

HS: GEOMETRY- SIMILARITY, RIGHT TRIANGLES, & TRIGONOMETRY

Cluster Statement: A: Understand similarity in terms of similarity transformations

<p>Standard Text</p> <p>HSG.SRT.A.1 Verify experimentally the properties of dilations given by a center and a scale factor:</p> <ul style="list-style-type: none"> • HSG.SRT.A.1.A: A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. • HSG.SRT.A.1.B: The dilation of a line segment is longer or shorter in the ratio given by the scale factor. 	<p>Standard for Mathematical Practices</p> <p>SMP2 Students reason abstractly and quantitatively by requiring students to make sense of quantities such as scale factor and their relationships to one another in problem situations</p> <p>SMP3 Students construct viable arguments and critique reasoning of other by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.</p> <p>SMP5 Students use appropriate tools strategically by expecting students to consider available tools when solving a mathematical problem. Tools might include pencil and paper, concrete models, rulers, protractors, compasses, calculators, and software or apps.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Determine the properties of dilation. Dilate when the center of dilation is in, on and out of the shape. Dilate when given a center of dilation and a scale factor. Determine the center of dilation and the scale factor from a diagram. Dilate using both positive and negative scale factors. Construct a dilation coordinate rules for dilations using any center of dilation. • Construct a dilated image which has corresponding line segments and is transformed along the same line from the center of the dilation. • Verify experimentally that a dilated image is similar to its pre-image by showing congruent, corresponding angles, and proportional sides. • Determine and apply the properties of dilation. <p>Webb’s Depth of Knowledge: 1-2</p> <p>Bloom’s Taxonomy: understand, apply</p>
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<p>Standard Text</p> <p>HSG.SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.</p>	<p>Standard for Mathematical Practices</p> <p>SMP3 Students construct viable arguments and critique reasoning of other by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.</p> <p>SMP5 Students use appropriate tools strategically by expecting students to consider available tools when solving a mathematical problem. Tools might include pencil and paper, concrete models, rulers, protractors, compasses, calculators, and software or apps.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Dilate figures using both positive and negative scale factors. • Identify corresponding angles and sides based on similarity statements. • Develop and write similarity statements for two polygons. • Determine if two triangles are similar based on their corresponding parts. • Establish a sequence of similarity transformations between two similar polygons. <p>Webb's Depth of Knowledge: 1-2</p> <p>Bloom's Taxonomy: understand, apply</p>
<p>Standard Text</p> <p>HSG.SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.</p>	<p>Standard for Mathematical Practices</p> <p>SMP7 Students look for and make use of structure by using the properties of similarity transformations to establish the AA criterion for two triangles to be similar.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Develop the Angle-angle criteria of similarity by expanding on previously learn properties of angles of Triangles • Use transformations as a tool to discover how AA similarity is derived and to make the process more efficient. • Express the properties of similarity transformations to explain the justification of AA similarity. <p>Webb's Depth of Knowledge: 1-2</p> <p>Bloom's Taxonomy: understand, apply</p>

Previous Learning Connections	Current Learning Connections	Future Learning Connections
<ul style="list-style-type: none"> In 8th grade, students perform transformations, including dilations, in a coordinate plane. They also identify a sequence of transformations that highlights the similarity of two figures. 	<ul style="list-style-type: none"> In later clusters within the Geometry course, students connect their conceptual understanding of similarity to explore trigonometric relationships including special right triangles and trigonometric ratios. 	<ul style="list-style-type: none"> Students will continue their work with similar figures in later courses when working with trigonometric ratios and the unit circle. They will use their understanding of dilations when working with functions to determine a stretch/shrink transformation.
<p>Clarification Statement This cluster establishes the basic criteria of similarity through an analysis of dilation transformations. Students formalize the similarity theorems and use the theorems to prove pairs of triangles are similar.</p>		
<p>Common Misconceptions A common misconception is thinking that the comparison of any pair of angles will be sufficient, when the comparison must be made using corresponding pairs.</p> <p>Students may incorrectly apply the scale factor. Some students often do not list the vertices of similar triangles in order. However, the order in which vertices are listed is preferred and especially important for similar triangles so that proportional sides can be correctly identified.</p>		
<p>Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies</p> <p>Pre-Teach</p> <p>Pre-teach (targeted): <i>What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?</i></p> <ul style="list-style-type: none"> For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when studying understanding similarity in terms of similarity transformation SRT. A. cluster because it is important for students to understand prior knowledge vocabularies as they are introduced to more complex one. <p>Pre-teach (intensive): <i>What critical understandings will prepare students to access the mathematics for this cluster?</i></p> <ul style="list-style-type: none"> Standard 8.G.A.4- Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two- dimensional figures, describe a sequence that exhibits the similarity between them. This standard provides a foundation for work with understanding similarity in terms of similarity transformation SRT. A. cluster because when students are not clear on the language structure of the mathematical problem at hand, it allows for a lot of misconceptions when the language has been presented in an advanced manner. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade-level instruction and assignments. <p>Core Instruction</p> <p><i>Access</i></p> <p>Interest: <i>How will the learning for students provide multiple options for recruiting student interest?</i></p> <ul style="list-style-type: none"> For example, learners engaging with understanding similarity in terms of similarity transformation benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because for students to understand how ratios and proportions expand into dilations of points and segments in a creative way for example with art can prove to be an effective cross-curricular activity to achieve the focus of this cluster. 		

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understanding similarity in terms of similarity transformation benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because incorporating a student's background into the lessons this cluster provides will prove to explain the connection between transformations in a cultural context. For example, with native American weaving practices and learning how the ratios and proportions of the lines bring out the intricate display of patterns.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with understanding similarity in terms of similarity transformation benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because student's misconceptions could arise from the lack of background knowledge, specifically in relation to the vocabulary used in this cluster.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding similarity in terms of similarity transformation benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing Computer-Aided-Design (CAD) or mathematical notation software because allowing students to connect this cluster with bigger ideas such as vectors in the field of computer science, helping students understand how dilations and extrapolations of specific segments aid in the creation of video games, phone applications etc.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with understanding similarity in terms of similarity transformation benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as making explicit cross-curricular connections because the practical approach of dilations can be visually expressed in the sciences and arts. Pattern recognition in these subjects when we explore physics (e.g. motion vectors) and art (e.g. pattern arrangement) can translate the big idea of how ratios are embedded in geometrical transformations.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understanding similarity in terms of similarity transformation SRT. A. cluster by

examining tasks from a different perspective through a short mini-lesson because allowing students to connect their knowledge of scale and transitions into more complex thought processes such as dilations can help re-shift the reason why this standard is important in this context.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on understanding similarity in terms of similarity transformation SRT. A. cluster by offering opportunities to understand and explore different strategies because students might be able to explore the concept of “same shape” much easier than the concept of congruence.

Extension

What type of extension will offer additional challenges to ‘broaden’ your student’s knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying understanding similarity in terms of similarity transformation SRT. A. cluster because introducing students to angle measurements and how they aid the process of transformation as well as congruence will help students avoid any misconceptions in the similarity cluster.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students’ home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion, we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn’t capable of mathematics, we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. “A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others.” For example, when studying understanding similarity in terms of similarity transformation, facilitating meaningful mathematical discourse is critical because instructors should be able to draw from student misconceptions and translate these into learning pieces which will engage students in building on each other’s ideas and deepen understanding of similarity transformation.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

SAT Item #: 421874 The linked assessment question addresses G-SRT.A, specifically the question requires students to analyze a composition of similar triangles to write a ratio of side lengths equivalent to a given ratio.

CollegeBoard Question ID 421874							
SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Right triangles and trigonometry	1. Solve problems in a variety of contexts using b. right triangle trigonometry;	Calculator



Triangles ABC and DEF are shown above. Which of the following is equal to the ratio $\frac{BC}{AB}$?

Question Difficulty: Medium

- A. $\frac{DE}{DF}$
- B. $\frac{DF}{DE}$
- C. $\frac{DF}{EF}$
- D. $\frac{EF}{DE}$

Choice B is correct. In right triangle ABC, the measure of angle B must be 58° because the sum of the measure of angle A, which is 32° , and the measure of angle B is 90° . Angle D in the right triangle DEF has measure 58° . Hence, triangles ABC and DEF are similar (by angle-angle similarity). Since $\frac{BC}{AB}$ is the side opposite to the angle with measure 32° and AB is the hypotenuse in right triangle ABC, the ratio $\frac{BC}{AB}$ is equal to $\frac{DF}{DE}$.

Alternate approach: The trigonometric ratios can be used to answer this question. In right triangle ABC, the ratio $\frac{BC}{AB} = \sin(32^\circ)$. The angle E in triangle DEF has measure 32° because $m\angle D + m\angle E = 90^\circ$. In triangle DEF, the ratio $\frac{DF}{DE} = \sin(32^\circ)$. Therefore, $\frac{DF}{DE} = \frac{BC}{AB}$.

Choice A is incorrect because $\frac{DE}{DF}$ is the reciprocal of the ratio $\frac{BC}{AB}$. Choice C is incorrect because $\frac{DF}{EF} = \frac{BC}{AC}$, not $\frac{BC}{AB}$. Choice D is incorrect because $\frac{EF}{DE} = \frac{AC}{AB}$, not $\frac{BC}{AB}$.

Additional Assessment:

Similar Triangles: <https://achievethecore.org/coherence-map/HS/G/116/611/611>

The linked assessment question addresses G-SRT.A, specifically the question requires students to look at two triangles with a given pair of congruent angles and state a series of transformations to map one onto the other. Students will apply rotation, translation and a generic dilation in this example. This assessment should be given to students after they've had time to work with concrete examples of dilations as this more complicated example requires abstract algebra in terms of the scale factor. Students will engage in SMP 1, SMP 2, and potentially SMP 3 depending on if students work in groups to share their solutions.

Relevance to families and communities:

During a unit focused on understanding similarity in terms of similarity transformation, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, For example: when looking at ancient pottery pattern samples, how can Mesopotamian pottery patterns relate to Native American or African pottery patterns displayed throughout various cultures.

Cross-Curricular Connections:

Drafting/Architecture: Connect to trusses, shadow lengths