HS: GEOMETRY- SIMILARITY, RIGHT TRIANGLES, & TRIGONOMETRY

Cluster Statement: B: Prove theorems involving similarity

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSG.SRT.B.4 **Prove theorems about triangles.** *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*	SMP3 Students construct viable arguments and critique reasoning of other by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments. SMP5 Students use appropriate tools strategically by expecting students to consider available tools when solving a mathematical problem. Tools might include pencil and paper, concrete models, rulers, protractors, compasses, calculators, and software or apps.	• Prove two triangles are similar using AA (could extend to SAS or SSS) similarity theorem. Use proportion to understand and justify logical claims. • Prove that two triangles are similar using the AA (could extend to SAS or SSS) similarity theorem. • Analyze a proof that two triangles are similar to determine if the argument is valid. • Prove various theorems about a triangle's properties. • Determine if two lines are parallel. • Set up and solve a proportion. • Apply the Pythagorean Theorem. • Organize and write a mathematical proof, including justification of my argument. **Webb's Depth of Knowledge: 1-3** **Bloom's Taxonomy:** understand, apply, analyze
Standard Text **HSG.SRT.B.5** **Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.**	**Standard for Mathematical Practices** SMP3 Students construct viable arguments and critique reasoning of other by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	**Students who demonstrate understanding can:** • Apply Theorems and postulates of triangle similarity to solve problems and prove relationships within and between geometric figures. • Use similar figures to find missing side lengths and missing angle measures. • Use congruent figures to find missing side lengths and missing angle measures.

	SMP5 Students use appropriate tools strategically by expecting students to consider available tools when solving a mathematical problem. Tools might include pencil and paper, concrete models, rulers, protractors, compasses, calculators, and software or apps.	• Determine if two geometric figures are congruent or similar. • Justify why two figures are congruent or similar using theorems from Geometry.
		Webb's Depth of Knowledge: 1-3
		Bloom's Taxonomy: understand, apply, analyze
<u>Previous Learning Connections</u> • In 8th grade, students developed the idea of "same shape" and "scale factor" as a definition of similarity. They will develop and connect these ideas when proving theorems within this cluster.	**<u>Current Learning Connections</u>** • Having previously studied dilations, students expand their definition of similarity to include congruence and dilation. These concepts lead to the criteria for triangle similarity. Students use proportional reasoning to approach problems involving similar figures. Trigonometric ratios will be developed using similar right triangles in connection to the work within this cluster.	**<u>Future Learning Connections</u>** • The trigonometric ratios (sine, cosine, tangent) will be founded on right triangles and similarity in subsequent learning. The Pythagorean theorem is generalized to non-right triangles by the Law of Cosines and Law of Sines.
Clarification Statement Students continue to develop their ability to create proofs while incorporating similarity. They will prove the Pythagorean Theorem based on similar triangles. They will then apply similarity to a variety of real world situations.		
Common Misconceptions Students may forget the importance of the order of vertices when making similarity statements. Students may confuse the alternate interior angle theorem and its converse as well as the Pythagorean Theorem and its converse.		
Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies **Pre-Teach** Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?* • For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time when studying proving theorems involving similarity because students can make connections between right triangles by drawing a perpendicular line to bisect a bigger right triangle to form two smaller ones.		

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.G.B.6- Explain a proof of the Pythagorean Theorem and its converse**.** This standard provides a foundation for work with proving theorems involving similarity because <when students understand the similarity between right triangles and the Pythagorean theorem, they will be able to make trigonometric connections between the two. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with proving theorems involving similarity benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information like the ones origami can visually conceptualize for students because origami allows students to visualize the concept of triangle proportionality and similarity.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with proving theorems involving similarity benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success because if students are able to use origami as a visual representation of triangle similarity proofs, students can quickly realize their patterns of error and pinpoint in an active classroom discourse with peers and facilitator.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with proving theorems involving similarity benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) because students will be able to see the relationship between right triangles and Pythagorean proofs. Understanding how it can be mapped onto a coordinate system can aid learners make connections between the similarities of transforming a right triangle in the different quadrants.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with proving theorems involving similarity benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because triangle similarity can be shown in more than one way for students to understand the similarities and differences.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with proving theorems involving similarity benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing templates, graphic organizers, concept maps to support note-taking because students will build connections from prior knowledge such as ratios, proportions, and scales. This will help students engage in the newer vocabulary and relate it to their prior knowledge.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on proving theorems involving similarity by revisiting student thinking through a short mini-lesson because it is important to understand where students are in terms of vocabulary such as similar and scale factors, so that when tackling the proofs students are not intimidated by the mathematical language expected.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on proving theorems involving similarity by offering opportunities to understand and explore different strategies because it might help clear up different misconceptions when students are allowed to display understanding in different ways. For example: with EL students one strategy would be to pair up individuals with native English-speaking classmates as they explore the task.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying proving theorems involving similarity because students might be able to broaden their knowledge of similarity into real world scenarios such as in the architectural field.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

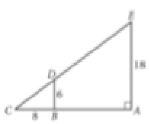
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying proving theorems involving similarity, the types of mathematical tasks are critical because the connections that can be utilized from an ELA standpoint (argumentative critical thinking) towards proving theorems in a mathematical world can be of instrumental value prior to introducing the procedural fluency of for example the Pythagorean theorem.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
SAT Item #: 422453 The linked assessment question addresses G-SRT.B, specifically the question requires students to solve for a side length in a series of composed right triangles.

CollegeBoard Question ID 422453

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Additional Topics in Math	Hard	Additional Topics in Math	Right triangles and trigonometry	1. Solve problems in a variety of contexts using c. properties of special right triangles.	No Calculator



In the figure above, $\overline{BD}$ is parallel to $\overline{AE}$. What is the length of $\overline{CE}$?

Question Difficulty: Hard

The correct answer is 30. In the figure given, since $\overline{BD}$ is parallel to $\overline{AE}$ and both segments are intersected by $\overline{CE}$, then angle BDC and angle AEC are corresponding angles and therefore congruent. Angle BCD and angle ACE are also congruent because they are the same angle. Triangle BCD and triangle ACE are similar because if two angles of one triangle are congruent to two angles of another triangle, the triangles are similar. Since triangle BCD and triangle ACE are similar, their corresponding sides are proportional. So in triangle BCD and triangle ACE, $\overline{BD}$ corresponds to $\overline{AE}$ and $\overline{CD}$ corresponds to $\overline{CE}$. Therefore, $\frac{BD}{CD} = \frac{AE}{CE}$. Since triangle BCD is a right triangle, the Pythagorean theorem can be used to give the value of CD: $6^2 + 8^2 = CD^2$. Taking the square root of each side gives $CD = 10$. Substituting the values in the proportion $\frac{BD}{CD} = \frac{AE}{CE}$ yields $\frac{6}{10} = \frac{18}{CE}$. Multiplying each side by CE, and then multiplying by $\frac{10}{6}$ yields $CE = 30$. Therefore, the length of $\overline{CE}$ is 30.

Additional Assessment:
http://tasks.illustrativemathematics.org/content-standards/HSG/SRT/B/4/tasks/1568

The linked assessment question addresses G-SRT.B, specifically the question requires students to show two triangles are similar and then use ratios of side lengths to derive the Pythagorean theorem. This assessment should be given to students after they've worked with setting up ratios for similar triangles. Students will engage in SMP 3, SMP 6 and, if asked to share and critique work of peers, SMP 3.

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on proving theorems involving similarity, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example in the field of forensics being able to determine the height of an individual contrasted with a fixed object in a video frame.	Physics- Connect to Vectors, particularly in resultants and to Dimensional Kinematics Art – Connect to drafting/architecture and to shapes and reflection within works of art