NEW MEXICO
Public Education Department

HS: STATISTICS & PROBABILITY- CONDITIONAL PROBABILITY & THE RULES OF PROBABILITY

Cluster Statement: A: Understand independence and conditional probability and use them to interpret data

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSS.CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	SMP2 Students reason abstractly and quantitatively by describing the probability of an event explicitly as well as describing what a given probability represents in context. SMP 6 Students attend to precision by using precise language, symbols and calculations when describing events and probabilities.	• Analyze a sample space to describe an event, union of events, intersection of events and complement of event • Use tree diagrams, organized lists, tables, and/or Venn diagrams to represent sample spaces. • Determine unions of sample spaces. • Determine intersections of sample spaces • Determine complements of sample sets. • Represent unions, intersections, and complements using set notation. **Webb's Depth of Knowledge:** 2-3 **Bloom's Taxonomy:** apply, analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSS.CP.A.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	SMP 2: Students reason abstractly and quantitatively by analyzing an event for independence (mathematically and logically) SMP 3: Students construct viable arguments and critique the reasoning of others by stating or defending the independence of an event using logical and/or mathematical arguments/claims.	• Explain and apply the formula to determine if two events are independent. • Test for independence using the definition of independent events. • State problems' independence and dependence contextually. **Webb's Depth of Knowledge: 1-3** **Bloom's Taxonomy:** understand, apply, analyze

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSS.CP.A.3 Understand the conditional probability of *A* given *B* as *P(A and B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.	SMP 2: Students reason abstractly and quantitatively by analyzing an event for independence (mathematically and logically) SMP 3: Students construct viable arguments and critique the reasoning of others by stating or defending the independence of an event using logical and/or mathematical arguments/claims. SMP 6: Students attend to precision by clearly stating the difference between p(a|b) and p(b|a)	• Read and state conditional probabilities of two events and explain how they are different. apply conditional probability to argue if two events are independent. • Calculate conditional probabilities. • Relate conditional probability to relative frequency tables and/or tree diagrams. • Use conditional probabilities to determine whether events are independent. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** understand, apply
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSS.CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*	SMP 2: Students reason abstractly and quantitatively by analyzing an event for independence (mathematically and logically) SMP 3: Students construct viable arguments and critique the reasoning of others by stating or defending the independence of an event using logical and/or mathematical arguments/claims. SMP 4: Students model with mathematics by creating two-way frequency tables from given information and by stating conditional probability from given information.	• Interpret and organize data to describe events and independence of events using 2-way frequency tables • Collect sample data from a real-world situation in order to examine conditional probabilities and independence of events. • Interpret and make sense of these in context of the situation. **Webb's Depth of Knowledge:** 2-3 **Bloom's Taxonomy:** apply, analyze

<table>
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<td>Standard Text

HSS.CP.A.5
Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</td>
<td>Standard for Mathematical Practices

SMP 1:
Students make sense of problems and persevere in solving them by reading a scenario closely to identify concepts of conditional probability.

SMP 4:
Students model with mathematics by applying a model/algorithm or logic to determine conditional probability of an event.</td>
<td>Students who demonstrate understanding can:
Analyze a scenario to describe conditional probability in terms of a real-life context
Use conditional probability to make decisions and justify claims of relationships to contextual situations.
Interpret conditional probability and independence across a variety of situations.
Distinguish between association and causality.
Webb’s Depth of Knowledge: 2-3

Bloom’s Taxonomy: apply, analyze</td>
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<td><u>Previous Learning Connections</u>
In previous years, learners used sample spaces to represent compound events in organized lists, tables, and tree diagrams. Learners are initially introduced to probability in 7th grade. They have investigated chance processes and developed probability models using experimental and theoretical probability.</td>
<td><u>Current Learning Connections</u>
Learners will use their knowledge of conditional probability and their skills of determining conditional probability to make decisions for real world situations. They will also expand the knowledge of this cluster to learn specific rules such as the Addition Rule. This knowledge will lead into permutations and combinations.</td>
<td><u>Future Learning Connections</u>
Learners will extend their learning to develop and make sense of the Multiplication Rule and Addition Rule. Future learning such as binomial distribution and statistical significance build upon conditional probability. Other applications are found in calculus, statistics, engineering, and the sciences.</td>
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<td colspan="3">Clarification Statement
A probability model may consist of a list or description of possible outcomes (the sample space) each of which is assigned a certain probability. Probability rules can be developed and understood through the use of the sample space. When events are independent, the outcome of the first event does not change the sample space for subsequent events. In dependent events, knowing one event has occurred affects the likelihood of another event occurring. Use of two-way frequency tables helps learners develop conceptual understanding of conditional probability. The use of tables, symbols, and real-world scenarios are emphasized. Learners consider the context of situations as they build mathematical models, interpret events, and explain results in terms of a probability model.</td>
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<td colspan="3">Common Misconceptions
Students may think that two events occurring is as simple as adding their probabilities.

Students may fail to check both parts of the algorithm. students may also assume if p(a)=p(a|b) then p(b) must equal p(b|a).</td>
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Students may also have an incomplete understanding of conditional probability.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying independence and conditional probability and use them to interpret data because students will need to develop an appropriate vocabulary usage for new subjects as well as tying it to previously learned material

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.SPC.7: This standard provides a foundation for work with independence and conditional probability and use them to interpret data Because this previously learned standard forms the foundation of understanding probability and probability models. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with Understanding independence and conditional probability and use them to interpret data benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features <give an example connected to this standard such as the size of text, images, graphs, tables, or other visual content; contrast between background and text or image; color used for information or emphasis; volume or rate of speech or sound; speed or timing of video, animation, sound, simulations, etc.; layout of visual or other elements; font used for print materials>because we can target individual learning styles for each student in order to master this concept.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with Understanding independence and conditional probability and use them to interpret data benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports (e.g., peer-tutors) because students will be able to gather and explore the reasoning of others and be able to include this into their own conclusions.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or*

puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with Understanding independence and conditional probability and use them to interpret data benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because newer terminology is built upon the older concepts in this case.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with Understanding independence and conditional probability and use them to interpret data benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because the terminology in this cluster may lead to confusion when solving and having multiple ways to approach the material will be beneficial.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with Understanding independence and conditional probability and use them to interpret data benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as making explicit cross-curricular connections (e.g., teaching literacy strategies in the social studies classroom) because there are numerous incidences of independent and conditional probability in real-world situations.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on independent events and conditional probability and use them to interpret data by clarifying mathematical ideas and/or concepts through a short mini-lesson because precise usage of terms in this cluster is the key to future understanding

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit independent events and conditional probability and use them to interpret data by confronting student misconceptions because there will be much confusion of terminology here and that will lead to errors in calculations later

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying independence and conditional probability and use them to interpret data because students will be able to analyze experiments and studies of their own choosing to further their understanding of independence and conditional probability

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying independence and conditional probability and use them to interpret data, the types of mathematical tasks are critical because the problems presented to students would need to reflect a relevance to the students' life experiences. This would offer a meaning to the student that the math can go beyond the classroom.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
SAT Item #: 1474700 The linked assessment question addresses S-SCPA.A, specifically the question requires students to read a two-way frequency table and state conditional probability in context.

CollegeBoard **Question ID 1474700**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Problem Solving and Data Analysis	Medium	Problem Solving and Data Analysis	Probability and conditional probability	1. Compute and interpret probability and conditional probability in simple contexts.	Calculator

Number of Adults Contracting Colds

	Cold	No cold	Total
Vitamin C	21	129	150
Sugar pill	33	117	150
Total	54	246	300

The table shows the results of a research study that investigated the therapeutic value of vitamin C in preventing colds. A random sample of 300 adults received either a vitamin C pill or a sugar pill each day during a 2-week period, and the adults reported whether they contracted a cold during that time period. What proportion of adults who received a sugar pill reported contracting a cold?

Question Difficulty: Medium

A. $\frac{11}{18}$

B. $\frac{11}{50}$

C. $\frac{9}{50}$

D. $\frac{11}{100}$

Choice B is correct. A total of 150 adults received the sugar pill. Of those, 33 reported contracting a cold. Therefore, $\frac{33}{150}$, or the equivalent $\frac{11}{50}$, is the proportion of adults receiving a sugar pill who reported contracting a cold.

Choice A is incorrect. This is the proportion of adults receiving a sugar pill and contracting a cold to all adults contracting a cold $\left(\frac{33}{54}\right)$. Choice C is incorrect. This is the proportion of adults who reported contracting a cold to all the participants in the study $\left(\frac{54}{300} = \frac{9}{50}\right)$. Choice D is incorrect. This is the proportion of adults who received a sugar pill and reported contracting a cold to all the participants in the study $\left(\frac{33}{300} = \frac{11}{100}\right)$.

Additional Assessment:
https://tasks.illustrativemathematics.org/content-standards/HSS/CP/A/4/tasks/2045

The linked assessment question addresses S.CPA.A, specifically the question requires students to look at data organized in a two-way frequency table and state probabilities. This assessment could be given to students after they've been introduced to the concept of conditional probabilities or as an exploration into finding conditional probabilities. Students will engage in SMP 4 and possibly SMP 3 if they are sharing or critiquing responses with peers.	
Relevance to families and communities: During a unit focused on independence and conditional probability and use them to interpret data, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, consider what types of conditional probability occur in the students' lives outside of school.	**Cross-Curricular Connections:** Social Studies: Connect to census data, voter demographics Forensic Science: Connect to crime scene analysis given suspect characteristics