New Mexico Public Education Department

New Mexico Mathematics Instructional Scope for Kindergarten

June 2020

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Overview

This mathematics instructional scope was created by a cohort of New Mexico educators and the New Mexico Public Education Department. This document is organized into three sections. Section 1 describes how to use this document to support equitable and excellent mathematics instruction. Section 2 contains planning support for each cluster of mathematics standards within the grade level or course. Section 3 provides additional resources, references, and glossary.

The intention of this document is to act as companion during the planning process alongside High Quality Instructional Materials (HQIM). A sample template is presented to show a quick snapshot of planning supports provided within each cluster of standards in section 2.

During the creation of this document, we leveraged the work of other states, organizations, and educators from across country and the world. This work would not have been possible without all that came before it and we wish to express our sincerest gratitude for everyone that contributed to the resources listed within our references. This document is a work in progress and in some circumstances, our team of New Mexico educators may have embedded content from resources that have yet to be cited, as these elements are discovered in the use of this tool the references in section 3 will be updated.

Section 1: New Mexico Instructional Scope for Supporting Equitable and Excellent Mathematics Instruction

To better understand the planning supports provided in section 2, for each cluster of standards, this section provides a brief description of each planning support including: *what* support is provided; *why* the planning support is critical for equitable and excellent mathematics instruction; and, *how* to use the planning support with HQIM.

Cluster Statement

What: The New Mexico Mathematics Standards are grouped by Domains with somewhere between 4 to 10 domains per grade level. Within each domain the standards are arranged around clusters. Cluster statements summarize groups of related standards. The cluster statement planning support also indicates if the clusters is major, supporting, or additional work of the grade.

Why: The New Mexico Mathematics Standards require a stronger *focus*[1] on the way time and energy are spent in the mathematics classroom. Students should spend the large majority of their time (65-85%) on the major clusters of the grade/course. Supporting clusters and, where appropriate, additional clusters should be connected to and engage students in the major work of the grade.

How: When planning with your HQIM consider the time being devoted to major versus additional or supporting clusters. Major Work of each grade should be designed to provide students with strong foundations for future mathematical work which will require more time than additional or supporting clusters. Consider also the ways the

[1] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

HQIM makes explicit for students the connections between additional and supporting clusters and the major work of the grade.

Standard Text

What: Each cluster level support document contains the text of each standard within the cluster.

Why: The cluster statement and standards are meant to be read together to understand the structure of the standards. By grouping the standards within the cluster the connectedness of the standards is reinforced.

How: The text of the standards should always ground all planning with HQIM. Reading the standards within a cluster intentionally focuses on the connections within and among the standards.

Standards for Mathematical Practice

What: The Standards for Mathematical Practice describe the varieties of expertise and habits of mind that mathematics educators at all levels should seek to develop in their students.

Why: Equitable and excellent mathematics instruction supports students in becoming confident and competent mathematicians. By engaging with the standards for mathematical practice students are engaging in the practice of doing mathematics and development of mathematical habits of mind—the ability to think mathematically, analyze situations, understand relationships, and adapt what they know to solve a wide range of problems, including problems they may not look like any they have encountered before.[2]

How: When planning with HQIM it is critical to consider the connections between the content standards and the standards for mathematical practice. The planning supports highlight a few practices in which students could engage when learning the content of the standard. Note it is not necessary or even appropriate to engage in all of the practices every day, rather choosing a few and spending time intentionally supporting students in learning both the what (content standards) and the how (standards for mathematical practice) will create a stronger foundation for ongoing learning.

Students Who Demonstrate Understanding Can (Webb's Depth of Knowledge and Bloom's Taxonomy)

What: The New Mexico Mathematics Standards include each aspect of mathematical rigor: conceptual understanding, procedural skill and fluency, and application to the real world.[3] This planning support considers which aspect(s) of rigor are within each standard and then identifies academics skills students need to demonstrate comprehension of the standard and associated mathematical practices. The statements also highlight both the receptive (listening and reading) and expressive (speaking and writing) parts of language by considering the types of mathematical representations (verbal, visual, symbolic, contextual, physical) within the standard and what students need to do with them. The planning supports also provide information about two common classifications on cognitive complexity, Webb's Depth of Knowledge and Bloom's Taxonomy.

Why: Analyzing standards alongside the standards for mathematical practice provide a fuller picture of the mathematical competencies demanded in the standard.

How: When planning for a cluster of standards with your HQIM a critical first step is to analyze the content and language demands of the standards and standards for mathematical practice. The analysis can be used to inform

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[2] Seeley, C. L. (2016). Math is Supposed to Make Sense. In *Making sense of math: How to help every student become a mathematical thinker and problem solver*. Alexandria, VA, USA: ASCD. (P. 13)
[3]Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

formative assessment, or it can be used to plan/design appropriate formative assessment.[4] The planning supports provide a possible break-down of the standard that can serve as the basis for this sort analysis.

Connections

What: The New Mexico Mathematics Standards are designed around coherent progressions of learning. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but an extension of previous learning.[5] The connections to previous, current and future learning make this coherence visible.

Why: Students build stronger foundations for learning when they see mathematics as an inter-connected discipline of relationships rather than discrete skills and knowledge. The intentional inclusion of connections to previous, current, and future learning can support a more inter-connected understanding of mathematics.

How: When planning with HQIM use the connection planning supports to find ways to support students in making explicit connections within their study of mathematics.

Clarification Statement

What: The clarification statement provides greater clarity for teachers in understanding the purpose of the standards within a cluster.

Why: The New Mexico Mathematics Standards illustrate how progressions support student learning within each major domain of mathematics. The clarification statement provides additional context about the ways each cluster of standards supports student learning of the larger learning progression.

How: When planning with HQIM use the clarification statement to support an understanding of how the materials use specific types of representations or change the learning sequence from instructional approaches not grounded in progressions of learning.

Common Misconceptions

What: This planning support identifies some of the common misconceptions students develop about a mathematical topic.

Why: Students create misconceptions based on an over generalization of patterns they notice or an over reliance on rules rather than underlying mathematics. Rules in mathematics expire[6] over time (e.g., you can't subtract 1-3) as students expand their knowledge of mathematics (e.g., from whole numbers to rational numbers). It is critical to understand some of the common misconceptions students can develop so we can address them directly with students and continue to build a strong foundation for their mathematical learning.

How: When planning with your HQIM look for ways to directly address with students some common misconceptions. The planning supports in this document provide some possible misconceptions and your HQIM might include additional ones. The goal is not to avoid misconceptions, they are a natural part of the learning process, but we want to support students in exploring the misconception and modifying incorrect or partial understandings.

Multi-Layered System of Supports/Suggested Instructional Strategies

What: The section on Multi-Layered Systems of Supports(MLSS)/Suggested Instructional Strategies is designed to support teachers in planning for the needs of all students. Each section includes options for pre-teaching, reteaching, extensions and core instructional supports for students. Targeted pre-teaching and reteaching support student's acquisition of the knowledge and skills identified in the New Mexico Mathematics Standards to support student success with high-quality differentiated instruction. Intensive supports may be provided for a longer duration, more

[4] English Learners Success Forum. (2020). ELSF | Resource: Analyzing Content and Language Demands. Retrieved from https://www.elsuccessforum.org/resources/math-analyzing-content-and-language-demands

[5] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

[6] Cardone, T. (n.d.). Nix the Tricks. Retrieved from https://nixthetricks.com/

frequently, smaller groups, or otherwise be more intensive than targeted supports. Progress monitoring should occur to assess students' responses to additional supports, see Standards Aligned Instructionally Embedded Formative Assessment Resources.

Why: MLSS is a holistic framework that guides educators, those closest to the student, to intervene quickly when students need additional supports. The framework moves away from the "wait to fail" model and empowers teachers to use their professional judgement to make data-informed decisions regarding the students in their classrooms to ensure academic success with the grade level expectations of the New Mexico Mathematics Standards.

How: When planning with your HQIM use the suggestions for pre-teaching as a starting point to determine if some or all of the students in your classroom may need targeted or intensive pre-teaching at the start of unit to ensure they can access the grade level material with the unit. The core-instruction and reteach sections work together to support planning within a unit, look for the ways the materials are supporting greater access for all students and providing options to revisit materials based on formative assessments. The planning supports for each cluster are grounded in the Universal Design Learning (UDL) Framework, additional planning supports based on this framework can be found in Section 3 of this document in the part titled, Planning Guidance for Multi-Layered Systems of Support: Core Instruction.

Culturally and Linguistically Responsive Instruction

What: Culturally and Linguistically Responsive Instruction (CLRI), or the practice of situational appropriateness, requires educators to contribute to a positive school climate by validating and affirming students' home languages and cultures. Validation is making the home culture and language legitimate, while affirmation is affirming or making clear that the home culture and language are positive assets. It is also the intentional effort to reverse negative stereotypes of non-dominant cultures and languages and must be intentional and purposeful, consistent and authentic, and proactive and reactive. Building and bridging is the extension of validation and affirmation. By building and bridging students learning to toggle between home culture and linguistic behaviors and expectations and the school culture and linguistic behaviors and expectations. The building component focuses on creating connections between the home culture and language and the expectations of school culture and language for success in school. The bridging component focuses on creating opportunities to practice situational appropriateness or utilizing appropriate cultural and linguistic behaviors.[7]

Why: The mathematical identities of students are shaped by the messages they receive about their ability to do mathematics and the power of mathematics in their lives outside of school.[8] Mathematics educators must intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages. In addition, create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to supports students in creating mathematical identities as capable mathematicians within school and society.

How: When planning instruction is critical to consider ways to validate/affirm and build/bridge from your students cultural and linguistic assets. The planning supports for each cluster provide an example of how to support equity-based teaching practices. Look for additional ways within your HQIM to ensure all students develop strong mathematical identities.

Standards Aligned Instructionally Embedded Formative Assessment Resources

What: Formative Assessment is the planned, ongoing process used by all students and teachers during learning and teaching to elicit and use evidence of student learning to improve student understanding of the outcomes and support students to become directed learners. All New Mexico educators have access to standards aligned instructionally embedded formative assessments: iStation at K-2; Cognia at 3-8, and the SAT Suite Question

[7] Hollie, S. (2011). *Culturally and linguistically responsive teaching and learning*. Teacher Created Materials.

[8] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics. (P. 14)

Bank at 9-12. These are intended to be used during instruction for each at each grade alongside assessments within your HQIM.

Why: When student thinking is made visible the teacher can examine the progression of learning towards the goals of the standards and adjust instruction as necessary. By including students in the assessment and analysis process students become strategic and goal-directed with their learning.

How: The planning supports at each cluster provide an example of a task that addresses one more aspect of the cluster of standards. This example can be used to discuss possible responses by students and next steps for instruction. A similar process can then be used to identify additional items from one of the formative assessment resources provided by NM PED and your HQIM.

Relevance to Families and Communities

What: Relevance to families and communities requires finding the relevance of mathematics outside of the classroom by connecting to families and communities and learning about varied and often unexpected ways they use mathematics.

Why: When school mathematics is connected to the mathematics outside of school students can build a bridge between their ways of thinking about quantities outside and inside school created a bridge between home and school.

How: When planning at the year and unit level with you HQIM find ways to intentionally learn from your families and communities the cultural and linguistic ways they use mathematics outside of school.

Cross-Curricular Connections

What: New Mexico defines cross-curricular connections as connections between two or more areas of study made by teachers or students within the structure of a subject.

Why: The purpose of planning cross-curricular connections in an instructional sequence is to ensure that students build connections and recognize the relevance of mathematics beyond the mathematics classroom.

How: When planning with HQIM look for opportunities to make explicit connections to other content areas such as the examples provided for each cluster.

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Template of the New Mexico Cluster Level Planning Support for the New Mexico Mathematics Standards

<GRADE/COURSE/DOMAIN ABBREVIATION: DOMAIN NAME>

Cluster Statement: Statement from New Mexico Mathematics Standards summarize a group of related standards.

Major/Additional/Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)
Identifies if the cluster is major, additional or supporting work of the grade.

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
Full text of the standard	The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.	The cognitive skills students perform to demonstrate to comprehension of a standard.
		Depth Of Knowledge: Correlation of standard to Webb's Depth of Knowledge
		Bloom's Taxonomy: Correlation of standard to Bloom's Taxonomy
Connections to Previous Learning: Supports student connections to learning from previous grade levels.	**Connections to Current Learning** Supports student connections to learning within the grade level.	**Connections to Future Learning** Supports student connections to learning in a future grade.

Clarification Statement: Clarifies the language of the standard.

Common Misconceptions: Guidance on where a student misconception or misunderstanding could potentially occur.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach
Pre-teach (targeted): Guidance for how to activate students' knowledge to support their learning.
Pre-teach (intensive): Guidance for how to use earlier grade standards to build a strong foundational understanding upon which to build grade level concepts.

Core Instruction
Access: Guidance for optimizing universal access to learning experiences.
Build: Guidance for supporting students build their understanding of the cluster.
Internalize: Guidance for ensuring student internalization of the learning goal.

Re-teach
Re-teach (targeted): Guidance for adjusting instruction during a unit by using formative assessment data.
Re-teach (intensive): Guidance for analyzing assessment data to identify content that would benefit from more intensive reteaching.
Extension Ideas: Suggestions that offer additional challenges to 'broaden' students' knowledge of the mathematics within the cluster.

Culturally and Linguistically Responsive Instruction: Provides equity based instructional suggestions aligned to the cluster of standards

Standards Aligned Instructionally Embedded Formative Assessment Resources: Includes reference to high-quality formative assessment resources, including examples from New Mexico's formative assessment banks.

Relevance to Families and Communities:	**Cross Curricular Connections:**
Connecting with families and communities to create relevant connections between mathematics inside and outside of school.	Includes examples of how the cluster provides opportunities to connect to other disciplines such as literacy, science, social studies, and the arts.

Section 2: Cluster Level Planning Support for the New Mexico Mathematics Standards

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<tr><td colspan="3">Cluster Statement: A: Know number names and the count sequence.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)</td></tr>
<tr><td rowspan="3">Standard Text

K.CC.A.1: Count to 100 by ones and by tens.</td><td rowspan="3">Standard for Mathematical Practices

SMP 6: Students can attend to precision by learning and using the number names when counting by ones and by tens.

SMP 8: Students can look for and express regularity in repeated reasoning by recognizing patterns that exist when counting by ones and by tens. We always use the same 10 digits.</td><td>Students who demonstrate understanding can:
• Count to 100 by ones, increasing their range with time
• Count to 100 by tens.</td></tr>
<tr><td>Depth of Knowledge: 1</td></tr>
<tr><td>Bloom's Taxonomy:
Remember</td></tr>
<tr><td>Standard Text

K.CC.A.2: Count forward beginning from a given number within the known sequence (instead of having to begin at 1).</td><td>Standard for Mathematical Practices

SMP 6: Students can attend to precision by learning and using the number names when counting by ones and by tens.

SMP 7: Students can look for and make use of structure by using the patterns of ones and decades to count forward from any given number.</td><td>Students who demonstrate understanding can:
• Count forward from a random starting number, instead of 1, increasing their range with time.</td></tr>
</table>

		Depth of Knowledge: 1-2 **Bloom's Taxonomy:** Remember and Understand
Standard Text **K.CC.A.3: Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).**	**Standard for Mathematical Practices** SMP 2: Students can reason abstractly and quantitatively by recognizing that each numeral symbol is connected to a specific quantity. SMP 8: Students look for and express regularity in repeated reasoning by recognizing patterns that exist when writing the numbers 1-20. We always use the same 10 digits and cycle through them in the same order for each place value.	**Students who demonstrate understanding can:** • Students can write numbers 1-20, increasing their range with time. • Represent up to 20 objects with written numerals, no matter the arrangement of the objects. • Recognize the relationship between 0 and no objects. **Depth of Knowledge:** 1 **Bloom's Taxonomy:** Remember
Previous Learning Connections • Connect to counting by ones to 10 and higher. • Connect to recognizing and naming numerals 1 to 5.	**Current Learning Connections** • Connect continuing in the Counting and Cardinality domain to use counting to tell the number of objects. **(K.CC.4)** • Connect to continuing to work with concepts of number meaning in the domains of **Operations and Algebraic Thinking**, as well as **Number and Operations in Base Ten**.	**Future Learning Connections** • Connect to extending the counting sequence, number recognition and writing to 120. **(1.NBT.1)**
Clarification Statement: The emphasis of this cluster is on the counting sequence. When counting by ones, students need to understand that the next number in the sequence is one more. When counting by tens, the next number in the sequence is "ten more" (or one more group of ten). Students should be able to count forward from any number, 1-99. Students should be given multiple opportunities to count objects and recognize that a number represents a specific quantity.		
Common Misconceptions ● Struggling with continuous counting and skipping numbers ● Being confused by the names for the teen numbers ● Believing that counting must always start at 1 ● Not seeing 0 as a number		

● Inverting and/or reversing numerals

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies
Pre-Teach

Pre-teach (targeted)
What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?
For example, some learners may benefit from targeted pre-teaching that introduces new representations when studying knowing the number names and the count sequence because students will need support when learning numbers and number sequences. Visual aids that give support create confidence and will stimulate thinking and improve the learning environment in a classroom.

Pre-teach (intensive)
What critical understandings will prepare students to access the mathematics for this cluster?
Indicator 9.3 of the "New Mexico Early Learning Guidelines, Essential Indicator" will provide some knowledge that is required. This standard provides a foundation for work with numbers and ways of representing numbers because numbers represent quantity or "how many". Children who develop number sense understand the order in math. They see the relationships that numbers have to one another; they understand how numbers are put together and taken apart; and they have an intuitive sense about our number system. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access: How will the learning for students provide multiple options for recruiting student interest?
For example, learners engaging with: Know number names and the count sequence benefit when learning experiences include ways to recruit interest such as utilizing classroom instructional routines to involve all students because as in other subjects, math students must be able to read, write, listen, speak, and discuss the subject at hand. Routines that are designed to support a variety of language-focused skill growth reinforcing mathematical terminology and providing opportunities for students to deepen their conceptual understanding by describing their work. Routines, done regularly, can benefit all students, though they are particularly supportive of English Language Learners or those struggling with the linguistic components of math.

Build:
Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?
For example, learners engaging with Counting to 100 by ones and by tens and counting forward beginning from a given number other than 1, benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as constructing communities of learners engaged in common interests or activities because learners must be able to communicate and collaborate effectively within a community of learners which introduces the processes and structures that form the basis of mathematics that establishes the mathematical community. Classroom routines develop the concept of community and shared interests and because routines are done often students have my opportunities to count and to see and hear others count.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)
For example, learners engaging with Counting to 100 by ones and by tens and counting forward beginning from a given number other than 1, benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners

such as making connections to previously learned structures because the words, symbols and numbers are differentially accessible to learners with varying backgrounds, languages, and lexical knowledge. To ensure accessibility for all, number names should be linked to, or associated with, alternate representations of their meaning (e.g., calendar, birthday chart or map). Providing different ways for students to see numbers helps them to draw on previous knowledge such as.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?
For example, learners engaging with Counting to 100 by ones and by tens and counting forward beginning from a given number other than 1 benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks, 3D models, base-ten blocks) because math manipulatives help make abstract ideas concrete for students as well as giving them a reason to test and confirm their reasoning. Concrete objects help to intrigue and motivate students.

Internalize:
Comprehension: How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?
For example, learners engaging with Counting to 100 by ones and by tens and counting forward beginning from a given number other than 1 benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because giving children experience with immediately recognizing and labeling quantities of a collection and having them answer the question "How many are there?" helps to solidify the concept. To help children construct a more abstract concept of number, teachers can use classroom routines and procedures that involve counting and numbers.

Re-teach

Re-teach (targeted)
What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?
Students may benefit from re-engaging with content during a unit on addition and subtraction by revisiting student thinking through a short mini-lesson because students should have a good understanding of number names and count sequence.

Re-teach (intensive)
What assessment data will help identify content needing to be revisited for intensive interventions?
Some students may benefit from intensive extra time during and after a unit counting to quantity by offering opportunities to understand and explore different strategies because it is important for students to have lots of opportunities to practice counting and hearing others count in order to develop fluency with place value patterns and allows students to become familiar with patterns through counting. ...

Extension
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?
Some learners may benefit from an extension such as the opportunity to explore links between various topics when studying knowing the number names and the count sequence because cross-curricular teaching, or instruction that intentionally applies multiple academic disciplines simultaneously, is an effective way to teach students transferable problem solving skills, give real-world meaning to school assignments, and increase engagement and rigor.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Task: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)." For example, when studying knowing number names and the counting in sequence the types of mathematical tasks are critical because Practices within a culture affect understanding. Some assessment tools may greatly underestimate the knowledge that students possess. Tools that are used in students' everyday lives may better capture student understanding. For example, a student who stops by the corner store to by snacks every day understands place value to some degree but may not be able to show that knowledge using cubes.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

http://s3.amazonaws.com/illustrativemathematics/attachments/000/009/254/original/public_task_1397.pdf?1462395701
The purpose of this task is to give students an opportunity to count real objects and write numbers. This activity can become a daily 10-minute routine, with the students counting as many bags of "stuff" as they can in that time period. Students can also work together in pairs. Students should focus on the numerals 1-10 before continuing with numerals 11-20. A number line or chart could be made available for those students who need support.

Relevance to families and communities:
During a unit focused on counting, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, incorporating how mathematical concepts were already being used at home. Then finding ways to incorporate this prior knowledge, such as familiarity with sports, into classroom lessons. Instead of using football merely as the context for a problem, the numbers inherent to football, like series of sevens and threes, could be used.

Cross-Curricular Connections:

Social Studies: In Kindergarten, the New Mexico Social Studies Standards state students should "identify classroom population". Consider providing a connection for students to count the classroom population in ways that change (such as number of students present and number of students absent each day).

Morning Meeting (or other morning routine): Consider providing a connection to counting various aspects related to the calendar, including the first 100 days of school.

K.CC: COUNTING & CARDINALITY

Cluster Statement: B: Count to tell the number of objects.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
K.CC.B.4: Understand the relationship between numbers and quantities; connect counting to cardinality. • **K.CC.B.4.A: When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.** • **K.CC.B.4.B: Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.** • **K.CC.B.4.C: Understand that each successive number name refers to a quantity that is one larger.**	SMP 2: Students can reason abstractly and quantitatively by representing sets of objects with the number of counts. SMP 6: Students can attend to precision by developing the idea of one-to-one correspondence and realizing that one number name goes with each item when counting objects.	• Count objects in a group (each object is counted only once) regardless of arrangement and order. • Determine "how many" are in a group after counting all the objects. • Indicate by counting that the last item said tells the number of objects. • Count on from a known number (without recounting the whole group) when one more object is added to the group. **Depth of Knowledge:** 2 **Bloom's Taxonomy:** Apply and Analyze

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
K.CC.B.5: Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.	SMP 7: Students can look for and make use of structure by rearranging scattered items and placing them into circles, a straight line or groups of two to help them count. SMP 8: Students look for and express regularity in repeated reasoning by recognizing that the total number of objects does not change regardless of the way the items/objects are arranged.	• Count objects up to 20 in a variety of arrangements (transition to dot cards, ten frames, dominos, and other representations) • Tell "how many" objects are in a group in a variety of arrangements. • Show the correct number of objects when I am told a number up to 20. • Show the correct number of objects, when told a number, in different arrangements. **Depth of Knowledge:** 2 **Bloom's Taxonomy:** Apply
<u>Previous Learning Connections</u> • Connect to counting the number of items in a group of up to 10 objects and knowing that the last number tells how many • Connect to giving up to 5 items when requested	**<u>Current Learning Connections</u>** • Connect to continuing to work with concepts of number meaning in the domains of **Order and Algebraic Thinking**, as well as **Number and Operations in Base Ten**. • Connect to comparing the size of sets to tell greater than, less than, or the same, including written numerals. **(K.CC.6, 7)**	**<u>Future Learning Connections</u>** • Connect to extending the counting sequence, number recognition and writing to 120. **(1.NBT.1)** • Connect to counting strategies to add and subtract within 20. **(1.OA.1)**

Clarification Statement:

K.CC.B.4: Experience with **counting** allows students to discuss and come to understand the second part of K.CC.4b—that the **number** of objects is the **same** regardless of their **arrangement** or the **order** in which they were counted.

K.CC.B.5: Counting objects arranged in a **line** is easiest; with more practice, students learn to count objects in more difficult arrangements, such as **rectangular arrays** (they need to ensure they reach every **row** or **column** and do not repeat rows or columns); **circles** (they need to stop just before the object they started with); and **scattered** configurations (they need to make a single path through all of the objects).

Common Misconceptions

● Not yet understanding one-to-one correspondence

● Believing that the arrangement of a set of objects affects the total count

● Believing that the tagged count is related to the object rather than its position (e.g., the triangle is always 4 even when it is first in a line)

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Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted)

What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?

For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying counting objects because not all students will make the connection between rote counting, numbers, and quantity/one-to-one correspondence.

Pre-teach (intensive)

What critical understandings will prepare students to access the mathematics for this cluster?

New Mexico Early Learning Guidelines, Essential Indicator 9.1, 9.3 a-b, and 12.1 and K.CC.A.12: These standards provide a foundation for work with counting objects because students need a foundational understanding of numbers and counting/labeling quantities prior to counting objects at higher quantities. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access:

How will the learning for students provide multiple options for recruiting student interest?

For example, learners engaging with materials of interest to them benefit when learning experiences include ways to recruit interest such as creating accepting and supportive classroom climate because students value respectful support of their interest for learning, such as allowing them to demonstrate skills learned with materials that are relevant to them culturally or personally. Bugs, beads, gems, and dinosaurs are much more interesting than primary colored circle counters.

Build:

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

For example, learners engaging with counting objects benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as using prompts or scaffolds for visualizing desired outcomes because students at this level have varying experiences and background knowledge, or needs support with academic language to understand the expectations and learn the skill. Modeling counting objects for students can support their understanding of the prompts and can be used in scaffolding students to perform counting objects in increasing quantities.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

For example, learners engaging with counting objects benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because connecting previous learning supports student to understand expectations and generalize skills and begin to understand application of the skill of counting in various activities or with various objects.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

For example, learners engaging with counting objects benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners) because student's respond to different types of feedback in different ways. A student may find great success or great frustration using computer-aided instruction to get feedback on correct/ incorrect responses and may respond differently to a teacher's verbal response to their counting.

Internalize: How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?
For example, learners engaging with counting objects benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as using cues and prompts to draw attention to critical features because supporting students to use the skill of counting in a functional and meaningful way will support them in their math learning and establishing an interest in learning long term. Supporting students to group larger quantities and count with one-to-one correspondence allows students to increase the quantity of items counted and may lead students to use their counting skills in novel situations, rather than in teacher directed or classroom learning.

Re-teach

Re-teach (targeted)
What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?
Examine assessments for evidence of lingering misconceptions (see common misconceptions). If students exhibit one more of these misconceptions, consider addressing the misconception. For example, students may benefit from re-engaging with content during a unit on counting objects by critiquing student approaches/solutions to make connection through a short mini-lesson because not all students have the functional ability and experience to develop a strategy or the perseverance to try until they develop a strategy that will encourage their success in the long term (e.g., counting 2 items is not likely to need more than one-to-one-correspondence, however, 20 items may require grouping, moving, recall and memory to sustain the task to completion and success).

Re-teach (intensive)
What assessment data will help identify content needing to be revisited for intensive interventions?
Some students may benefit from intensive extra time during and after a unit counting objects by offering opportunities to understand and explore different strategies because not all students have the functional ability and experience to develop a strategy or the perseverance to try until they develop a strategy that will encourage their success in the long term (e.g., counting 2 items is not likely to need more than one-to-one-correspondence, however, 20 items may require grouping, moving, recall and memory to sustain the task to completion and success). ...

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?
To extend students learning about counting objects, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying counting objects because standard K.CC.C.6, Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies is taught in tandem and allows for the natural extension and linking of concepts around grouping and sorting objects.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying counting objects goal setting is critical because students come to Kindergarten with a variety of early experiences and different developmental levels and rate of learning differs depending on the needs of individual students. When students know the expectations and can establish goals as targets there is a development of intrinsic motivation that encourages student progress in the development of the skill.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: http://tasks.illustrativemathematics.org/content-standards/K/CC/B/5/tasks/1420

The purpose of this task is for students to build fluency in counting. Fluency is about being able to quickly and efficiently use the knowledge that is stored in one's brain. The timer is used so that students will use their most efficient counting strategies.

Relevance to families and communities:

During a unit focused on counting objects, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, families may find value in teaching students about counting activities in their everyday activities and filming them to share with the class. Students and families can share the names for numbers in their heritage language and active connection to the learning in the classroom and learning in their culture.

Cross-Curricular Connections:

Science: In Kindergarten, the NGSS states students should "use and share observations of local weather conditions to describe patterns over time. Consider providing opportunities for students to track on a calendar and then count the number of cloudy, sunny or rainy days.

Language Arts: Literature can offer connections about measurement such as: *Ten Black Dots* by Donald Crews and *The Very Hungry Caterpillar* by Eric Carle.

K.CC: COUNTING & CARDINALITY

Cluster Statement: C: Compare numbers.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
K.CC.C.6: Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.	SMP 6: Students can attend to precision by using clear vocabulary to describe the relative difference between sizes of sets. SMP 7: Students can look for and make use of structure by using one-to-one correspondence when comparing the groups to see which group has more, less, or if they are the same.	• Tell which has more by matching or counting the number of objects in both groups. • Tell which has less by matching or counting the number of objects in both groups. • Tell when groups are equal by matching or counting. • Create equal groups in different arrangements. **Depth Of Knowledge:** 2 **Bloom's Taxonomy:** Apply and Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
K.CC.C.7: Compare two numbers between 1 and 10 presented as written numerals.	SMP 2: Students can reason abstractly and quantitatively by connecting the comparison of physical objects to the number names in describing the comparison. SMP 6: Students can attend to precision by labeling a set of concrete materials with the appropriate numeral.	• Read numerals to 10. • Tell the values of numbers to 10. • Determine if a set is greater or less than another set (up to 10). • Compare two numerals between 1 and 10 and say which has a greater value.

		Depth Of Knowledge: 1-2 **Bloom's Taxonomy:** Remember, Apply and Analyze
Previous Learning Connections • Connect to recognizing and naming numerals 1 to 5 • Connect to comparing two groups (containing up to 5 objects each) and describing them using comparative words, such as, less, fewer, or equal • Connect to looking at a group of up to 4 objects and quickly seeing and saying the number of objects	**Current Learning Connections** • Connect to continuing in the Counting and Cardinality domain to use counting to tell the number of objects. **(K.CC.4-5)** • Connect to classifying objects and counting the number of objects in each category. **(K.MD.3)**	**Future Learning Connections** • Connect to comparing two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, = and <. **(1.NBT.3)** • Connect to organizing, representing, and interpreting data with up to three categories; asking and answering questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. **(1.MD.4)**

Clarification Statement:
K.CC.C.6: Students first learn to **match** the objects in the two groups to see if there are any extra and then to **count** the objects in each group and use their knowledge of the **count sequence** to decide which number is **greater** than the other (the **number** farther along in the count sequence).

Common Misconceptions

- Lack of one-to-one correspondence
- Believing that the arrangement of a set of objects affects the total count
- Believing that a longer line of objects automatically contains more objects
- Struggling with the language of comparison

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted)

What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?

For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., number lines) when studying comparing numbers because experience and exposure to numbers and the concepts required for a comparison may not be familiar and may require tools and new vocabulary for students to access the content required to learn and demonstrate knowledge of the standard.

Pre-teach (intensive)

What critical understandings will prepare students to access the mathematics for this cluster?

New Mexico Early Learning Guidelines, Essential Indicator 9.1, 9.3 a-b, and 12.1 and K.CC.A.12: This standard provides a foundation for work with comparing numbers because students must have a foundation in numbers to engage in comparison taxonomy. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access:

Physical Action: How will the learning for students provide a variety of methods for navigation to support access?
For example, learners engaging with comparing numbers benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as varying methods for response and navigation by providing alternatives to written response and allowing physically responding or indicating selections; physically interacting with materials by hand, voice, single switch, joystick, keyboard, or adapted keyboard because a variety of physical actions engage learners and support active understanding of comparison.

Build:

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?
For example, learners engaging with comparing numbers benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities because students can share knowledge and understanding and learn from peers as well as receive peer scaffolding and support to demonstrate the skill.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)
For example, learners engaging with comparing numbers benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) because not all learners will be able to grasp the concept of comparison without support and vocabulary to identify the differences and similarities in numbers.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?
For example, learners engaging with comparing numbers benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing different approaches to motivate, guide, feedback or inform students of progress towards fluency because comparing numbers can be communicated in a variety of ways and varying answer types will sustain engagement in practicing this skill and developing a depth of understanding in application through routine practice and regular application.

Internalize:

Self-Regulation: How will the design of the learning strategically support students to effectively cope and engage with the environment?
For example, learners engaging with comparing numbers benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as increasing the length of on-task orientation in the face of distractions because developing the skills around comparing

numbers requires repeated application and practice requiring students to engage for sustained periods of time and repeatedly working with this skill/concept.

Re-teach

Re-teach (targeted)
What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?
Examine assessments for evidence of lingering misconceptions (see common misconceptions). If students exhibit one more of these misconceptions, consider addressing the misconception by For example, students may benefit from re-engaging with content during a unit on comparing numbers by clarifying mathematical ideas and/or concepts through a short mini-lesson because differences in language acquisition, exposure to vocabulary and higher level taxonomy may not be areas of strength or familiarity for young students.
Re-teach (intensive)
What assessment data will help identify content needing to be revisited for intensive interventions?
Examine assessments for evidence of students still developing the underlying ideas For example, some students may benefit from intensive extra time during and after a unit comparing numbers by addressing conceptual understanding because comparison requires a level of understanding of numbers that some students may need more time to develop and may need support to begin to understand.

Extension
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?
To extend students learning, for example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying comparing numbers because students come to Kindergarten with varying levels of experience and understanding of numbers and should be encouraged to explore numbers of higher value or develop deeper comparisons of numbers based on their developmental levels.

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying comparing numbers the types of mathematical tasks are critical because students may benefit from a routine and ritual practice and process to develop their comparison skills and build fluency to compare a variety of numbers.

Standards Aligned Instructionally Embedded Formative Assessment Resources: Source: *http://tasks.illustrativemathematics.org/content-standards/K/CC/C/7/tasks/697* The iteration of greater than, less than, and equal to with a specific "target number" will help strengthen the concept. It is important that all the numerals used in the game are written down both to aid in comparison and to meet the standard.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on comparing numbers, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, learning about relevance of numbers and the value associated with numbers in the culture of the student/family may provide relevance for the student in learning this skill.	Social Studies: In Kindergarten, the New Mexico Social Studies Standards state students should "identify classroom population". Consider providing a connection for students to count the classroom population in ways that change (such as number of students present and number of students absent each day) and then compare those numbers. Language Arts: Literature can offer connections about measurement such as: *More or Less?* by Stuart J. Murphy and *Albert Keeps Score* by Daphne Skinner.

NEW MEXICO
Public Education Department

K.OA: OPERATIONS & ALGEBRAIC THINKING		
Cluster Statement: A: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. **Major Cluster** (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)		
Standard Text **K.OA.A.1: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.**	**Standard for Mathematical Practices** SMP 2: Students can reason abstractly and quantitatively by making connections between their representations and addition and subtraction. SMP 5: Students can use tools by using fingers, drawings, expressions, and/or equations to represent addition and subtraction.	**Students who demonstrate understanding can:** • Represent addition as putting together and adding to with objects, fingers, drawings, sounds, acting out situations, or verbal explanations. • Represent subtraction as taking apart and taking from with objects, fingers, drawings, sounds, acting out situations, or verbal explanations. • Identify the mathematical symbols used to show addition and subtraction. • Relate an expression or equation for addition or subtraction to a situation. **Depth Of Knowledge:** 2 **Bloom's Taxonomy:** Apply and Analyze
Standard Text **K.OA.A.2: Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.**	**Standard for Mathematical Practices** SMP 1: Students can make sense of problems and persevere in solving them by solving addition and subtraction word problems within 10. SMP 4: Students can model with mathematics by using objects and drawings to represent addition and subtraction problems.	**Students who demonstrate understanding can:** • Represent addition word problems with objects or drawings. • Represent subtraction word problems with objects or drawings. • Add within 10. • Subtract within 10. • Solve addition and subtraction word problems using objects and drawings.

		Depth of Knowledge: 2
		Bloom's Taxonomy: Apply and Analyze
Standard Text **K.OA.A.3: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).**	**Standard for Mathematical Practices** SMP 2: Students can reason abstractly and quantitatively by symbolically representing a pair of numbers less than 10 modeled concretely or pictorially with numerals and/or vice-versa. SMP 7: Students can look for and make use of structure by recognizing the commutative property (but not needing to know it by name) (e.g., because 5 = 2 + 3, 5 = 3 + 2 also).	**Students who demonstrate understanding can:** • Decompose (break apart) numbers to 10 using objects or drawings, increasing their range with time. • Decompose a number to 10 in more than one way (e.g., 5 = 2 + 3 and 5 = 4 + 1). • Identify an equation for a decomposed number.
		Depth of Knowledge: 2-3
		Bloom's Taxonomy: Analyze and Evaluate
Standard Text **K.OA.A.4: For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.**	**Standard for Mathematical Practices** SMP 2: Students can reason abstractly and quantitatively by symbolically representing combinations of 10 modeled concretely or pictorially with numerals and/or vice-versa. SMP 6: Students can attend to precision by using the equals sign accurately and appropriately.	**Students who demonstrate understanding can:** • Determine the number to add a given number 1-9 to make 10. • Represent combinations of 10 with a drawing or equation.
		Depth of Knowledge: 1-2

		Bloom's Taxonomy: Remember and Apply
Standard Text **K.OA.A.5: Fluently add and subtract within 5.**	**Standard for Mathematical Practices** SMP 6: Students can attend to precision by accurately, automatically and flexibly knowing their addition and subtraction facts within 5. SMP 7: Students can look for and make use of structure by using the patterns they found when composing and decomposing numbers to help them add and subtract.	**Students who demonstrate understanding can:** • Consistently add within 5 with accurate and efficient results • Consistently subtract within 5 with accurate and efficient results **Depth of Knowledge:** 1 **Bloom's Taxonomy:** Remember
<u>Previous Learning Connections</u> • Connect to students work with counting. (**K.CC**)	**<u>Current Learning Connections</u>** • Connect to decomposing larger numbers in the range of 11-19 to gain foundations for place value by composing and decomposing into "ten ones and some more." (e.g., 18 is ten ones and eight more). **(K.NBT.1)**	**<u>Future Learning Connections</u>** • Connect to represent and solving problems with addition and subtraction within 20, including a new type of problem situation (compare). **(1.OA.1)** • Connect to understanding and applying properties of operations and the relationship between addition and subtraction. **(1.OA.3)** • Connect to adding and subtracting within 20. **(1.OA.6)** • Connect to working with addition and subtraction equation. **(1.OA.7)**

Clarification Statement:

K.OA.A.1:

- **Math drawings** facilitate reflection and discussion because they remain after the problem is solved.
- The teacher can write **expressions** (e.g., 3 - 1) to **represent operations**, as well as writing **equation**s that represent the whole **situation** before the solution (e.g., 3 - 1 = ?) or after (e.g., 3 - 1 = 2). Expressions like 3 - 1 or 2 + 1 show the operation, and it is helpful for students to have experience just with the expression so they can conceptually chunk this part of an equation.
- Students may bring from home different ways to show **numbers** with their fingers and to raise (or lower) them when **counting**. The three major ways used around the world are starting with the thumb, the little finger, or the pointing finger (ending with the thumb in the latter two cases). Each way has advantages physically or mathematically, so students can use whatever is familiar to them. The teacher can use the range of methods present in the classroom, and these methods can be compared by students to expand their understanding of numbers.

K.OA.A.2:

- In **Put Together/Take Apart situations**, two **quantities** jointly **compose** a third quantity (the **total**), or a quantity can be **decomposed** into two quantities (the **addends**). This composition/decomposition may be **physical** or **conceptual**. These situations are acted out with objects initially and later children begin to move to conceptual mental actions of shifting between seeing the addends and seeing the total (e.g., seeing children or seeing boys and girls, or seeing red and green apples or all the apples).

- **Addition and Subtraction Situations** by Grade Level

sign (=, here with the meaning of "becomes," rather than the more general "equals").

Table 2: Addition and subtraction situations by grade level.

	Result Unknown	Change Unknown	Start Unknown
Add To	*A* bunnies sat on the grass. *B* more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	*A* bunnies were sitting on the grass. Some more bunnies hopped there. Then there were *C* bunnies. How many bunnies hopped over to the first *A* bunnies? $A + \square = C$	Some bunnies were sitting on the grass. *B* more bunnies hopped there. Then there were *C* bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	*C* apples were on the table. I ate *B* apples. How many apples are on the table now? $C - B = \square$	*C* apples were on the table. I ate some apples. Then there were *A* apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate *B* apples. Then there were *A* apples. How many apples were on the table before? $\square - B = A$
	Total Unknown	Both Addends Unknown[1]	Addend Unknown[2]
Put Together /Take Apart	*A* red apples and *B* green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has *C* flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	*C* apples are on the table. *A* are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	*"How many more?" version.* Lucy has *A* apples. Julie has *C* apples. How many more apples does Julie have than Lucy? *"How many fewer?" version.* Lucy has *A* apples. Julie has *C* apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$	*"More" version suggests operation.* Julie has *B* more apples than Lucy. Lucy has *A* apples. How many apples does Julie have? *"Fewer" version suggests wrong operation.* Lucy has *B* fewer apples than Julie. Lucy has *A* apples. How many apples does Julie have? $A + B = \square$	*"Fewer" version suggests operation.* Lucy has *B* fewer apples than Julie. Julie has *C* apples. How many apples does Lucy have? *"More" version suggests wrong operation.* Julie has *B* more apples than Lucy. Julie has *C* apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

Darker shading indicates the four Kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Unshaded (white) problems are the four difficult subtypes or variants that students should work with in Grade 1 but need not master until Grade 2. Adapted from CCSS, p. 88, which is based on *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

[1] This can be used to show all decompositions of a given number, especially important for numbers within 10. Equations with totals on the left help children understand that = does not always mean "makes" or "results in" but always means "is the same number as." Such problems are not a problem subtype with one unknown, as is the Addend Unknown subtype to the right. These problems are a productive variation with two unknowns that give experience with finding all of the decompositions of a number and reflecting on the patterns involved.

[2] Either addend can be unknown; both variations should be included.

K.OA.A.3:

- Put Together/Take Apart situations with **Both Addends Unknown** play an important role in Kindergarten because they allow students to explore various compositions that make each number.
- Addition and Subtraction Situations by Grade Level

sign (=, here with the meaning of "becomes," rather than the more general "equals").

Table 2: Addition and subtraction situations by grade level.

	Result Unknown	Change Unknown	Start Unknown
Add To	*A* bunnies sat on the grass. *B* more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	*A* bunnies were sitting on the grass. Some more bunnies hopped there. Then there were *C* bunnies. How many bunnies hopped over to the first *A* bunnies? $A + \square = C$	Some bunnies were sitting on the grass. *B* more bunnies hopped there. Then there were *C* bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	*C* apples were on the table. I ate *B* apples. How many apples are on the table now? $C - B = \square$	*C* apples were on the table. I ate some apples. Then there were *A* apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate *B* apples. Then there were *A* apples. How many apples were on the table before? $\square - B = A$
	Total Unknown	**Both Addends Unknown[1]**	**Addend Unknown[2]**
Put Together /Take Apart	*A* red apples and *B* green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has *C* flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	*C* apples are on the table. *A* are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	*"How many more?" version.* Lucy has *A* apples. Julie has *C* apples. How many more apples does Julie have than Lucy? *"How many fewer?" version.* Lucy has *A* apples. Julie has *C* apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$	*"More" version suggests operation.* Julie has *B* more apples than Lucy. Lucy has *A* apples. How many apples does Julie have? *"Fewer" version suggests wrong operation.* Lucy has *B* fewer apples than Julie. Lucy has *A* apples. How many apples does Julie have? $A + B = \square$	*"Fewer" version suggests operation.* Lucy has *B* fewer apples than Julie. Julie has *C* apples. How many apples does Lucy have? *"More" version suggests wrong operation.* Julie has *B* more apples than Lucy. Julie has *C* apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

Darker shading indicates the four Kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Unshaded (white) problems are the four difficult subtypes or variants that students should work with in Grade 1 but need not master until Grade 2. Adapted from CCSS, p. 88, which is based on *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

[1] This can be used to show all decompositions of a given number, especially important for numbers within 10. Equations with totals on the left help children understand that = does not always mean "makes" or "results in" but always means "is the same number as." Such problems are not a problem subtype with one unknown, as is the Addend Unknown subtype to the right. These problems are a productive variation with two unknowns that give experience with finding all of the decompositions of a number and reflecting on the patterns involved.

[2] Either addend can be unknown; both variations should be included.

K.OA.A.5: Experience with decompositions of numbers and with **Add To and Take From situations** enables students to begin to **fluently add** and **subtract** within 5.

Common Misconceptions

- Believing that certain words always indicate a particular operation

5

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted)
What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?
For example, some learners may benefit from targeted pre-teaching that introduces new representations when studying the understanding of addition and subtraction because new symbols and concepts, such as the plus, minus, and equal sign will be introduced. Students at this point will more than likely have not been exposed to the understanding of combining numbers.

Pre-teach (intensive)
What critical understandings will prepare students to access the mathematics for this cluster?
Indicator 9.3 (New Mexico Early Learning Guidelines, Essential Indicator): This standard provides a foundation for work with understanding addition as putting together and adding to, and subtraction as taking apart and taking away from because students need foundational skills relating to initial understanding of numbers and rote counting. Also, students learn that numbers are associated with words and numeral symbols. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction
Access:
Interest: How will the learning for students provide multiple options for recruiting student interest?
For example, learners engaging with understanding addition as putting together and adding to, and understanding subtraction as taking apart and taking from, benefit when learning experiences include ways to recruit interest such as providing choices in their learning (such as using manipulatives or visuals), because students may be more so intrigued and motivated to learn the new concept; for example students may use hands-on objects of their choice, such as bear shaped counters or buttons to work to combine new numbers to make one, or to take apart from.

Build:
Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?
For example, learners engaging with understanding addition and subtraction, benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports because the opportunity for students to work amongst each other and collaborate will allow for students to exchange ideas. This may be done during "center time" in small groups, which also allows the teacher to better understand individual learning needs by getting to work with each student in a smaller setting.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)
For example, learners engaging with understanding addition and subtraction benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form because students will be able to use multiple means to better their individual understanding such as using different visuals or manipulatives to represent equations. For example, rather than writing out an equation using numerals (i.e., 1+2=3, students can view or illustrate the

same equation using visual representations, such as drawing one circle, then two circles, and counting all circles to combine for a number of 3. This method ensures students are engaging in and understanding the overall concept of addition.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?
For example, learners engaging with understanding addition and subtraction benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because students will have the opportunity to engage with multiple means of learning such as using visuals, manipulatives, physical movement or oral word problems (ex: Dad gave Timmy three pencils, and Timmy lost one, how many pencils does Timmy now have?" to better develop understanding of the concept of addition and subtraction>.

Internalize:
Self-Regulation: How will the design of the learning strategically support students to effectively cope and engage with the environment?
For example, learners engaging with understanding addition and subtraction benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as <offering devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the mathematical practices for the purpose of monitoring and improving because students can refer back to these tools throughout their work to remain on task and assist with eliminating confusion. For example, an anchor chart with important symbols and vocabulary such as the plus sign, subtraction sign, and equal sign, can be posted so that students can refer back to which symbol corresponds with which problems when working in small groups or independently. This will assist in eliminating confusion with symbols.

Re-teach

Re-teach (targeted)
What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?
Examine assessments for evidence of lingering misconceptions (see common misconceptions). If students exhibit one more of these misconceptions, consider addressing the misconception by: For example, students may benefit from re-engaging with content during a unit on understanding addition and subtraction by examining tasks from a different perspective through a short mini-lesson because students may be able to learn addition or subtraction concepts in multiple ways such as learning with the use of visuals or manipulatives.
Re-teach (intensive)
What assessment data will help identify content needing to be revisited for intensive interventions?
Examine assessments for evidence of students still developing the underlying ideas, for example, some students may benefit from intensive extra time during and after a unit on being able to represent addition and subtraction with objects, fingers, mental images, drawings*, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations by offering opportunities to understand and explore different strategies through the use of concrete manipulative or fingers and to accommodate various learning styles because some students may need to practice the concept by using more than one modality of learning to then progress from concrete to pictorial representations of the models.

Extension Ideas
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?
To extend students learning on understanding addition as putting together and adding to, and understanding subtraction as taking apart and taking from, some learners may benefit from an

extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students, when studying the skill to represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations because students may benefit from learning to reframe a certain problem in a new way, such as moving away from using physical and visual cues to add and subtract and start using word based, or oral problems.	
Culturally and Linguistically Responsive Instruction: **Validate/Affirm:** How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? **Build/Bridge:** How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society? Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time." For example, when studying understanding addition as putting together and adding to, and understanding subtraction as taking apart and taking from eliciting and using student thinking is critical because providing opportunities for instructional conversations as students work through conceptualizing addition and subtraction helps build equity of participation and develops active listening skills.	
Standards Aligned Instructionally Embedded Formative Assessment Resources: Source: http://tasks.illustrativemathematics.org/content-standards/K/OA/A/3/tasks/165 The purpose of this task is for students to decompose a number as a sum of two other numbers in more than one way. The teacher should demonstrate how to "shake and spill" the counters as well as how to represent the sum using pictures or equations. The word "sum" is easily confused with "some," especially for young children; take care to use language that the students understand. However, make sure that they understand that they are representing, for example, 3+2, not just 3 and 2 separately. Language like, "How many red? How many yellow? How many altogether?" might be appropriate. Although this task uses 5 counters, it can be repeated using any number through 10 to address K.OA.3.	
Relevance to families and communities: During a unit focused on understanding addition as putting together and adding to, and understanding subtraction as taking apart and taking from, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example Students may bring from home different ways to show numbers with their fingers and to raise (or lower) them when counting. The three major ways used around the world are starting with the thumb, the little finger, or the pointing finger (ending with the thumb in the latter two cases).	**Cross-Curricular Connections:** Social Studies: In Kindergarten, the New Mexico Social Studies Standards state students should "describe trade (e.g., buying and selling, bartering, simple exchange).". Consider providing a connection for students to add and subtract related to buying and selling. Language Arts: Literature can offer connections about addition and subtraction such as: *Making Tens* by John Burstein and *Ten Little Caterpillars* by Bill Martin, Jr.

K.NBT: NUMBER & OPERATIONS IN BASE TEN

Cluster Statement: A: Work with numbers 11-19 to gain foundations for place value.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
K.NBT.A.1: Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	SMP 2: Students can reason abstractly and quantitatively by symbolically representing a teen quantity modeled concretely or pictorially with numerals and/or vice-versa. SMP 4: Students can model with mathematics by using a variety of groupable objects such as Unifix cubes, bundles of popsicle sticks, and tens frames to compose and decompose teen numbers. SMP 7: Students can look for and make use of structure by shifting from counting every object one at a time to recognizing a group of ten ones and some additional ones.	• Describe a representation as ten ones and some additional ones, such as describing a bundle of 10 popsicle sticks and 4 additional popsicle sticks as 10 ones and 4 ones. • Connect equivalent representations for the numbers 11 to 19, such as knowing that the number 14 means to count out and bundle 10 popsicle sticks and then to grab 4 additional popsicle sticks and that a pictorial representation of a full tens frame and a second tens frame with four additional dots can be represented symbolically using the numeral "14". • Write equations based on concrete and pictorial models that show how a teen number is composed of 10 ones and some additional ones, such as 14 = 10 + 4.
		Depth Of Knowledge: 2
		Bloom's Taxonomy: Apply and Analyze
<u>Previous Learning Connections</u> • Connect to counting by one to 10 and higher. • Connect to counting the number of items in a group of up to 10 objects and knowing that the last number tells how many.	**<u>Current Learning Connections</u>** • Connect to decomposing numbers to ten into pairs in more than one way. **(K.OA.3)**	**<u>Future Learning Connections</u>** • Connect to thinking of 10 ones as "a ten". **(1.NBT.2a)** • Connect to understanding the numbers 10, 20, 30, 40, 50,60,70, 80, and 90 refer to one, two, three, four, five, six, seven, eight, and nine tens and 0 ones. **(1.NBT.2c)**

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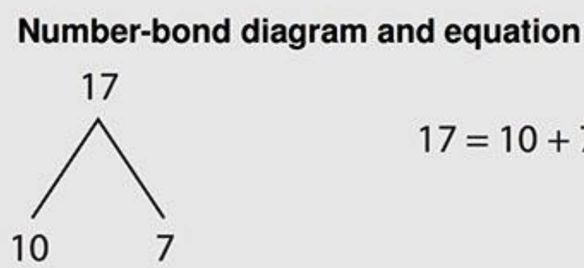
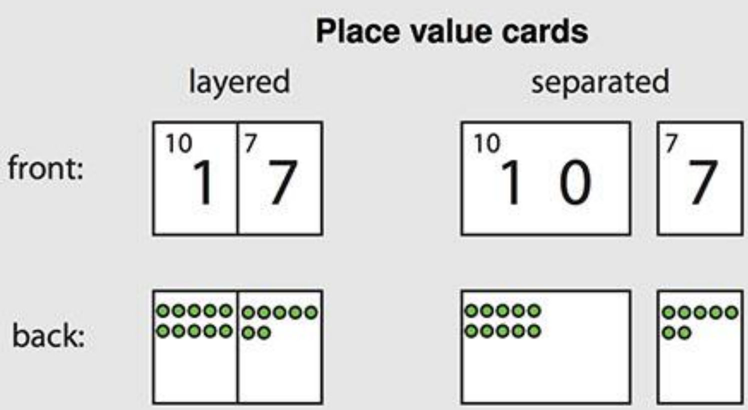
Clarification Statement:
K.NBT.A.1: Math drawings are simple drawings that make essential mathematical features and relationships salient while suppressing details that are not relevant ot the mathematical ideas.

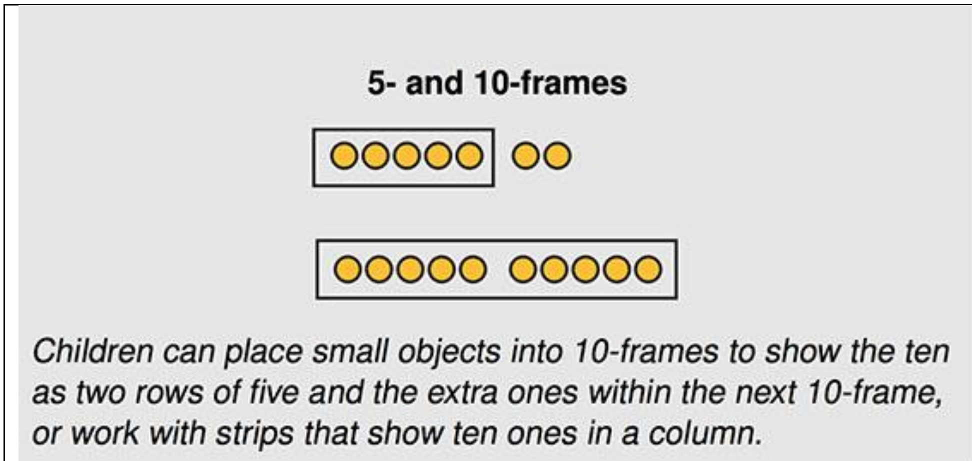
The **numerals** 11, 12, 13, ..., 19 need special attention for children to understand them. The first nine numerals 1, 2, 3, ..., 9, and 0 are essentially arbitrary marks. These same marks are used again to represent larger numbers. Children need to learn the differences in the ways these marks are used. For example, initially, a numeral such as 16 looks like "one, six," not "1 **ten** and 6 **ones**." Layered **place value** cards can help children see the 0 "hiding" under the **ones plac**e and that the 1 in the **tens place** really is 10 (ten ones).



Decompositions of teen numbers can be recorded with diagrams or equations.



Children can use layered place value cards to see the 10 "hiding" inside any teen number. Such decompositions can be connected to numbers represented with objects and math drawings. When any of the number arrangements is turned over, the one card is hidden under the tens card. Children can see this and that they need to move the ones dots above and on the right side of the tens card.



Common Misconceptions

- Being confused by the names for the teen numbers
- Connecting representations to number names
- Struggling with the concept of unitizing (seeing ten ones as one ten)

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted)

What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?

For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying work with numbers 11-19 to gain foundations for place value because this targeted instruction will support greater access to grade level instruction and assignments through the integration and early exposure to vocabulary words within the actual mini lesson for the upcoming place value lesson. Illustrations with the oral integration of the vocabulary and modeling will give these students a head start for the actual work with teen numbers and ten-frames.

Pre-teach (intensive)

What critical understandings will prepare students to access the mathematics for this cluster?

K.OA.A.3: This standard provides a foundation for work with work with numbers 11-19 to gain foundations for place value because students need to have the basic foundation of counting numbers and also comparing numbers which one is bigger or smaller. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access:

Interest: How will the learning for students provide multiple options for recruiting student interest?

For example, learners engaging with work with numbers 11-19 to gain foundations for place value benefit when learning experiences include ways to recruit interest such as creating an accepting and supportive classroom climate because students need to feel they can take risks in the classroom without being judged. This feeling is very important because as teachers we want all students to take risks and make mistakes because we can pinpoint where the connection is being broken and we can provide interventions or pre-teach to the students before the actual lesson (setting students up for success).

Build:

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?
For example, learners engaging with working with numbers 11-19 to gain foundations for place value benefit when learning experiences attend to students' attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities because kindergarteners need lots of clear goals and one thing teachers can do is to model. Modeling what to do for students is crucial, and students will have a clear understanding of what they should be doing with counting numbers and understanding the place value of numbers through the use of ten-frames. This should become a daily routine in order for students to really get to understand place value and how addition is putting together.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)
For example, learners engaging with work with numbers 11-19 to gain foundations for place value benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because students in kindergarten need repeated exposure to content vocabulary and the teacher has to be intentional when teacher lessons in regards to this cluster because students need to identify and know, explain what the words mean when learning about place value.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?
For example, learners engaging with work with numbers 11-19 to gain foundations for place value benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) because students at this age need to be exposed to a lot of modeling of concepts through manipulatives when building numeracy. Teachers can provide the whole group but can begin teaching in small groups in order to begin these activities orally and the teacher will have more control and be able to see and redirect students when working with teen numbers. For example, before students get to work on numbers 11-19, they must have a good foundation of numbers up to 10 and should know that ones can be shown on a ten-frame, but when the ten-frame is full of objects that represent the number 10. Students will be working from the concrete to the abstract and this progression needs to be developed and worked throughout kindergarten. Some students will catch on faster than others and here is where the teacher needs to have an extra eye to see who is ready and who needs more practice on previous building skills. Again, in kindergarten there needs to be lots of modeling and practicing taking place orally and manipulating objects. Having ten-frames available for each child and objects as counters would help with the practice of the teen numbers. This has to become a daily routine not just a one-time lesson.

Internalize
Comprehension: How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?
For example, learners engaging with work with numbers 11-19 to gain foundations for place value benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because students in kindergarten will need lots of practice with working with numbers 11-19 to gain the place value foundation. Students will have to have different ways of composing teen numbers and all this practice will lead students to understand that numbers are related and they can compose and decompose numbers. This is only possible with providing students continuous practice like a

daily review and providing more opportunities for students to work with teen numbers, there are some great place value games for kindergarteners to build fluency with teen numbers (dice and ten-frames) .

Re-teach

Re-teach (targeted)
What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?
Examine assessments for evidence of lingering misconceptions (see common misconceptions). If students exhibit one more of these misconceptions, consider addressing the misconception. For example, students may benefit from re-engaging with content during a unit on work with numbers 11-19 to gain foundations for place value by critiquing student approaches/solutions to make connections through a short mini-lesson because as students are able to give each other feedback this helps them with their critical thinking and examining whether they are correct or make changes to their work.
Re-teach (intensive)
What assessment data will help identify content needing to be revisited for intensive interventions?
Examine assessments for evidence of students still developing the underlying ideas, for example, some students may benefit from intensive extra time during and after a unit work with numbers 11-19 to gain foundations for place value by offering opportunities to understand and explore different strategies because students need to have multiple opportunities to count numbers, know numbers and be able to decompose numbers. Working with numbers at this level means knowing that addition is putting together therefore a number line should be accessible to students but most importantly the use of ten-frames and markers should be used to show composition of numbers.

Extension
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?
To extend students learning about working with numbers 11-19, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying work with numbers 11-19 to gain foundations for place value because students will benefit from having to relate multiple skills/strategies like counting, adding smaller numbers to get bigger ones, understanding the value of 10 and some ones that come after 10 (11, 12, 13, 14, ...). Having a ten-frame to work with and to show the work is essential specially when the task becomes more abstract as to utilizing symbols for addition.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying work with numbers 11-19 to gain foundations for place value the use of mathematical representations within the classroom is critical because five-year old children rely on visuals once a skill has been introduced in a concrete manner. Teacher's need to plan strategically, foreseeing that vocabulary in kindergarten will be a major issue with all the different experiences students come with. Not all students had the opportunity to attend pre-school, therefore teaching the foundations of place value should be in the progression of difficulty using the model of concrete to abstract representations and the teacher has to provide modeling using objects in order for students to gain understanding and benign to have a foundation with place value.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source:
http://s3.amazonaws.com/illustrativemathematics/attachments/000/009/257/original/public_task_1404.pdf?14623
The purpose of this task is to help students understand the base-ten structure of teen numbers. This task was designed specifically to support students in developing fluency with tens and teen numbers.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on working with numbers 11-19 to gain foundations for place value, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about number names and the place value across the languages in the classroom will help students get a better understanding of numbers in general making connections of number names in other languages. For kindergarten, there are lots of games that can be sent home for parents to play with their child and increase the child's understanding of numeracy and gain the language communication will increase while playing the games. Math nights would be another way of including parent and community involvement in which teachers will model how to play math games in different grade levels. Also, taking advantage of teachers' second languages to model games in the children's home language will be very powerful for parents whose second language is developing.	Social Studies: In Kindergarten, the New Mexico Social Studies Standards state students should "understand the concept of product". Consider providing a connection for students to see the idea of unitizing in products that are individual items packaged together and sold as a single unit, such as a box of crayons or a box of popsicles. Morning Meeting (or other morning routine): Consider providing a connection to tracking the number of days in school in a way that makes the number efficient to count, such as full groups of tens frames and an additional partially filled tens frame.

K.MD: MEASUREMENT & DATA

Cluster Statement: A: Describe and compare measurable attributes.

Additional Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
K.MD.A.1: Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.	SMP 3: Students can construct viable arguments by describing measurable attributes of objects. SMP 6: Students can attend to precision by using specific and accurate language when describing attributes of objects. SMP 7: Students can look for and make use of structure by learning to discriminate among different measurable attributes.	• Describe measurable attributes of objects, including length, weight, and size. • Recognize that a single object has more than one measurable attribute. **Depth Of Knowledge:** 2 **Bloom's Taxonomy:** Remember and Understand
Standard Text K.MD.A.2: Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*	**Standard for Mathematical Practices** SMP 3: Students can construct viable arguments by describing measurable attributes of objects. SMP 6: Students can attend to precision by using specific and accurate language when describing attributes of objects.	**Students who demonstrate understanding can:** • Compare two objects directly by placing them next to one another to determine which is longer or bigger. • Compare two objects directly by holding one in each hand to determine which is heavier. • Describe which of two objects has more or less of an attribute using vocabulary such as taller, longer, shorter, heavier and lighter **Depth Of Knowledge:** 2-3 **Bloom's Taxonomy:** Understand, Apply and Analyze

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• Connect to comparing length and other attributes of objects, using the terms bigger, longer, and taller • Connect to comparing two objects by placing one on top of another and indicating which objects takes up more space • Connect to arranging objects in order according to characteristics or attributes, such as height	• Connect to classifying objects into given categories; count the numbers of objects in each category and sort the categories by count **(K.MD.3)** • Connect to analyzing, describing, and comparing shapes to investigate measurable attributes **(K.G)**	• Connect to ordering three objects by length; comparing the lengths of two objects indirectly by using a third object **(1.MD.A.1)**

Clarification Statement:

- K.MD.A.1: Students often initially hold undifferentiated views of **measurable attributes**, saying that one object is "bigger" than another whether it is **longer**, or greater in **area**, or greater in **volume**, and so forth. For example, two students might both claim their block building is "the biggest." Conversations about how they are comparing—one building may be **taller** (greater in length) and another may have a larger base (greater in area)—help students learn to discriminate and name these measurable attributes. As they discuss these situations and compare objects using different attributes, they learn to distinguish, label, and describe several measurable attributes of a single object.
- K.MD.A.2: Kindergartners easily **directly compare** lengths in simple situations, such as comparing people's heights, because standing next to each other automatically aligns one endpoint. However, in other situations they may initially compare only one **endpoint** of objects to say which is longer. Discussing such situations (e.g., when a child claims that he is "tallest" because he is standing on a chair) can help students resolve and coordinate perceptual and conceptual information when it conflicts.

Common Misconceptions

● Believing that a larger object is automatically heavier.

● Not understanding conservation of length (when an object is moved away from a second object it is being compared to, the length does not change)

● Believing that an object is "bigger" or "smaller" based on a single attribute (e.g., a student stating that one book is bigger than another because it is longer when the other book may be wider and heavier)

● Not lining up the ends of objects being compared

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies
Pre-Teach

Pre-teach (targeted)

What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?

For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when studying describe and compare measurable attributes because students at this level need manipulatives and actual objects to explore and use for comparing measurable attributes of objects. Students need to hold and feel the objects. The exposure to objects being used in a lesson will benefit students when describing and comparing measurable attributes. These students will have a chance at feeling the mass of the objects, seeing which ones are shorter and longer, which objects are lighter and which ones are heavier. This will also influence their learning of vocabulary words since the teacher will be sort of front-loading for the actual lesson.

Pre-teach (intensive)

What critical understandings will prepare students to access the mathematics for this cluster? Indicator 11.3 of the "New Mexico Early Learning Guidelines, Essential Indicator:" Demonstrates emerging knowledge of measurement: This standard provides a foundation for work with describe and compare measurable attributes because the student demonstrates an understanding of non-standard units to measure and make comparisons. It is important for students to have the foundation for measurement so they can move on in their learning continuum to describe the comparison of objects and their measurable attributes. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access:

Interest: How will the learning for students provide multiple options for recruiting student interest? For example, learners engaging with Describe and Compare measurable attributes benefit when learning experiences include ways to recruit interest such as creating socially relevant tasks because students at this level will need lots of experiences with objects that they have at hand in the classroom (scissors, glue, pencils, crayons, etc.) and manipulate them in the tasks, also the use of different manipulatives for measuring would be fun and gain their interest because some measuring tools might be shorter than others and the final measurements will be different if using different manipulatives. These students would definitely benefit from having them come up and compare each other's height etc.

Build:

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

For example, learners engaging with Describe and Compare measurable attributes benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities because by providing students the opportunity to work in small groups students will be able to talk to each other and describe and compare objects and this will allow all students to work through the assignment and get a deeper understanding that objects length and weight are separate measurements. Therefore the teachers main objective is to give the groups objects of different lengths and weight but at the same time be intentional in labeling them as well so that when sharing out students will be able to see that length and weight are different just because one object is longer does not mean it is heavier (keep in mind to provide multiple solutions to the task.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

For example, learners engaging with Describe and Compare measurable attributes benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc.) because describing and comparing objects is challenging for students when it comes to length and weight. If we provide students with physical representations of objects that are longer and shorter but the weight of the larger object is less than the shorter object through these experiences, students will be able to understand and come to the conclusion in which length and weight are separate measurements. As teachers we have to be intentional in the delivery of the lesson, we have to provide an array of supports for our diverse classroom. Some students have different learning modalities and by providing visual, video and tangible experiences the students will be able to describe and compare objects.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?
For example, learners engaging with Describe and Compare measurable attributes benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks, 3D models, base-ten blocks) because < in Kindergarten students need to have lots of experiences with tangible not abstract material at this time. Lots of hands on activities as much as possible and being intentional in the planning of lessons. Having a variety of different length objects and a variety of different weight objects and giving groups a variety of objects to compare the length and weight is imperative for the kiddos to begin to understand measurement and weight are separate and then being able to articulate their thinking about the objects for the task.

Internalize:
Comprehension: How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?
For example, learners engaging with Describe and Compare measurable attributes benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because the understanding of measurable attributes is difficult to learn when you are a 5 year old. It would be necessary for a continuation of more tasks related to measurement, length vs. width, and capacity vs. weight. Students would need lots of practice with measurement in describing and comparing them. The task (activity) should have multiple solutions and before these there should be lots of teacher demonstrations.

Re-teach

Re-teach (targeted)
What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?
Examine assessments for evidence of lingering misconceptions (see common misconceptions). If students exhibit one more of these misconceptions, consider addressing the misconception by, for example, students may benefit from re-engaging with content during a unit on describe and compare measurable attributes by providing specific feedback to students on their work through a short mini-lesson because students at this level will have to work in partners or individually on a white board show their work, the teacher can quickly scan the room and see misunderstanding. Teachers can quickly have the child orally explain how they organize and compare the object's attributes. The child's thinking process might be exposed when explaining and the teacher will be able to help the child on the spot or in a small group. Targeted re-engagement can support students as they internalize the content while still maintaining the flow of the unit because they might be missing just a little piece, there must be a misunderstanding but if the rest of the students in the small group start questioning and the teacher providing assistance by providing sentence frames for the students to use this will feel less intrusive.
Re-teach (intensive)
What assessment data will help identify content needing to be revisited for intensive interventions?
Examine assessments for evidence of students still developing the underlying ideas for example, some students may benefit from intensive extra time during and after a unit Describe and Compare measurable attributes by confronting student misconceptions because five-year old children have misconceptions of measurement, they see things as smaller and bigger. This is a hard concept to learn for the little ones. Therefore, students will need lots of hands-on activities and experiences with measurement: weight, length and volume to begin to understand measuring.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM? To extend students learning about describing and comparing measurable attributes, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying to describe and compare measurable attributes because students will have the opportunity to explore other objects that they can measure specially the object permanence is hard for students to understand. Exploration time with liquids and different size flasks to pour in.
Culturally and Linguistically Responsive Instruction: **Validate/Affirm:** How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? **Build/Bridge:** How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society? When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)." For example, when studying describing and comparing measurable attributes the types of mathematical tasks are critical because students will have a different understanding on what attributes the teacher is referring to. The vocabulary will be a major key component and modeling will also be crucial for students at this level since they all come with different levels of mathematics. Language could also be a factor to consider, so lots of pictures and actual objects will enhance and will aid in students moving in the learning of the math continuum.
Standards Aligned Instructionally Embedded Formative Assessment Resources: Source: http://s3.amazonaws.com/illustrativemathematics/attachments/000/008/744/original/public_task_455.pdf?146239 The purpose of this task is for students to understand and practice using comparison language for height. Kindergarten students will often use the words "littler" and "bigger" when they compare themselves, but this could be weight rather than height. When students use comparison language specific to the attribute being measured (such as shorter/taller) rather than more generic comparison language (such as smaller/bigger) they are engaging in MP6, Attend to precision.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on describing and comparing measurable attributes, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, students can go on a shape hunt and draw 4 shapes they found at home and orally tell the parents which shapes they found and this interaction will lead to finding more shapes at home in their backyard etc. which in turn will help all students with the language aspect of the shapes but also with their own home language because this activity can be done in any language.	Science: In Kindergarten, the NGSS states students should "make observations (firsthand or from media) to collect data that can be used to make comparisons." Consider providing a connection for students to make direct comparisons based on length, width or size. Language Arts: Literature can offer connections about measurement such as: *The Giant Carrot* by Jan Peck and *Size* by Henry Pluckrose.

K.MD: MEASUREMENT & DATA

Cluster Statement: B: Classify objects and count the number of objects in each category.

Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
K.MD.B.3: Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.	SMP 2: Students can reason abstractly and quantitatively by analyzing shapes/objects to create groups and identifying the number in the group based on identified attributes. SMP 6: Students can attend to precision by using specific and accurate language when describing how objects are sorted into categories.	• Identify similarities and differences between objects (e.g., size, color, shape) • Use identified attributes to sort a collection of objects. • Count the number of objects in each collection. • Group the collections by the amount in each one.
		Depth of Knowledge: 1-2
		Bloom's Taxonomy: Remember, Apply and Analyze

Previous Learning Connections	**Current Learning Connections**	**Future Learning Connections**
• Connect to sorting objects onto a large graph according to one attribute, such as size, shape or color. • Connect to sorting, classifying, and ordering objects by size and other properties. • Connect to arranging objects in order according to characteristics or attributes, such as height.	• Connect to using understanding of counting and cardinality to accurately count to tell how many. Connect to recognizing whether the number in a group greater than, less than, or equal to the number in another group. **(K.CC.4, 5, 6)**	• Connect to organizing, representing, interpreting, and comparing data with up to three categories. **(1.MD.4)**

Clarification Statement:
- K.MD.B.3: Students in Kindergarten **classify** objects into **categories**, initially specified by the teacher and perhaps eventually elicited from students. For example, in a science context, the teacher might ask students in the class to sort pictures of various organisms into two piles: organisms with wings and those without wings. Students can then **count** the number of specimens in each pile. Students can use these category counts and their understanding of **cardinality** to say whether there are more specimens with wings or without wings.

Common Misconceptions

- Not yet counting each object in a set once, and only once with one touch per object (one-to-one correspondence)

- Not yet realizing that objects can be sorted into multiple categories

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted)

What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?

For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., number lines) when studying Classify objects and count the number of objects in each category because students might have familiarity with counting on a number line and will need the number line as a point of reference especially if the child is at the count all stage in which the child has to recount everything from one. The number line will help the child classify numbers and determine which number is smaller, or larger than the others for organizing groups of objects when counting.

Pre-teach (intensive)

What critical understandings will prepare students to access the mathematics for this cluster?

K.CC.C.6: This standard provides a foundation for work with classify objects and count the number of objects in each category because comparing numbers is a foundational skill which is critical to learning to classify groups of objects. Students must have previous experience with identifying groups of objects in groups as less than, equal to or greater than a number of objects in another group. Students must have the counting and matching skills. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access:

Interest: How will the learning for students provide multiple options for recruiting student interest?

For example, learners engaging with Classify objects and count the number of objects in each category benefit when learning experiences include ways to recruit interest such as creating accepting and supportive classroom climate because when students feel a sense of belonging and know that it's okay to make mistakes and there is this mutual understanding of making mistakes is a big part in learning all students will have an equal accessibility to learning.

Build:

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

For example, learners engaging with Classify objects and count the number of objects in each category benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as creating expectations for group work (e.g., rubrics, norms, etc.) because in kindergarten students need to be able to work in small groups and or partners therefore students need to be cleared on when working together how and what to specifically do. For example, students are working in pairs and are given 3 bags with different counts of objects in them they need to know to count the objects in each bag and order the bags from least to greatest. Next, they get another set and continue working. Students at this level need very explicit directions and lots of modeling on how to do the activities (process of actually completing the task).

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)
For example, learners engaging with Classify objects and count the number of objects in each category benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) because the five-year old children need lots of demonstrations and they also need to have that progression from the concrete to the abstract level, and don't forget to challenge and provide additional assistance to those that need the extra explanation. Students need to count sets of objects and order them from least to greatest. You can provide challenges by providing higher numbers to 20.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?
For example, learners engaging with Classify objects and count the number of objects in each category benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks, 3D models, base-ten blocks) because kindergarteners need to count objects they need to physically manipulate them, therefore they need lots of different manipulatives and sets of them to count. Students can also classify the objects if they are given 3 sets of buttons of different color (5 blue, 6 red and 7 yellow) students would have to sort them out according to the color then count the buttons and organize them according to the number of buttons from least to greatest. Students will need lots of exposure to counting sets of objects and sorting in order to become fluent and to understand counting, and classifying, etc. All these tasks (activities) have to be intentional, eventually these activities can be moved from teacher directed to centers for students to gain more experience with classifying and counting objects.

Internalize:
Self-Regulation: How will the design of the learning strategically support students to effectively cope and engage with the environment?
For example, learners engaging with Classify objects and count the number of objects in each category benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as increasing the length of on-task orientation in the face of distractions because 5 year old children have a short attention span as teachers we have to be intentional in planning and foreseeing the need to decrease or increase the length of on-task orientation in the face of distractions. We are working with kindergarteners and distractions are inevitable so flexibility is key in addressing distractions by increasing or decreasing certain tasks and activities.

Re-teach

Re-teach (targeted)
What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?
Examine assessments for evidence of lingering misconceptions (see common misconceptions). If students exhibit one more of these misconceptions, consider addressing the misconception, for example, students may benefit from re-engaging with content during a unit on classify objects and count the number of objects in each category by critiquing student approaches/solutions to make connections through a short mini-lesson because in kindergarten is all about kid-watch approach the teacher is everywhere and has ears and eyes everywhere, this means we have to be in constant

interaction with the class and actively surveying and checking for understanding. You know when they give you that look or their board is blank you can quickly be responsive to students who are struggling. It might mean the child needs access to a number line, number chart or just needs one more push to get it. You will know once you are roaming the room and seeing what the kids are demonstrating with manipulatives, paper and pencil etc.

Re-teach (intensive)
What assessment data will help identify content needing to be revisited for intensive interventions? Examine assessments for evidence of students still developing the underlying ideas, for example, some students may benefit from intensive extra time during and after a unit classify objects and count the number of objects in each category by addressing conceptual understanding because students must have a foundation in counting and also comparing numbers which one is bigger, smaller or if the numbers are the same. Students will need intensive reteach in the foundational skills in order to support students as they internalize the content. This support will mean maybe going back to counting and cardinality in order to build on the numeracy in order to move on to counting groups of objects and organizing them in order from least to greatest.

Extension
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?
To extend students learning about classifying objects and counting the number of objects in each category, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying classify objects and count the number of objects in each category because the students can count shapes and categorize them, the students can classify the different shapes by color size or sides. Students can make a presentation of their work; they can model their work with pictures

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying classifying objects and counting the number of objects in each category the types of mathematical tasks are critical because students at this level will have difficulty remembering number names and understanding new vocabulary for example identifying and classifying and categorizing. Therefore, the tasks associated with this cluster must be ongoing, it should be part of the kindergarten daily routine.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/K/MD/B/3/tasks/990
The purpose of this task is for students to sort the same set of objects according to different attributes and to practice counting to tell the number of objects in a set (K.MD.B). The teacher can extend the task by asking the students which group has the most and which group has the least and if any of the groups have the same number.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on classifying objects and counting the number of objects in each category, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, the home school connection activity can be send home and the objective will be for students to help their mother with pairing up socks. The students will have to categorize white socks, black socks etc., then they would have to pair socks according to size. Once students are done, they will have to put the socks away etc. This will also elicit conversation with parents and other siblings when helping with laundry.	Science: Consider providing opportunities to sort various organisms or animals into two piles, such as organisms with wings and those without wings. Students can then count the number of specimens in each pile. Finally, students can use these category counts and their understanding of cardinality to say whether there are more specimens with wings or without wings. Language Arts: Consider providing opportunities for students to sort words spelling pattern or word families. Follow up with questions related to category counts and count comparisons.

NEW MEXICO
Public Education Department

K.G: GEOMETRY

Cluster Statement: A: Identify and describe shapes.

Additional Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
K.G.A.1: Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to*.	SMP 4: Students can model with mathematics by describing real word objects using the names of shapes. SMP 6: Students can attend to precision by using position words to indicate the location of shapes.	• Describe the position of objects as above, below, beside, in front of, and next to. • Identify shapes in my environment regardless of their orientation or overall size (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres). **Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Remember and Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
K.G.A.2: Correctly name shapes regardless of their orientations or overall size.	SMP 6: Students can attend to precision by using clear language to analyze and name two- and three-dimensional shapes. SMP 7: Students can look for and make use of structure by recognizing that shapes with a particular set of attributes will have the same name.	• Identify shapes regardless of their orientation or overall size (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres) • Identify shapes correctly even when their size and orientation is unusual or different.

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		Depth of Knowledge: 1 **Bloom's Taxonomy:** Remember
Standard Text K.G.A.3: Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").	**Standard for Mathematical Practices** SMP 3: Students can construct viable arguments by explaining their decisions about shape names. SMP 6: Students can attend to precision by using clear language to analyze and identify two- and three-dimensional shapes. SMP 7: Students can look for and make use of structure by describing and defining shapes in terms of attributes (properties.	**Students who demonstrate understanding can:** • Define two-dimensional as being flat. • Define three-dimensional as being solid. • Identify two-dimensional shapes. • Identify three-dimensional shapes. **Depth of Knowledge:** 1 **Bloom's Taxonomy:** Remember
<u>Previous Learning Connections</u> • Connect to recognizing circle, triangle, and rectangle which includes squares. • Connect to recognizing that a shape remains the same shape when it changes position. • Connect to demonstrating and beginning to use the language of the relative position of objects in the environment and play situations, such as up, down, over, under, top, bottom, inside, outside, in front, behind, between, next to. • Connect to comparing length and other attributes of objects, using the terms bigger, longer, and taller. • Connect to arranging objects in order according to characteristics or attributes, such as height.	**<u>Current Learning Connections</u>** • Connect to sorting by attributes to investigate measurement and data. **(K.MD.1-3)**	**<u>Future Learning Connections</u>** • Connect to reason with shapes and their defining attributes. **(1.G.1)** • Connect to identification of additional shapes (trapezoids, half-circles, quarter-circles) and combining three-dimensional shapes to create larger shapes. **(1.G.2)**

Clarification Statement:
K.G.A.1: Students refine their informal language by learning mathematical concepts and vocabulary so as to increasingly describe their physical world from geometric perspectives, e.g., **shape**, **orientation**, **spatial relations** (MP4). They increase their knowledge of a variety of shapes, including **circles**, **triangles**, **squares**, **rectangles**, and special cases of other shapes such as **regular hexagons**, and **trapezoids** with **unequal base**s and **non-parallel sides** of **equal length**. Students also begin to name and describe **three-dimensional shapes** with mathematical vocabulary, such as "**sphere**," "**cube**," "**cylinder**," and "**cone**." Finally, in the domain of **spatial reasoning**, students discuss not only shape and orientation, but also the **relative positions** of objects, using terms such as "**above**," "**below**," "**next to**," "**behind**," "**in front of**," and "**beside**."

K.G.A.2: Students learn to name shapes such as circles, triangles, and squares, whose names occur in everyday language, and distinguish them from **nonexamples** of these **categories**, often based initially on visual prototypes.

KG.A.3: In the domain of shape, students learn to match t**wo-dimensional shapes** even when the shapes have different orientations. The need to explain their decisions about shape names or classifications prompts students to attend to and describe certain features of the shapes. That is, concept images and names they have learned for the shapes are the raw material from which they can abstract common features. They identify **faces** of three-dimensional shapes as two-dimensional **geometric figures** and explicitly identify shapes as two-dimensional ("**flat"** or lying in a **plane**) or three-dimensional ("**solid**").

Common Misconceptions
● Using informal names for shapes

● Incorrectly identifying figures that visually "resemble" shapes but don't possess all the needed attributes as that shape (such as an upside-down heart as a triangle)

● Not recognizing inverted or upside-down shapes as being that shape (especially upside-down triangles)

● Mixing up the terminology for two- and three-dimensional shapes (such as calling a cube a square)

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies
Pre-Teach

Pre-teach (targeted)
What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?
For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when studying shapes (describing/identifying) because this is the first time that they have seen the shape or the concept of a shape. A visual representation is the best option.

Pre-teach (intensive)
What critical understandings will prepare students to access the mathematics for this cluster?
K.G.A.1- Identify and Describe Shapes (Squares, Circles, Triangles, Rectangles, Hexagons, Cubes, Cones, Cylinders, And Spheres). This standard provides a foundation for work with shapes (identifying and describing because it is the starting point for shapes. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access:

Physical Action: How will the learning for students provide a variety of methods for navigation to support access?

For example, learners engaging with identifying and describing shapes will benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as physically responding or indicating selections because students can physically pick random shapes out of a bag and describe and identify the shape. Or the teacher can give a description of the shape and the student has to find the physical shape.

Build:

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?
For example, learners engaging with identifying and describing shapes will benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as creating expectations for group work (e.g., rubrics, norms, etc.) because this motivates students to regulate their learning and provide a working environment for small groups. The students in the small groups can help guide each other's learning.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)
For example, learners engaging with describing and identifying shapes will benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because students will need vocabulary words such as (sides, vertices, face, 3D/2D) in order to describe and identify the shapes.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?
For example, learners engaging with describing and identifying shapes will benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks, 3D models, base-ten blocks) because instead of using verbal representations students can build the shapes or identify them through physical representations.

Internalize:

Self-Regulation: How will the design of the learning strategically support students to effectively cope and engage with the environment?
For example, learners engaging with identifying and describing shapes will benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as using activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely because students can track their learning and see where they made the mistake. A chart can be used to identify what shapes still need to be learned.

Re-teach

Re-teach (targeted)
What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?
Examine assessments for evidence of lingering misconceptions (see common misconceptions). If students exhibit one more of these misconceptions, consider addressing the misconception by …For example, students may benefit from re-engaging with content during a unit on shapes (identifying and

describing) by providing specific feedback to students on their work through a short mini-lesson because seeing mistakes or good work will help the student analyze their thinking.

Re-teach (intensive)

What assessment data will help identify content needing to be revisited for intensive interventions? Examine assessments for evidence of students still developing the underlying ideas For example, some students may benefit from intensive extra time during and after a unit Identify And Describe Shapes by <helping students move from specific answers to generalizations for certain types of problems because <students will begin to understand about the attributes that makes a shape as a general, for example what makes a rectangle a rectangle, or a triangle a triangle. ...

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

To extend students learning about, for example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying shapes (describing and identifying) because they might need deeper thinking in order to better understand the topic.

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying Identifying and describing shapes, the types of mathematical tasks are critical because although rote practice increases fluency it usually does not engage children for long because they are based on students' recall or memorization of facts. When students are placed in situations in which recall speed determines success, they may infer that being "smart" in mathematics means getting the correct answer quickly instead of valuing the process of thinking.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

https://achievethecore.org/content/upload/Gr%20K.P.5%20Recognizing%20Squares_Final.pdf

Student should have time to explore and engage with shapes that are similar and different throughout the environment as part of developing an understanding of the attributes used in classifying objects.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on identifying and describing shapes, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning the different names for shapes in other languages could bring interest and awareness to student cultures and families.	Science: In Kindergarten, the NGSS state students should "develop a simple sketch, drawing, or physical model to illustrate how the shape of an object helps it function as needed to solve a given problem." Consider providing a connection for students to identify the shapes of the objects and whether they are two- or three-dimensional.

	Language Arts: Literature can offer connections about shapes such as: *Shape by Shape* by Suze MacDonald and *Perfect Square* by Michael Hall.

K.G: GEOMETRY

Cluster Statement: B: Analyze, compare, create, and compose shapes.

Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
K.G.B.4: Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).	SMP 3: Students can construct viable arguments by explaining how they sorted or compared a set of shapes. SMP 7: Students can look for and make use of structure by sorting objects into categories based on attributes and similarities and differences.	• Describe a shape by telling things like the number of sides, number of vertices (corners), and other special qualities. • Describe two-dimensional shapes (circles, triangles, rectangles, and squares) by the number of sides and corners. • Compare two-dimensional shapes and describe their similarities and differences. • Compare three-dimensional shapes and describe their similarities and differences. **Depth Of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand and Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
K.G.B.5: Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.	SMP 4: Students can model with mathematics by drawing and building shapes to represent real-world objects. SMP 7: Students can look for and make use of structure by perceiving a variety of shapes in their environment and identifying and describing these shapes.	• Build shapes from materials in their environment. • Draw shapes in their environment.

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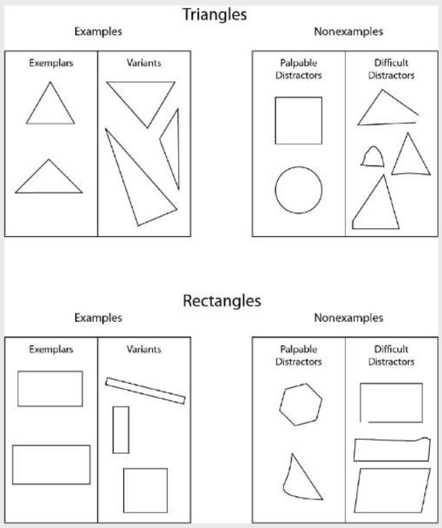
		Depth Of Knowledge: 2-3 **Bloom's Taxonomy:** Analyze and Create
Standard Text K.G.B.6: Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*	**Standard for Mathematical Practices** SMP 1: Students can make sense of problems and persevere in solving them by composing larger shapes and pictures from smaller shapes. SMP 6: Students can attend to precision by using geometry vocabular to describe the shapes and pictures that have composed.	**Students who demonstrate understanding can:** • Put shapes together to make new shapes (compose shapes). • Name the new shape that results from composing two simple shapes. • Decide which piece will fit into a space in a puzzle. **Depth Of Knowledge:** 2-3 **Bloom's Taxonomy:** Analyze and Create
Previous Learning Connections • Connect to building and describing two-dimensional shapes, such as making circles and triangles with blocks and play dough. • Connect to sorting and matching objects with the same shape and size, and lay an object of the same shape and size on top of another to show they are the same. • Connect to making a picture by combining shapes. • Connect to comparing length and other attributes of objects, using the terms bigger, longer, and taller. • Connect to arranging objects in order according to characteristics or attributes, such as height.	**Current Learning Connections** • Connect to building upon students' knowledge of identifying and describing shapes. **(K.G.1-3)** • Connect to students using their knowledge of sorting by attributes to investigate measurement and data. **(K.MD.1-3)**	**Future Learning Connections** • Connect to reason with shapes and their defining attributes. **(1.G.1)** • Connect to identification of additional shapes (trapezoids, half-circles, quarter-circles) and combining three-dimensional shapes to create larger shapes. **(1.G.2)**
Clarification Statement: K.G.B.4:		

K.G.4 Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).



Exemplars are the typical visual prototypes of the shape category.

Variants are other examples of the shape category.

Palpable distractors are nonexamples with little or no overall resemblance to the exemplars.

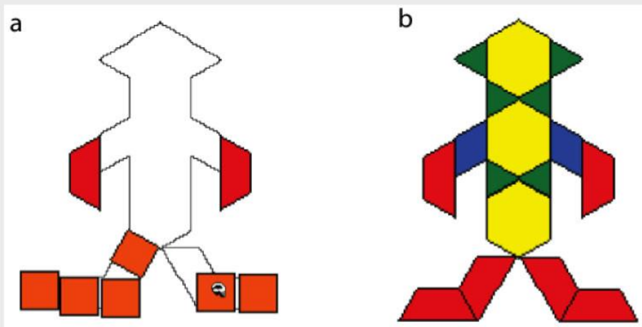
Difficult distractors are visually similar to examples but lack at least one defining attribute.

K.G.B.5: The need to explain their decisions about **shape names** or **classifications** prompts students to attend to and describe certain **features** of the **shapes**. That is, concept images and names they have learned for the shapes are the raw material from which they can abstract common features. This also supports their learning to represent shapes informally with drawings and by building them from components (e.g., manipulatives such as sticks). With repeated experiences such as these, students become more precise (MP6).

K.G.B.6:

K.G.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Combining shapes to build pictures



Students first use trial and error (part a) and gradually consider components (part b).

A second important area for kindergartners is the **composition** of **geometric figures**. Students not only build shapes from **components**, but also compose shapes to build pictures and designs. Initially lacking competence in composing geometric shapes, they gain abilities to combine shapes–first by trial and error and gradually by considering components–into pictures. At first, **side lengt**h is the only component considered. Later experience brings an intuitive appreciation of **angle size**. Students combine **two-dimensional shapes** and solve problems such as deciding which piece will fit into a space in a puzzle, intuitively using **geometric motions** (slides, flips, and turns, the informal names for translations, reflections, and rotations, respectively). They can construct their own outline puzzles and exchange them, solving each other's.

Common Misconceptions

● Not realizing that triangles can be inverted or rotated

● Not considering the properties of two-dimensional shapes (such as identifying all quadrilaterals as rectangles)

● Mixing up the terminology for two- and three-dimensional shapes (such as calling a cube a square)

● Not being able to see shapes from different perspectives and struggling to "move" shapes through slides, flips and turns

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies
Pre-Teach

Pre-teach (targeted)

What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?

For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time)when studying to analyze, compare, create, and compose shapes because the students were previously taught the names and some attributes of the different 2-dimensional as well as 3-dimensional shapes, for example if the window and the door are compared students can see that both are rectangles but one is bigger than the other.

Pre-teach (intensive)

What critical understandings will prepare students to access the mathematics for this cluster?

K.G.A., These standards provide a foundation for work with the analyzing, comparing, creating, and the composition of shapes because students need to be able to identify and describe the shapes in order for them to analyze, compare, create and or compose shares. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access:

Interest: How will the learning for students provide multiple options for recruiting student interest?
For example, learners engaging with analyzing, comparing, creating and composing shapes benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because students will get motivated by playing with blocks with different shapes, and students will be imitating what they play with at home.

Build:

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?
For example, learners engaging with analyzing, comparing, creating and composing shapes benefit when learning experiences attend to students' attention and affect to support sustained effort and concentration such as providing feedback that is frequent, timely, and specific because some students might lose concentration and they will move to something different. At the same time, it will give you information of who is moving with ease, having difficulty or unable to perform the task.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)
For example, learners engaging with analyzing, comparing, creating and composing shapes benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because students will concentrate only on the learning task and not occupy their minds with vocabulary, and or necessary symbols.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?
For example, learners engaging with analyzing, comparing, creating and composing shapes benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) because many students are used to playing with blocks either at home or at school, making the manipulation easier when students combine learning with playing and experimenting.

Internalize:

Comprehension: How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?
For example, learners engaging with analyzing, comparing, creating and composing shapes benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as pre-teaching critical prerequisite concepts through demonstration or representations because students will see the direction that you want to take them and also some structure to a task is necessary to promote learning.

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Re-teach

Re-teach (targeted)
What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?
Examine assessments for evidence of lingering misconceptions (see common misconceptions). If students exhibit one more of these misconceptions, consider addressing the misconception by: For example, students may benefit from re-engaging with content during a unit on analyzing, comparing, creating, and composing shapes by clarifying mathematical ideas and/or concepts through a short mini-lesson because if students are struggling with the names, and their attributes they will also struggle when they are required to analyze and compare shapes

Re-teach (intensive)
What assessment data will help identify content needing to be revisited for intensive interventions?
Examine assessments for evidence of students still developing the underlying ideas For example, some students may benefit from intensive extra time during and after a unit analyzing, comparing, creating, and composing shapes by offering opportunities to understand and explore different strategies because by using different strategies the students might get to the conceptual understanding for example by overlapping some shapes and finding how they are different the students might start to see the attributes of the different shapes.

Extension Ideas
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?
To extend students learning about analyzing, comparing, creating, and composing shapes, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying analyze, compare, create, and compose shapes because by asking questions like what would you need to do to this square to make it a rectangle, and or can you decompose a shape into different shapes?

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying shapes (analyzing, comparing, creating, and composing shapes) the types of mathematical tasks are critical because tasks should have many entry points, and should be opened ended so our students can and will make sense of the problems according to the background knowledge they bring. Tasks should be wide, then slowly tasks should start getting narrow into what the goal of the cluster is. In other words, refining the learning so our students can meet the cluster.

Standards Aligned Instructionally Embedded Formative Assessment Resources:	
Source: http://s3.amazonaws.com/illustrativemathematics/attachments/000/008/783/original/public_task_515.pdf?1462392632 Explicit connection: Student should have multiple opportunities to explore and engage with shapes in the context of developing analysis and comparative skills, modeling and making connections to the real world and using shapes to compose larger shapes.	
Relevance to families and communities: During a unit focused on analyzing, comparing, creating, and composing shapes, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, by asking the students or send a letter home to ask parents to talk about students on how shapes (analyzing, comparing, creating, and composing) are utilized in their work or around the house/family life.	**Cross-Curricular Connections:** Social Studies: Students should "recognize and name symbols and activities of the United States, New Mexico, and tribes." Consider providing a connection for students to model these symbols and pictures related to the activities in terms of shapes. Language Arts: Literature can offer connections about composing and decomposing shapes such as: *Changes, Changes* by Pat Hutchins.

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Section 3: Resources, References, and Glossary

Resources

Evidence-Based Resources	English Learner Resources	MLSS Resources	Mathematics Standard Resources
What Works Clearinghouse Best Evidence Encyclopedia Evidence for Every Students Succeeds Act Evidence in Education Lab	World-Class Instructional Design and Assessment (WIDA) Standards USCALE Language Routines for Mathematics English Language Development Standards Spanish Language Development Standards	NM Multi-Layered System of Supports (MLSS) Universal Design for Learning Guidelines Achieve the Core: Instructional Routines for Mathematics Project Zero Thinking Routines	Focus by Grade Level and Widely Applicable Prerequisites High school Coherence Map College-and Career Ready Math Shifts Fostering Math Practices: Routines for the Mathematical Practices

Planning Guidance for Multi-Layered Systems of Support: Core Instruction[9]

Core Instructional Planning must reflect and leverage scientific insights into how humans learn in order to ensure all students are ready for success, thus the following guidance for optimizing teaching and learning is grounded in the **Universal Design Learning (UDL) Framework**

Key design questions, planning actions, and potential strategies are provided below, with respect to guidance for minimizing barriers to learning and optimizing (1) universal ACCESS to learning experiences, (2) opportunities for students to BUILD their understanding of the **Learning Goal**, and (3) INTERNALIZATION of the Learning Goal.

Optimizing Universal ACCESS to Learning Experiences	
ENGAGEMENT ? How will you provide multiple options for recruiting interest?	**Recruiting Student Interest**: ? What do you anticipate in the range of student interest for this lesson? ▯ Plan for options for recruiting student interest: ❒ provide choice (e.g. sequence or timing of task completion) ❒ set personal academic goals ❒ provide contextualized examples connected to their lives ❒ support culturally relevant connections (i.e home culture) ❒ create socially relevant tasks ❒ provide novel & relevant problems to make sense of complex ideas in creative ways

[9] Adapted from: CAST (2018). *Universal Design for Learning Guidelines version 2.2*. Retrieved from http://udlguidelines.cast.org

	❒ provide time for self-reflection about content & activities ❒ create accepting and supportive classroom climate ❒ utilize **instructional routines** to involve all students
REPRESENTATION ? How will you reduce barriers to perceiving the information presented in this lesson?	**Perception**: ? What do you anticipate about the range in how students will perceive information presented in this lesson? ▯ Plan for different modalities and formats to reduce barriers to learning: ❒ display information in a flexible format to vary perceptual features ❒ offer alternatives for auditory information ❒ offer alternatives for visual information
ACTION & EXPRESSION ? How will the learning for students provide a variety of methods for navigation to support access?	**Physical Action**: ? What do you anticipate about the range in how students will physically navigate and respond to the learning experience? ▯ Plan a variety of methods for response and navigation of learning experiences by offering alternatives to: ❒ requirements for rate, timing, speed, and range of motor action with instructional materials, manipulatives, and technologies ❒ physically indicating selections ❒ interacting with materials by hand, voice, keyboard, etc.

Opportunities for Students to BUILD their Understanding

ENGAGEMENT ? How will the learning for students provide options for sustaining effort and persistence?	**Sustaining Effort & Persistence**: ? What do you anticipate about the range in student effort? ▯ Plan multiple methods for attending to student attention and affect by: ❒ prompting learners to explicitly formulate or restate learning goals ❒ displaying the learning goals in multiple ways ❒ using prompts or scaffolds for visualizing desired outcomes ❒ engaging assessment discussions of what constitutes excellence ❒ generating relevant examples with students that connect to their cultural background and interests ❒ providing alternatives in the math representations and scaffolds ❒ creating cooperative groups with clear goals, roles, responsibilities ❒ providing prompts to guide when and how to ask for help ❒ supporting opportunities for peer interactions and supports (e.g. peer tutors) ❒ constructing communities of learners engaged in common interests ❒ creating expectations for group work (e.g., rubrics, norms, etc.) ❒ providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge ❒ providing feedback that: ❒ emphasizes effort, improvement, and achieving a standard rather than on relative performance ❒ is frequent, timely, and specific ❒ is informative rather than comparative or competitive

	❒ models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success
REPRESENTATION ⍰ How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners?	**Language & Symbols:** ⍰ What do you anticipate about the range of student background experience and vocabulary? ▯ Plan multiple methods for attending to linguistic and nonlinguistic representations of mathematics to ensure universal clarity by: ❒ pre-teaching vocabulary and symbols in ways that promote connection to the learners' experience and prior knowledge ❒ graphic symbols with alternative text descriptions ❒ highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to structure ❒ embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) ❒ embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) ❒ highlighting structural relations or make them more explicit ❒ making connections to previously learned structures ❒ making relationships between elements explicit (e.g., highlighting the transition words in an argument, links between ideas, etc.) ❒ allowing the use of text-to-speech and automatic voicing with digital mathematical notation (math ml) ❒ allowing flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) ❒ clarification of notation through lists of key terms ❒ making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf ❒ linking key vocabulary words to definitions and pronunciations in both dominant and heritage languages ❒ defining domain-specific vocabulary (e.g., "map key" in social studies) using both domain-specific and common terms ❒ electronic translation tools or links to multilingual web glossaries ❒ embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc) ❒ presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) ❒ making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams
ACTION & EXPRESSION ⍰ How will the learning provide multiple	**Expression & Communication:** ⍰ What do you anticipate about the range in how students will express their thinking in the learning environment? ▯ Plan multiple methods for attending to the various ways in which students can express knowledge, ideas, and concepts by providing:

modalities for students to easily express knowledge, ideas, and concepts in the learning environment?	❒ options to compose in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video ❒ use of social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) ❒ flexibility in using a variety of problem solving strategies ❒ spell or grammar checkers, word prediction software ❒ text-to-speech software, human dictation, recording ❒ calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper ❒ sentence starters or sentence strips ❒ concept mapping tools ❒ Computer-Aided-Design (CAD) or mathematical notation software ❒ virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) ❒ multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches) ❒ multiple examples of novel solutions to authentic problems ❒ different approaches to motivate, guide, feedback or inform students of progress towards fluency ❒ scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) ❒ differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners)

Optimizing INTERNALIZATION of the Learning Goal	
ENGAGEMENT [?] How will the design of the learning strategically support students to effectively cope and engage with the environment?	**Self-Regulation**: [?] What do you anticipate about barriers to student engagement? ▯ Plan to address barriers to engagement by promoting healthy responses and interactions, and ownership of learning goals: ❒ metacognitive approaches to frustration when doing mathematics ❒ increase length of on-task orientation through distractions ❒ frequent self-reflection and self-reinforcements ❒ address subject specific phobias and judgments of “natural” aptitude (e.g., “how can I improve on the areas I am struggling in?” rather than “I am not good at math”) ❒ offer devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the math practices for the purpose of monitoring and improving ❒ use activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely
REPRESENTATION [?] How will the learning support transforming accessible information into usable knowledge	**Comprehension**: [?] What do you anticipate about barriers to student comprehension? ▯ Plan to address barriers to comprehension by intentionally building connections to prior understandings and experiences, relating meaningful information to learning goals,

that is accessible for future learning and decision-making?	providing a process for meaning making of new learning, and applying learning to new contexts: ❒ incorporate explicit opportunities for review and practice ❒ note-taking templates, graphic organizers, concept maps ❒ scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) ❒ explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) ❒ opportunities over time to revisit key ideas and connections ❒ make explicit cross-curricular connections ❒ highlight key elements in tasks, graphics, diagrams, formulas ❒ outlines, graphic organizers, unit organizer routines, concept organizer routines, and concept mastery routines to emphasize key ideas and relationships ❒ multiple examples & non-examples ❒ cues and prompts to draw attention to critical features ❒ highlight previously learned skills that can be used to solve unfamiliar problems ❒ options for organizing and possible approaches (tables and representations for processing mathematical operations) ❒ interactive representations that guide exploration and new understandings ❒ introduce graduated scaffolds that support information processing strategies ❒ tasks with multiple entry points and optional pathways ❒ "Chunk" information into smaller elements ❒ remove unnecessary distractions unless essential to learning goal ❒ anchor instruction by linking to and activating relevant prior knowledge (e.g., using visual imagery, concept anchoring, or concept mastery routines) ❒ pre-teach critical prerequisite concepts via demonstration or representations ❒ embed new ideas in familiar ideas and contexts (e.g., use of analogy, metaphor, drama, music, film, etc.) ❒ advanced organizers (e.g., KWL methods, concept maps) ❒ bridge concepts with relevant analogies and metaphors
ACCESS ACTION & EXPRESSION ? How will the learning for students support the development of executive functions to allow them to take advantage of their environment?	**Executive Functions**: ? What do you anticipate about barriers to students demonstrating what they know? ▯ Plan to address barriers to demonstrating understanding by providing opportunities for students to set goals, formulate plans, use tools and processes to support organization and memory, and analyze their growth in learning and how to build from it: ❒ prompts and scaffolds to estimate effort, resources, difficulty ❒ models and examples of process and product of goal-setting ❒ guides and checklists for scaffolding goal-setting ❒ post goals, objectives, and schedules in an obvious place ❒ embed prompts to "show and explain your work" ❒ checklists and project plan templates for understanding the problem, prioritization, sequences, and schedules of steps ❒ embed coaches/mentors to demonstrate think-alouds of process ❒ guides to break long-term goals into short-term objectives ❒ graphic organizers/templates for organizing information & data ❒ embed prompts for categorizing and systematizing ❒ checklists and guides for note-taking ❒ asking questions to guide self-monitoring and reflection ❒ showing representations of progress (e.g., before and after photos, graphs/charts showing progress, process portfolios)

	❒ prompt learners to identify type of feedback or advice they seek ❒ templates to guide self-reflection on quality & completeness ❒ differentiated models of self-assessment strategies (e.g., role-playing, video reviews, peer feedback) ❒ assessment checklists, scoring rubrics, and multiple examples of annotated student work/performance examples

Planning Guidance for Culturally and Linguistically Responsive Instruction[10]

In order to ensure our students from marginalized cultures and languages view themselves as confident and competent learners and doers of mathematics within and outside of the classroom, educators must intentionally plan ways to counteract the negative or missing images and representations that exist in our curricular resources. The guiding questions below support the design of lessons that validate, affirm, build, and bridge home and school culture for learners of mathematics:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

In addition, Aguirre and her colleagues[11] define **mathematical identities** as the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives. Many students see themselves as "not good at math" and approach math with fear and lack of confidence. Their identity, developed through earlier years of schooling, has the potential to affect their school and career choices.

Five Equity-Based Mathematics Teaching Practices[12]

Go deep with mathematics. Develop students' conceptual understanding, procedural fluency, and problem solving and reasoning.

Leverage multiple mathematical competencies. Use students' different mathematical strengths as a resource for learning.

Affirm mathematics learners' identities. Promote student participation and value different ways of contributing.

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[10] This resource relied heavily on the work of: Hollie, S. (2011). Culturally and linguistically responsive teaching and learning. Teacher Created Materials. (see also, https://www.culturallyresponsive.org/vabb)

[11] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices. Reston, VA: National Council of Teachers of Mathematics (p. 14).

[12] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc. (p.6). (adapted from Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013) (p. 43).

Challenge spaces of marginality. Embrace student competencies, value multiple mathematical contributions, and position students as sources of expertise.

Draw on multiple resources of knowledge (mathematics, language, culture, family). Tap students' knowledge and experiences as resources for mathematics learning.

The following lesson design strategies support Culturally and Linguistically Responsive Instruction, specific examples for each cluster of standards can be found in part 2 of the document. These were adapted from the Promoting Equity section of the Taking Action series published by NCTM.[13]

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.

Mathematical Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher.

Modifying Mathematical Tasks: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)."

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics.

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages.

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we

[13] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians.

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others."

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time."

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students.

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Glossary[14]

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (- 3/4) = (- 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[15]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. See: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances
from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten.
For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

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[14] Glossary and tables taken from: Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/

[15] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[16] See also: median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.
Integer. A number expressible in the form a or -a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line.
Also known as a dot plot.[17]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[18] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 Ã· 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 Ã— 4/3 = 4/3 Ã— 3/4 = 1.

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[16] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

[17] Adapted from Wisconsin Department of Public Instruction, op. cit.

[18] To be more precise, this defines the arithmetic mean.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin,
selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also*: uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form *a/b* or – *a/b* for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also*: terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[19]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar
model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

[19] Adapted from Wisconsin Department of Public Instruction, op. cit.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also*: median, first quartile, interquartile range.

Table 1: Common addition and subtraction.[1]

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 =5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now?5-2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?? -2 = 3
	TOTAL UNKNOWN	ADDEND UNKNOWN	BOTH ADDENDS UNKNOWN[2]
PUT TOGETHER / TAKE APART[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5-3 = ?	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? 5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1, 5 = 2 + 3, 5 = 3 + 2
COMPARE	DIFFERENCE UNKNOWN	BIGGER UNKNOWN	SMALLER UNKNOWN
	("How many more?" version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2: Common multiplication and division situations.[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	3 x ? = 18, and 18 ÷ 3 = ?	? x 6 = 18, and 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	a x b = ?	a x ? = p and p ÷ a = ?	? x b = p, and p ÷ b = ?

[1]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3: The properties of operations.

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	(a +b) + c = a + (b+c)
Commutative property of addition	a + b = b + a

Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists -*a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) x c = a x (b x c)
Commutative property of multiplication	a x b = b x a
Multiplicative identity property 1	a x 1 = 1 x a = a
Existence of multiplicative inverses	For every *a* ≠ 0 there exists 1/*a* so that *a* x 1/*a* = 1/*a* x *a* = 1
Distributive property of multiplication over additions	a x (b + c) = a x b + a x c

Table 4: The properties of equality.

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a.
Symmetric property of equality	If a = b, then b = a.
Transitive property of equality	If a = b and b = c , then a = c.
Addition property of equality	If a = b, then a +c = b + c.
Subtraction property of equality	If a = b then a – c = b – c.
Multiplication property of equality	If a = b, then a x c = b x c.
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c.
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality.

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b.
If a > b and b > c then a > c.
If a > b, b < a.
If a > b, then -a <-b.
If a > b, then a ± c > b ± c.
If a > b and c > 0, then a x c > b x c.
If a > b and c < 0, then a x c < b x c.
If a > b and c > 0, then a ÷ c > b ÷ c.
If a > b and c < 0, then a ÷ c < b ÷ c.