NEW MEXICO
Public Education Department

HS: GEOMETRY-EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS

Cluster Statement: B: Use coordinates to prove simple geometric theorems algebraically

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSG.GPE.B.4 **Use coordinates to prove simple geometric theorems algebraically.** *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*	SMP 2 Students reason abstractly and quantitively by using formulas such as the distance formula, midpoint formula, equation of circles, equation of ellipses, and slope formula to prove figures on a coordinate plane are qualified shapes such as rectangles, squares, circles and ellipses SMP 6 Students attend to precision by using correct and appropriate theorems and formulae for the given shape to prove its existence SMP 7 Students look for and make use of structure by utilizing slope to prove parallelism and perpendicularity when working with quadrilaterals on the coordinate plane, and the Pythagorean theorem to prove triangles, circles and ellipses	• Use the distance formula to find the distance between coordinates. • Find the slope of a line connecting two coordinates. • Determine if a point lies on a specific circle. • Use coordinates to prove that a quadrilateral is, or is not, a parallelogram, rectangle, rhombus, square, or trapezoid. • Use coordinates to prove a triangle's classification by its sides. **Webb's Depth Of Knowledge: 1-2** **Bloom's Taxonomy:** understand, apply
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSG.GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	SMP 2 Students reason abstractly and quantitatively by interpreting the meaning of parallel and perpendicular lines graphically, numerically, and to generalize their findings SMP 5 Students use appropriate tools strategically by using straightedges, protractors, geometry and graphing software/apps. SMP 7	• Prove how parallel lines increase at the same rate of change. • Explain that perpendicular lines intersect at a right angle. • Construct an equation of a line that is parallel or perpendicular to a given line. • Calculate slope from given ordered pairs. • Classify lines or segments as parallel or perpendicular given slopes, graphs, and/or equations of lines. • Write equations for parallel lines and perpendicular lines

	Students look for and make use of structure by using patterns relating the slopes of parallel and perpendicular lines to generalize to form rules about these pairs of lines.	given a point and an equation of a line. **Webb's Depth of Knowledge: 1-3** **Bloom's Taxonomy:** understand, analyze
Standard Text HSG.GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	**Standard for Mathematical Practices** SMP 1 Students make sense of problems and persevere in solving them by using a variety of methods such as graphing, algebra, and proportional reasoning to make sense of the given information and the task at hand SMP 3 Students construct viable arguments and critique the reasoning of others by justifying using graphing, mathematics and logic why two lines are parallel, perpendicular, or neither SMP 7 Students look for and make use of structure by using patterns relating the slopes of parallel and perpendicular line to form general rules about pairs of lines	**Students who demonstrate understanding can:** • Determine the coordinates of a point of a given partition on a directed segment. • Use the midpoint formula, the section formula, and the distance formula to find the partition point of a given line segment. • Determine the ratio of a partition using the distance formula. • Given two points, find the point on a line segment between the two points that divides the segment into a given ratio **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** understand, apply

Standard Text **HSG.GPE.B.7** **Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.**	**Standard for Mathematical Practices** SMP 2 Students reason abstractly and quantitatively by working problem piecewise as separate distances and partial areas and synthesizing into perimeters and areas of Polygons. SMP 4 Students model with mathematics by using mathematical properties and graphic representations to solve problems. SMP 7 Students look for and make use of structure by using distance, slope, and areas of prior learned shapes to prove properties of geometric figures.	**Students who demonstrate understanding can:** • Use the distance formula to find the length of sides of a polygon. • Choose the appropriate formula for perimeter or area of a given polygon. • Calculate areas and perimeters of polygons. • Use appropriate labels for the areas and perimeters. **Webb's Depth of Knowledge: 1-2** **Bloom's Taxonomy:** understand, apply
Previous Learning Connections • In 6th grade, learners find the area of polygons by composing into rectangles or decomposing into triangles and other shapes. They also draw polygons in the coordinate plane given coordinates for the vertices and find the length of horizontal and vertical sides. In 7th grade, learners solve real-world and mathematical problems involving area of triangles, quadrilaterals, and polygons. In 8th grade, learners apply the Pythagorean Theorem to find the distance between two points in a coordinate system. In Algebra I, learners write equations of lines given a slope and point.	**Current Learning Connections** • Learners have already had experience with properties of quadrilaterals, equations of circles, and finding area and perimeter earlier in the course. They now apply this knowledge to working with coordinates. Learners will use the concept of distance and midpoint throughout the rest of the geometry course They apply the concepts later when calculating volumes and surface areas or when proving types of quadrilaterals given the ordered pairs of their vertices. They also use distance and midpoint when writing and deriving the equation of circles.	**Future Learning Connections** • Distance is an application important for many future concepts. For example, when writing equations of conic sections or converting between polar and rectangular coordinates or finding the magnitude of vectors.

Clarification Statement
The focus of this cluster is coordinate geometry. Students work with coordinates to find slope, distances, midpoints, and locations that are at a specified ratio from an endpoint. They then use this information to prove geometric relationships such as properties of quadrilaterals or location of a point on a circle. Using slope criteria for parallel and perpendicular lines, students write equations of lines. Using lengths computed from coordinates, students find perimeters and areas of polygons.

Common Misconceptions

Students may misunderstand the negative reciprocal slope with perpendicular lines.

Students commonly forget to take the square root of the constant to find the radius in the equation of a circle.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying the use of coordinates to prove simple geometric theorems algebraically because students will need to be familiar with discovering geometric properties before they can make sense of how to use coordinates to understand the properties.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- <HS.G-GPE.A.1>: This standard provides a foundation for work with using coordinates to prove simple geometric theorems algebraically because students should be able to work with and derive equations of geometric shapes before proving theorems. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with the use of coordinates to prove simple geometric theorems algebraically benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because students can then make connections between their learning and real-life applications.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with the use of coordinates to prove simple geometric theorems algebraically benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports (e.g., peer-tutors) because students may understand the properties but struggle formalizing a proof. Further, some students may feel mathematically sound using the coordinate plane but struggle connecting the algebra to a specific geometric property. In either case, working with a peer can reveal possible next steps and allows students time to critique the thinking of others and well as reflect on their own ideas.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with the use of coordinates to prove simple geometric theorems algebraically benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as providing graphic symbols with alternative text descriptions because this cluster requires a firm grasp of simple geometric properties, the coordinate plane, and the structures of proof writing. Each of these areas contains concepts that students may need help recalling. Consider a vocabulary strategy, word wall or other resource which includes terms, visual examples and student friendly definitions.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with the use of coordinates to prove simple geometric theorems algebraically benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because students are able to use a variety of tools such as graph paper and dynamic geometry software to make connections between characteristics of geometric figures and the coordinate plane. As a result, students may show their understanding through a variety of visual means, a written explanation and/or algebraic expressions.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with the use of coordinates to prove simple geometric theorems algebraically benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review because students must have a firm grasp of the geometric properties conceptually before they can apply them to algebraic proofs using coordinate math. Reviewing key properties students struggle with before and/or throughout this cluster can help students build the connection between prior studies and algebraic proofs.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on using coordinate to prove simple geometric theorems algebraically by clarifying mathematical ideas and/or concepts through a short mini-lesson because students may struggle to apply knowledge of geometric figures to coordinate algebra or vice-versa.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit of using coordinates to prove simple geometric theorems algebraically by addressing conceptual understanding because revisiting basic geometric shapes in the coordinate plane will assist in discovering geometric properties, which in turn will help them understand how coordinates can help prove theorems.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying the use of coordinates to prove simple geometric theorems algebraically because students will have the autonomy to make connections to personalized real-life situations in connection to this standard.

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying the use coordinates to prove simple geometric theorems algebraically the pattern of questions within the classroom is critical because it can allow students to build upon each other's ideas. Further, teachers can tap into student's prior knowledge and use it to promote learning for all students. Teachers can utilize strategic sequencing and questioning to encourage all students to participate in engaging with the content and seeing connections between multiple representations and solution methods. This can facilitate cross-content connections. When posing purposeful questions to the whole group, teachers should have protocols in place (classroom management) that tend to how students will respond to and discuss questions. Finally, teacher can use activities in which students are able to share (partners or groups) their thoughts and ideas in a judgement-free zone.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
SAT Item #: 421901 The linked assessment question addresses G-GPE.B., specifically the question requires students to analyze a given an equation and determine if a point is within the circle.

CollegeBoard | Question ID 421901

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Additional Topics in Math	Hard	Additional Topics in Math	Circles	1. Use definitions, properties, and theorems relating to circles and parts of circles, such as radii, diameters, tangents, angles, arcs, arc lengths, and sector areas to solve problems.	Calculator

A circle in the xy-plane has equation $(x+3)^2+(y-1)^2=25$. Which of the following points does NOT lie in the interior of the circle?

Question Difficulty: Hard

A. $(-7, 3)$

B. $(-3, 1)$

C. $(0, 0)$

D. $(3, 2)$

Choice D is correct. The circle with equation $(x+3)^2+(y-1)^2=25$ has center $(-3,1)$ and radius 5. For a point to be inside of the circle, the distance from that point to the center must be less than the radius, 5. The distance between $(3,2)$ and $(-3,1)$ is $\sqrt{(-3-3)^2+(1-2)^2}=\sqrt{(-6)^2+(-1)^2}=\sqrt{37}$, which is greater than 5. Therefore, $(3,2)$ does NOT lie in the interior of the circle.

Choice A is incorrect. The distance between $(-7,3)$ and $(-3,1)$ is $\sqrt{(-7+3)^2+(3-1)^2}=\sqrt{(-4)^2+(2)^2}=\sqrt{20}$, which is less than 5, and therefore $(-7,3)$ lies in the interior of the circle. Choice B is incorrect because it is the center of the circle. Choice C is incorrect because the distance between $(0,0)$ and $(-3,1)$ is $\sqrt{(0+3)^2+(0-1)^2}=\sqrt{(3)^2+(1)^2}=\sqrt{8}$, which is less than 5, and therefore $(0,0)$ in the interior of the circle.

Additional Assessment
http://tasks.illustrativemathematics.org/content-standards/HSG/GPE/B/4/tasks/605

The linked assessment question addresses G-GPE.B, specifically the question requires students to create their own quadrilateral and state verbally what observations they can make. Students then apply generic coordinate algebra to prove their observation. This assessment should be given to students after they've had practice applying coordinate math to prove conjectures about geometric figures. Students will engage in SMP 2, SMP 6 and possibly SMP 3 if they are asked to share and critique other's work.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on the use coordinates to prove simple geometric theorems algebraically, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example allowing students to relate this math to their home and community by plotting points to create an approximate map of a sectioned off area at home or parking lots in the community and finding the amount of fencing needed.	Home Economics: Connect to construction and agriculture.