

K.OA: OPERATIONS & ALGEBRAIC THINKING

Cluster Statement: A: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

<p>Standard Text</p> <p>K.OA.A.1: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.</p>	<p>Standard for Mathematical Practices</p> <p>SMP 2: Students can reason abstractly and quantitatively by making connections between their representations and addition and subtraction.</p> <p>SMP 5: Students can use tools by using fingers, drawings, expressions, and/or equations to represent addition and subtraction.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Represent addition as putting together and adding to with objects, fingers, drawings, sounds, acting out situations, or verbal explanations. • Represent subtraction as taking apart and taking from with objects, fingers, drawings, sounds, acting out situations, or verbal explanations. • Identify the mathematical symbols used to show addition and subtraction. • Relate an expression or equation for addition or subtraction to a situation.
		<p>Depth Of Knowledge: 2</p>
		<p>Bloom's Taxonomy: Apply and Analyze</p>
<p>Standard Text</p> <p>K.OA.A.2: Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.</p>	<p>Standard for Mathematical Practices</p> <p>SMP 1: Students can make sense of problems and persevere in solving them by solving addition and subtraction word problems within 10.</p> <p>SMP 4: Students can model with mathematics by using objects and drawings to represent addition and subtraction problems.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Represent addition word problems with objects or drawings. • Represent subtraction word problems with objects or drawings. • Add within 10. • Subtract within 10. • Solve addition and subtraction word problems using objects and drawings.

		<p>Depth of Knowledge: 2</p>
<p>Standard Text</p> <p>K.OA.A.3: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$).</p>	<p>Standard for Mathematical Practices</p> <p>SMP 2: Students can reason abstractly and quantitatively by symbolically representing a pair of numbers less than 10 modeled concretely or pictorially with numerals and/or vice-versa.</p> <p>SMP 7: Students can look for and make use of structure by recognizing the commutative property (but not needing to know it by name) (e.g., because $5 = 2 + 3$, $5 = 3 + 2$ also).</p>	<p>Bloom's Taxonomy: Apply and Analyze</p> <p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Decompose (break apart) numbers to 10 using objects or drawings, increasing their range with time. • Decompose a number to 10 in more than one way (e.g., $5 = 2 + 3$ and $5 = 4 + 1$). • Identify an equation for a decomposed number. <p>Depth of Knowledge: 2-3</p> <p>Bloom's Taxonomy: Analyze and Evaluate</p>
<p>Standard Text</p> <p>K.OA.A.4: For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.</p>	<p>Standard for Mathematical Practices</p> <p>SMP 2: Students can reason abstractly and quantitatively by symbolically representing combinations of 10 modeled concretely or pictorially with numerals and/or vice-versa.</p> <p>SMP 6: Students can attend to precision by using the equals sign accurately and appropriately.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Determine the number to add a given number 1-9 to make 10. • Represent combinations of 10 with a drawing or equation. <p>Depth of Knowledge: 1-2</p>

		Bloom's Taxonomy: Remember and Apply
Standard Text K.OA.A.5: Fluently add and subtract within 5.	Standard for Mathematical Practices SMP 6: Students can attend to precision by accurately, automatically and flexibly knowing their addition and subtraction facts within 5. SMP 7: Students can look for and make use of structure by using the patterns they found when composing and decomposing numbers to help them add and subtract.	Students who demonstrate understanding can: <ul style="list-style-type: none"> Consistently add within 5 with accurate and efficient results Consistently subtract within 5 with accurate and efficient results
		Depth of Knowledge: 1
		Bloom's Taxonomy: Remember
Previous Learning Connections <ul style="list-style-type: none"> Connect to students work with counting. (K.CC) 	Current Learning Connections <ul style="list-style-type: none"> Connect to decomposing larger numbers in the range of 11-19 to gain foundations for place value by composing and decomposing into "ten ones and some more." (e.g., 18 is ten ones and eight more). (K.NBT.1) 	Future Learning Connections <ul style="list-style-type: none"> Connect to represent and solving problems with addition and subtraction within 20, including a new type of problem situation (compare). (1.OA.1) Connect to understanding and applying properties of operations and the relationship between addition and subtraction. (1.OA.3) Connect to adding and subtracting within 20. (1.OA.6) Connect to working with addition and subtraction equation. (1.OA.7)
Clarification Statement: K.OA.A.1: <ul style="list-style-type: none"> Math drawings facilitate reflection and discussion because they remain after the problem is solved. The teacher can write expressions (e.g., $3 - 1$) to represent operations, as well as writing equations that represent the whole situation before the solution (e.g., $3 - 1 = ?$) or after (e.g., $3 - 1 = 2$). Expressions like $3 - 1$ or $2 + 1$ show the operation, and it is helpful for students to have experience just with the expression so they can conceptually chunk this part of an equation. Students may bring from home different ways to show numbers with their fingers and to raise (or lower) them when counting. The three major ways used around the world are starting with the thumb, the little finger, or the pointing finger (ending with the thumb in the latter two cases). Each way has advantages physically or mathematically, so students can use whatever is familiar to them. The teacher can use the range of methods present in the classroom, and these methods can be compared by students to expand their understanding of numbers. K.OA.A.2: <ul style="list-style-type: none"> In Put Together/Take Apart situations, two quantities jointly compose a third quantity (the total), or a quantity can be decomposed into two quantities (the addends). This composition/decomposition may be physical or conceptual. These situations are acted out with objects initially and later children 		

begin to move to conceptual mental actions of shifting between seeing the addends and seeing the total (e.g., seeing children or seeing boys and girls, or seeing red and green apples or all the apples).

- **Addition and Subtraction Situations by Grade Level**

sign (=, here with the meaning of “becomes,” rather than the more general “equals”).

Table 2: Addition and subtraction situations by grade level.

	Result Unknown	Change Unknown	Start Unknown
Add To	<p>A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now?</p> $A + B = \square$	<p>A bunnies were sitting on the grass. Some more bunnies hopped there. Then there were C bunnies. How many bunnies hopped over to the first A bunnies?</p> $A + \square = C$	<p>Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before?</p> $\square + B = C$
Take From	<p>C apples were on the table. I ate B apples. How many apples are on the table now?</p> $C - B = \square$	<p>C apples were on the table. I ate some apples. Then there were A apples. How many apples did I eat?</p> $C - \square = A$	<p>Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before?</p> $\square - B = A$
Put Together / Take Apart	<p>Total Unknown</p> <p>A red apples and B green apples are on the table. How many apples are on the table?</p> $A + B = \square$	<p>Both Addends Unknown¹</p> <p>Grandma has C flowers. How many can she put in her red vase and how many in her blue vase?</p> $C = \square + \square$	<p>Addend Unknown²</p> <p>C apples are on the table. A are red and the rest are green. How many apples are green?</p> $A + \square = C$ $C - A = \square$
Compare	<p>Difference Unknown</p> <p>“How many more?” version. Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy?</p> <p>“How many fewer?” version. Lucy has A apples. Julie has C apples. How many fewer apples does Lucy have than Julie?</p> $A + \square = C$ $C - A = \square$	<p>Bigger Unknown</p> <p>“More” version suggests operation. Julie has B more apples than Lucy. Lucy has A apples. How many apples does Julie have?</p> <p>“Fewer” version suggests wrong operation. Lucy has B fewer apples than Julie. Lucy has A apples. How many apples does Julie have?</p> $A + B = \square$	<p>Smaller Unknown</p> <p>“Fewer” version suggests operation. Lucy has B fewer apples than Julie. Julie has C apples. How many apples does Lucy have?</p> <p>“More” version suggests wrong operation. Julie has B more apples than Lucy. Julie has C apples. How many apples does Lucy have?</p> $C - B = \square$ $\square + B = C$

Darker shading indicates the four Kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Unshaded (white) problems are the four difficult subtypes or variants that students should work with in Grade 1 but need not master until Grade 2. Adapted from CCSS, p. 88, which is based on *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

¹ This can be used to show all decompositions of a given number, especially important for numbers within 10. Equations with totals on the left help children understand that = does not always mean “makes” or “results in” but always means “is the same number as.” Such problems are not a problem subtype with one unknown, as is the Addend Unknown subtype to the right. These problems are a productive variation with two unknowns that give experience with finding all of the decompositions of a number and reflecting on the patterns involved.

² Either addend can be unknown; both variations should be included.

K.OA.A.3:

- Put Together/Take Apart situations with **Both Addends Unknown** play an important role in Kindergarten because they allow students to explore various compositions that make each number.
- Addition and Subtraction Situations by Grade Level

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Add To	<p><i>A</i> bunnies sat on the grass. <i>B</i> more bunnies hopped there. How many bunnies are on the grass now?</p> $A + B = \square$	<p><i>A</i> bunnies were sitting on the grass. Some more bunnies hopped there. Then there were <i>C</i> bunnies. How many bunnies hopped over to the first <i>A</i> bunnies?</p> $A + \square = C$	<p>Some bunnies were sitting on the grass. <i>B</i> more bunnies hopped there. Then there were <i>C</i> bunnies. How many bunnies were on the grass before?</p> $\square + B = C$
Take From	<p><i>C</i> apples were on the table. I ate <i>B</i> apples. How many apples are on the table now?</p> $C - B = \square$	<p><i>C</i> apples were on the table. I ate some apples. Then there were <i>A</i> apples. How many apples did I eat?</p> $C - \square = A$	<p>Some apples were on the table. I ate <i>B</i> apples. Then there were <i>A</i> apples. How many apples were on the table before?</p> $\square - B = A$
	Total Unknown	Both Addends Unknown ¹	Addend Unknown ²
Put Together /Take Apart	<p><i>A</i> red apples and <i>B</i> green apples are on the table. How many apples are on the table?</p> $A + B = \square$	<p>Grandma has <i>C</i> flowers. How many can she put in her red vase and how many in her blue vase?</p> $C = \square + \square$	<p><i>C</i> apples are on the table. <i>A</i> are red and the rest are green. How many apples are green?</p> $A + \square = C$ $C - A = \square$
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	<p><i>“How many more?”</i> version. Lucy has <i>A</i> apples. Julie has <i>C</i> apples. How many more apples does Julie have than Lucy?</p> <p><i>“How many fewer?”</i> version. Lucy has <i>A</i> apples. Julie has <i>C</i> apples. How many fewer apples does Lucy have than Julie?</p> $A + \square = C$ $C - A = \square$	<p><i>“More”</i> version suggests operation. Julie has <i>B</i> more apples than Lucy. Lucy has <i>A</i> apples. How many apples does Julie have?</p> <p><i>“Fewer”</i> version suggests wrong operation. Lucy has <i>B</i> fewer apples than Julie. Lucy has <i>A</i> apples. How many apples does Julie have?</p> $A + B = \square$	<p><i>“Fewer”</i> version suggests operation. Lucy has <i>B</i> fewer apples than Julie. Julie has <i>C</i> apples. How many apples does Lucy have?</p> <p><i>“More”</i> version suggests wrong operation. Julie has <i>B</i> more apples than Lucy. Julie has <i>C</i> apples. How many apples does Lucy have?</p> $C - B = \square$ $\square + B = C$

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² Either addend can be unknown; both variations should be included.

K.OA.A.5: Experience with decompositions of numbers and with **Add To and Take From situations** enables students to begin to **fluently add** and **subtract** within 5.

Common Misconceptions

- Believing that certain words always indicate a particular operation

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted)

What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?

For example, some learners may benefit from targeted pre-teaching that introduces new representations when studying the understanding of addition and subtraction because new symbols and concepts, such as the plus, minus, and equal sign will be introduced. Students at this point will more than likely have not been exposed to the understanding of combining numbers.

Pre-teach (intensive)

What critical understandings will prepare students to access the mathematics for this cluster?

Indicator 9.3 (New Mexico Early Learning Guidelines, Essential Indicator): This standard provides a foundation for work with understanding addition as putting together and adding to, and subtraction as taking apart and taking away from because students need foundational skills relating to initial understanding of numbers and rote counting. Also, students learn that numbers are associated with words and numeral symbols. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access:

Interest: How will the learning for students provide multiple options for recruiting student interest?

For example, learners engaging with understanding addition as putting together and adding to, and understanding subtraction as taking apart and taking from, benefit when learning experiences include ways to recruit interest such as providing choices in their learning (such as using manipulatives or visuals), because students may be more so intrigued and motivated to learn the new concept; for example students may use hands-on objects of their choice, such as bear shaped counters or buttons to work to combine new numbers to make one, or to take apart from.

Build:

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

For example, learners engaging with understanding addition and subtraction, benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports because the opportunity for students to work amongst each other and collaborate will allow for students to exchange ideas. This may be done during “center time” in small groups, which also allows the teacher to better understand individual learning needs by getting to work with each student in a smaller setting.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

For example, learners engaging with understanding addition and subtraction benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form because students will be able to use multiple means to better their individual understanding such as using different visuals or manipulatives to represent equations. For example, rather than writing out an equation using numerals (i.e., $1+2=3$, students can view or illustrate the

same equation using visual representations, such as drawing one circle, then two circles, and counting all circles to combine for a number of 3. This method ensures students are engaging in and understanding the overall concept of addition.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

For example, learners engaging with understanding addition and subtraction benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because students will have the opportunity to engage with multiple means of learning such as using visuals, manipulatives, physical movement or oral word problems (ex: Dad gave Timmy three pencils, and Timmy lost one, how many pencils does Timmy now have?" to better develop understanding of the concept of addition and subtraction>.

Internalize:

Self-Regulation: How will the design of the learning strategically support students to effectively cope and engage with the environment?

For example, learners engaging with understanding addition and subtraction benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as <offering devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the mathematical practices for the purpose of monitoring and improving because students can refer back to these tools throughout their work to remain on task and assist with eliminating confusion. For example, an anchor chart with important symbols and vocabulary such as the plus sign, subtraction sign, and equal sign, can be posted so that students can refer back to which symbol corresponds with which problems when working in small groups or independently. This will assist in eliminating confusion with symbols.

Re-teach

Re-teach (targeted)

What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?

Examine assessments for evidence of lingering misconceptions (see common misconceptions). If students exhibit one more of these misconceptions, consider addressing the misconception by: For example, students may benefit from re-engaging with content during a unit on understanding addition and subtraction by examining tasks from a different perspective through a short mini-lesson because students may be able to learn addition or subtraction concepts in multiple ways such as learning with the use of visuals or manipulatives.

Re-teach (intensive)

What assessment data will help identify content needing to be revisited for intensive interventions?

Examine assessments for evidence of students still developing the underlying ideas, for example, some students may benefit from intensive extra time during and after a unit on being able to represent addition and subtraction with objects, fingers, mental images, drawings*, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations by offering opportunities to understand and explore different strategies through the use of concrete manipulative or fingers and to accommodate various learning styles because some students may need to practice the concept by using more than one modality of learning to then progress from concrete to pictorial representations of the models.

Extension Ideas

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

To extend students learning on understanding addition as putting together and adding to, and understanding subtraction as taking apart and taking from, some learners may benefit from an

extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students, when studying the skill to represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations because students may benefit from learning to reframe a certain problem in a new way, such as moving away from using physical and visual cues to add and subtract and start using word based, or oral problems.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time." For example, when studying understanding addition as putting together and adding to, and understanding subtraction as taking apart and taking from eliciting and using student thinking is critical because providing opportunities for instructional conversations as students work through conceptualizing addition and subtraction helps build equity of participation and develops active listening skills.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: <http://tasks.illustrativemathematics.org/content-standards/K/OA/A/3/tasks/165>

The purpose of this task is for students to decompose a number as a sum of two other numbers in more than one way. The teacher should demonstrate how to "shake and spill" the counters as well as how to represent the sum using pictures or equations. The word "sum" is easily confused with "some," especially for young children; take care to use language that the students understand. However, make sure that they understand that they are representing, for example, 3+2, not just 3 and 2 separately. Language like, "How many red? How many yellow? How many altogether?" might be appropriate. Although this task uses 5 counters, it can be repeated using any number through 10 to address K.OA.3.

Relevance to families and communities:

During a unit focused on understanding addition as putting together and adding to, and understanding subtraction as taking apart and taking from, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example Students may bring from home different ways to show numbers with their fingers and to raise (or lower) them when counting. The three major ways used around the world are starting with the thumb, the little finger, or the pointing finger (ending with the thumb in the latter two cases).

Cross-Curricular Connections:

Social Studies: In Kindergarten, the New Mexico Social Studies Standards state students should "describe trade (e.g., buying and selling, bartering, simple exchange)". Consider providing a connection for students to add and subtract related to buying and selling.

Language Arts: Literature can offer connections about addition and subtraction such as: *Making Tens* by John Burstein and *Ten Little Caterpillars* by Bill Martin, Jr.