

HS: STATISTICS & PROBABILITY- CONDITIONAL PROBABILITY & THE RULES OF PROBABILITY

Cluster Statement: B: Use the rules of probability to compute probabilities of compound events.

<p>Standard Text</p> <p>HSS.CP.B.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.</p>	<p>Standard for Mathematical Practices</p> <p>SMP 1: Students make sense of problems and persevere in solving them by reading a scenario closely to identify concepts of conditional probability.</p> <p>SMP 2: Students reason abstractly and quantitatively by using specific calculations and general description of events to describe a scenario.</p> <p>SMP 4: Students model with mathematics by applying a model/algorithm or logic to determine conditional probability of an event.</p> <p>SMP 6: Students attend to precision by using precise language and application of formulas when finding conditional probability of events.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Describe how to find conditional probabilities Calculate conditional probabilities Explain conditional probability in context of a scenario Interpret a given scenario and relate context to conditional probability, both abstractly and mathematically Justify reasoning in making conditional probability arguments <p>Webb's Depth of Knowledge: 1-3</p> <p>Bloom's Taxonomy: understand, apply, evaluate</p>
<p>Standard Text</p> <p>HSS.CP.B.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.</p>	<p>Standard for Mathematical Practices</p> <p>SMP 2: Students reason abstractly and quantitatively by explaining how to find the union of two events, both generally and in a given scenario.</p> <p>SMP 3: Students construct viable arguments and critique the reasoning of others by defending the probability of a union occurring in a given context.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Calculate the union of two events Explain the union of two events in terms of the context of the problem Given a scenario, interpret what the union of two events represents and calculate the probability <p>Webb's Depth of Knowledge: 1-2</p>

	<p>SMP 4: Students model with mathematics by applying the formula for unions to a contextual probability problem.</p> <p>SMP 6: Students attend to precision by describing overlapping events in context and accounting for the overlap in calculations.</p>	<p>Bloom’s Taxonomy: understand, apply</p>
<p>Previous Learning Connections</p> <ul style="list-style-type: none"> In grade 7, learners have investigated chance processes, and developed, used, and evaluated probability models. They have learned that probability of a chance event is a number between 0 and 1 (7.SP.5) and found probabilities of compound events (7.SP.8). 	<p>Current Learning Connections</p> <ul style="list-style-type: none"> Learners are expanding their understanding and skills explored and learned in the G.SP.A cluster. They are discovering that conditional probability can be found from a narrowed subset of the original sample space. 	<p>Future Learning Connections</p> <ul style="list-style-type: none"> Future learning such as binomial distribution and statistical significance build upon conditional probability. Other applications are found in calculus, statistics, engineering, and the sciences.
<p>Clarification Statement</p> <p>The development and uses of algorithms are built on conceptual understanding as concepts of sample spaces are explored and deepened. Probabilities are described in terms of the intersections and unions of events. Venn diagrams and two-way frequency tables will be generalized to discover patterns and create algorithms and formulas that can be used in routine fashion. Although learners will use these formulas strategically to determine different values, the use of tree diagrams, organized lists, and other tools will help make sense of these abstractions.</p>		
<p>Common Misconceptions</p> <p>Students may struggle with determining the correct denominator. They may use the total rather than the specified event.</p> <p>Students may struggle to understand the “overlap” in compound events.</p>		
<p>Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies</p> <p>Pre-Teach</p> <p>Pre-teach (targeted): <i>What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?</i></p> <ul style="list-style-type: none"> For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying the rules of probability to compute probabilities of compound events because this cluster focuses on compound probability with the introduction of combinations and permutations which take practice and perseverance to master <p>Pre-teach (intensive): <i>What critical understandings will prepare students to access the mathematics for this cluster?</i></p> <ul style="list-style-type: none"> 7.SPC.8: This standard provides a foundation for work with the rules of probability to compute probabilities of compound events because this older standard introduces the formal definitions of compound events and calls for modeling of the standard to represent its situations. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support 		

prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Physical Action: How will the learning for students provide a variety of methods for navigation to support access?

- For example, learners engaging with Using the rules of probability to compute probabilities of compound events benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as varying methods for response and navigation by providing alternatives to <requirements for rate, timing, speed, and range of motor action with instructional materials, physical manipulatives, and technologies; physically responding or indicating selections; physically interacting with materials by hand, voice, single switch, joystick, keyboard, or adapted keyboard because studies prove learning is active and activity during learning accelerates concept acquisition

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with Using the rules of probability to compute probabilities of compound events, benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that is frequent, timely, and specific because it shows a care for what your students are achieving and immediate feedback is essential before students errors become what they believe to be the concept.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with Using the rules of probability to compute probabilities of compound events, benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as highlighting structural relations or make them more explicit because this aids in organizing and contextualizing the increasingly complex structure of intermediate and advanced probability.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with Using the rules of probability to compute probabilities of compound events, benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because this will aid in the use of computation of advanced probabilities.

Internalize

Executive Functions: How will the learning for students support the development of executive functions to allow them to take advantage of their environment?

- For example, learners engaging with Using the rules of probability to compute probabilities of compound events, benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to

build from it such as providing graphic organizers and templates for data collection and organizing information because the amount of information provided in this cluster will need to be organized by the student in order to avoid confusion of concepts and equations to be used here.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on computing probabilities of compound events by providing specific feedback to students on their work through a short mini-lesson because by pinpointing minor errors in a multistep process we can perfect our processes.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on computing probabilities of compound events by addressing conceptual understanding because by sitting down and helping a student analyze their process, we can bring them to a deeper level of understanding of their errors as well as the content.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students. When studying the rules of probability to compute probabilities of compound events because students working together opens up new paths of thinking and reasoning for them.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying the rules of probability to compute probabilities of compound events, goal setting is critical because in this cluster of Statistics and probability it necessary to be organized and complete in which procedure must be used at a given time. Helping a student set a piecewise organizational goal will assist. This can be organized linearly, as a graphic organizer, or any method of the students choosing.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

SAT Item #: 4168721 The linked assessment question addresses S-CPA.A, specifically the question requires students to read a two-way frequency table and state compound probability in context.

CollegeBoard		Question ID 4168721					
Assessment SAT	Test Math	Cross-Test and Subscore Problem Solving and Data Analysis	Difficulty Medium	Primary Dimension Problem Solving and Data Analysis	Secondary Dimension Probability and conditional probability	Tertiary Dimension 1. Compute and interpret probability and conditional probability in simple contexts.	Calculator Calculator

Observed Matings among Fruit Flies

		Female fruit fly group		Total
		Female raised on starch	Female raised on maltose	
Male fruit fly group	Male raised on starch	22	9	31
	Male raised on maltose	8	20	28
Total		30	29	59

The table above shows the observed mating frequencies among a group of fruit flies raised on either a starch medium or a maltose medium. What fraction of the observed matings were between fruit flies that were raised on the same medium?

Question Difficulty: Medium

- A. $\frac{9}{31}$
- B. $\frac{17}{59}$
- C. $\frac{31}{59}$
- D. $\frac{42}{59}$

Choice D is correct. According to the table, a total of 59 fruit fly matings were observed. Of these, 22 matings were between males and females who were both raised on starch and 20 were between males and females who were both raised on maltose. Therefore, a total of $22 + 20$ or 42 of the 59 observed matings were between fruit flies raised on the same medium. This situation is represented by the fraction $\frac{42}{59}$.

Choice A is incorrect. This represents the fraction of observed fruit fly matings between females raised on maltose and males raised on starch. Choice B is incorrect. This represents the fraction of observed fruit fly matings between fruit flies raised on different mediums. Choice C is incorrect. This represents the fraction of observed fruit fly matings with males raised on starch.

Additional Assessment:

<http://tasks.illustrativemathematics.org/content-standards/HSS/CP/B/7/tasks/1112>

The linked assessment question addresses S.CPA.B, specifically the question requires students to apply a variety of more complex probability theorems to a contextual problem (independence, conditional probabilities and union of events). This assessment should be given to students after they've been introduced to these concepts. Students will engage in SMP 4, SMP 6 and possibly SMP 3 if they are asked to share and critique solutions.

Relevance to families and communities:

During a unit focused on the rules of probability to compute probabilities of compound events, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, consider what types of compound probability a student will experience on a daily basis to form a foundation for this concept

Cross-Curricular Connections:

Social Studies: Connect to voter demographics

Science: Connect to crime science investigation/analysis