NEW MEXICO
Public Education Department

HS: GEOMETRY-EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS

Cluster Statement: A: Translate between the geometric description and the equation for a conic section

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSG.GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	SMP 2 Students reason abstractly and quantitatively by using the Pythagorean Theorem to derive the equation of a circle and complete the square to find the center of a circle. SMP 3 Students construct viable arguments and critique the reasoning of others by discussion of why and how the Pythagorean theorem relates to the equation of a circle and why this is important. SMP 7 Students look for and make use of structure by expecting students to apply rules, look for patterns, and analyze structure.	• Explain how the Pythagorean Theorem can be used to derive the equation of a circle. • Write the equation of a circle, given the center and radius. • Complete the square within the equation of a circle in order to find the center and radius. **Webb's Depth of Knowledge: 2**-3 **Bloom's Taxonomy:** apply, analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSG.GPE.A.2 Derive the equation of a parabola given a focus and directrix.	SMP 1 Students make sense of problems and persevere in solving them by taken a given set of values such the focus and directrix and using that info to derive the equation of a parabola. SMP 8 Students look for and express regularity in repeated reasoning by utilizing given information and rules about parabolas to make	• Describe the characteristics of a parabola given its equation. • Derive the equation for a parabola given the focus and directrix. **Webb's Depth of Knowledge: 1-3**

	general extrapolations about the equations of a parabola	**Bloom's Taxonomy: understand, analyze**
Previous Learning Connections • Students have worked with coordinates, slope, and the Pythagorean Theorem in 8th grade math. This work exploring facts about right triangles connects to the foundational formulas in analytic geometry. Additionally, in Algebra I, students have been rewriting expressions in different forms (factoring and completing the square) which directly correlates to the work they will complete in this cluster when creating algebraic proofs of the theorems.	**Current Learning Connections** • Students will connect the information in this cluster to learning later in the course by extending the precise definitions of circles and polygons to work with coordinates on the plane.	**Future Learning Connections** Learners will continue with graphing quadratic functions, showing vertices, intercepts, and identifying maxima or minima in the Algebra II course.

Clarification Statement
The introduction of coordinates into geometry connects geometry and algebra, allowing algebraic proofs of geometric theorems.

Common Misconceptions
Students commonly swap h and k when working with the equations for the circle.

Students will make similar mistakes with h and k, when finding the vertex of a parabola.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying translating between geometric descriptions and the equations for a conic section because students will need to know the distance formula which is learned in Grade 8. A review of distance between two points on the coordinate grid will help students in this cluster.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.G.B.8: This standard provides a foundation for work with translating between the geometric descriptions and the equation for a conic section because students have to apply the Pythagorean theorem to find the distance between two points. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Physical Action: *How will the learning for students provide a variety of methods for navigation to support access?*

- For example, learners engaging with translating between the geometric description and the equation for a conic section benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as relating formulas and written equations to graphical representations because students may benefit from seeing the connection between changes made in equations and the visual outcome.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with prompts or scaffolds for visualizing desired outcomes benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as working in cooperative learning groups with clear goals, roles and responsibilities because some students may need extra help when writing equations, using technology, or incorporating formulas.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with translating between the geometric description and the equation for a conic section benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc.) because students may need support in recalling what specific terms mean in a definition, how it appears in a visual representation and how this is shown through an equation.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with translating between the geometric description and the equation for a conic section benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches, strategies, skills, etc.) because this standard calls for students to explore the relationships and patterns to reason about their derived equations.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because we want students to feel supported in their learning and empowered in their thinking. Providing student supports will minimize frustration and give them a means to cope in difficult situations.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on translating between the geometric description and the equation for a conic section by critiquing student approaches/solutions to make connections through a short mini-lesson because this cluster requires students to generalize patterns they see through exploration. These patterns may not be the same for every student but connecting the different patterns can reveal opportunities to deepen understanding and/or correct misunderstandings.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit of translating between the geometric description and the equation for a conic section by helping students move from specific answers to generalizations for certain types of problems because this cluster calls on students to recognize and generalize patterns. Students may need extra support in moving from concrete examples to generic patterns.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying translating between the geometric description and the equation for a conic section because some students may see generalizations easier than others. Allowing these students to move faster through the concrete examples to get to the abstract generalizations will allow them to stretch their expression of mathematical reasoning from concrete to abstract.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying translating between the geometric description and the equation for a conic section, goal setting is critical because students may be at varying levels of academic and language proficiency. To help students identify goals, teachers can use strategies such as writing prompts to gauge their thinking. Teachers can also provide students with a means to track their personal data (this assists in knowing where you currently are in attaining your goal). Teachers should work to build rapport, relationships and respect with their students and amongst each other to create that positive classroom culture in which students are willing to share/monitor their goals with one another without being judged.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
SAT Item #: 4170563 The linked assessment question addresses G-GPE.A., specifically the question requires students to use a given radius and point to write equation of a circle.

CollegeBoard **Question ID 4170563**

SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Circles	3. Create an equation to represent a circle in the xy-plane.	Calculator

In the xy-plane, a circle with radius 5 has center $(-8,6)$. Which of the following is an equation of the circle?

Question Difficulty: Medium

A. $(x-8)^2+(y+6)^2=25$

B. $(x+8)^2+(y-6)^2=25$

C. $(x-8)^2+(y+6)^2=5$

D. $(x+8)^2+(y-6)^2=5$

Choice B is correct. An equation of a circle is $(x-h)^2+(y-k)^2=r^2$, where the center of the circle is (h,k) and the radius is r. It's given that the center of this circle is $(-8,6)$ and the radius is 5. Substituting these values into the equation gives $(x-(-8))^2+(y-6)^2=5^2$, or $(x+8)^2+(y-6)^2=25$.

Choice A is incorrect. This is an equation of a circle that has center $(8,-6)$. Choice C is incorrect. This is an equation of a circle that has center $(8,-6)$ and radius $\sqrt{5}$. Choice D is incorrect. This is an equation of a circle that has radius $\sqrt{5}$.

SAT Item #: 1474672 The linked assessment question addresses G-GPE.A., specifically the question requires students to use a given equation and point to find diameter of a circle.

CollegeBoard Question ID 1474672

SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Circles	5. Understand that the ordered pairs that satisfy an equation of the form (x − h) ² + (y − k) ² = r² form a circle when plotted in the xy-plane.	No Calculator

$(x-6)^2+(y+5)^2=16$

In the xy-plane, the graph of the equation above is a circle. Point P is on the circle and has coordinates $(10,-5)$. If $\overline{PQ}$ is a diameter of the circle, what are the coordinates of point Q ?

Question Difficulty: Medium

A. $(2,-5)$

B. $(6,-1)$

C. $(6,-5)$

D. $(6,-9)$

Choice A is correct. The standard form for the equation of a circle is $(x-h)^2+(y-k)^2=r^2$, where (h,k) are the coordinates of the center and r is the length of the radius. According to the given equation, the center of the circle is $(6,-5)$. Let (x_1,y_1) represent the coordinates of point Q. Since point P $(10,-5)$ and point Q (x_1,y_1) are the endpoints of a diameter of the circle, the center $(6,-5)$ lies on the diameter, halfway between P and Q. Therefore, the following relationships hold: $\frac{x_1+10}{2}=6$ and $\frac{y_1+(-5)}{2}=-5$. Solving the equations for x_1 and y_1, respectively, yields $x_1=2$ and $y_1=-5$. Therefore, the coordinates of point Q are $(2,-5)$.

Alternate approach: Since point P $(10,-5)$ on the circle and the center of the circle $(6,-5)$ have the same y-coordinate, it follows that the radius of the circle is $10-6=4$. In addition, the opposite end of the diameter $\overline{PQ}$ must have the same y-coordinate as P and be 4 units away from the center. Hence, the coordinates of point Q must be $(2,-5)$.

Choices B and D are incorrect because the points given in these choices lie on a diameter that is perpendicular to the diameter $\overline{PQ}$. If either of these points were point Q, then $\overline{PQ}$ would not be the diameter of the circle. Choice C is incorrect because $(6,-5)$ is the center of the circle and does not lie on the circle.

Additional Assessment:

http://tasks.illustrativemathematics.org/content-standards/HSG/GPE/A/1/tasks/1425
The linked assessment question addresses G-GPE.A, specifically the question requires students to develop a generic equation for a circle using Pythagorean theorem after proving a specific case works. Students begin with a specific point and radius. Once the connection is made in this specific case, students apply the same logic to a general point. This assessment should be given to students after they've been introduced to the Pythagorean theorem and had time to work with its use in the coordinate plane. Students will engage in SMP 2 and SMP 6.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on translating between the geometric description and the equation for a conic section, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, relating these standards to real-life context such as finding the distance from one building to another in the community will solidify	Consider linking using circles in cartography as a means of utilizing central point to look at an area of interest. This may relate to social studies as well as forensic science.

the school learning to the actual application of standards/skills.	