New Mexico Mathematics Instructional Scope for Geometry

June 2020

TABLE OF CONTENTS:

Overview

This mathematics instructional scope was created by a cohort of New Mexico educators and the New Mexico Public Education Department. This document is organized into three sections. Section 1 describes how to use this document to support equitable and excellent mathematics instruction. Section 2 contains planning support for each cluster of mathematics standards within the grade level or course. Section 3 provides additional resources, references, and glossary.

The intention of this document is to act as companion during the planning process alongside High Quality Instructional Materials (HQIM). A sample template is presented to show a quick snapshot of planning supports provided within each cluster of standards in section 2.

During the creation of this document, we leveraged the work of other states, organizations, and educators from across country and the world. This work would not have been possible without all that came before it and we wish to express our sincerest gratitude for everyone that contributed to the resources listed within our references. This document is a work in progress and in some circumstances, our team of New Mexico educators may have embedded content from resources that have yet to be cited, as these elements are discovered in the use of this tool the references in section 3 will be updated.

Section 1: New Mexico Instructional Scope for Supporting Equitable and Excellent Mathematics Instruction

To better understand the planning supports provided in section 2, for each cluster of standards, this section provides a brief description of each planning support including: *what* support is provided; *why* the planning support is critical for equitable and excellent mathematics instruction; and, *how* to use the planning support with HQIM.

Cluster Statement

What: The New Mexico Mathematics Standards are grouped by Domains with somewhere between 4 to 10 domains per grade level. Within each domain the standards are arranged around clusters. Cluster statements summarize groups of related standards. The cluster statement planning support also indicates if the clusters is major, supporting, or additional work of the grade.

Why: The New Mexico Mathematics Standards require a stronger *focus*[1] on the way time and energy are spent in the mathematics classroom. Students should spend the large majority of their time (65-85%) on the major clusters of the grade/course. Supporting clusters and, where appropriate, additional clusters should be connected to and engage students in the major work of the grade.

How: When planning with your HQIM consider the time being devoted to major versus additional or supporting clusters. Major Work of each grade should be designed to provide students with strong foundations for future mathematical work which will require more time than additional or supporting clusters. Consider also the ways the HQIM makes explicit for students the connections between additional and supporting clusters and the major work of the grade.

Standard Text

What: Each cluster level support document contains the text of each standard within the cluster.

Why: The cluster statement and standards are meant to be read together to understand the structure of the standards. By grouping the standards within the cluster the connectedness of the standards is reinforced.

How: The text of the standards should always ground all planning with HQIM. Reading the standards within a cluster intentionally focuses on the connections within and among the standards.

Standards for Mathematical Practice

What: The Standards for Mathematical Practice describe the varieties of expertise and habits of mind that mathematics educators at all levels should seek to develop in their students.

Why: Equitable and excellent mathematics instruction supports students in becoming confident and competent mathematicians. By engaging with the standards for mathematical practice students are engaging in the practice of doing mathematics and development of mathematical habits of mind—the ability to think mathematically, analyze situations, understand relationships, and adapt what they know to solve a wide range of problems, including problems they may not look like any they have encountered before.[2]

How: When planning with HQIM it is critical to consider the connections between the content standards and the standards for mathematical practice. The planning supports highlight a few practices in which students could engage when learning the content of the standard. Note it is not necessary or even appropriate to engage in all of the practices every day, rather choosing a few and spending time intentionally supporting students in learning both the what (content standards) and the how (standards for mathematical practice) will create a stronger foundation for ongoing learning.

Students Who Demonstrate Understanding Can (Webb's Depth of Knowledge and Bloom's Taxonomy)

What: The New Mexico Mathematics Standards include each aspect of mathematical rigor: conceptual understanding, procedural skill and fluency, and application to the real world.[3] This planning support considers which aspect(s) of rigor are within each standard and then identifies academics skills students need to demonstrate comprehension of the standard and associated mathematical practices. The statements also highlight both the receptive (listening and reading) and expressive (speaking and writing) parts of language by considering the types of mathematical representations (verbal, visual, symbolic, contextual, physical) within the standard and what students need to do with them. The planning supports also provide information about two common classifications on cognitive complexity, Webb's Depth of Knowledge and Bloom's Taxonomy.

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[1] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

[2] Seeley, C. L. (2016). Math is Supposed to Make Sense. In *Making sense of math: How to help every student become a mathematical thinker and problem solver*. Alexandria, VA, USA: ASCD. (P. 13)

[3] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

Why: Analyzing standards alongside the standards for mathematical practice provide a fuller picture of the mathematical competencies demanded in the standard.

How: When planning for a cluster of standards with your HQIM a critical first step is to analyze the content and language demands of the standards and standards for mathematical practice. The analysis can be used to inform formative assessment, or it can be used to plan/design appropriate formative assessment.[4] The planning supports provide a possible break-down of the standard that can serve as the basis for this sort analysis.

Connections

What: The New Mexico Mathematics Standards are designed around coherent progressions of learning. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but an extension of previous learning.[5] The connections to previous, current and future learning make this coherence visible.

Why: Students build stronger foundations for learning when they see mathematics as an inter-connected discipline of relationships rather than discrete skills and knowledge. The intentional inclusion of connections to previous, current, and future learning can support a more inter-connected understanding of mathematics.

How: When planning with HQIM use the connection planning supports to find ways to support students in making explicit connections within their study of mathematics.

Clarification Statement

What: The clarification statement provides greater clarity for teachers in understanding the purpose of the standards within a cluster.

Why: The New Mexico Mathematics Standards illustrate how progressions support student learning within each major domain of mathematics. The clarification statement provides additional context about the ways each cluster of standards supports student learning of the larger learning progression.

How: When planning with HQIM use the clarification statement to support an understanding of how the materials use specific types of representations or change the learning sequence from instructional approaches not grounded in progressions of learning.

Common Misconceptions

What: This planning support identifies some of the common misconceptions students develop about a mathematical topic.

Why: Students create misconceptions based on an over generalization of patterns they notice or an over reliance on rules rather than underlying mathematics. Rules in mathematics expire[6] over time (e.g., you can't subtract 1-3) as students expand their knowledge of mathematics (e.g., from whole numbers to rational numbers). It is critical to understand some of the common misconceptions students can develop so we can address them directly with students and continue to build a strong foundation for their mathematical learning.

How: When planning with your HQIM look for ways to directly address with students some common misconceptions. The planning supports in this document provide some possible misconceptions and your HQIM might include additional ones. The goal is not to avoid misconceptions, they are a natural part of the learning process, but we want to support students in exploring the misconception and modifying incorrect or partial understandings.

[4] English Learners Success Forum. (2020). ELSF | Resource: Analyzing Content and Language Demands. Retrieved from https://www.elsuccessforum.org/resources/math-analyzing-content-and-language-demands
[5] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics
[6] Cardone, T. (n.d.). Nix the Tricks. Retrieved from https://nixthetricks.com/

Multi-Layered System of Supports/Suggested Instructional Strategies

What: The section on Multi-Layered Systems of Supports(MLSS)/Suggested Instructional Strategies is designed to support teachers in planning for the needs of all students. Each section includes options for pre-teaching, reteaching, extensions and core instructional supports for students. Targeted pre-teaching and reteaching support student's acquisition of the knowledge and skills identified in the New Mexico Mathematics Standards to support student success with high-quality differentiated instruction. Intensive supports may be provided for a longer duration, more frequently, smaller groups, or otherwise be more intensive than targeted supports. Progress monitoring should occur to assess students' responses to additional supports, see Standards Aligned Instructionally Embedded Formative Assessment Resources.

Why: MLSS is a holistic framework that guides educators, those closest to the student, to intervene quickly when students need additional supports. The framework moves away from the "wait to fail" model and empowers teachers to use their professional judgement to make data-informed decisions regarding the students in their classrooms to ensure academic success with the grade level expectations of the New Mexico Mathematics Standards.

How: When planning with your HQIM use the suggestions for pre-teaching as a starting point to determine if some or all of the students in your classroom may need targeted or intensive pre-teaching at the start of unit to ensure they can access the grade level material with the unit. The core-instruction and reteach sections work together to support planning within a unit, look for the ways the materials are supporting greater access for all students and providing options to revisit materials based on formative assessments. The planning supports for each cluster are grounded in the Universal Design Learning (UDL) Framework, additional planning supports based on this framework can be found in Section 3 of this document in the part titled, Planning Guidance for Multi-Layered Systems of Support: Core Instruction.

Culturally and Linguistically Responsive Instruction

What: Culturally and Linguistically Responsive Instruction (CLRI), or the practice of situational appropriateness, requires educators to contribute to a positive school climate by validating and affirming students' home languages and cultures. Validation is making the home culture and language legitimate, while affirmation is affirming or making clear that the home culture and language are positive assets. It is also the intentional effort to reverse negative stereotypes of non-dominant cultures and languages and must be intentional and purposeful, consistent and authentic, and proactive and reactive. Building and bridging is the extension of validation and affirmation. By building and bridging students learning to toggle between home culture and linguistic behaviors and expectations and the school culture and linguistic behaviors and expectations. The building component focuses on creating connections between the home culture and language and the expectations of school culture and language for success in school. The bridging component focuses on creating opportunities to practice situational appropriateness or utilizing appropriate cultural and linguistic behaviors.[7]

Why: The mathematical identities of students are shaped by the messages they receive about their ability to do mathematics and the power of mathematics in their lives outside of school.[8] Mathematics educators must intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages. In addition, create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to supports students in creating mathematical identities as capable mathematicians within school and society.

How: When planning instruction is critical to consider ways to validate/affirm and build/bridge from your students cultural and linguistic assets. The planning supports for each cluster provide an example of how to support equity-

[7] Hollie, S. (2011). *Culturally and linguistically responsive teaching and learning*. Teacher Created Materials.

[8] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics. (P. 14)

based teaching practices. Look for additional ways within your HQIM to ensure all students develop strong mathematical identities.

Standards Aligned Instructionally Embedded Formative Assessment Resources

What: Formative Assessment is the planned, ongoing process used by all students and teachers during learning and teaching to elicit and use evidence of student learning to improve student understanding of the outcomes and support students to become directed learners. All New Mexico educators have access to standards aligned instructionally embedded formative assessments: iStation at K-2; Cognia at 3-8, and the SAT Suite Question Bank at 9-12. These are intended to be used during instruction for each at each grade alongside assessments within your HQIM.

Why: When student thinking is made visible the teacher can examine the progression of learning towards the goals of the standards and adjust instruction as necessary. By including students in the assessment and analysis process students become strategic and goal-directed with their learning.

How: The planning supports at each cluster provide an example of a task that addresses one more aspect of the cluster of standards. This example can be used to discuss possible responses by students and next steps for instruction. A similar process can then be used to identify additional items from one of the formative assessment resources provided by NM PED and your HQIM.

Relevance to Families and Communities

What: Relevance to families and communities requires finding the relevance of mathematics outside of the classroom by connecting to families and communities and learning about varied and often unexpected ways they use mathematics.

Why: When school mathematics is connected to the mathematics outside of school students can build a bridge between their ways of thinking about quantities outside and inside school created a bridge between home and school.

How: When planning at the year and unit level with you HQIM find ways to intentionally learn from your families and communities the cultural and linguistic ways they use mathematics outside of school.

Cross-Curricular Connections

What: New Mexico defines cross-curricular connections as connections between two or more areas of study made by teachers or students within the structure of a subject.

Why: The purpose of planning cross-curricular connections in an instructional sequence is to ensure that students build connections and recognize the relevance of mathematics beyond the mathematics classroom.

How: When planning with HQIM look for opportunities to make explicit connections to other content areas such as the examples provided for each cluster.

New Mexico Public Education Department

Template of the New Mexico Cluster Level Planning Support for the New Mexico Mathematics Standards

<GRADE/COURSE/DOMAIN ABBREVIATION: DOMAIN NAME>

Cluster Statement: Statement from New Mexico Mathematics Standards summarize a group of related standards.

Major/Additional/Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)
Identifies if the cluster is major, additional or supporting work of the grade.

Standard Text Full text of the standard	**Standard for Mathematical Practices** The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.	**Students who demonstrate understanding can:** The cognitive skills students perform to demonstrate to comprehension of a standard. **Depth Of Knowledge:** Correlation of standard to Webb's Depth of Knowledge **Bloom's Taxonomy:** Correlation of standard to Bloom's Taxonomy
Connections to Previous Learning: Supports student connections to learning from previous grade levels.	**Connections to Current Learning** Supports student connections to learning within the grade level.	**Connections to Future Learning** Supports student connections to learning in a future grade.

Clarification Statement: Clarifies the language of the standard.

Common Misconceptions: Guidance on where a student misconception or misunderstanding could potentially occur.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach
Pre-teach (targeted): Guidance for how to activate students' knowledge to support their learning.
Pre-teach (intensive): Guidance for how to use earlier grade standards to build a strong foundational understanding upon which to build grade level concepts.

Core Instruction
Access: Guidance for optimizing universal access to learning experiences.
Build: Guidance for supporting students build their understanding of the cluster.
Internalize: Guidance for ensuring student internalization of the learning goal.

Re-teach
Re-teach (targeted): Guidance for adjusting instruction during a unit by using formative assessment data.
Re-teach (intensive): Guidance for analyzing assessment data to identify content that would benefit from more intensive reteaching.
Extension Ideas: Suggestions that offer additional challenges to 'broaden' students' knowledge of the mathematics within the cluster.

Culturally and Linguistically Responsive Instruction: Provides equity based instructional suggestions aligned to the cluster of standards

Standards Aligned Instructionally Embedded Formative Assessment Resources: Includes reference to high-quality formative assessment resources, including examples from New Mexico's formative assessment banks.

Relevance to Families and Communities: Connecting with families and communities to create relevant connections between mathematics inside and outside of school.	**Cross Curricular Connections:** Includes examples of how the cluster provides opportunities to connect to other disciplines such as literacy, science, social studies, and the arts.

Section 2: Cluster Level Planning Support for the New Mexico Mathematics Standards

TABLE OF CONTENTS:

[9] Appendix A of the Common Core State Standards was used to determine the standards within each high school course, (+) were not included in this version of the instructional scope.

Public Education Department

HS: GEOMETRY-CONGRUENCE

Cluster Statement: Experiment with transformations in the plane

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
G.CO.A.1 **State and apply precise definitions of angle, circle, perpendicular, parallel, ray, line segment, and distance based on the undefined notions of point, line, and plane.**	SMP1 Students make sense of problems and persevere in solving them by making sense of definable and undefinable terms that exist in geometry. SMP7 Students look for and make use of structure by using a basic understanding of definitions and being able to apply them to generalizations of the rule.	• Demonstrate the knowledge of precise definitions of angles, line, point, plane, circles, perpendicular and parallel lines, and line segments. • Calculate the linear distance and arc length. • Demonstrate the use of proper notation. • Make connections with rigid motions in relation to the definitions of words above. **Webb's Depth of Knowledge:** 2-3 **Bloom's Taxonomy:** Understand, Apply
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
G.CO.A.2 **Represent transformations in the plane. (e.g., using transparencies and/or geometry software) a. Describe transformations as functions that take points in the plane as inputs and give other points as outputs. b. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus dilation).**	SMP 1 Students can make sense of problems and persevere in solving them by representing the transformations in the plane while also describing and comparing them in order to solve problems. SMP 3 Students make arguments and critique the arguments of others when they compare strategies for finding sequences of rigid transformations that take one figure onto another SMP 6 Students can attend to precision by utilizing precise language when describing the transformations with the appropriate mathematics vocabulary.	• Write a function that maps a preimage to its image from an image or a description of a transformation. • Understand which transformations result in figures with congruent sides and angles and which do not. • Represent transformations in the plane. • Compare rigid motions that preserve distance and angle measure (translations, reflections, rotations) to transformations that do not preserve both distance and angle measure (e.g. stretches, dilations). • Understand that rigid motions produce congruent figures while dilations produce similar figures. **Webb's Depth of Knowledge:** 2-3

		Bloom's Taxonomy: Understand, Apply
Standard Text G.CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and/or reflections that map the figure onto itself.	**Standard for Mathematical Practices** SMP1 Students make sense of problems and persevere in solving them by applying transformations to a given shape SMP4 Students model with mathematics by constructing transformations SMP5 Students use appropriate tools strategically by expecting students to consider available tools when solving a mathematical problem. Tools might include pencil and paper, concrete models, rulers, protractors, compasses, calculators, and software or apps. SMP7 Students look for and make use of structure by understanding different types of rotations and/or reflections and be able to generalize these understandings to other objects	**Students who demonstrate understanding can:** • Identify and describe the different symmetries (line symmetry, rotational symmetry, point symmetry) of a figure. • Determine the maximum possible lines of symmetries that exist for a given polygon. • Determine the order and angle of a rotational symmetry. • Determine the symmetries of a parallelogram, rectangle, rhombus, square, trapezoid and regular polygon. • Understand symmetry in terms of transformations" (The Common Core Mathematics Companion). • Explore which shapes are symmetric and what symmetries they will have" (The Common Core Mathematics Companion). • Develop generalizations for the symmetries held by various geometric shapes" (The Common Core Mathematics Companion). • Determine the properties of a shape based on its symmetries" (The Common Core Mathematics Companion).
		Webb's Depth of Knowledge: 2-3
		Bloom's Taxonomy: Understand, Apply

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
G.CO.A.4 **Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.**	SMP3 Students construct viable arguments and critique reasoning of other by justification of method of transformations. SMP7 Students look for and make use of structure by applying the rules and definitions of transformations.	• Describe rotations, reflections and translations. • Determine and apply the properties of the isometric transformations. • Identify which transformation has taken place based on the properties found between the preimage and image. • Identify the orientation relationship between the preimage and image. • Explore properties of transformations using common geometric relationships (e.g., parallel, perpendicular, and congruence)" (The Common Core Mathematics Companion). • Develop definitions of the transformations in terms of their properties" (The Common Core Mathematics Companion).
		Webb's Depth of Knowledge: 2-3
		Bloom's Taxonomy: understand, apply
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
G.CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure	SMP1 Students make sense of problems and persevere in solving them by applying the various types of transformations SMP3 Students construct viable arguments and critique reasoning of other by justification of method of transformations SMP5 Students use appropriate tools strategically by expecting students to consider available tools when	• Model a sequence of transformations including reflections, rotations, and translations with a geometric figure. • Determine the sequence of transformations performed between a given preimage and image. • Describe which single transformation is the result of two reflections over parallel lines. • Describe which single transformation is the result of two reflections over intersecting lines.

	solving a mathematical problem. Tools might include pencil and paper, concrete models, rulers, protractors, compasses, calculators, and software or apps.	• Identify a transformation by its coordinate rule and then apply it to transform the shape. • Demonstrate how some composite transformations are not commutative. **Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply
Previous Learning Connections • This cluster aligns directly to the learning in 8th grade, when students were introduced to congruence and similarity using physical models, transparencies, or geometry software. They have described sequences of rigid motions informally and in terms of coordinates. Learners have verified experimentally the properties of transformations, and describe their effects on two-dimensional figures using coordinates.	**Current Learning Connections** • Transformation definitions will serve as the basis for theorems that will be proven later in the year.	**Future Learning Connections** • In Algebra II, students will connect their knowledge of transformations to functions. They will use transformation language to compare a function to its parent function, identify lines of symmetry, and other characteristics of functions.

Clarification Statement:
Students formalize their transformation language by building precise definitions based on properties. They use formal notation and precise descriptions of transformations and sequences of transformations. Rotational and reflection symmetry are identified with a specific degree of rotation or line(s) of symmetry.

Common Misconceptions
Students may confuse transformation and translation and/or not know how to express the differences between the two terms. Other misconceptions often lie within the connection between the terms (angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.) in the standard G.CO.A.1 and their application in this standard.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying transformations in the plane because students are experimenting with transformations and precise language will be important.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 4.GA.1: This standard provides a foundation for work with transformations in the plane because students should have already built a firm grasp of key vocabulary such as angle, point, line, parallel, perpendicular, etc. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception*: How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with experimenting with transformations in the plane benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as descriptions (text or spoken) for transformations, auditory cues and/or vocabulary word wall for key terms because the section is vocabulary heavy and students may not have a satisfactory grasp of prior skills and/or may have a different first language and by providing the visual meaning along with a written description and re-explained orally provides multiple means of access for the students. Providing auditory cues and/or a vocabulary word wall for key terms can reinforce this. Further, using patty paper/trace paper/geometric software to create transformations allows for interaction between the student and the material because it provides opportunities for the student to physically perform transformations, along with seeing a visual, hearing a description, and reading a definition.

Build

Effort and Persistence*: How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with transformations in the plane benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics representations and scaffolds because students may struggle with visualizing a series of transformations and/or have a hard time abstractly connecting the concept of transformations to angles, circles, parallel and perpendicular lines etc.

Language and Symbols*: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with transformations in the plane benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because most, if not all, of these terms have been introduced at prior grades and students will have some incoming concept of some of the terms. Building off their prior knowledge and/or identifying misunderstandings at this point can ensure precise use of language throughout the learning.

Expression and Communication*: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with transformations in the plane benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., cut out objects, geometric software, patty paper/trace paper) because students can physically create the transformations and/or series of transformations which may deepen their understanding.

Internalize

Executive Functions*: How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with transformations in the plane benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as embedding prompts to "show and explain your work" because students may see transformations in a variety of ways (e.g. one student may see a reflection followed by a rotation where another student sees a series of reflections, etc.).

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on transformations in the plane by clarifying mathematical ideas and/or concepts through a short mini lesson because students may struggle with rotations or reflections about a point/line that is not at the origin.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on transformations in the plane by offering opportunities to understand and explore different strategies because there may be several approaches to map one object onto another.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying transformations in the plane because geometric software can allow students to experiment with combinations of transformations and using such software can allow for abstract application of knowledge of transformations.

Culturally and Linguistically Responsive Instruction:[1]

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

[1] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics.
Boston, M., Dillon, F., & Miller, S. (2017). *TAKING ACTION: IMPLEMENTING EFFECTIVE MATHEMATICS TEACHING PRACTICES IN Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.
Los Angeles, CAUSA. (n.d.). VABB™. Retrieved from https://www.culturallyresponsive.org/vabb

<table>
<tr><td colspan="2">Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Equity Based Practices (Tasks): The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying Exploring Transformations in the Plane the types of mathematical tasks are critical because the purpose of the standard is for students to explore and experience the transformations for themselves. This requires teachers to select tasks and activities that provide for students to experiment with the transformations in the plane rather than explicitly tell students what steps to take.</td></tr>
<tr><td colspan="2">Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/HSG/CO/A/tasks/1468

The linked assessment question addresses G-CO.A, specifically the question requires students to look at lines of symmetry using reflections. Two different arguments are presented using triangle congruence and another which uses rotations and reflections. This assessment should be given to students after they've been introduced to the formal definition of reflections. Students will engage in SMP 1, SMP 7, and potentially SMP 3 depending on if students work in groups to share their solutions.</td></tr>
<tr><td>Relevance to families and communities:

During a unit focused on Exploring Transformations in the Plane, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, viewing artwork from a variety of cultures or having students select and bring in artwork that shows specific transformations can build the bridge between things they may have seen at home to what they are seeing in school. These cultural connections can be made explicitly or can be incorporated into word problems and other abstract application style questions.</td><td>Cross-Curricular Connections:

Art: Support students in making connections in between geometric transformation and art, especially in relation to the idea of repetition and perspective in artwork.[2]</td></tr>
</table>

[2] https://uwm.edu/arts/wp-content/uploads/sites/71/2020/01/Peterson_ArtsECOFellowsLesson2019.pdf

NEW MEXICO
Public Education Department

HS: GEOMETRY-CONGRUENCE		
Cluster Statement: B: Understand congruence in terms of rigid motions		
Standard Text **HSG.CO.B.6** **Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.**	**Standard for Mathematical Practices** MP3 Students construct viable arguments and critique reasoning of other when proving congruency and describe transformations. MP6 Students attend to Precision by using accurate transformations to map shapes onto other shapes. MP7 Students look for and make use of structure by applying rules of transformations to prove congruence of objects.	**Students who demonstrate understanding can:** • Demonstrate that two figures are congruent if there is a sequence of rigid motions that map one figure to another. • Express in words and in writing that two figures are congruent if and only if they have the same shape and size. • Model composite transformations to map one figure onto another. • Recognize and explain the effects of rigid motion on orientation and location of a figure. • Define congruence as a test to see if two figures are congruent.
		Webb's Depth of Knowledge: 1-2
		Bloom's Taxonomy: understand, apply
Standard Text HSG.CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	**Standard for Mathematical Practices** MP3 Students construct viable arguments and critique reasoning of other when proving congruency and describe transformations. MP6 Students attend to Precision by using accurate transformations to map shapes onto other shapes. MP7 Students look for and make use of structure by applying rules of transformations to prove congruence of objects.	**Students who demonstrate understanding can:** • Identify corresponding angles and sides based on congruence statements. • Develop and write congruence statements for two congruent triangles. • Determine if two triangles are congruent based on their corresponding parts. • Compare given figures to determine congruence and indicate whether the figure went through a rigid transformation. • Explain, using rigid motions, why in congruent triangles, corresponding parts must be congruent.

		Webb's Depth of Knowledge: 1-2 **Bloom's Taxonomy:** understand, apply
Standard Text **HSG.CO.B.8** **Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.**	**Standard for Mathematical Practices** MP3 Students construct viable arguments and critique reasoning of other when proving congruency and describe transformations. MP6 Students attend to Precision by using accurate transformations to map shapes onto other shapes. MP7 Students look for and make use of structure by applying rules of transformations to prove congruence of objects.	**Students who demonstrate understanding can:** • Create a method to determine unknown measurements of congruent triangles. • Explain the approach that was used to determine to congruency of two triangles given limited parts of triangles. • Explain the approach that was used to determine the congruence of two triangles. • Apply the criteria of SSS, SAS, ASA to prove triangle congruency. **Webb's Depth of Knowledge:1-2** **Bloom's Taxonomy: understand, apply**
Previous Learning Connections • In 7th grade, students focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. In 8th grade, students develop understanding of congruence using physical models, transparencies, or geometry software. Students also understand that figures are congruent if the second can be obtained from the first by a sequence of rotations, reflections, and/or translations. These foundational skills are applied within this standard to continue to explore congruence and the impact of rigid motions on geometric shapes.	**Current Learning Connections** • Students will use triangle congruence concepts to develop future postulates and theorems. Concepts of triangle congruence serve to build a foundation for work with triangle proofs in future clusters.	**Future Learning Connections** • In later courses, students consider triangle congruence and the ambiguous case when working with the Law of Sines and Law of Cosines.

Clarification Statement:
Students create a definition of triangle congruence in terms of rigid motions. They work to develop a set of criteria for triangle congruence and build a foundation for geometric proofs.

Common Misconceptions
Combinations such as SSA or AAA are also a congruence criterion for triangles. All transformations, including dilations, are rigid motions. Any two figures that have the same area represent a rigid transformation. Students should recognize that the areas remain the same, but preservation of side and angle lengths determine that the transformation is rigid.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying understanding congruence in terms of rigid motion because students may incorrectly interchange congruence and similarity.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.GA.2: This standard provides a foundation for work with understanding congruence in terms of rigid motion because this is where the concept of congruence is solidified in terms of one object being able to be moved directly on top of another and match perfectly. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with experimenting with transformations in the plane benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as descriptions (text or spoken) for transformations, auditory cues and/or vocabulary word wall for key terms because the section is vocabulary heavy and students may not have a satisfactory grasp of prior skills and/or may have a different first language and by providing the visual meaning along with a written description and re-explained orally provides multiple means of access for the students. Providing auditory cues and/or a vocabulary word wall for key terms can reinforce this. Further, using patty paper/trace paper/geometric software to create transformations allows for interaction between the student and the material because it provides opportunities for the student to physically perform transformations, along with seeing a visual, hearing a description, and reading a definition.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with transformations in the plane benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics representations and scaffolds because students may struggle with visualizing a series of transformations and/or have a hard time abstractly connecting the concept of transformations to angles, circles, parallel and perpendicular lines etc.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with transformations in the plane benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because most, if not all, of these terms have been introduced at prior grades and students will have some incoming concept of some of the terms. Building off their prior knowledge and/or identifying misunderstandings at this point can ensure precise use of language throughout the learning.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with transformations in the plane benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., cut out objects, geometric software, patty paper/trace paper) because students can physically create the transformations and/or series of transformations which may deepen their understanding.

Internalize

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with transformations in the plane benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as embedding prompts to "show and explain your work" because students may see transformations in a variety of ways (eg. one student may see a reflection followed by a rotation where another student sees a series of reflections, etc.).

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

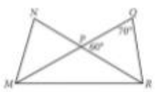
- For example, students may benefit from re-engaging with content during a unit on understanding congruence in terms of rigid motion by clarifying mathematical ideas and/or concepts through a short mini-lesson because students may incorrectly apply the triangle congruence theorems (ASA, SSS, etc) and could benefit from clarifying these and connecting them to the concept of congruence.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit understanding congruence in terms of rigid motion by helping students move from specific answers to generalizations for certain types of problems because this cluster requires students to prove, generally speaking, if two triangles are congruent

by applying knowledge of congruence rather than using specific triangles with concrete measurements. **Extension** *What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?* • For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying understanding congruence in terms of rigid motion because their established knowledge base of congruence could allow them to generate examples and non-examples of congruent shapes.
Culturally and Linguistically Responsive Instruction: **Validate/Affirm**: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? **Build/Bridge**: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society? Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying Understanding Congruence in terms of Rigid Motion the types of mathematical tasks are critical because students need the time and experience in connecting prior knowledge of congruence to rigid motions in the plane. Tasks should activate knowledge of both congruence and rigid motions, and build the bridge between them.
Standards Aligned Instructionally Embedded Formative Assessment Resources: SAT Item #: 422659: The linked assessment question addresses G-CO.B., specifically the question requires students to identify and use congruent angles to find the angle measure in a triangle.

CollegeBoard	Question ID 422659						
SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Lines, angles, and triangles	1. Use concepts and theorems relating to congruence and similarity of triangles to solve problems.	No Calculator



In the figure above, $\overline{MQ}$ and $\overline{NR}$ intersect at point P, $NP = QP$, and $MP = PR$. What is the measure, in degrees, of $\angle QMR$? (Disregard the degree symbol when gridding your answer.)

Question Difficulty: Medium

The correct answer is 30. It is given that the measure of $\angle QPR$ is 60°. Angle MPR and $\angle QPR$ are collinear and therefore are supplementary angles. This means that the sum of the two angle measures is 180°, and so the measure of $\angle MPR$ is 120°. The sum of the angles in a triangle is 180°. Subtracting the measure of $\angle MPR$ from 180° yields the sum of the other angles in the triangle MPR. Since $180 - 120 = 60$, the sum of the measures of $\angle QMR$ and $\angle NRM$ is 60°. It is given that $MP = PR$, so it follows that triangle MPR is isosceles. Therefore $\angle QMR$ and $\angle NRM$ must be congruent. Since the sum of the measure of these two angles is 60°, it follows that the measure of each angle is 30°.

An alternate approach would be to use the exterior angle theorem, noting that the measure of $\angle QPR$ is equal to the sum of the measures of $\angle QMR$ and $\angle NRM$. Since both angles are equal, each of them has a measure of 30°.

Additional Assessment:
Properties of Congruent Triangles: https://tasks.illustrativemathematics.org/content-standards/HSG/CO/B/7/tasks/1637
The linked assessment question addresses G-CO.B, specifically the question requires students to look at two triangles and connect the concept of congruence of corresponding parts to congruence of shape in terms of rigid motion. Two different approaches are prompted: one assuming the triangles can be mapped on each other and asking students to explain which parts are congruent, and another assuming the triangles are congruent and asking students to create the sequence of transformations to map one onto the other. This assessment should be given to students after they've had time to work with rigid motion and have a firm grasp of naming parts of triangles. Students will engage in SMP 3, SMP 6 and SMP 7.

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on Congruence in terms of Rigid Motion, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, teachers can create connections from native language to English by focusing on cognates (Words that sound the same in two different languages). Incorporating the	Computer Science: program to create visual demo of transformations

usage of cognates throughout a unit validates and affirms all languages and can encourage students to explore these terms in language other than their native language.	

<table>
<tr><th colspan="3">HS: GEOMETRY-CONGRUENCE</th></tr>
<tr><td colspan="3">Cluster Statement: C: Prove geometric theorems

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers</td></tr>
<tr>
<td rowspan="3">Standard Text

HSG.CO.C.9
Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.</td>
<td rowspan="3">Standard for Mathematical Practices

SMP 2
Students reason abstractly and quantitatively by using Theorems and postulates to prove relationships between angles formed by intersecting lines and systems of intersecting lines

SMP 5
Students use appropriate tools strategically by using dynamic geometry tools to solve problems. These tools might include pencil and paper, concrete models, rulers, protractors, compasses, software, apps, and calculators

SMP 7
Students look for and make use of structure by learning the language of geometric theorems and postulates to frame problem solving

SMP 8
Students look for and express regularity in repeated reasoning by generalizing rules, theorems, and postulates to similar problems and situations</td>
<td>Students who demonstrate understanding can:
• Prove the vertical angle theorem and alternate interior angle theorem.
• Prove corresponding angles are congruent.
• Prove the converse of the alternate interior angle theorem and the corresponding angle theorem and use it to show that two lines are parallel.
• Use perpendicular bisectors to locate the circumcenter of a triangle and to find the center of a circle given three points on the circle.
• Express proofs both in writing and orally by using precise mathematical language
• Examine and critique proofs produced by other students as well as their own</td>
</tr>
<tr><td>Webb's Depth of Knowledge: 2-3</td></tr>
<tr><td>Bloom's Taxonomy: apply, analyze</td></tr>
</table>

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSG.CO.C.10 **Prove theorems about triangles.** *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	SMP 2 Students reason abstractly and quantitatively by using Theorems and postulates to prove relationships between angles formed by intersecting lines and systems of intersecting lines SMP 5 Students use appropriate tools strategically by using dynamic geometry tools to solve problems. These tools might include pencil and paper, concrete models, rulers, protractors, compasses, software, apps, and calculators SMP 7 Students look for and make use of structure by learning the language of geometric theorems and postulates to frame problem solving SMP 8 Students look for and express regularity in repeated reasoning by generalizing rules, theorems, and postulates to similar problems and situations	• Prove and apply that the sum of the interior angles of a triangle is 180°. • Prove and apply that the base angles of an isosceles triangle are congruent. • Prove and apply the midsegment (midline) of triangle theorem. • Prove that the medians of a triangle meet at a point, a point of concurrency. • Prove and apply that the exterior angle theorem. • Determine the conditions for forming a triangle, when given three lengths. **Webb's Depth of Knowledge: 1-2** **Bloom's Taxonomy:** understand, apply
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSG.CO.C.11 **Prove theorems about parallelograms.** *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	SMP 2 Students reason abstractly and quantitatively by using Theorems and postulates to prove relationships between angles formed by intersecting lines and systems of intersecting lines SMP 5 Students use appropriate tools strategically by using dynamic geometry tools to solve problems. These tools might include pencil and paper, concrete models, rulers,	• Prove properties of parallelograms and then apply them. • Prove the properties of rectangles and then apply them. • Prove the properties of rhombi and then apply them. • Prove the properties of squares and then apply them. • Classify a quadrilateral by its properties. • Identify the conditions necessary to prove that a quadrilateral is a parallelogram.

	protractors, compasses, software, apps, and calculators SMP 7 Students look for and make use of structure by learning the language of geometric theorems and postulates to frame problem solving SMP 8 Students look for and express regularity in repeated reasoning by generalizing rules, theorems, and postulates to similar problems and situations	**Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** understand, apply
Previous Learning Connections In 7th grade, students use facts about supplementary, complementary, vertical, and adjacent angles. In 8th grade, learners use informal arguments to establish facts about the angle sum and exterior angle of triangles, and about the angles created when parallel lines are cut by a transversal. These angle facts connect this cluster as students apply them when creating proofs.	**Current Learning Connections** The formalized theorems within this cluster will be used to build theorems and proofs for concepts in future clusters within the Geometry course.	**Future Learning Connections** Understanding the logical flow of developing a proof will be used in future courses such as when proving trigonometric identities.

Clarification Statement
Students focus on formalizing geometric proof structure and language. They write formal proofs focusing on angle relationships, triangle segment and angle relationships, and parallelogram properties.

Common Misconceptions
Students may have a hard time generalizing and spend unnecessary time looking for multiple counterexamples to prove or disprove a proof, or, they may assume a conjecture is always true because it worked in all examples that were explored. Additionally, they may assume the converse of a statement is always true.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., structured proofs) when studying proving geometric theorems because students may be unfamiliar with traditionally structured mathematical proofs.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- G.CO.B6: This standard provides a foundation for work with proving geometric theorems because this is where students have formalized an understanding of congruence in terms of rigid motion, which is required for every aspect of this cluster. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with proving geometric theorems benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for auditory information (e.g. connecting an oral argument for a theorem to a written proof to an image/series of images that display the argument, and in some cases, a written proof in native language) because students may be able to articulate their thoughts using one of these methods, but may not see how to produce a formal proof.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with proving geometric theorems benefit when learning experiences attend to student's attention and affect to support sustained effort and concentration such as using prompts or scaffolds for visualizing desired outcomes (e.g. two-column proof outline) because students may fail to recognize what is being asked of them when constructing a proof, or how to properly get from point A to point B.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with proving geometric theorems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) because students may be able to explain their mathematical thinking using one particular modality as opposed to always creating formal proofs, etc.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with proving geometric theorems benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches, strategies, skills, etc.) because students see arguments/proofs in different ways (visual, oral, physical proof, written, etc.). Eliciting a variety of examples can help students to see math from a different perspective.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with proving geometric theorems benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because formal mathematical proofs can feel overwhelming to students. Modeling for students that choosing a starting point and following it through, whether it arrives at the desired outcome or not, can still be helpful and meaningful can encourage students to pick up a proof from wherever they are comfortable and try it when they may have otherwise not attempted it to begin with.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on proving geometric theorems by <critiquing student approaches/solutions to make connections through a short mini-lesson because students will often see different methods of progressing through a proof which can benefit all students to see. Further, students may make specific claims or statements that are unsupported or incorrect, and critiquing this reasoning will strengthen all students' understanding of the content.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit about proving geometric theorems by offering opportunities to understand and explore different strategies because students may struggle with writing their own proofs, particularly in the beginning. It may be helpful to provide many examples and ask students to analyze each statement as to whether it can be mathematically supported or not. This could also be an opportunity to allow students to show their thinking with pictures rather than a formal proof structure.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying proving geometric theorems because giving students the opportunity to select certain theorems to prove can be both challenging and engaging for those with a firm grasp of the process.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

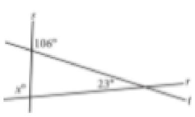
Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion, we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics, we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying Proving Geometric Theorems facilitating meaningful mathematical discourse is critical because proofs of theorems can frequently be seen from multiple perspectives. Purposefully sequencing, discussing and validating these different perspectives can help students internalize their worth in the classroom. It can also show students that knowing a piece of the larger puzzle often "unlocks" the door to the next step in a proof. The key is in selecting and sequencing a variety of perspectives to share out and focus on how it helps the process, rather than whether it is complete and correct.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
SAT Item #: 422005 The linked assessment question addresses G-CO.C., specifically the question requires students to know and apply theorems about exterior and interior angles.

CollegeBoard **Question ID 422005**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	**Math**	**Additional Topics in Math**	**Hard**	**Additional Topics in Math**	**Lines, angles, and triangles**	**4. Know and directly apply relevant theorems such as b. triangle similarity and congruence criteria;**	**No Calculator**

Intersecting lines r, s, and t are shown below.



What is the value of x ?

Question Difficulty: Hard

The correct answer is 97. The intersecting lines form a triangle, and the angle with measure of $x°$ is an exterior angle of this triangle. The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles of the triangle. One of these angles has measure of $23°$ and the other, which is supplementary to the angle with measure $106°$, has measure of $180° - 106° = 74°$. Therefore, the value of x is $23 + 74 = 97$.

Additional Assessment:

Midpoints of the Sides of a Parallelogram: https://tasks.illustrativemathematics.org/content-standards/HSG/CO/C/11/tasks/35
The linked assessment question addresses G-CO.C, specifically the question requires students to prove segments of a parallelogram are congruent. Students use knowledge of corresponding parts of congruent triangles are congruent to form arguments. This assessment should be given to students after they've worked with corresponding parts of triangles. Students will engage in SMP 2 and SMP 7.

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on proving geometric theorems, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, the history of geometry spans across cultures. This history can be incorporated as students learn how to build and create formal proofs from informal diagrams, experimentation and/or oral arguments.	Links to history can be made by exploring how geometric proofs developed over time in different cultures. Further, links to computer science can be made by discussing and displaying how coding reflects a variety of steps to get from one point to another, as is mirrored in proofs.

NEW MEXICO
Public Education Department

HS: GEOMETRY-CONGRUENCE		
Cluster Statement: D: Make geometric constructions		
Standard Text HSG.CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*	**Standard for Mathematical Practices** SMP 1 Students make sense of problems and persevere in solving them by making constructions of situations. Students reason and experiment with rigid motions, determining a correct sequence of transformations with perseverance. SMP 4 Students will apply ideas about transformations to model real-world contexts SMP 5 Students use appropriate tools strategically by using dynamic geometry tools to solve problems. These tools might include pencil and paper, concrete models, rulers, protractors, compasses, software, apps, and calculators. SMP 6 Students attend to precision by using precise geometric mathematical language to thoroughly explain the reasoning behind their work and when formalizing definitions. Precision is of crucial importance in constructions, since even small errors in executing a construction may lead to results that don't work.	**Students who demonstrate understanding can:** • Construction techniques (compass, straight edge, software, etc.) are used to create figures. • Perform constructions including: copy a segment, copy an angle, bisect segments and angles, construct perpendicular lines/segments, construct parallel lines. **Webb's Depth of Knowledge: 1-2** **Bloom's Taxonomy:** understand, apply

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSG.CO.D.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	SMP 1 Students make sense of problems and persevere in solving them by making constructions of situations. Students reason and experiment with rigid motions, determining a correct sequence of transformations with perseverance. SMP 4 Students will apply ideas about transformations to model real-world contexts SMP 5 Students use appropriate tools strategically by using dynamic geometry tools to solve problems. These tools might include pencil and paper, concrete models, rulers, protractors, compasses, software, apps, and calculators. SMP 6 Students attend to precision by using precise geometric mathematical language to thoroughly explain the reasoning behind their work and when formalizing definitions. Precision is of crucial importance in constructions, since even small errors in executing a construction may lead to results that don't work.	• Construct an equilateral triangle inscribed within a circle using construction techniques • Construct a square inscribed within a circle using construction techniques • Construct a regular hexagon inscribed within a circle using construction techniques. **Webb's Depth of Knowledge:** 3-4 **Bloom's Taxonomy:** apply, create
<u>Previous Learning Connections</u> • This cluster connects to students previously taught skills from 7th grade when they constructed geometric shapes when given certain conditions in the 7.G.A cluster. Additionally, in 8th grade within the 8.G.A cluster, students worked with two-dimensional figures and verified their properties.	**<u>Current Learning Connections</u>** • Construction connects and adds on to learning from previous clusters within the Geometry course by building on triangle congruence theorems (SSS, SAS), properties of parallel and perpendicular lines, and polygons and their properties.	**<u>Future Learning Connections</u>** • Construction techniques could be applied to unit circle, and conic sections in future courses.

Clarification Statement
This cluster focuses on hands-on basic constructions. Students use geometric tools (compass, straightedge, software, etc.) to generate foundational pieces of geometry

Common Misconceptions

Some students may believe that a construction is the same as a sketch or drawing. Emphasize the need for precision and accuracy when doing constructions. Stress the idea that a compass and straightedge are identical to a protractor and ruler. Explain the difference between measurement and construction.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying making geometric constructions because students should have experience working with geometric construction tools (straight edge, ruler, compass, etc) but may not use them consistently correctly or may need a refresher.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.GA.2: This standard provides a foundation for work with making geometric constructions because this standard called for students to use a variety of tools to draw shapes that fit given constraints. This is where they should have mastered use of ruler/straight edge and compass. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with making geometric constructions benefit when learning experiences include ways to recruit interest such as providing choices in their learning (eg. using rule/straightedge, compass, geometric software, etc) because students may have a varied background knowledge of, and interest in, these construction tools.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with making geometric constructions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports (e.g., peer-tutors) because students may have different backgrounds in appropriate use of these tools and harnessing student knowledge in said tools can help establish/build/promote a positive learning environment.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with making geometric constructions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) because some students will not have prior experience with using all tools. These supports can help expand students' repertoire of mathematical tools.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with making geometric constructions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., trace paper, compass, straightedge, geometric software) because students may see how to formalize a construction better from one perspective than another. Further, it may allow for students to stretch their comfort from using one explicit tool to using a variety of tools strategically.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with making geometric constructions benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because complex constructions require a firm graphs of basic construction skills. Explicitly and routinely practicing these skills ensures students have entry-points to more complex constructions.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on making geometric constructions by clarifying mathematical ideas and/or concepts through a short mini-lesson because students may have simple errors (e.g. copying a segment without measurement, etc.) that require short and direct instruction.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on making geometric constructions by offering opportunities to understand and explore different strategies> because some students may work more efficiently with one consistent tool, while others may benefit from having a variety of tools at their disposal. Further, some students benefit from using geometric software to visualize their constructions rather than pencil and paper.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying making geometric constructions because this can challenge students to construct any shape, explaining or showing their methods for each.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying Making Geometric Constructions the use of mathematical representations within the classroom is critical because students may have a varied background in reading/writing technical written directions and/or using rulers, straightedges, compasses, geometric software, etc. This background could be established in prior schooling or in specific cultural/home usage. Connecting tools they are familiar with to tools that may be new or uncomfortable to them shows the value of their current knowledge at the same time as expanding that knowledge base.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://tasks.illustrativemathematics.org/content-standards/HSG/CO/D/12/tasks/966
The linked assessment question addresses G-CO.D, specifically the question requires students to prove that a specific segment is a perpendicular bisector to another segment. Students may use knowledge of corresponding parts of congruent triangles are congruent to form arguments, or may work through explanation using congruence as it follows from rigid motion transformations. This assessment should be given to students after they've been introduced to geometric construction tools and have a firm grasp of congruence. Students will engage in SMP 1, SMP 5 and SMP 6.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on Making Geometric Constructions, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, architects and other technical careers make regular use of the same tools used in geometric constructions. Connecting the use of mathematical tools with the real world can solidify the importance and relevance of material being learned, as well as encourage students to goal-set for future careers and interests.	Art: drafting, geometric shape work

NEW MEXICO
Public Education Department

<table>
<tr><th colspan="3">HS: GEOMETRY- SIMILARITY, RIGHT TRIANGLES, & TRIGONOMETRY</th></tr>
<tr><td colspan="3">Cluster Statement: A: Understand similarity in terms of similarity transformations</td></tr>
<tr><td rowspan="3">Standard Text

HSG.SRT.A.1
Verify experimentally the properties of dilations given by a center and a scale factor:
HSG.SRT.A.1.A: A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
HSG.SRT.A.1.B: The dilation of a line segment is longer or shorter in the ratio given by the scale factor.</td><td rowspan="3">Standard for Mathematical Practices

SMP2
Students reason abstractly and quantitatively by requiring students to make sense of quantities such as scale factor and their relationships to one another in problem situations

SMP3
Students construct viable arguments and critique reasoning of other by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

SMP5
Students use appropriate tools strategically by expecting students to consider available tools when solving a mathematical problem. Tools might include pencil and paper, concrete models, rulers, protractors, compasses, calculators, and software or apps.</td><td>Students who demonstrate understanding can:
Determine the properties of dilation. Dilate when the center of dilation is in, on and out of the shape. Dilate when given a center of dilation and a scale factor. Determine the center of dilation and the scale factor from a diagram. Dilate using both positive and negative scale factors. Construct a dilation. Dilation coordinate rules for dilations using any center of dilation.
Construct a dilated image which has corresponding line segments and is transformed along the same line from the center of the dilation.
Verify experimentally that a dilated image is similar to its pre-image by showing congruent, corresponding angles, and proportional sides.
Determine and apply the properties of dilation.</td></tr>
<tr><td>Webb's Depth of Knowledge: 1-2</td></tr>
<tr><td>Bloom's Taxonomy: understand, apply</td></tr>
</table>

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSG.SRT.A.2 **Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.**	SMP3 Students construct viable arguments and critique reasoning of other by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments. SMP5 Students use appropriate tools strategically by expecting students to consider available tools when solving a mathematical problem. Tools might include pencil and paper, concrete models, rulers, protractors, compasses, calculators, and software or apps.	• Dilate figures using both positive and negative scale factors. • Identify corresponding angles and sides based on similarity statements. • Develop and write similarity statements for two polygons. • Determine if two triangles are similar based on their corresponding parts. • Establish a sequence of similarity transformations between two similar polygons. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** understand, apply
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSG.SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	SMP7 Students look for and make use of structure by using the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	• Develop the Angle-angle criteria of similarity by expanding on previously learn properties of angles of Triangles • Use transformations as a tool to discover how AA similarity is derived and to make the process more efficient. • Express the properties of similarity transformations to explain the justification of AA similarity. **Webb's Depth of Knowledge: 1-2** **Bloom's Taxonomy:** understand, apply

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• In 8th grade, students perform transformations, including dilations, in a coordinate plane. They also identify a sequence of transformations that highlights the similarity of two figures.	• In later clusters within the Geometry course, students connect their conceptual understanding of similarity to explore trigonometric relationships including special right triangles and trigonometric ratios.	• Students will continue their work with similar figures in later courses when working with trigonometric ratios and the unit circle. They will use their understanding of dilations when working with functions to determine a stretch/shrink transformation.

Clarification Statement

This cluster establishes the basic criteria of similarity through an analysis of dilation transformations. Students formalize the similarity theorems and use the theorems to prove pairs of triangles are similar.

Common Misconceptions

A common misconception is thinking that the comparison of any pair of angles will be sufficient, when the comparison must be made using corresponding pairs.

Students may incorrectly apply the scale factor. Some students often do not list the vertices of similar triangles in order. However, the order in which vertices are listed is preferred and especially important for similar triangles so that proportional sides can be correctly identified.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when studying understanding similarity in terms of similarity transformation SRT. A. cluster because it is important for students to understand prior knowledge vocabularies as they are introduced to more complex one.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- Standard 8.G.A.4- Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two- dimensional figures, describe a sequence that exhibits the similarity between them. This standard provides a foundation for work with understanding similarity in terms of similarity transformation SRT. A. cluster because when students are not clear on the language structure of the mathematical problem at hand, it allows for a lot of misconceptions when the language has been presented in an advanced manner. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade-level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with understanding similarity in terms of similarity transformation benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because for students to understand how ratios and proportions expand into dilations of points and segments in a creative way for example with art can prove to be an effective cross-curricular activity to achieve the focus of this cluster.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understanding similarity in terms of similarity transformation benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because incorporating a student's background into the lessons this cluster provides will prove to explain the connection between transformations in a cultural context. For example, with native American weaving practices and learning how the rations and proportions of the lines bring out the intricate display of patterns.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with understanding similarity in terms of similarity transformation benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because student's misconceptions could arise from the lack of background knowledge, specifically in relation to the vocabulary used in this cluster.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding similarity in terms of similarity transformation benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing Computer-Aided-Design (CAD) or mathematical notation software because allowing students to connect this cluster with bigger ideas such as vectors in the field of computer science, helping students understand how dilations and extrapolations of specific segments aid in the creation of video games, phone applications etc.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with understanding similarity in terms of similarity transformation benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as making explicit cross-curricular connections because the practical approach of dilations can be visually expressed in the sciences and arts. Pattern recognition in these subjects when we explore physics (e.g. motion vectors) and art (e.g. pattern arrangement) can translate the big idea of how ratios are embedded in geometrical transformations.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understanding similarity in terms of similarity transformation SRT. A. cluster by examining tasks from a different perspective through a short mini-lesson because allowing students to connect their knowledge of scale and transitions into more complex thought processes such as dilations can help re-shift the reason why this standard is important in this context.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on understanding similarity in terms of similarity transformation SRT. A. cluster by offering opportunities to understand and explore different strategies because students might be able to explore the concept of "same shape" much easier than the concept of congruence.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying understanding similarity in terms of similarity transformation SRT. A. cluster because introducing students to angle measurements and how they aid the process of transformation as well as congruence will help students avoid any misconceptions in the similarity cluster.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

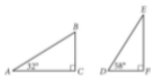
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion, we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics, we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying understanding similarity in terms of similarity transformation, facilitating meaningful mathematical discourse is critical because instructors should be able to draw from student misconceptions and translate these into learning pieces which will engage students in building on each other's ideas and deepen understanding of similarity transformation.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

SAT Item #: 421874 The linked assessment question addresses G-SRT.A, specifically the question requires students to analyze a composition of similar triangles to write a ratio of side lengths equivalent to a given ratio.

CollegeBoard	Question ID 421874						
SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Right triangles and trigonometry	1. Solve problems in a variety of contexts using b. right triangle trigonometry;	Calculator



Triangles ABC and DEF are shown above. Which of the following is equal to the ratio $\frac{BC}{AB}$?

Question Difficulty: Medium

A. $\frac{DE}{DF}$

B. $\frac{DF}{DE}$

C. $\frac{DF}{EF}$

D. $\frac{EF}{DE}$

Choice B is correct. In right triangle ABC, the measure of angle B must be 58° because the sum of the measure of angle A, which is 32°, and the measure of angle B is 90°. Angle D in the right triangle DEF has measure 58°. Hence, triangles ABC and DEF are similar (by angle-angle similarity). Since $\overline{BC}$ is the side opposite to the angle with measure 32° and AB is the hypotenuse in right triangle ABC, the ratio $\frac{BC}{AB}$ is equal to $\frac{DF}{DE}$.

Alternate approach: The trigonometric ratios can be used to answer this question. In right triangle ABC, the ratio $\frac{BC}{AB} = \sin(32^\circ)$. The angle E in triangle DEF has measure 32° because $m(\angle D) + m(\angle E) = 90^\circ$. In triangle DEF, the ratio $\frac{DF}{DE} = \sin(32^\circ)$. Therefore, $\frac{DF}{DE} = \frac{BC}{AB}$.

Choice A is incorrect because $\frac{DE}{DF}$ is the reciprocal of the ratio $\frac{BC}{AB}$. Choice C is incorrect because $\frac{DF}{EF} = \frac{BC}{AC}$, not $\frac{BC}{AB}$. Choice D is incorrect because $\frac{EF}{DE} = \frac{AC}{AB}$, not $\frac{BC}{AB}$.

Additional Assessment:
Similar Triangles: https://achievethecore.org/coherence-map/HS/G/116/611/611

The linked assessment question addresses G-SRT.A, specifically the question requires students to look at two triangles with a given pair of congruent angles and state a series of transformations to map one onto the other. Students will apply rotation, translation and a generic dilation in this example. This assessment should be given to students after they've had time to work with concrete examples of dilations as this more complicated example requires abstract algebra in terms of the scale factor. Students will engage in SMP 1, SMP 2, and potentially SMP 3 depending on if students work in groups to share their solutions.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on understanding similarity in terms of similarity transformation, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, For example: when looking at ancient pottery pattern samples, how can Mesopotamian pottery patterns relate to Native American or African pottery patterns displayed throughout various cultures.	Drafting/Architecture: Connect to trusses, shadow lengths

HS: GEOMETRY- SIMILARITY, RIGHT TRIANGLES, & TRIGONOMETRY		
Cluster Statement: B: Prove theorems involving similarity Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers		
Standard Text **HSG.SRT.B.4** **Prove theorems about triangles.** *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*	**Standard for Mathematical Practices** SMP3 Students construct viable arguments and critique reasoning of other by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments. SMP5 Students use appropriate tools strategically by expecting students to consider available tools when solving a mathematical problem. Tools might include pencil and paper, concrete models, rulers, protractors, compasses, calculators, and software or apps.	**Students who demonstrate understanding can:** • Prove two triangles are similar using AA (could extend to SAS or SSS) similarity theorem. Use proportion to understand and justify logical claims. • Prove that two triangles are similar using the AA (could extend to SAS or SSS) similarity theorem. • Analyze a proof that two triangles are similar to determine if the argument is valid. • Prove various theorems about a triangle's properties. • Determine if two lines are parallel. • Set up and solve a proportion. • Apply the Pythagorean Theorem. • Organize and write a mathematical proof, including justification of my argument. **Webb's Depth of Knowledge: 1-3** **Bloom's Taxonomy:** understand, apply, analyze
Standard Text **HSG.SRT.B.5** **Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.**	**Standard for Mathematical Practices** SMP3 Students construct viable arguments and critique reasoning of other by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	**Students who demonstrate understanding can:** • Apply Theorems and postulates of triangle similarity to solve problems and prove relationships within and between geometric figures. • Use similar figures to find missing side lengths and missing angle measures. • Use congruent figures to find missing side lengths and missing angle measures.

	SMP5 Students use appropriate tools strategically by expecting students to consider available tools when solving a mathematical problem. Tools might include pencil and paper, concrete models, rulers, protractors, compasses, calculators, and software or apps.	• Determine if two geometric figures are congruent or similar. • Justify why two figures are congruent or similar using theorems from Geometry. **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy:** understand, apply, analyze
Previous Learning Connections • In 8th grade, students developed the idea of "same shape" and "scale factor" as a definition of similarity. They will develop and connect these ideas when proving theorems within this cluster.	**Current Learning Connections** • Having previously studied dilations, students expand their definition of similarity to include congruence and dilation. These concepts lead to the criteria for triangle similarity. Students use proportional reasoning to approach problems involving similar figures. Trigonometric ratios will be developed using similar right triangles in connection to the work within this cluster.	**Future Learning Connections** • The trigonometric ratios (sine, cosine, tangent) will be founded on right triangles and similarity in subsequent learning. The Pythagorean theorem is generalized to non-right triangles by the Law of Cosines and Law of Sines.

Clarification Statement

Students continue to develop their ability to create proofs while incorporating similarity. They will prove the Pythagorean Theorem based on similar triangles. They will then apply similarity to a variety of real world situations.

Common Misconceptions

Students may forget the importance of the order of vertices when making similarity statements.
Students may confuse the alternate interior angle theorem and its converse as well as the Pythagorean Theorem and its converse.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time when studying proving theorems involving similarity because students can make connections between right triangles by drawing a perpendicular line to bisect a bigger right triangle to form two smaller ones.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.G.B.6- Explain a proof of the Pythagorean Theorem and its converse**.** This standard provides a foundation for work with proving theorems involving similarity because <when students understand the similarity between right triangles and the Pythagorean theorem, they will be able to make trigonometric connections between the two. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with proving theorems involving similarity benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information like the ones origami can visually conceptualize for students because origami allows students to visualize the concept of triangle proportionality and similarity.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with proving theorems involving similarity benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success because if students are able to use origami as a visual representation of triangle similarity proofs, students can quickly realize their patterns of error and pinpoint in an active classroom discourse with peers and facilitator.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with proving theorems involving similarity benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) because students will be able to see the relationship between right triangles and Pythagorean proofs. Understanding how it can be mapped onto a coordinate system can aid learners make connections between the similarities of transforming a right triangle in the different quadrants.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with proving theorems involving similarity benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because triangle similarity can be shown in more than one way for students to understand the similarities and differences.

Internalize
Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with proving theorems involving similarity benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing templates, graphic organizers, concept maps to support note-taking because students will build connections from prior knowledge such as ratios, proportions, and scales. This will help students engage in the newer vocabulary and relate it to their prior knowledge.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on proving theorems involving similarity by revisiting student thinking through a short mini-lesson because it is important to understand where students are in terms of vocabulary such as similar and scale factors, so that when tackling the proofs students are not intimidated by the mathematical language expected.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on proving theorems involving similarity by offering opportunities to understand and explore different strategies because it might help clear up different misconceptions when students are allowed to display understanding in different ways. For example: with EL students one strategy would be to pair up individuals with native English-speaking classmates as they explore the task.

Extension
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying proving theorems involving similarity because students might be able to broaden their knowledge of similarity into real world scenarios such as in the architectural field.

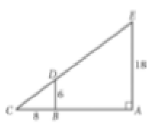
Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying proving theorems involving similarity, the types of mathematical tasks are critical because the connections that can be utilized from an ELA standpoint (argumentative critical thinking) towards proving theorems in a mathematical world can be of instrumental value prior to introducing the procedural fluency of for example the Pythagorean theorem.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
SAT Item #: 422453 The linked assessment question addresses G-SRT.B, specifically the question requires students to solve for a side length in a series of composed right triangles.

CollegeBoard | **Question ID 422453**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	**Math**	**Additional Topics in Math**	**Hard**	**Additional Topics in Math**	**Right triangles and trigonometry**	**1. Solve problems in a variety of contexts using c. properties of special right triangles.**	**No Calculator**



In the figure above, $\overline{BD}$ is parallel to $\overline{AE}$. What is the length of $\overline{CE}$?

Question Difficulty: Hard

The correct answer is 30. In the figure given, since $\overline{BD}$ is parallel to $\overline{AE}$ and both segments are intersected by $\overline{CE}$, then angle BDC and angle AEC are corresponding angles and therefore congruent. Angle BCD and angle ACE are also congruent because they are the same angle. Triangle BCD and triangle ACE are similar because if two angles of one triangle are congruent to two angles of another triangle, the triangles are similar. Since triangle BCD and triangle ACE are similar, their corresponding sides are proportional. So in triangle BCD and triangle ACE, $\overline{BD}$ corresponds to $\overline{AE}$ and $\overline{CD}$ corresponds to $\overline{CE}$. Therefore, $\frac{BD}{CD} = \frac{AE}{CE}$. Since triangle BCD is a right triangle, the Pythagorean theorem can be used to give the value of CD: $6^2 + 8^2 = CD^2$. Taking the square root of each side gives $CD = 10$. Substituting the values in the proportion $\frac{BD}{CD} = \frac{AE}{CE}$ yields $\frac{6}{10} = \frac{18}{CE}$. Multiplying each side by CE, and then multiplying by $\frac{10}{6}$ yields $CE = 30$. Therefore, the length of $\overline{CE}$ is 30.

Additional Assessment:
http://tasks.illustrativemathematics.org/content-standards/HSG/SRT/B/4/tasks/1568

The linked assessment question addresses G-SRT.B, specifically the question requires students to show two triangles are similar and then use ratios of side lengths to derive the Pythagorean theorem. This assessment should be given to students after they've worked with setting up ratios for similar triangles. Students will engage in SMP 3, SMP 6 and, if asked to share and critique work of peers, SMP 3.

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on proving theorems involving similarity, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example in the field of forensics being able to determine the height of an individual contrasted with a fixed object in a video frame.	Physics- Connect to Vectors, particularly in resultants and to Dimensional Kinematics Art – Connect to drafting/architecture and to shapes and reflection within works of art

HS: GEOMETRY- SIMILARITY, RIGHT TRIANGLES, & TRIGONOMETRY

Cluster Statement: C: Define trigonometric ratios and solve problems involving right triangles

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSG.SRT.C.6 **Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.**	SMP3 Students construct viable arguments and critique reasoning of other by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments. Students will examine proofs using properties, definitions, and theorems. SMP4 Students model with mathematics by modeling with right triangles and determine corresponding parts of similar figures when calculating indirect measurements in the context of a given real-world scenario. SMP6 Students attend to precision by requiring students to calculate efficiently and accurately and to communicate precisely using correct mathematical language SMP7 Students look for and make use of structure by expecting students to apply rules, look for patterns, and analyze structure. Students identify patterns in tables of values to formulate generalizations about relationships within and between trigonometric ratios. They will also determine how complementary angles and their trigonometric functions are related.	• Use similarity, side ratios, and angles in right triangles to develop and define trigonometric ratios to help in completion of triangles • Identify the side opposite to and adjacent to an acute angle in a right triangle. • Write and simplify ratios using the sides of a right triangle. • Compare side ratios of similar right triangles and identify if they are equivalent. • Use the definition of sine, cosine, tangent, secant, cosecant, and cotangent to write those trigonometric ratios for a given triangle. **Webb's Depth Of Knowledge:** 1-2 **Bloom's Taxonomy:** understand, apply

<table>
<tr>
<td rowspan="3">Standard Text

HSG.SRT.C.7
Explain and use the relationship between the sine and cosine of complementary angles.</td>
<td rowspan="3">Standard for Mathematical Practices

SMP3
Students construct viable arguments and critique reasoning of other by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments. Students will examine proofs using properties, definitions, and theorems.

SMP4
Students model with mathematics by modeling with right triangles and determine corresponding parts of similar figures when calculating indirect measurements in the context of a given real-world scenario.

SMP6
Students attend to precision by requiring students to calculate efficiently and accurately and to communicate precisely using correct mathematical language

SMP7
Students look for and make use of structure by expecting students to apply rules, look for patterns, and analyze structure. Students identify patterns in tables of values to formulate generalizations about relationships within and between trigonometric ratios. They will also determine how complementary angles and their trigonometric functions are related.</td>
<td>Students who demonstrate understanding can:
Use the concept of complementary angles to show how sine and cosine are related
Identify the opposite leg, adjacent leg, and hypotenuse with respect to an angle in a right triangle.
Explain the relationship between sine and cosine of complementary angles of right triangles.</td>
</tr>
<tr>
<td>Webb’s Depth of Knowledge: 1</td>
</tr>
<tr>
<td>Bloom’s Taxonomy: understand</td>
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<table>
<tr>
<td>Standard Text

HSG.SRT.C.8
Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.</td>
<td>Standard for Mathematical Practices

SMP3
Students construct viable arguments and critique reasoning of other by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments. Students will examine proofs using properties, definitions, and theorems.

SMP4
Students model with mathematics by modeling with right triangles and determine corresponding parts of similar figures when calculating indirect measurements in the context of a given real-world scenario.

SMP6
Students attend to precision by requiring students to calculate efficiently and accurately and to communicate precisely using correct mathematical language

SMP7
Students look for and make use of structure by expecting students to apply rules, look for patterns, and analyze structure. Students identify patterns in tables of values to formulate generalizations about relationships within and between trigonometric ratios. They will also determine how complementary angles and their trigonometric functions are related.</td>
<td>Students who demonstrate understanding can:
• Apply the trig ratios and the Pythagorean theorem to solve right triangle models
• Identify the unknown parts of a right triangle using the sine/cosine/tangent ratios.
• Solve for the unknown angle measures of a right triangle using inverse sine, inverse cosine, and inverse tangent.
• Solve for the unknown parts of a right triangle using Pythagorean Theorem.
• Solve real world problems using trigonometric ratios and the Pythagorean Theorem.

Webb's Depth of Knowledge: 1-2

Bloom's Taxonomy: understand, apply</td>
</tr>
<tr>
<td><u>Previous Learning Connections</u>
• In 8th grade, students applied the Pythagorean Theorem to find unknown side length in right triangles and distance between two points. They will make connections between Pythagorean Theorem and</td>
<td><u>Current Learning Connections</u>
• Students will continue to use trigonometric ratios throughout the remainder of the course. A strong procedural fluency is necessary for individuals to apply these ratios to items within future</td>
<td><u>Future Learning Connections</u>
• In future courses, trigonometric ratios are used to develop more complex concepts such as relationships within the unit circle. Students will graph the trigonometric functions and observe the</td>
</tr>
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trigonometric ratios to continue solving right triangles in this cluster.	clusters. Pythagorean Theorem and the trigonometric ratios are used to find lengths necessary for finding surface areas and volumes. Students use similarity concepts when defining properties of circles, arc lengths, and sector areas.	cyclic patterns that arise from the trigonometric ratio relationships.

Clarification Statement
This cluster builds on the concepts of similarity to define the trigonometric ratios. Using Pythagorean Theorem and trigonometric ratios, students solve for unknown side lengths and angle measures in right triangles.

Common Misconceptions
Students may confuse side lengths with angle measurements and will place values as the wrong substitutions in the ratios.
Students may think that right triangles must be oriented a particular way. They may not realize that opposite and adjacent sides need to be identified with reference to a particular acute angle in a right triangle.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that introduces new representations when studying trigonometric ratios because < students will be able to draw prior knowledge of trigonometric ratios by representing proportional relationships between quantities learned prior to this cluster.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.RP.A.2- Recognize and represent proportional relationships between quantities: This standard provides a foundation for work with trigonometric ratios because the ratios explored in graphing linear relationships can be explored by now exposing students to trigonometric ratios within a right triangle. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with defining trigonometric ratios and solving problems involving right triangles benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because right triangles can be seen any a variety of applications (surveying, architecture, construction working, estimating height, etc.). Providing a variety of contexts can help attend to student interest.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with defining trigonometric ratios and solving problems involving right triangles benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success because in setting up the initial ratio, students may write a reciprocal relationship and/or incorrectly place an angle measurement as a side length (or vice versa). These are simple errors that students will make repeatedly. Providing feedback on that pattern of errors will allow students to progress with success.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds*

- For example, learners engaging with defining trigonometric ratios and solving problems involving right triangles benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because students frequently struggle with correctly identifying "opposite" and "adjacent" sides in a triangle. Without that key piece of understanding mastered, students cannot correctly set up or identify appropriate trigonometric ratios to model scenarios. Pre-teaching what opposite and adjacent mean as well as how that relationship is dependent upon a reference angle can help students easily progress to identifying and writing trigonometric ratios.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with defining trigonometric ratios and solving problems involving right triangles benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) because students may initially struggle to create their own visual representation of a scenario/visualize a scenario. Beginning with problems that provide a visual and then progressing into modelling creating a visual can encourage students to eventually create their own visuals.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with defining trigonometric ratios and solving problems involving right triangles benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) because complex scenarios can often be broken down into right triangles. Once students comprehend this structure, they can apply their understanding of right triangles efficiently in new contexts.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on solving problems involving right triangles by clarifying mathematical ideas and/or concepts through a short mini-lesson because polygons other than triangles are not necessarily similar if each pair of corresponding angles is congruent. For example, all rectangles have congruent corresponding angles, but the corresponding sides of all rectangles do not have the same ratio.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on solving problems involving right triangles by offering opportunities to understand and explore different strategies because by investigating patterns of association in bivariate data students can use scatter plots and linear models

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when studying to define trigonometric ratios because students can make connections between engineering practices such as building electronics such as TVs. Understanding how trigonometric ratios are an intricate part of the development of tv screens will create a real-life extension for students.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying trigonometric ratios and solving problems involving right triangles the use of mathematical representations within the classroom is critical because students will relate the background knowledge within cross-curricular activities and relate it to the different mathematical representations needed for this cluster.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

SAT Item #4169029: The linked assessment question addresses G-SRT.C, specifically the question requires students to take a provided trigonometric ratio and use it to find another ratio.

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CollegeBoard | Question ID 4169029

SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Right triangles and trigonometry	4. Solve problems using the relationship between sine and cosine of complementary angles.	Calculator

In a right triangle, the tangent of one of the two acute angles is $\frac{\sqrt{3}}{3}$. What is the tangent of the other acute angle?

Question Difficulty: Medium

A. $-\frac{\sqrt{3}}{3}$

B. $-\frac{3}{\sqrt{3}}$

C. $\frac{\sqrt{3}}{3}$

D. $\frac{3}{\sqrt{3}}$

Choice D is correct. The tangent of a nonright angle in a right triangle is defined as the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle. Using that definition for tangent, in a right triangle with legs that have lengths a and b, the tangent of one acute angle is $\frac{a}{b}$ and the tangent for the other acute angle is $\frac{b}{a}$. It follows that the tangents of the acute angles in a right triangle are reciprocals of each other. Therefore, the tangent of the other acute angle in the given triangle is the reciprocal of $\frac{\sqrt{3}}{3}$ or $\frac{3}{\sqrt{3}}$.

Choice A is incorrect and may result from assuming that the tangent of the other acute angle is the negative of the tangent of the angle described. Choice B is incorrect and may result from assuming that the tangent of the other acute angle is the negative of the reciprocal of the tangent of the angle described. Choice C is incorrect and may result from interpreting the tangent of the other acute angle as equal to the tangent of the angle described.

Additional Assessment:
http://tasks.illustrativemathematics.org/content-standards/HSG/SRT/C/tasks/1316
The linked assessment question addresses G-SRT.C, specifically the question requires students to apply right triangle geometry to the context of points on a map. Students will need to visualize points on a map forming a right triangle and then apply formulas and concepts they are familiar with to solve contextual problems. This assessment should be given to students after they've been introduced to the formal definition of trigonometric ratios and applications of Pythagorean theorem and similar triangles. Students will engage in SMP 1, SMP 4, and potentially SMP 5 if students are required to generate their own maps using tools.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on trigonometric ratios and solving problems involving right triangles, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, For example: when looking at trigonometric ratios of right triangles, students can relate the ratios of the triangle if we focus on sports. Shooting a basketball from 5 feet away vs. shooting a basketball from 10 ft away will show you congruence. Scaling down the basket by ½ the	STEM: Connect to engineering and construction use of trigonometry to determine accurate angles and/or missing lengths.

height can provide a transition into trigonometric ratios.	

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HS: GEOMETRY-MODELING WITH GEOMETRY		
Cluster Statement: A: Apply geometric concepts in modeling situations		
Standard Text **HSG.MG.A.1** **Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).**	**Standard for Mathematical Practices** SMP 1 Students make sense of problems and persevere in solving them by analyzing a scenario to determine a geometric shape that fits the context. SMP 4 Students model with mathematics by applying an appropriate geometric formula to solve a contextual problem.	**Students who demonstrate understanding can:** • Recognize the geometric shape that corresponds to a real object. • Utilize geometric shapes, measures, and properties to describe objects. **Webb's Depth of Knowledge: 1-2** **Bloom's Taxonomy:** understand, apply
Standard Text HSG.MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	**Standard for Mathematical Practices** SMP 1 Students make sense of problems and persevere in solving them by analyzing a scenario to determine a geometric shape that fits the context. SMP 4 Students model with mathematics by applying an appropriate geometric formula to solve a contextual problem.	**Students who demonstrate understanding can:** • Construct the different volume and area formulas for shapes and figures. • Explain how to find density for different types of information. • Apply formulas to find density for different types of information. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** understand, apply

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSG.MG.A.3 **Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).**	SMP 1 Students make sense of problems and persevere in solving them by analyzing a scenario to determine a geometric shape that fits the context. SMP 4 Students model with mathematics by applying an appropriate geometric formula to solve a contextual problem.	• Determine which geometric concepts/figures best model a given situation. • Apply an array of formulas to determine the appropriate geometric solutions. • Design a model of a real-life object using geometric figures. **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy: understand, apply, create**
Previous Learning Connections	**Current Learning Connections**	**Future Learning Connections**
• In grades 7 and 8, learners worked with formulas for area, perimeter, surface area and volume, solving real world and mathematical problems.	• Students have been modeling throughout the Geometry course with many of the clusters with focus on using skills to model the real-world situations in this cluster.	• More complex modeling will be used in statistics, physics, trigonometry, and calculus when approaching real-world problems analytically.

Clarification Statement

Modeling is the process of choosing and using appropriate mathematics to analyze situations, to understand them better, and to improve decisions. Modeling links classroom mathematics to everyday life, work, and decision making. Mathematical objects that represent a situation from outside mathematics can be used to model and solve problems. Modeling often involves making simplifying assumptions and sometimes minimizes or disregards some features of the situation being modeled. Modeling is best interpreted not as a collection of isolated topics, but in relation to other standards as well.

Common Misconceptions

Students may struggle identifying approximate shapes to model scenarios.
Students may struggle breaking complex shapes into a combination of simpler shapes.
Students may struggle applying concepts like volume and surface area to language and contexts such as "has a capacity of" or "wraps around."

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that introduces new representations when studying applications of geometric concepts in modeling situation because students need a strong foundation on geometric methods such as solving for the area and volume and use the area and volume to solve for the density of a given geometric shape.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. This standard provides a foundation for work with applications of geometric concepts in modeling situation because students need to have a strong foundation on basic formulas and how to properly use it to solve problems and use it to justify their answers. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with modeling with Geometry will have the chance to link geometry to everyday life, work and decision making. In this specific cluster where students have to apply geometric concepts in modeling situations, students can choose several ways to model geometric concepts, such as, identifying different geometric shapes around them and discuss the use of the said shape benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features. Integration of this cluster to Science, Engineering and Technology, where you can challenge your students to design a bridge, building or any other structure using geometric shapes will allow students to see the use of geometry in real world and so that students can also relate. because letting our students be engaged in analyzing and applying their understanding through projects, illustrations and computations, we are encouraging them to make ways in solving real world problems. You can access different tasks on this link: https://achievethecore.org/coherence-map/HS/M/tasks

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with modelling with geometry benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports (e.g., peer-tutors) because students will benefit from hearing the perspectives of their peers, whether said perspective is a solution or a point of frustration. Establishing appropriate guidelines for these interactions will ensure that students are encouraged and supported by their peers. Consider grouping strategies based on student current level of understanding (homogenous or heterogeneous), native language (homogenous or heterogeneous), as well as student self-selected groups. Discuss and model appropriate guidelines for these interactions so that students have structure within their interactions.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with modelling with geometry benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams because students can often identify key information but may struggle with what to do next. Encouraging students to link information they know to formulas they know and/or creating sketches to show what is happening in the problem can help students make progress in solving the problem.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with modelling with geometry benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches, strategies, skills, etc.) because students may see solution methods in different ways. Showing students multiple perspectives, whether correct or incorrect, encourages students to think critically about their thinking and the reasoning of others. Further, showing the connections between the different solution methods can encourage students to think about mathematics in a variety of ways.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with modelling with geometry benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because the solution path is often not explicit. Students will need to pull from their background knowledge and determine what information is useful and how they can apply geometric concepts to arrive at a solution. Because this thinking process is abstract and vague, modelling strategies to persevere through frustration can benefit students. Consider strategies such as providing students with questions they can ask themselves (What do I know? What do I want to solve? etc.), encouraging students to draw/sketch a visual of the problem scenario, code the text of the problem. If students get stuck, help them to verbalize their "sticking point" and work with peers to get "unstuck". Each of these strategies teaches students to think about their thinking and encourages them to make progress in problem solving.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on applications of geometric concepts in modeling situations by critiquing student approaches/solutions to make connections through a short mini-lesson because you want to highlight and model how to decompose a problem and/or image to the apply characteristics of geometric figures. This initial step may be the hardest for students in solving real world problems.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on applications of geometric concepts in modeling situations by offering opportunities to understand and explore different strategies because it is very important that before moving to the next lesson, students must demonstrate understanding on the wide range of application of geometric shape and use in real world, such as solving for volume, area, and density specifically population density of a given area.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying applications of geometric concepts in modeling situations because students will have the opportunity to use their own foundation on solving geometric shapes the way they understand it as long as they can justify it mathematically.

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying applications of geometric concepts in modeling situations the use of mathematical representations within the classroom is critical because of the diverse cultural representation of every single student, however, if we let our students draw their own understanding on specific problems, where students can relate and they can justify their claim mathematically then we can say that learning took place by making connections.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
SAT Item #: 1053899 The linked assessment question addresses G-MG.A., specifically the question requires students to use the surface area given for a cube to find volume

CollegeBoard	Question ID 1053899						
SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Area and volume	1. Solve real-world and mathematical problems about a geometric figure or an object that can be modeled by a geometric figure using given information such as length, area, surface area, or volume.	No Calculator

A cube has a surface area of 54 square meters. What is the volume, in cubic meters, of the cube?

Question Difficulty: Medium

A. 18

B. 27

C. 36

D. 81

Choice B is correct. The surface area of a cube with side length s is equal to $6s^2$. Since the surface area is given as 54 square meters, the equation $54 = 6s^2$ can be used to solve for s. Dividing both sides of the equation by 6 yields $9 = s^2$. Taking the square root of both sides of this equation yields $3 = s$ and $-3 = s$. Since the side length of a cube must be a positive value, $s = -3$ can be discarded as a possible solution, leaving $s = 3$. The volume of a cube with side length s is equal to s^3. Therefore, the volume of this cube, in cubic meters, is 3^3, or 27.

Choices A, C, and D are incorrect and may result from calculation errors.

Additional Assessment:
http://tasks.illustrativemathematics.org/content-standards/HSG/MG/A/2/tasks/1146
The linked assessment question addresses G-MG.A, specifically the question requires students to apply the relationship among density, volume and mass to reasonably estimate the number of cells in a human body. In this approach, we assume that a cell is a sphere and use that fact, along with the provided density of a cell to determine the mass of a cell. We then divide an individual's mass by the mass of a single cell. This assessment should be given to students after they've had opportunity to work with this relationship as well as had time to work with numbers in scientific notation. Students will engage in SMP 1 and SMP 6.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on applications of geometric concepts in modeling situation, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning the different geometric shape in school, home, and community can be a great way to connect school task with home task, such as letting the students identify geometric shape around them in school, home, or community. Let them describe the use and how helpful that shape is to the structure or building.	Business: Connect to minimizing waste, maximizing volume. Social Studies: Connect to census data/population density

NEW MEXICO
Public Education Department

HS: GEOMETRY-GEOMETRIC MEASUREMENT & DIMENSION		
Cluster Statement: A: Explain volume formulas and use them to solve problems		
Standard Text HSG.GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	**Standard for Mathematical Practices** SMP 2 Students reason abstractly and quantitatively by making conjectures about volume of objects with similar dimensions SMP 3 Students construct viable arguments and critique the reasoning of others by constructing and analyzing arguments about volumes SMP 4 Students model with mathematics by constructing multiple representations of given situation in order to make valid arguments and conclusions	**Students who demonstrate understanding can:** • Demonstrate Cavalieri's Principle concretely. • Give an informal argument for circumference and area formulas for circles. • Give an informal argument for volume formulas of cylinders, pyramids, and cones. • Construct viable arguments to validate the circumference of a circle, volume of a cylinder, volume of a pyramid, and volume of a cube by using Cavalieri's Principle. **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy:** understand, apply, analyze
Standard Text **HSG.GMD.A.3** **Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.**	**Standard for Mathematical Practices** SMP 1 Students make sense of problems and persevere in solving them by analyzing the problem and choosing the correct model in which to proceed SMP 4 Students model with mathematics by using derived equations, formulae, and theorems to complete problems SMP 7 Students look for and make use of structure by noting that volume formulae are constructed from the products of the area of the base of	**Students who demonstrate understanding can:** • Identify these geometric shapes: cylinders, pyramids, cones, and spheres. • Calculate volume for cylinders, pyramids, cones and spheres. • Use formulas to solve problems involving three-dimensional figures. • Apply volume to real world problems. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** understand, apply

	an object times its height or some variation on that theme	
Previous Learning Connections In 7th grade, learners worked with area, and circumference which extended to find components needed for surface area and volume. Throughout grades 6, 7, and 8 students calculated the volumes and surface areas of prisms, cones, cylinders, and spheres which will connect to their work figures within this cluster.	**Current Learning Connections** Students will continue to expand their work to include composite figures. They will also justify volume formulas and other constructions.	**Future Learning Connections** In Calculus, students will apply Cavalieri's principle to calculate volumes for solids of rotation.

Clarification Statement

Students move from applying volume formulas to justifying them. Students will be exposed to advanced concepts in an informal setting. Learners will deconstruct complex geometric shapes into basic three-dimensional shapes to calculate their surface areas and volumes.

Common Misconceptions

Students may mix up which formula to use for a given figure.

Students may have difficulty identifying the base of a figure.

When considering units, students may struggle to interpret inches cubed or forget these units in the final answer.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying explaining volume formulas and using them to solve problems because students have to use different formulas that are necessary to support them on solving problems involving volume of three-dimensional figures, such as, cylinder, pyramid, and cone.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. This standard provides a foundation for work with explaining volume formulas and using them to solve problems because students need to master the different formulas and use them to solve real-world problems. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging in explaining volume formulas and use them to solve problems because it will help the student understand the concept at a deeper level than just plugging in numbers and making calculations. When the student can give an informal argument for what makes up a volume formula, their understanding will be at a greater depth of knowledge.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with volume formulas and use them to solve problems benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because allowing students to become self-directed learners will foster in them a need to explore concepts deeply. to become investigators

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging volume formulas and use them to solve problems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) because when using volume formulas the background knowledge gives so much understanding of how these formulas are derived. If a student understands the language and symbology of volume as a concept, they do not need to even remember the exact formula. The student will be able to construct a volume calculation using the parts of the object.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging volume formulas and use them to solve problems benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as composing in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video because in this cluster the focus on the volume of an object can be very easily adapted into large scale representations such as constructions or by measuring everyday objects. By allowing students to represent using multiple media of their choosing, we are jumping into interest categories enjoyed by each student.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging volume formulas and use them to solve problems benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as addressing subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math")because there is a need to create a culture of positive math experiences. So many students do not have that, they have a history of feeling as if they were failures.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on GMD.A: Explain Volume Formulas and Use Them to Solve Problems by providing specific feedback to students on their work through a short mini-lesson because teachers need to monitor students on how they use different formulas and use them precisely to answer a given problem. Providing students specific feedback will help students to create strong justifications of their answers.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit GMD.A: Explain Volume Formulas and Use Them to Solve Problems by helping students move from specific answers to generalizations for certain types of problems because students need to master how they justify answers mathematically and base from specific concepts to come up with a general, accurate and precise answers.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying explaining volume formulas and using them to solve problems because these topics are interconnected and it will help students to master the whole concepts. Students need to have a strong foundation of different formulas and let them use it to formulate and come up with justifications on volumes.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students. For example, when studying, explaining volume formulas and use them to solve problems supporting productive struggle is critical because students may not see the connections of other formulas to volume and they need those to solve problems involving volume. However, students may see the connection by presenting the actual image which is relevant and relatable to them, such as, car tires, basketball, and any other shape that is visible to them.

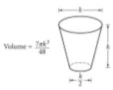
Standards Aligned Instructionally Embedded Formative Assessment Resources:

SAT Item #: 1474145 The linked assessment question addresses G-GMD.A., specifically the question requires students to use a specified volume for an object to determine the radius.

NEW MEXICO
Public Education Department

CollegeBoard | Question ID 1474145

SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Area and volume	1. Solve real-world and mathematical problems about a geometric figure or an object that can be modeled by a geometric figure using given information such as length, area, surface area, or volume.	Calculator



The glass pictured above can hold a maximum volume of 473 cubic centimeters, which is approximately 16 fluid ounces.

What is the value of k, in centimeters?

Question Difficulty: Medium

A. 2.52

B. 7.67

C. 7.79

D. 10.11

Choice D is correct. Using the volume formula $V = \frac{7\pi k^3}{48}$ and the given information that the volume of the glass is 473 cubic centimeters, the value of k can be found as follows:

$$473 = \frac{7\pi k^3}{48}$$

$$k^3 = \frac{473(48)}{7\pi}$$

$$k = \sqrt[3]{\frac{473(48)}{7\pi}} \approx 10.10690$$

Therefore, the value of k is approximately 10.11 centimeters.

Choices A, B, and C are incorrect. Substituting the values of k from these choices in the formula results in volumes of approximately 7 cubic centimeters, 207 cubic centimeters, and 217 cubic centimeters, respectively, all of which contradict the given information that the volume of the glass is 473 cubic centimeters.

Additional Assessment:
http://tasks.illustrativemathematics.org/content-standards/HSG/GMD/A/3/tasks/1899
The linked assessment question addresses G-GMD.A, specifically the question requires students to apply volume formulas to model the Egyptian Pyramids and find missing information. Students will utilize the formula for volume by rearranging the formula to highlight to value of interest. This assessment should be given to students after they've been introduced to volume formulas and had opportunity to reason with the formulas when a variety of information is provided. Students will engage in SMP 2 and SMP 4.

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on explaining volume formulas and use them to solve problems , consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning the Cavalieri's principle is a great entry point and support students to create arguments and use it to solve volume of a given geometric shapes.	Because volume can be found for any given item, this connection can be made to a variety of areas: science-beakers, social studies-coffins, art-paint bottles, music-variety of instruments

NEW MEXICO
Public Education Department

HS: GEOMETRY-GEOMETRIC MEASUREMENT & DIMENSION

Cluster Statement: B: Visualize relationships between two-dimensional and three-dimensional objects

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSG.GMD.B.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	SMP 1 Students make sense of problems and persevere in solving them by using visual representations and or 3D manipulatives to make sense of unfolding a 3D object.	• Explain why a cross section is a two-dimensional representation of a slice of a three-dimensional object. • Realize that the cross section can be different depending on location and angle where the three-dimensional object is cut. • Identify possible cross sections in a given object • Identify three-dimensional objects generated by the rotations of two-dimensional objects
		Webb's Depth Of Knowledge: 1-2
		Bloom's Taxonomy: apply, analyze
<u>Previous Learning Connections</u> In 6th grade, students represented three-dimensional figures using nets made up of rectangles and triangles to calculate surface area. In 7th grade students moved on to describe the two-dimensional figures that result from slicing three-dimensional figures, focusing on right rectangular prisms and pyramids. These skills prepare students to address the content within this cluster.	**<u>Current Learning Connections</u>** Students use cross section dimensions in volume calculations (i.e. the height of the triangle when calculating the volume of the cone)	**<u>Future Learning Connections</u>** In later courses, students study conic sections which can be described as cross sections of a cone. Calculus concepts will build on the volume of solids of rotation.

Clarification Statement
The focus of this cluster is to reinforce the relationship between a three-dimensional object and the dimensions of its two-dimensional cross section. Focusing on the two-dimensional cross-sections helps learners visualize dimensions needed later for finding volume and surface area of solids. For example, the height of the triangle in the cross section of a cone.

Common Misconceptions
Students may struggle to visualize cross sections and rotations.
Students may need support with the concept of "slicing" a three-dimensional figure.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when studying visualize relationships between two-dimensional and three-dimensional objects because students need to have a visual representation of the object. As much as possible use objects that are familiar to students.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.G.A.3: Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. This standard provides a foundation for work with visualize relationships between two-dimensional and three-dimensional objects because students need to visualize how it looks like when a three-dimensional object is being cut. What are the different products after cutting? If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Physical Action: *How will the learning for students provide a variety of methods for navigation to support access?*

- For example, learners engaging with visualize relationships between 2-d and 3-d representations benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as varying methods for response and navigation by providing alternatives to hand-drawn cross-sections because students may struggle to visualize these cross-sections. Providing hands-on manipulatives to rotate and/or using geometric software can aide students in visualizing and creating 2-d cross-sections for 3-d objects.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with visualize relationships between 2-d and 3-d representation benefit when learning experiences attend to student attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics representations and scaffolds because students learn in various ways. Attend to visual, auditory, kinesthetic, interpersonal, intrapersonal, etc. learning modalities by offering opportunities for structured learning and/or independent learning, and allow students to demonstrate their understanding using a variety of tools strategically.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with visualize relationships between 2-d and 3-d representation benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as linking key vocabulary words to definitions and pronunciations in both dominant and heritage languages because terms like circumscribed and inscribed can be seen as cognates between Spanish/English. Drawing the connection between languages can help students whose first language is not English.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with visualize relationships between 2-d and 3-d representation benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing sentence starters or sentence strips because the cluster requires students to communicate (in writing and/or orally) about circles. Students may struggle with a starting point in stating a proof, or describing a characteristic they see. Students may also struggle with using the vocabulary of this cluster.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with visualize relationships between 2-d and 3-d representation benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as using multiple examples and non-examples to emphasize critical features because the terminology of this cluster can be easy for students to mix up (circumscribed V inscribed, radius V diameter, etc.). Providing regular pictorial examples and non-examples of these can help to solidify student conceptual understanding and allow them to apply this knowledge in communication and problem solving.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on visualize relationships between two-dimensional and three-dimensional objects by providing specific feedback to students on their work through a short mini-lesson because students have to properly visualize the object, it's movement to the plane so students can accurate use the object to solve problems.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit visualize relationships between two-dimensional and three-dimensional objects by offering opportunities to understand and explore different strategies because offering students different ways of visualizing two-dimensional and three-dimensional objects will give them the opportunity to express their thinking through illustrations and analysis.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

	• For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying visualizing relationships between two-dimensional and three-dimensional objects because students will deepen their understanding of the topic and they can use this in real-world. Challenge the students to do frustum of a two-dimensional and three-dimensional objects.

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying visualize relationships between two-dimensional and three-dimensional objects the use of mathematical representations within the classroom is critical because students need to critically think about the relationship of two-dimensional and three-dimensional objects. We can help our students by presenting them the actual image and image that they can relate to, like a ball, car tires, cellphone and any other object that is visible to them.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/HSG/GMD/B/4/tasks/512
The linked assessment question addresses G-GMD.B, specifically the question requires students to consider an object being passed through an x-ray machine. Students are describing the shapes they would see. Later parts of this example approach more complex connections to future work in calculus as students are applying Cavalieri's principle. The first 3 parts of this assessment should be given to students after they've had opportunity to practice visualizing cross sections and possibly after use of tools to create cross sections. Students will engage in SMP 1 and SMP 3. If tools are used, SMP 5 may be addressed as well.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on visualize relationships between two-dimensional and three-dimensional objects, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, let the students identify two-dimensional and three-dimensional objects and let them discuss the difference and relate it to the question "what if this two-dimensional object is not present to the three-dimensional object, what will happen to the image?"	Students can produce interesting pieces of art by rotating a two-dimensional object that has been dipped in ink. This can also link to Language Arts and Science by having students verbally describe a conjecture of what they think an end result may be and then experimentally verifying and reflecting upon their initial thoughts. Stressing the importance of precise language in writing descriptions deepens the connection between ELA and math.

NEW MEXICO
Public Education Department

HS: GEOMETRY-EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS		
Cluster Statement: A: Translate between the geometric description and the equation for a conic section		
Standard Text HSG.GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	**Standard for Mathematical Practices** SMP 2 Students reason abstractly and quantitatively by using the Pythagorean Theorem to derive the equation of a circle and complete the square to find the center of a circle. SMP 3 Students construct viable arguments and critique the reasoning of others by discussion of why and how the Pythagorean theorem relates to the equation of a circle and why this is important. SMP 7 Students look for and make use of structure by expecting students to apply rules, look for patterns, and analyze structure.	**Students who demonstrate understanding can:** • Explain how the Pythagorean Theorem can be used to derive the equation of a circle. • Write the equation of a circle, given the center and radius. • Complete the square within the equation of a circle in order to find the center and radius. **Webb's Depth of Knowledge: 2**-3 **Bloom's Taxonomy:** apply, analyze
Standard Text HSG.GPE.A.2 Derive the equation of a parabola given a focus and directrix.	**Standard for Mathematical Practices** SMP 1 Students make sense of problems and persevere in solving them by taken a given set of values such the focus and directrix and using that info to derive the equation of a parabola. SMP 8 Students look for and express regularity in repeated reasoning by utilizing given information and rules about parabolas to make	**Students who demonstrate understanding can:** • Describe the characteristics of a parabola given its equation. • Derive the equation for a parabola given the focus and directrix. **Webb's Depth of Knowledge: 1-3**

	general extrapolations about the equations of a parabola	**Bloom's Taxonomy: understand, analyze**
<u>**Previous Learning Connections**</u> • Students have worked with coordinates, slope, and the Pythagorean Theorem in 8th grade math. This work exploring facts about right triangles connects to the foundational formulas in analytic geometry. Additionally, in Algebra I, students have been rewriting expressions in different forms (factoring and completing the square) which directly correlates to the work they will complete in this cluster when creating algebraic proofs of the theorems.	<u>**Current Learning Connections**</u> • Students will connect the information in this cluster to learning later in the course by extending the precise definitions of circles and polygons to work with coordinates on the plane.	<u>**Future Learning Connections**</u> Learners will continue with graphing quadratic functions, showing vertices, intercepts, and identifying maxima or minima in the Algebra II course.

Clarification Statement
The introduction of coordinates into geometry connects geometry and algebra, allowing algebraic proofs of geometric theorems.

Common Misconceptions
Students commonly swap h and k when working with the equations for the circle.

Students will make similar mistakes with h and k, when finding the vertex of a parabola.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying translating between geometric descriptions and the equations for a conic section because students will need to know the distance formula which is learned in Grade 8. A review of distance between two points on the coordinate grid will help students in this cluster.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.G.B.8: This standard provides a foundation for work with translating between the geometric descriptions and the equation for a conic section because students have to apply the Pythagorean theorem to find the distance between two points. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Physical Action: *How will the learning for students provide a variety of methods for navigation to support access?*

- For example, learners engaging with translating between the geometric description and the equation for a conic section benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as relating formulas and written equations to graphical representations because students may benefit from seeing the connection between changes made in equations and the visual outcome.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with prompts or scaffolds for visualizing desired outcomes benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as working in cooperative learning groups with clear goals, roles and responsibilities because some students may need extra help when writing equations, using technology, or incorporating formulas.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with translating between the geometric description and the equation for a conic section benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc.) because students may need support in recalling what specific terms mean in a definition, how it appears in a visual representation and how this is shown through an equation.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with translating between the geometric description and the equation for a conic section benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches, strategies, skills, etc.) because this standard calls for students to explore the relationships and patterns to reason about their derived equations.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because we want students to feel supported in their learning and empowered in their thinking. Providing student supports will minimize frustration and give them a means to cope in difficult situations.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on translating between the geometric description and the equation for a conic section by critiquing student approaches/solutions to make connections through a short mini-lesson because this cluster requires students to generalize patterns they see through exploration. These patterns may not be the same for every student but connecting the different patterns can reveal opportunities to deepen understanding and/or correct misunderstandings.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit of translating between the geometric description and the equation for a conic section by helping students move from specific answers to generalizations for certain types of problems because this cluster calls on students to recognize and generalize patterns. Students may need extra support in moving from concrete examples to generic patterns.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying translating between the geometric description and the equation for a conic section because some students may see generalizations easier than others. Allowing these students to move faster through the concrete examples to get to the abstract generalizations will allow them to stretch their expression of mathematical reasoning from concrete to abstract.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying translating between the geometric description and the equation for a conic section, goal setting is critical because students may be at varying levels of academic and language proficiency. To help students identify goals, teachers can use strategies such as writing prompts to gauge their thinking. Teachers can also provide students with a means to track their personal data (this assists in knowing where you currently are in attaining your goal). Teachers should work to build rapport, relationships and respect with their students and amongst each other to create that positive classroom culture in which students are willing to share/monitor their goals with one another without being judged.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
SAT Item #: 4170563 The linked assessment question addresses G-GPE.A., specifically the question requires students to use a given radius and point to write equation of a circle.

CollegeBoard **Question ID 4170563**

SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Circles	3. Create an equation to represent a circle in the xy-plane.	Calculator

In the xy-plane, a circle with radius 5 has center $(-8,6)$. Which of the following is an equation of the circle?

Question Difficulty: Medium

A. $(x-8)^2+(y+6)^2=25$

B. $(x+8)^2+(y-6)^2=25$

C. $(x-8)^2+(y+6)^2=5$

D. $(x+8)^2+(y-6)^2=5$

Choice B is correct. An equation of a circle is $(x-h)^2+(y-k)^2=r^2$, where the center of the circle is (h,k) and the radius is r. It's given that the center of this circle is $(-8,6)$ and the radius is 5. Substituting these values into the equation gives $(x-(-8))^2+(y-6)^2=5^2$, or $(x+8)^2+(y-6)^2=25$.

Choice A is incorrect. This is an equation of a circle that has center $(8,-6)$. Choice C is incorrect. This is an equation of a circle that has center $(8,-6)$ and radius $\sqrt{5}$. Choice D is incorrect. This is an equation of a circle that has radius $\sqrt{5}$.

SAT Item #: 1474672 The linked assessment question addresses G-GPE.A., specifically the question requires students to use a given equation and point to find diameter of a circle.

CollegeBoard **Question ID 1474672**

SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Circles	5. Understand that the ordered pairs that satisfy an equation of the form (x − h) ² + (y − k) ² = r² form a circle when plotted in the xy-plane.	No Calculator

$(x-6)^2+(y+5)^2=16$

In the xy-plane, the graph of the equation above is a circle. Point P is on the circle and has coordinates $(10,-5)$. If $\overline{PQ}$ is a diameter of the circle, what are the coordinates of point Q ?

Question Difficulty: Medium

A. $(2,-5)$

B. $(6,-1)$

C. $(6,-5)$

D. $(6,-9)$

Choice A is correct. The standard form for the equation of a circle is $(x-h)^2+(y-k)^2=r^2$, where (h,k) are the coordinates of the center and r is the length of the radius. According to the given equation, the center of the circle is $(6,-5)$. Let (x_1,y_1) represent the coordinates of point Q. Since point P $(10,-5)$ and point Q (x_1,y_1) are the endpoints of a diameter of the circle, the center $(6,-5)$ lies on the diameter, halfway between P and Q. Therefore, the following relationships hold: $\frac{x_1+10}{2}=6$ and $\frac{y_1+(-5)}{2}=-5$. Solving the equations for x_1 and y_1, respectively, yields $x_1=2$ and $y_1=-5$. Therefore, the coordinates of point Q are $(2,-5)$.

Alternate approach: Since point P $(10,-5)$ on the circle and the center of the circle $(6,-5)$ have the same y-coordinate, it follows that the radius of the circle is $10-6=4$. In addition, the opposite end of the diameter $\overline{PQ}$ must have the same y-coordinate as P and be 4 units away from the center. Hence, the coordinates of point Q must be $(2,-5)$.

Choices B and D are incorrect because the points given in these choices lie on a diameter that is perpendicular to the diameter $\overline{PQ}$. If either of these points were point Q, then $\overline{PQ}$ would not be the diameter of the circle. Choice C is incorrect because $(6,-5)$ is the center of the circle and does not lie on the circle.

Additional Assessment:

http://tasks.illustrativemathematics.org/content-standards/HSG/GPE/A/1/tasks/1425

The linked assessment question addresses G-GPE.A, specifically the question requires students to develop a generic equation for a circle using Pythagorean theorem after proving a specific case works. Students begin with a specific point and radius. Once the connection is made in this specific case, students apply the same logic to a general point. This assessment should be given to students after they've been introduced to the Pythagorean theorem and had time to work with its use in the coordinate plane. Students will engage in SMP 2 and SMP 6.

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on translating between the geometric description and the equation for a conic section, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, relating these standards to real-life context such as finding the distance from one building to another in the community will solidify	Consider linking using circles in cartography as a means of utilizing central point to look at an area of interest. This may relate to social studies as well as forensic science.

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Public Education Department

the school learning to the actual application of standards/skills.	

HS: GEOMETRY-EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS

Cluster Statement: B: Use coordinates to prove simple geometric theorems algebraically

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSG.GPE.B.4 **Use coordinates to prove simple geometric theorems algebraically.** *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*	SMP 2 Students reason abstractly and quantitively by using formulas such as the distance formula, midpoint formula, equation of circles, equation of ellipses, and slope formula to prove figures on a coordinate plane are qualified shapes such as rectangles, squares, circles and ellipses SMP 6 Students attend to precision by using correct and appropriate theorems and formulae for the given shape to prove its existence SMP 7 Students look for and make use of structure by utilizing slope to prove parallelism and perpendicularity when working with quadrilaterals on the coordinate plane, and the Pythagorean theorem to prove triangles, circles and ellipses	• Use the distance formula to find the distance between coordinates. • Find the slope of a line connecting two coordinates. • Determine if a point lies on a specific circle. • Use coordinates to prove that a quadrilateral is, or is not, a parallelogram, rectangle, rhombus, square, or trapezoid. • Use coordinates to prove a triangle's classification by its sides. **Webb's Depth Of Knowledge: 1-2** **Bloom's Taxonomy:** understand, apply
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSG.GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	SMP 2 Students reason abstractly and quantitatively by interpreting the meaning of parallel and perpendicular lines graphically, numerically, and to generalize their findings SMP 5 Students use appropriate tools strategically by using straightedges, protractors, geometry and graphing software/apps. SMP 7	• Prove how parallel lines increase at the same rate of change. • Explain that perpendicular lines intersect at a right angle. • Construct an equation of a line that is parallel or perpendicular to a given line. • Calculate slope from given ordered pairs. • Classify lines or segments as parallel or perpendicular given slopes, graphs, and/or equations of lines. • Write equations for parallel lines and perpendicular lines

	Students look for and make use of structure by using patterns relating the slopes of parallel and perpendicular lines to generalize to form rules about these pairs of lines.	given a point and an equation of a line.
		Webb's Depth of Knowledge: 1-3
		Bloom's Taxonomy: understand, analyze
Standard Text HSG.GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	**Standard for Mathematical Practices** SMP 1 Students make sense of problems and persevere in solving them by using a variety of methods such as graphing, algebra, and proportional reasoning to make sense of the given information and the task at hand SMP 3 Students construct viable arguments and critique the reasoning of others by justifying using graphing, mathematics and logic why two lines are parallel, perpendicular, or neither SMP 7 Students look for and make use of structure by using patterns relating the slopes of parallel and perpendicular line to form general rules about pairs of lines	**Students who demonstrate understanding can:** • Determine the coordinates of a point of a given partition on a directed segment. • Use the midpoint formula, the section formula, and the distance formula to find the partition point of a given line segment. • Determine the ratio of a partition using the distance formula. • Given two points, find the point on a line segment between the two points that divides the segment into a given ratio
		Webb's Depth of Knowledge: 1-2
		Bloom's Taxonomy: understand, apply

<table>
<tr>
<td rowspan="3">Standard Text

HSG.GPE.B.7
Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.</td>
<td rowspan="3">Standard for Mathematical Practices
SMP 2
Students reason abstractly and quantitatively by working problem piecewise as separate distances and partial areas and synthesizing into perimeters and areas of Polygons.

SMP 4
Students model with mathematics by using mathematical properties and graphic representations to solve problems.

SMP 7
Students look for and make use of structure by using distance, slope, and areas of prior learned shapes to prove properties of geometric figures.</td>
<td>Students who demonstrate understanding can:
Use the distance formula to find the length of sides of a polygon.
Choose the appropriate formula for perimeter or area of a given polygon.
Calculate areas and perimeters of polygons.
Use appropriate labels for the areas and perimeters.</td>
</tr>
<tr>
<td>Webb's Depth of Knowledge: 1-2</td>
</tr>
<tr>
<td>Bloom's Taxonomy: understand, apply</td>
</tr>
<tr>
<td><u>Previous Learning Connections</u>
In 6th grade, learners find the area of polygons by composing into rectangles or decomposing into triangles and other shapes. They also draw polygons in the coordinate plane given coordinates for the vertices and find the length of horizontal and vertical sides. In 7th grade, learners solve real-world and mathematical problems involving area of triangles, quadrilaterals, and polygons. In 8th grade, learners apply the Pythagorean Theorem to find the distance between two points in a coordinate system. In Algebra I, learners write equations of lines given a slope and point.</td>
<td><u>Current Learning Connections</u>
Learners have already had experience with properties of quadrilaterals, equations of circles, and finding area and perimeter earlier in the course. They now apply this knowledge to working with coordinates. Learners will use the concept of distance and midpoint throughout the rest of the geometry course They apply the concepts later when calculating volumes and surface areas or when proving types of quadrilaterals given the ordered pairs of their vertices. They also use distance and midpoint when writing and deriving the equation of circles.</td>
<td><u>Future Learning Connections</u>
Distance is an application important for many future concepts. For example, when writing equations of conic sections or converting between polar and rectangular coordinates or finding the magnitude of vectors.</td>
</tr>
<tr>
<td colspan="3">Clarification Statement
The focus of this cluster is coordinate geometry. Students work with coordinates to find slope, distances, midpoints, and locations that are at a specified ratio from an endpoint. They then use this information to prove geometric relationships such as properties of quadrilaterals or location of a point on a circle. Using slope criteria for parallel and perpendicular lines, students write equations of lines. Using lengths computed from coordinates, students find perimeters and areas of polygons.</td>
</tr>
</table>

Common Misconceptions
Students may misunderstand the negative reciprocal slope with perpendicular lines.

Students commonly forget to take the square root of the constant to find the radius in the equation of a circle.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying the use of coordinates to prove simple geometric theorems algebraically because students will need to be familiar with discovering geometric properties before they can make sense of how to use coordinates to understand the properties.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- <HS.G-GPE.A.1>: This standard provides a foundation for work with using coordinates to prove simple geometric theorems algebraically because students should be able to work with and derive equations of geometric shapes before proving theorems. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with the use of coordinates to prove simple geometric theorems algebraically benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because students can then make connections between their learning and real-life applications.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with the use of coordinates to prove simple geometric theorems algebraically benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports (e.g., peer-tutors) because students may understand the properties but struggle formalizing a proof. Further, some students may feel mathematically sound using the coordinate plane but struggle connecting the algebra to a specific geometric property. In either case, working with a peer can reveal possible next steps and allows students time to critique the thinking of others and well as reflect on their own ideas.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with the use of coordinates to prove simple geometric theorems algebraically benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as providing graphic symbols with alternative text descriptions because this cluster requires a firm grasp of simple geometric properties, the coordinate plane, and the structures of proof writing. Each of these areas contains concepts that students may need help recalling. Consider a vocabulary strategy, word wall or other resource which includes terms, visual examples and student friendly definitions.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with the use of coordinates to prove simple geometric theorems algebraically benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because students are able to use a variety of tools such as graph paper and dynamic geometry software to make connections between characteristics of geometric figures and the coordinate plane. As a result, students may show their understanding through a variety of visual means, a written explanation and/or algebraic expressions.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with the use of coordinates to prove simple geometric theorems algebraically benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review because students must have a firm grasp of the geometric properties conceptually before they can apply them to algebraic proofs using coordinate math. Reviewing key properties students struggle with before and/or throughout this cluster can help students build the connection between prior studies and algebraic proofs.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on using coordinate to prove simple geometric theorems algebraically by clarifying mathematical ideas and/or concepts through a short mini-lesson because students may struggle to apply knowledge of geometric figures to coordinate algebra or vice-versa.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit of using coordinates to prove simple geometric theorems algebraically by addressing conceptual understanding because revisiting basic geometric shapes in the coordinate plane will assist in discovering geometric properties, which in turn will help them understand how coordinates can help prove theorems.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

	• For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying the use of coordinates to prove simple geometric theorems algebraically because students will have the autonomy to make connections to personalized real-life situations in connection to this standard.

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying the use coordinates to prove simple geometric theorems algebraically the pattern of questions within the classroom is critical because it can allow students to build upon each other's ideas. Further, teachers can tap into student's prior knowledge and use it to promote learning for all students. Teachers can utilize strategic sequencing and questioning to encourage all students to participate in engaging with the content and seeing connections between multiple representations and solution methods. This can facilitate cross-content connections. When posing purposeful questions to the whole group, teachers should have protocols in place (classroom management) that tend to how students will respond to and discuss questions. Finally, teacher can use activities in which students are able to share (partners or groups) their thoughts and ideas in a judgement-free zone.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
SAT Item #: 421901 The linked assessment question addresses G-GPE.B., specifically the question requires students to analyze a given an equation and determine if a point is within the circle.

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CollegeBoard	Question ID 421901						
Assessment **SAT**	Test **Math**	Cross-Test and Subscore **Additional Topics in Math**	Difficulty **Hard**	Primary Dimension **Additional Topics in Math**	Secondary Dimension **Circles**	Tertiary Dimension **1. Use definitions, properties, and theorems relating to circles and parts of circles, such as radii, diameters, tangents, angles, arcs, arc lengths, and sector areas to solve problems.**	Calculator **Calculator**

A circle in the xy-plane has equation $(x+3)^2+(y-1)^2=25$. Which of the following points does NOT lie in the interior of the circle?

Question Difficulty: Hard

A. $(-7, 3)$

B. $(-3, 1)$

C. $(0, 0)$

D. $(3, 2)$

Choice D is correct. The circle with equation $(x+3)^2+(y-1)^2=25$ has center $(-3,1)$ and radius 5. For a point to be inside of the circle, the distance from that point to the center must be less than the radius, 5. The distance between $(3,2)$ and $(-3,1)$ is $\sqrt{(-3-3)^2+(1-2)^2}=\sqrt{(-6)^2+(-1)^2}=\sqrt{37}$, which is greater than 5. Therefore, $(3,2)$ does NOT lie in the interior of the circle.

Choice A is incorrect. The distance between $(-7,3)$ and $(-3,1)$ is $\sqrt{(-7+3)^2+(3-1)^2}=\sqrt{(-4)^2+(2)^2}=\sqrt{20}$, which is less than 5, and therefore $(-7,3)$ lies in the interior of the circle. Choice B is incorrect because it is the center of the circle. Choice C is incorrect because the distance between $(0,0)$ and $(-3,1)$ is $\sqrt{(0+3)^2+(0-1)^2}=\sqrt{(3)^2+(1)^2}=\sqrt{8}$, which is less than 5, and therefore $(0,0)$ in the interior of the circle.

Additional Assessment
http://tasks.illustrativemathematics.org/content-standards/HSG/GPE/B/4/tasks/605

The linked assessment question addresses G-GPE.B, specifically the question requires students to create their own quadrilateral and state verbally what observations they can make. Students then apply generic coordinate algebra to prove their observation. This assessment should be given to students after they've had practice applying coordinate math to prove conjectures about geometric figures. Students will engage in SMP 2, SMP 6 and possibly SMP 3 if they are asked to share and critique other's work.

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on the use coordinates to prove simple geometric theorems algebraically, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example allowing students to relate this math to their home and community by plotting points to create an approximate map of a sectioned off area at home or parking lots in the community and finding the amount of fencing needed.	Home Economics: Connect to construction and agriculture.

HS: GEOMETRY- CIRCLES		
Cluster Statement: A: Understand and apply theorems about circles		
Standard Text HSG.C.A.1 Prove that all circles are similar.	**Standard for Mathematical Practices** MP3 Students construct viable arguments and critique reasoning of other by engaging in discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments when discussing circles. MP5 Students use appropriate tools strategically by utilizing tools to prove circle similarity. These tools might include pencil and paper, concrete models, protractors, compasses, calculators, and software or apps.	**Students who demonstrate understanding can:** • Show that all circles are similar by proving that the ratio of a circle's circumference to its diameter for different sized circles is a constant • Calculate the circumference of a circle, given the diameter or radius. • Calculate angles inside and outside of a circle. • Prove that circles are similar. • Compare the ratios of the radius and circumference of multiple circles to determine similarity. **Webb's Depth of Knowledge:** 3-4 **Bloom's Taxonomy:** analyze, evaluate
Standard Text **HSG.C.A.2** **Identify and describe relationships among inscribed angles, radii, and chords.** *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	**Standard for Mathematical Practices** MP1 Students make sense of problems and persevere in solving them by interpreting and make meaning of a problem. Students monitor their progress and change their approach to solving if necessary MP3 Students construct viable arguments and critique reasoning of other by engaging in discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments when discussing circles.	**Students who demonstrate understanding can:** • Identify central angles, inscribed angles, circumscribed angles, tangent line and chords on a circle from a drawing. • Construct and explain examples of central angles, inscribed angles, circumscribed angles, tangent line and chords on a circle. • Describe the relationship between central angles, inscribed angles, circumscribed angles, tangent lines, and chords. **Webb's Depth of Knowledge: 1-2**

	MP5 Students use appropriate tools strategically by utilizing tools to prove circle similarity. These tools might include pencil and paper, concrete models, protractors, compasses, calculators, and software or apps.	**Bloom's Taxonomy: understand, apply**
Standard Text HSG.C.A.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	**Standard for Mathematical Practices** MP3 Students construct viable arguments and critique reasoning of other by engaging in discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments when discussing circles. MP5 Students use appropriate tools strategically by utilizing tools to prove circle similarity. These tools might include pencil and paper, concrete models, protractors, compasses, calculators, and software or apps.	**Students who demonstrate understanding can:** • Construct angle bisectors, perpendicular bisectors, inscribed circle of a triangle and circumscribed circle about a triangle. • Prove that opposite angles of an inscribed quadrilateral are supplementary. **Webb's Depth of Knowledge: 1-3** **Bloom's Taxonomy:** apply, analyze
Previous Learning Connections In 8th grade, learners have worked with two-dimensional figures and verified their properties. These skills from 8.G.A. will support students in their ability to work with circle while working within this standard.	**Current Learning Connections** Previous work with similarity will be applied to circles. Construction adds to learning from previous clusters by increasing skills with formal construction, building on angle congruence, perpendicular lines/segments, and properties of polygons. This will lead to work with arcs and areas of sectors, as well as prepare learners for future work in the Geometry course with 3-dimensional geometry and cross-sections.	**Future Learning Connections** Unit circles, their central angles, and reference angles will build on foundational skills learned in this cluster.

Clarification Statement
Learners will apply concepts of similarity to circles and their related components, explore inscribed and circumscribed circles and their associated polygons through constructions. This cluster builds many of the basic properties for angles, lines, and segments related to circles. Learners explore those properties and form conjectures. They should then be encouraged to create justifications regarding why their conjectures are correct. Learners refer back to their work with transformations to better understand these relationships.

Common Misconceptions

Students may try to solve by sketching the circles instead of ensuring precision by using a compass or appropriate tool to construct an accurate circle.

Student might confuse the relationships of central angle, inscribe angles, circumscribe angles, as well as tangent line and chords of a circle.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying understand and apply theorems about circles because new concepts (inscribed, circumscribed and central angles and tangent lines) are connected with prior held concepts (parallel, perpendicular, radii, etc). Students may need extra time to wrestle with the differences between these types of angles/lines and the work they have already mastered.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- SRTA2/CO.C.9: This standard provides a foundation for work with understand and apply theorems about circles because these lay the groundwork for understanding similarity and congruence, both of which are the foundational concepts of this cluster. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with understanding and applying theorems about circles> benefit when learning experiences include ways to recruit interest such as <providing contextualized examples to their lives> because <this will help them identify the relevance of the topics in this cluster to math in the real-world. For this cluster students.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understanding and applying theorems about circles benefit when learning experiences attend to student's attention and affect to support sustained effort and concentration such as using prompts or scaffolds for visualizing desired outcomes because some proofs may not be obvious. Teacher can revisit transformations with students in order to help students make the connections between them and circles.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with <understanding and applying theorems about circles benefit when learning experiences attend to the linguistic and nonlinguistic

representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) because this cluster requires the precise use of academic vocabulary in the area of math. The use of this academic vocabulary will assist students in developing and supporting their rationale of: 1) whether or not proofs are accurate; or 2) identifying the missing step in a proof.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding and applying theorems about circles benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners) because part of this cluster allows students to determine how theorems are related and then justify their reasoning. Students may have varying justifications for their rationale (and several might be right), therefore individual feedback is essential to honor the thinking of all learners in the classroom. In addition, different students may be missing different steps in the proofs so the personalized feedback individualizes the learning per student.

Internalize
Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with understanding and applying theorems about circles benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because this cluster does require the knowledge of material from other content standards. For example, students may find themselves needing to review material on transformations, arcs, central angles, measures of inscribed angles, etc. Providing students with practice to review these skills will lead to greater success in the cluster overall.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understand and apply theorems about circles by critiquing student approaches/solutions to make connections through a short mini-lesson because proofs and constructions can be seen from a variety of perspectives, some correct and others incorrect but will offer insight into common misconceptions all students may have.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit understand and apply theorems about circles by confronting student misconceptions because when students do not have a firm grasp of congruence and similarity, progressing forward and applying those to circles and proofs will be impossible.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

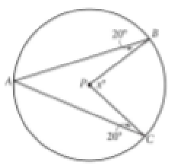
- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying understand and apply theorems about circles because proofs with circles can be challenging and require students to pull facts from a variety of places, some implied and others explicitly stated. This develops and pushes students' abstract thinking skills.

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying <understand and apply theorems about circles> the types of mathematical tasks are critical because they will help students make connections to how the math in this cluster is applicable to real-life context. In some regards, tasks can be designed to highlight various cultures which in turn allows for students to learn about the diversity amongst their peers' cultures. Tasks should be cognizant of culturally responsive academic vocabulary, language, and literacy.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
SAT Item # 422459: The linked assessment question addresses G-C.A., specifically the question requires students to use knowledge of inscribed and central angles.

CollegeBoard	Question ID 422459						
Assessment **SAT**	Test **Math**	Cross-Test and Subscore **Additional Topics in Math**	Difficulty **Hard**	Primary Dimension **Additional Topics in Math**	Secondary Dimension **Circles**	Tertiary Dimension **1. Use definitions, properties, and theorems relating to circles and parts of circles, such as radii, diameters, tangents, angles, arcs, arc lengths, and sector areas to solve problems.**	Calculator **Calculator**



Point P is the center of the circle in the figure above. What is the value of x ?

Question Difficulty: Hard

> The correct answer is 80. If points A and P are joined, then the triangles that will be formed, APB and APC, are isosceles because $PA = PB = PC$. It follows that the base angles on both triangles each measure $20°$. Angle BAC consists of two base angles; therefore, the measure of angle BAC = $40°$. Since the measure of an angle inscribed in a circle is half the measure of the central angle that intercepts the same arc, it follows that the value of x is $80°$.

Additional Assessment:
http://tasks.illustrativemathematics.org/content-standards/HSG/C/A/1/tasks/1368
The linked assessment question addresses G-C.A, specifically the question requires students to make use of visualizing transformations as well as knowledge of equations for a circle. This assessment should be given to students after they've been introduced to these concepts. Students will engage in SMP 1 and SMP 8.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on understanding and apply theorems about circles, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, providing a task that requires students to calculate the dimensions needed for a new restaurant to build a triangular deck (with one side being the restaurant building) will relate school learning to community/home application.	Art: Consider discussing how inscribed and circumscribed angles may be used in calculating specific designs in landscape, apparel, etc. Designers are often given constraints in which to create an image and may use knowledge of these angles to help design an appropriate image.

<table>
<tr><td colspan="3">HS: GEOMETRY- CIRCLES</td></tr>
<tr><td colspan="3">Cluster Statement: B: Find arc lengths and areas of sectors of circles</td></tr>
<tr><td rowspan="3">Standard Text

HSG.C.B.5
Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.</td><td rowspan="3">Standard for Mathematical Practices

SMP 2
Students reason abstractly and quantitatively by calculating the length of an intercepted arc.

SMP 4
Students model with mathematics by deriving the area of a sector using similarity.

SMP7
Students look for and make use of structure by expecting students to apply rules, look for patterns, and analyze structure.</td><td>Students who demonstrate understanding can:
• Calculate the length of an intercepted arc.
• Demonstrate that the constant of proportionality between arc length and the radius of the circle is the radian measure of the central angle.
• Derive the formula for the area of a sector using similarity.
• Calculate the area of a sector.</td></tr>
<tr><td>Webb’s Depth of Knowledge: 1-2</td></tr>
<tr><td>Bloom’s Taxonomy: understand, apply</td></tr>
<tr><td><u>Previous Learning Connections</u>
In 7th grade, the formulas for the area and circumference of a circle are learned and then applied to solve problems. They give an informal derivation of the relationship between the circumference and area of a circle.</td><td><u>Current Learning Connections</u>
Later in the Geometry course when calculating geometric probabilities, students will need to know how to calculate the area of a sector which is taught within this cluster.</td><td><u>Future Learning Connections</u>
In future courses, students expand on their basic understanding of the radian measure of an angle. They apply radian measures when discovering relationships within the unit circle and while learning trigonometric relationships.</td></tr>
<tr><td colspan="3">Clarification Statement
This cluster explores the relationship between the length of an arc and the measure of a central angle. Learners develop a definition for the radian measure of an angle and apply radians to find the area of sectors.</td></tr>
<tr><td colspan="3">Common Misconceptions
Students often struggle with precision while working within this cluster. Small errors in constructions will lead to results that do not work.</td></tr>
</table>

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying arc Lengths and areas of sectors of circles because understanding here depends on how deeply a student understands earlier concepts such as area of a circle and the terminology involved.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.GB.4 and 6.RP.A2: : This standard provides a foundation for work with arc lengths and areas of sectors of circles because it deals with concept of area of circles and the concept of proportionality. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with finding arc lengths and areas of sectors of circles benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because it will help facilitate the application of importance of the concept by giving the students an opportunity for hands on tactile learning

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with finding arc lengths and areas of sectors of circles benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because allowing students to become self-actualized persistent learners will foster in them a need and a desire for tackling new and challenging areas in their lives. It's more about becoming a life-long learner.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with finding arc lengths and areas of sectors of circles benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) because at this level of geometry this cluster is built upon previously learned concepts and because of that, there are numerous ways to achieve a solution. By encouraging multiple pathways, students can become very well versed in the cluster's concepts.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with finding arc lengths and areas of sectors of circles benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as composing in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video because this cluster focuses on parts of circles and can be very easily adapted into large scale representations. By allowing students to represent using multiple media of their choosing, we are crossing over into other interest categories enjoyed by each student.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with finding arc lengths and areas of sectors of circles benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as addressing subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math")because when students start to feel successful at math, they begin to shed those beliefs of inadequacy. There is a need to create a culture of positive math experiences. Some students enjoy large leaps in improvement, some need recognition for the small, daily victories. It's a matter of knowing your students' needs and abilities.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on Arc Lengths And Areas Of Sectors Of Circles by examining tasks from a different perspective through a short mini-lesson because we are welding concepts together to form a new concept, this process is not automatic and by backing up and looking at the problem from a different point of view

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on arc length and areas of sectors of circles by offering opportunities to understand and explore different strategies because what works for one student may not work for another.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying Arc Lengths And Areas Of Sectors Of Circles because linking the concept to something that a student will experience in their own lives will add depth to their experience regarding this concept

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying arc lengths and areas of sectors of circles the types of mathematical tasks are critical because the struggle to be all-inclusive can be an issue. Where students with strong procedural knowledge will easily follow a process, some students will struggle and need adaptation and accommodations. Some ways to address this would be to adapt procedures into the students spoken language, apply terminology and problems from the student's daily life, use hands-on demonstrations, and use bi-lingual grouping of students.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
SAT Item # 5209208: The linked assessment question addresses G-C.B., specifically the question requires students to use knowledge of arc length.

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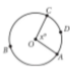
CollegeBoard	Question ID 5209208						
SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Circles	1. Use definitions, properties, and theorems relating to circles and parts of circles, such as radii, diameters, tangents, angles, arcs, arc lengths, and sector areas to solve problems.	No Calculator



The circle above has center O, the length of arc $\overparen{ADC}$ is 5π, and $x = 100$. What is the length of arc $\overparen{ABC}$?

Question Difficulty: Medium

A. 9π

B. 13π

C. 18π

D. $\frac{13}{2}\pi$

Choice B is correct. The ratio of the lengths of two arcs of a circle is equal to the ratio of the measures of the central angles that subtend the arcs. It's given that arc $\overparen{ADC}$ is subtended by a central angle with measure 100°. Since the sum of the measures of the angles about a point is 360°, it follows that arc $\overparen{ABC}$ is subtended by a central angle with measure $360° - 100° = 260°$. If s is the length of arc $\overparen{ABC}$, then s must satisfy the ratio $\frac{s}{5\pi} = \frac{260}{100}$. Reducing the fraction $\frac{260}{100}$ to its simplest form gives $\frac{13}{5}$. Therefore, $\frac{s}{5\pi} = \frac{13}{5}$. Multiplying both sides of $\frac{s}{5\pi} = \frac{13}{5}$ by 5π yields $s = 13\pi$.

Choice A is incorrect. This is the length of an arc consisting of exactly half of the circle, but arc $\overparen{ABC}$ is greater than half of the circle. Choice C is incorrect. This is the total circumference of the circle. Choice D is incorrect. This is half the length of arc $\overparen{ABC}$, not its full length.

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on arc lengths and areas of sectors of circles , consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, such as using everyday circular items to show how sectors are part of the entire circle.	Economics: Connect to a variety of circular foods, talking about maximizing crust or finding the largest slice.

HS: STATISTICS & PROBABILITY- CONDITIONAL PROBABILITY & THE RULES OF PROBABILITY

Cluster Statement: A: Understand independence and conditional probability and use them to interpret data

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSS.CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	SMP2 Students reason abstractly and quantitatively by describing the probability of an event explicitly as well as describing what a given probability represents in context. SMP 6 Students attend to precision by using precise language, symbols and calculations when describing events and probabilities.	• Analyze a sample space to describe an event, union of events, intersection of events and complement of event • Use tree diagrams, organized lists, tables, and/or Venn diagrams to represent sample spaces. • Determine unions of sample spaces. • Determine intersections of sample spaces • Determine complements of sample sets. • Represent unions, intersections, and complements using set notation. **Webb's Depth of Knowledge:** 2-3 **Bloom's Taxonomy:** apply, analyze
Standard Text HSS.CP.A.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	**Standard for Mathematical Practices** SMP 2: Students reason abstractly and quantitatively by analyzing an event for independence (mathematically and logically) SMP 3: Students construct viable arguments and critique the reasoning of others by stating or defending the independence of an event using logical and/or mathematical arguments/claims.	**Students who demonstrate understanding can:** • Explain and apply the formula to determine if two events are independent. • Test for independence using the definition of independent events. • State problems' independence and dependence contextually. **Webb's Depth of Knowledge: 1-3** **Bloom's Taxonomy:** understand, apply, analyze

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSS.CP.A.3 Understand the conditional probability of *A* given *B* as *P(A and B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.	SMP 2: Students reason abstractly and quantitatively by analyzing an event for independence (mathematically and logically) SMP 3: Students construct viable arguments and critique the reasoning of others by stating or defending the independence of an event using logical and/or mathematical arguments/claims. SMP 6: Students attend to precision by clearly stating the difference between p(a|b) and p(b|a)	• Read and state conditional probabilities of two events and explain how they are different. apply conditional probability to argue if two events are independent. • Calculate conditional probabilities. • Relate conditional probability to relative frequency tables and/or tree diagrams. • Use conditional probabilities to determine whether events are independent. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** understand, apply
Standard Text HSS.CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*	**Standard for Mathematical Practices** SMP 2: Students reason abstractly and quantitatively by analyzing an event for independence (mathematically and logically) SMP 3: Students construct viable arguments and critique the reasoning of others by stating or defending the independence of an event using logical and/or mathematical arguments/claims. SMP 4: Students model with mathematics by creating two-way frequency tables from given information and by stating conditional probability from given information.	**Students who demonstrate understanding can:** • Interpret and organize data to describe events and independence of events using 2-way frequency tables • Collect sample data from a real-world situation in order to examine conditional probabilities and independence of events. • Interpret and make sense of these in context of the situation. **Webb's Depth of Knowledge:** 2-3 **Bloom's Taxonomy:** apply, analyze

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<table>
<tr>
<td rowspan="3">Standard Text

HSS.CP.A.5
Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</td>
<td rowspan="3">Standard for Mathematical Practices

SMP 1:
Students make sense of problems and persevere in solving them by reading a scenario closely to identify concepts of conditional probability.

SMP 4:
Students model with mathematics by applying a model/algorithm or logic to determine conditional probability of an event.</td>
<td>Students who demonstrate understanding can:
Analyze a scenario to describe conditional probability in terms of a real-life context
Use conditional probability to make decisions and justify claims of relationships to contextual situations.
Interpret conditional probability and independence across a variety of situations.
Distinguish between association and causality.</td>
</tr>
<tr>
<td>Webb’s Depth of Knowledge: 2-3</td>
</tr>
<tr>
<td>Bloom’s Taxonomy: apply, analyze</td>
</tr>
<tr>
<td><u>Previous Learning Connections</u>
In previous years, learners used sample spaces to represent compound events in organized lists, tables, and tree diagrams. Learners are initially introduced to probability in 7th grade. They have investigated chance processes and developed probability models using experimental and theoretical probability.</td>
<td><u>Current Learning Connections</u>
Learners will use their knowledge of conditional probability and their skills of determining conditional probability to make decisions for real world situations. They will also expand the knowledge of this cluster to learn specific rules such as the Addition Rule. This knowledge will lead into permutations and combinations.</td>
<td><u>Future Learning Connections</u>
Learners will extend their learning to develop and make sense of the Multiplication Rule and Addition Rule. Future learning such as binomial distribution and statistical significance build upon conditional probability. Other applications are found in calculus, statistics, engineering, and the sciences.</td>
</tr>
<tr>
<td colspan="3">Clarification Statement
A probability model may consist of a list or description of possible outcomes (the sample space) each of which is assigned a certain probability. Probability rules can be developed and understood through the use of the sample space. When events are independent, the outcome of the first event does not change the sample space for subsequent events. In dependent events, knowing one event has occurred affects the likelihood of another event occurring. Use of two-way frequency tables helps learners develop conceptual understanding of conditional probability. The use of tables, symbols, and real-world scenarios are emphasized. Learners consider the context of situations as they build mathematical models, interpret events, and explain results in terms of a probability model.</td>
</tr>
<tr>
<td colspan="3">Common Misconceptions
Students may think that two events occurring is as simple as adding their probabilities.

Students may fail to check both parts of the algorithm. students may also assume if p(a)=p(a|b) then p(b) must equal p(b|a).</td>
</tr>
</table>

Students may also have an incomplete understanding of conditional probability.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying independence and conditional probability and use them to interpret data because students will need to develop an appropriate vocabulary usage for new subjects as well as tying it to previously learned material

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.SPC.7: This standard provides a foundation for work with independence and conditional probability and use them to interpret data Because this previously learned standard forms the foundation of understanding probability and probability models. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with Understanding independence and conditional probability and use them to interpret data benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features <give an example connected to this standard such as the size of text, images, graphs, tables, or other visual content; contrast between background and text or image; color used for information or emphasis; volume or rate of speech or sound; speed or timing of video, animation, sound, simulations, etc.; layout of visual or other elements; font used for print materials>because we can target individual learning styles for each student in order to master this concept.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with Understanding independence and conditional probability and use them to interpret data benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports (e.g., peer-tutors) because students will be able to gather and explore the reasoning of others and be able to include this into their own conclusions.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or* *puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with Understanding independence and conditional probability and use them to interpret data benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because newer terminology is built upon the older concepts in this case.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with Understanding independence and conditional probability and use them to interpret data benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because the terminology in this cluster may lead to confusion when solving and having multiple ways to approach the material will be beneficial.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with Understanding independence and conditional probability and use them to interpret data benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as making explicit cross-curricular connections (e.g., teaching literacy strategies in the social studies classroom) because there are numerous incidences of independent and conditional probability in real-world situations.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on independent events and conditional probability and use them to interpret data by clarifying mathematical ideas and/or concepts through a short mini-lesson because precise usage of terms in this cluster is the key to future understanding

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit independent events and conditional probability and use them to interpret data by confronting student misconceptions because there will be much confusion of terminology here and that will lead to errors in calculations later

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying independence and conditional probability and use them to interpret data because students will be able to analyze experiments and studies of their own choosing to further their understanding of independence and conditional probability

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Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying independence and conditional probability and use them to interpret data, the types of mathematical tasks are critical because the problems presented to students would need to reflect a relevance to the students' life experiences. This would offer a meaning to the student that the math can go beyond the classroom.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
SAT Item #: 1474700 The linked assessment question addresses S-SCPA.A, specifically the question requires students to read a two-way frequency table and state conditional probability in context.

CollegeBoard **Question ID 1474700**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	**Math**	**Problem Solving and Data Analysis**	**Medium**	**Problem Solving and Data Analysis**	**Probability and conditional probability**	**1. Compute and interpret probability and conditional probability in simple contexts.**	**Calculator**

Number of Adults Contracting Colds

	Cold	No cold	Total
Vitamin C	21	129	150
Sugar pill	33	117	150
Total	54	246	300

The table shows the results of a research study that investigated the therapeutic value of vitamin C in preventing colds. A random sample of 300 adults received either a vitamin C pill or a sugar pill each day during a 2-week period, and the adults reported whether they contracted a cold during that time period. What proportion of adults who received a sugar pill reported contracting a cold?

uestion Difficulty: Medium

A. $\frac{11}{18}$

B. $\frac{11}{50}$

C. $\frac{9}{50}$

D. $\frac{11}{100}$

Choice B is correct. A total of 150 adults received the sugar pill. Of those, 33 reported contracting a cold. Therefore, $\frac{33}{150}$, or the equivalent $\frac{11}{50}$, is the proportion of adults receiving a sugar pill who reported contracting a cold.

Choice A is incorrect. This is the proportion of adults receiving a sugar pill and contracting a cold to all adults contracting a cold $\left(\frac{33}{54}\right)$. Choice C is incorrect. This is the proportion of adults who reported contracting a cold to all the participants in the study $\left(\frac{54}{300} = \frac{9}{50}\right)$. Choice D is incorrect. This is the proportion of adults who received a sugar pill and reported contracting a cold to all the participants in the study $\left(\frac{33}{300} = \frac{11}{100}\right)$.

Additional Assessment:
https://tasks.illustrativemathematics.org/content-standards/HSS/CP/A/4/tasks/2045

The linked assessment question addresses S.CPA.A, specifically the question requires students to look at data organized in a two-way frequency table and state probabilities. This assessment could be given to students after they've been introduced to the concept of conditional probabilities or as an exploration into finding conditional probabilities. Students will engage in SMP 4 and possibly SMP 3 if they are sharing or critiquing responses with peers.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on independence and conditional probability and use them to interpret data, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, consider what types of conditional probability occur in the students' lives outside of school.	Social Studies: Connect to census data, voter demographics Forensic Science: Connect to crime scene analysis given suspect characteristics

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HS: STATISTICS & PROBABILITY- CONDITIONAL PROBABILITY & THE RULES OF PROBABILITY

Cluster Statement: B: Use the rules of probability to compute probabilities of compound events.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSS.CP.B.6 Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.	SMP 1: Students make sense of problems and persevere in solving them by reading a scenario closely to identify concepts of conditional probability. SMP 2: Students reason abstractly and quantitatively by using specific calculations and general description of events to describe a scenario. SMP 4: Students model with mathematics by applying a model/algorithm or logic to determine conditional probability of an event. SMP 6: Students attend to precision by using precise language and application of formulas when finding conditional probability of events.	• Describe how to find conditional probabilities • Calculate conditional probabilities • Explain conditional probability in context of a scenario • Interpret a given scenario and relate context to conditional probability, both abstractly and mathematically • Justify reasoning in making conditional probability arguments **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy: u**nderstand, apply, evaluate
Standard Text HSS.CP.B.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model.	**Standard for Mathematical Practices** SMP 2: Students reason abstractly and quantitatively by explaining how to find the union of two events, both generally and in a given scenario. SMP 3: Students construct viable arguments and critique the reasoning of others by defending the probability of a union occurring in a given context.	**Students who demonstrate understanding can:** • Calculate the union of two events • Explain the union of two events in terms of the context of the problem • Given a scenario, interpret what the union of two events represents and calculate the probability **Webb's Depth of Knowledge:** 1-2

	SMP 4: Students model with mathematics by applying the formula for unions to a contextual probability problem. SMP 6: Students attend to precision by describing overlapping events in context and accounting for the overlap in calculations.	**Bloom's Taxonomy: understand, apply**
Previous Learning Connections • In grade 7, learners have investigated chance processes, and developed, used, and evaluated probability models. They have learned that probability of a chance event is a number between 0 and 1 (7.SP.5) and found probabilities of compound events (7.SP.8).	**Current Learning Connections** • Learners are expanding their understanding and skills explored and learned in the G.SP.A cluster. They are discovering that conditional probability can be found from a narrowed subset of the original sample space.	**Future Learning Connections** • Future learning such as binomial distribution and statistical significance build upon conditional probability. Other applications are found in calculus, statistics, engineering, and the sciences.

Clarification Statement

The development and uses of algorithms are built on conceptual understanding as concepts of sample spaces are explored and deepened. Probabilities are described in terms of the intersections and unions of events. Venn diagrams and two-way frequency tables will be generalized to discover patterns and create algorithms and formulas that can be used in routine fashion. Although learners will use these formulas strategically to determine different values, the use of tree diagrams, organized lists, and other tools will help make sense of these abstractions.

Common Misconceptions

Students may struggle with determining the correct denominator. They may use the total rather than the specified event.

Students may struggle to understand the "overlap" in compound events.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying the rules of probability to compute probabilities of compound events because this cluster focuses on compound probability with the introduction of combinations and permutations which take practice and perseverance to master

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.SPC.8: This standard provides a foundation for work with the rules of probability to compute probabilities of compound events because this older standard introduces the formal definitions of compound events and calls for modeling of the standard to represent its situations. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Physical Action: *How will the learning for students provide a variety of methods for navigation to support access?*

- For example, learners engaging with Using the rules of probability to compute probabilities of compound events benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as varying methods for response and navigation by providing alternatives to <requirements for rate, timing, speed, and range of motor action with instructional materials, physical manipulatives, and technologies; physically responding or indicating selections; physically interacting with materials by hand, voice, single switch, joystick, keyboard, or adapted keyboard because studies prove learning is active and activity during learning accelerates concept acquisition

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with Using the rules of probability to compute probabilities of compound events, benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that is frequent, timely, and specific because it shows a care for what your students are achieving and immediate feedback is essential before students errors become what they believe to be the concept.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with Using the rules of probability to compute probabilities of compound events, benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as highlighting structural relations or make them more explicit because this aids in organizing and contextualizing the increasingly complex structure of intermediate and advanced probability.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with Using the rules of probability to compute probabilities of compound events, benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because this will aid in the use of computation of advanced probabilities.

Internalize

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with Using the rules of probability to compute probabilities of compound events, benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as providing graphic organizers and templates for data collection and organizing information because the amount of information provided in this cluster will need to organized by the student in order to avoid confusion of concepts and equations to be used here.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on computing probabilities of compound events by providing specific feedback to students on their work through a short mini-lesson because by pinpointing minor errors in a multistep process we can perfect our processes.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on computing probabilities of compound events by addressing conceptual understanding because by sitting down and helping a student analyze their process, we can bring them to a deeper level of understanding of their errors as well as the content.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students. when studying the rules of probability to compute probabilities of compound events because Students working together opens up new paths of thinking and reasoning for them.

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying the rules of probability to compute probabilities of compound events, goal setting is critical because in this cluster of Statistics and probability it necessary to be organized and complete in which procedure must be used at a given time. Helping a student set a piecewise organizational goal will assist. This can be organized linearly, as a graphic organizer, or any method of the students choosing.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
SAT Item #: 4168721 The linked assessment question addresses S-SCPA.A, specifically the question requires students to read a two-way frequency table and state compound probability in context.

CollegeBoard **Question ID 4168721**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	**Math**	**Problem Solving and Data Analysis**	**Medium**	**Problem Solving and Data Analysis**	**Probability and conditional probability**	**1. Compute and interpret probability and conditional probability in simple contexts.**	**Calculator**

Observed Matings among Fruit Flies

		Female fruit fly group		
		Female raised on starch	Female raised on maltose	Total
Male fruit fly group	Male raised on starch	22	9	31
	Male raised on maltose	8	20	28
	Total	30	29	59

The table above shows the observed mating frequencies among a group of fruit flies raised on either a starch medium or a maltose medium. What fraction of the observed matings were between fruit flies that were raised on the same medium?

Question Difficulty: Medium

A. $\frac{9}{31}$

B. $\frac{17}{59}$

C. $\frac{31}{59}$

D. $\frac{42}{59}$

Choice D is correct. According to the table, a total of 59 fruit fly matings were observed. Of these, 22 matings were between males and females who were both raised on starch and 20 were between males and females who were both raised on maltose. Therefore, a total of $22+20$ or 42 of the 59 observed matings were between fruit flies raised on the same medium. This situation is represented by the fraction $\frac{42}{59}$.

Choice A is incorrect. This represents the fraction of observed fruit fly matings between females raised on maltose and males raised on starch. Choice B is incorrect. This represents the fraction of observed fruit fly matings between fruit flies raised on different mediums. Choice C is incorrect. This represents the fraction of observed fruit fly matings with males raised on starch.

Additional Assessment:

http://tasks.illustrativemathematics.org/content-standards/HSS/CP/B/7/tasks/1112
The linked assessment question addresses S.CPA.B, specifically the question requires students to apply a variety of more complex probability theorems to a contextual problem (independence, conditional probabilities and union of events). This assessment should be given to students after they've been introduced to these concepts. Students will engage in SMP 4, SMP 6 and possibly SMP 3 if they are asked to share and critique solutions.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on the rules of probability to compute probabilities of compound events., consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, consider what types of compound probability a student will experience on a daily basis to form a foundation for this concept	Social Studies: Connect to voter demographics Science: Connect to crime science investigation/analysis

Section 3: Resources, References, and Glossary

Resources

Evidence-Based Resources	English Learner Resources	MLSS Resources	Mathematics Standard Resources
What Works Clearinghouse Best Evidence Encyclopedia Evidence for Every Students Succeeds Act Evidence in Education Lab	World-Class Instructional Design and Assessment (WIDA) Standards USCALE Language Routines for Mathematics English Language Development Standards Spanish Language Development Standards	NM Multi-Layered System of Supports (MLSS) Universal Design for Learning Guidelines Achieve the Core: Instructional Routines for Mathematics Project Zero Thinking Routines	Focus by Grade Level and Widely Applicable Prerequisites High school Coherence Map College-and Career Ready Math Shifts Fostering Math Practices: Routines for the Mathematical Practices

Planning Guidance for Multi-Layered Systems of Support: Core Instruction[10]

Core Instructional Planning must reflect and leverage scientific insights into how humans learn in order to ensure all students are ready for success, thus the following guidance for optimizing teaching and learning is grounded in the **Universal Design Learning (UDL) Framework**

Key design questions, planning actions, and potential strategies are provided below, with respect to guidance for minimizing barriers to learning and optimizing (1) universal ACCESS to learning experiences, (2) opportunities for students to BUILD their understanding of the **Learning Goal**, and (3) INTERNALIZATION of the Learning Goal.

Optimizing Universal ACCESS to Learning Experiences	
ENGAGEMENT ⍰ How will you provide multiple options for recruiting interest?	**Recruiting Student Interest**: ⍰ What do you anticipate in the range of student interest for this lesson? ▯ Plan for options for recruiting student interest: ❒ provide choice (e.g. sequence or timing of task completion) ❒ set personal academic goals ❒ provide contextualized examples connected to their lives ❒ support culturally relevant connections (i.e home culture) ❒ create socially relevant tasks ❒ provide novel & relevant problems to make sense of complex ideas in creative ways

[10] Adapted from: CAST (2018). *Universal Design for Learning Guidelines version 2.2*. Retrieved from http://udlguidelines.cast.org

	❒ provide time for self-reflection about content & activities ❒ create accepting and supportive classroom climate ❒ utilize **instructional routines** to involve all students
REPRESENTATION ? How will you reduce barriers to perceiving the information presented in this lesson?	**Perception**: ? What do you anticipate about the range in how students will perceive information presented in this lesson? ▯ Plan for different modalities and formats to reduce barriers to learning: ❒ display information in a flexible format to vary perceptual features ❒ offer alternatives for auditory information ❒ offer alternatives for visual information
ACTION & EXPRESSION ? How will the learning for students provide a variety of methods for navigation to support access?	**Physical Action**: ? What do you anticipate about the range in how students will physically navigate and respond to the learning experience? ▯ Plan a variety of methods for response and navigation of learning experiences by offering alternatives to: ❒ requirements for rate, timing, speed, and range of motor action with instructional materials, manipulatives, and technologies ❒ physically indicating selections ❒ interacting with materials by hand, voice, keyboard, etc.

Opportunities for Students to BUILD their Understanding	
ENGAGEMENT ? How will the learning for students provide options for sustaining effort and persistence?	**Sustaining Effort & Persistence**: ? What do you anticipate about the range in student effort? ▯ Plan multiple methods for attending to student attention and affect by: ❒ prompting learners to explicitly formulate or restate learning goals ❒ displaying the learning goals in multiple ways ❒ using prompts or scaffolds for visualizing desired outcomes ❒ engaging assessment discussions of what constitutes excellence ❒ generating relevant examples with students that connect to their cultural background and interests ❒ providing alternatives in the math representations and scaffolds ❒ creating cooperative groups with clear goals, roles, responsibilities ❒ providing prompts to guide when and how to ask for help ❒ supporting opportunities for peer interactions and supports (e.g. peer tutors) ❒ constructing communities of learners engaged in common interests ❒ creating expectations for group work (e.g., rubrics, norms, etc.) ❒ providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge ❒ providing feedback that: ❒ emphasizes effort, improvement, and achieving a standard rather than on relative performance ❒ is frequent, timely, and specific ❒ is informative rather than comparative or competitive

	❒ models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success
REPRESENTATION ⍰ How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners?	**Language & Symbols**: ⍰ What do you anticipate about the range of student background experience and vocabulary? ▯ Plan multiple methods for attending to linguistic and nonlinguistic representations of mathematics to ensure universal clarity by: ❒ pre-teaching vocabulary and symbols in ways that promote connection to the learners' experience and prior knowledge ❒ graphic symbols with alternative text descriptions ❒ highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to structure ❒ embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) ❒ embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) ❒ highlighting structural relations or make them more explicit ❒ making connections to previously learned structures ❒ making relationships between elements explicit (e.g., highlighting the transition words in an argument, links between ideas, etc.) ❒ allowing the use of text-to-speech and automatic voicing with digital mathematical notation (math ml) ❒ allowing flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) ❒ clarification of notation through lists of key terms ❒ making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf ❒ linking key vocabulary words to definitions and pronunciations in both dominant and heritage languages ❒ defining domain-specific vocabulary (e.g., "map key" in social studies) using both domain-specific and common terms ❒ electronic translation tools or links to multilingual web glossaries ❒ embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc) ❒ presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) ❒ making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams
ACTION & EXPRESSION ⍰ How will the learning provide multiple	**Expression & Communication**: ⍰ What do you anticipate about the range in how students will express their thinking in the learning environment? ▯ Plan multiple methods for attending to the various ways in which students can express knowledge, ideas, and concepts by providing:

modalities for students to easily express knowledge, ideas, and concepts in the learning environment?	❒ options to compose in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video ❒ use of social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) ❒ flexibility in using a variety of problem solving strategies ❒ spell or grammar checkers, word prediction software ❒ text-to-speech software, human dictation, recording ❒ calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper ❒ sentence starters or sentence strips ❒ concept mapping tools ❒ Computer-Aided-Design (CAD) or mathematical notation software ❒ virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) ❒ multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches) ❒ multiple examples of novel solutions to authentic problems ❒ different approaches to motivate, guide, feedback or inform students of progress towards fluency ❒ scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) ❒ differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners)

Optimizing INTERNALIZATION of the Learning Goal	
ENGAGEMENT ⍰ How will the design of the learning strategically support students to effectively cope and engage with the environment?	**Self-Regulation**: ⍰ What do you anticipate about barriers to student engagement? ▯ Plan to address barriers to engagement by promoting healthy responses and interactions, and ownership of learning goals: ❒ metacognitive approaches to frustration when doing mathematics ❒ increase length of on-task orientation through distractions ❒ frequent self-reflection and self-reinforcements ❒ address subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") ❒ offer devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the math practices for the purpose of monitoring and improving ❒ use activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely
REPRESENTATION ⍰ How will the learning support transforming accessible information into usable knowledge	**Comprehension**: ⍰ What do you anticipate about barriers to student comprehension? ▯ Plan to address barriers to comprehension by intentionally building connections to prior understandings and experiences, relating meaningful information to learning goals,

that is accessible for future learning and decision-making?	providing a process for meaning making of new learning, and applying learning to new contexts: ❒ incorporate explicit opportunities for review and practice ❒ note-taking templates, graphic organizers, concept maps ❒ scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) ❒ explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) ❒ opportunities over time to revisit key ideas and connections ❒ make explicit cross-curricular connections ❒ highlight key elements in tasks, graphics, diagrams, formulas ❒ outlines, graphic organizers, unit organizer routines, concept organizer routines, and concept mastery routines to emphasize key ideas and relationships ❒ multiple examples & non-examples ❒ cues and prompts to draw attention to critical features ❒ highlight previously learned skills that can be used to solve unfamiliar problems ❒ options for organizing and possible approaches (tables and representations for processing mathematical operations) ❒ interactive representations that guide exploration and new understandings ❒ introduce graduated scaffolds that support information processing strategies ❒ tasks with multiple entry points and optional pathways ❒ "Chunk" information into smaller elements ❒ remove unnecessary distractions unless essential to learning goal ❒ anchor instruction by linking to and activating relevant prior knowledge (e.g., using visual imagery, concept anchoring, or concept mastery routines) ❒ pre-teach critical prerequisite concepts via demonstration or representations ❒ embed new ideas in familiar ideas and contexts (e.g., use of analogy, metaphor, drama, music, film, etc.) ❒ advanced organizers (e.g., KWL methods, concept maps) ❒ bridge concepts with relevant analogies and metaphors
ACCESS ACTION & EXPRESSION ? How will the learning for students support the development of executive functions to allow them to take advantage of their environment?	**Executive Functions**: ? What do you anticipate about barriers to students demonstrating what they know? ▯ Plan to address barriers to demonstrating understanding by providing opportunities for students to set goals, formulate plans, use tools and processes to support organization and memory, and analyze their growth in learning and how to build from it: ❒ prompts and scaffolds to estimate effort, resources, difficulty ❒ models and examples of process and product of goal-setting ❒ guides and checklists for scaffolding goal-setting ❒ post goals, objectives, and schedules in an obvious place ❒ embed prompts to "show and explain your work" ❒ checklists and project plan templates for understanding the problem, prioritization, sequences, and schedules of steps ❒ embed coaches/mentors to demonstrate think-alouds of process ❒ guides to break long-term goals into short-term objectives ❒ graphic organizers/templates for organizing information & data ❒ embed prompts for categorizing and systematizing ❒ checklists and guides for note-taking ❒ asking questions to guide self-monitoring and reflection ❒ showing representations of progress (e.g., before and after photos, graphs/charts showing progress, process portfolios)

	❒ prompt learners to identify type of feedback or advice they seek ❒ templates to guide self-reflection on quality & completeness ❒ differentiated models of self-assessment strategies (e.g., role-playing, video reviews, peer feedback) ❒ assessment checklists, scoring rubrics, and multiple examples of annotated student work/performance examples

Planning Guidance for Culturally and Linguistically Responsive Instruction[11]

In order to ensure our students from marginalized cultures and languages view themselves as confident and competent learners and doers of mathematics within and outside of the classroom, educators must intentionally plan ways to counteract the negative or missing images and representations that exist in our curricular resources. The guiding questions below support the design of lessons that validate, affirm, build, and bridge home and school culture for learners of mathematics:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

In addition, Aguirre and her colleagues[12] define **mathematical identities** as the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives. Many students see themselves as "not good at math" and approach math with fear and lack of confidence. Their identity, developed through earlier years of schooling, has the potential to affect their school and career choices.

Five Equity-Based Mathematics Teaching Practices[13]

Go deep with mathematics. Develop students' conceptual understanding, procedural fluency, and problem solving and reasoning.

Leverage multiple mathematical competencies. Use students' different mathematical strengths as a resource for learning.

Affirm mathematics learners' identities. Promote student participation and value different ways of contributing.

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[11] This resource relied heavily on the work of: Hollie, S. (2011). Culturally and linguistically responsive teaching and learning. Teacher Created Materials. (see also, https://www.culturallyresponsive.org/vabb)

[12] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices. Reston, VA: National Council of Teachers of Mathematics (p. 14).

[13] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc. (p.6). (adapted from Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013) (p. 43).

Challenge spaces of marginality. Embrace student competencies, value multiple mathematical contributions, and position students as sources of expertise.

Draw on multiple resources of knowledge (mathematics, language, culture, family). Tap students' knowledge and experiences as resources for mathematics learning.

The following lesson design strategies support Culturally and Linguistically Responsive Instruction, specific examples for each cluster of standards can be found in part 2 of the document. These were adapted from the Promoting Equity section of the Taking Action series published by NCTM.[14]

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.

Mathematical Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher.

Modifying Mathematical Tasks: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)."

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics.

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages.

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we

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[14] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians.

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others."

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time."

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students.

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Glossary[15]

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (- 3/4) = (- 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[16]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. See: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances
from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten.
For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

[15] Glossary and tables taken from: Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/
[16] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[17] See also: median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.
Integer. A number expressible in the form a or -a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line.
Also known as a dot plot.[18]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[19] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 Ã· 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 Ã— 4/3 = 4/3 Ã— 3/4 = 1.

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[17] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

[18] Adapted from Wisconsin Department of Public Instruction, op. cit.

[19] To be more precise, this defines the arithmetic mean.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin,
selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also*: uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form *a/b* or – *a/b* for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also*: terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[20]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar
model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

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[20] Adapted from Wisconsin Department of Public Instruction, op. cit.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also*: median, first quartile, interquartile range.

Table 1: Common addition and subtraction.[1]

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 =5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now?5-2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?? -2 = 3
	TOTAL UNKNOWN	ADDEND UNKNOWN	BOTH ADDENDS UNKNOWN[2]
PUT TOGETHER / TAKE APART[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5-3 = ?	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? 5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1, 5 = 2 + 3, 5 = 3 + 2
COMPARE	DIFFERENCE UNKNOWN	BIGGER UNKNOWN	SMALLER UNKNOWN
	(“How many more?” version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?(“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? 2 + ? = 5, 5 – 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with “more”):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2: Common multiplication and division situations.[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	3 x ? = 18, and 18 ÷ 3 = ?	? x 6 = 18, and 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	a x b = ?	a x ? = p and p ÷ a = ?	? x b = p, and p ÷ b = ?

[1]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3: The properties of operations.

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	(a +b) + c = a + (b+c)
Commutative property of addition	a + b = b + a

Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists -*a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) x c = a x (b x c)
Commutative property of multiplication	a x b = b x a
Multiplicative identity property 1	a x 1 = 1 x a = a
Existence of multiplicative inverses	For every *a* ≠ 0 there exists 1/*a* so that *a* x 1/*a* = 1/*a* x *a* = 1
Distributive property of multiplication over additions	a x (b + c) = a x b + a x c

Table 4: The properties of equality.

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a.
Symmetric property of equality	If a = b, then b = a.
Transitive property of equality	If a = b and b = c , then a = c.
Addition property of equality	If a = b, then a +c = b + c.
Subtraction property of equality	If a = b then a – c = b – c.
Multiplication property of equality	If a = b, then a x c = b x c.
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c.
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality.

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b.
If a > b and b > c then a > c.
If a > b, b < a.
If a > b, then -a <-b.
If a > b, then a ± c > b ± c.
If a > b and c > 0, then a x c > b x c.
If a > b and c < 0, then a x c < b x c.
If a > b and c > 0, then a ÷ c > b ÷ c.
If a > b and c < 0, then a ÷ c < b ÷ c.