, students may see the connection by presenting the actual image which is relevant and relatable to them, such as, car tires, basketball, and any other shape that is visible to them.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

SAT Item #: 1474145 The linked assessment question addresses G-GMD.A., specifically the question requires students to use a specified volume for an object to determine the radius.

CollegeBoard Question ID 1474145

SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Area and volume	1. Solve real-world and mathematical problems about a geometric figure or an object that can be modeled by a geometric figure using given information such as length, area, surface area, or volume.	Calculator



The glass pictured above can hold a maximum volume of 473 cubic centimeters, which is approximately 16 fluid ounces.

What is the value of k, in centimeters?

Question Difficulty: Medium

A. 2.52

B. 7.67

C. 7.79

D. 10.11

Choice D is correct. Using the volume formula $V = \frac{7\pi k^3}{48}$ and the given information that the volume of the glass is 473 cubic centimeters, the value of k can be found as follows:

$$473 = \frac{7\pi k^3}{48}$$

$$k^3 = \frac{473(48)}{7\pi}$$

$$k = \sqrt[3]{\frac{473(48)}{7\pi}} \approx 10.10690$$

Therefore, the value of k is approximately 10.11 centimeters.

Choices A, B, and C are incorrect. Substituting the values of k from these choices in the formula results in volumes of approximately 7 cubic centimeters, 207 cubic centimeters, and 217 cubic centimeters, respectively, all of which contradict the given information that the volume of the glass is 473 cubic centimeters.

Additional Assessment:
http://tasks.illustrativemathematics.org/content-standards/HSG/GMD/A/3/tasks/1899
The linked assessment question addresses G-GMD.A, specifically the question requires students to apply volume formulas to model the Egyptian Pyramids and find missing information. Students will utilize the formula for volume by rearranging the formula to highlight to value of interest. This assessment should be given to students after they've been introduced to volume formulas and had opportunity to reason with the formulas when a variety of information is provided. Students will engage in SMP 2 and SMP 4.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on explaining volume formulas and use them to solve problems , consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning the Cavalieri's principle is a great entry point and support students to create arguments and use it to solve volume of a given geometric shapes.	Because volume can be found for any given item, this connection can be made to a variety of areas: science-beakers, social studies-coffins, art-paint bottles, music-variety of instruments