New Mexico Public Education Department

New Mexico Instructional Scope

Algebra 1 Building Functions Guide

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards of Mathematical Practice
    - Common Misconceptions
    - Identification of Priority Standards, as identified by NMPED.
    - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

New Mexico Instructional Scope

Algebra 1 Building Functions Guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	Conceptual Understanding	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	Application	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	Procedural Skill and Fluency	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Build a function that models a relationship between two quantities.
  - HSF.BF.A.1
  - HSF.BF.A.2
- Build new functions from existing functions.
  - HSF.BF.B.3
  - HSF.BF.B.4

Grade	CCSS Domain	CCSS Cluster
A1	Building Functions	Build a function that models a relationship between two quantities

Cluster Standard: HSF.BF.A.1

Standard	Standards for Mathematical Practice
Write a function that describes a relationship between two quantities. • HSF.BF.A.1.A Determine an explicit expression, a recursive process, or steps for calculation from a context. • HSF.BF.A.1.B Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	• **SMP 4:** Model with mathematics. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should write functions for given relationships between quantities. Students can use functions to model real-life situations and make predictions. Students should be able to use functions to describe relationships between two quantities, usually x and $f(x)$, where $f(x)$ is some output value that depends on the input value x. Within a context, students should be able to express a given relationship as a function.	• Write an explicit expression to model linear, exponential and quadratic relationships. • Determine and explain which arithmetic operation(s) should be performed to build the desired combined function given a context or scenario. • Combine two functions by adding, subtracting, multiplying, or dividing, and evaluate the domain of the combined functions related to the context of the problem.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	Building Functions	Build a function that models a relationship between two quantities

Cluster Standard: HSF.BF.A.2

Standard	Standards for Mathematical Practice
Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	• **SMP 4:** Model with mathematics. • **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students should write **formulas** for **arithmetic** and **geometric sequences** with both **explicit** and **recursive formulas**. They should be able to relate these to a context they represent and be able to **transition** from one form to the other. Students should know that they can write explicit functions recursively, too. For instance, with every year that passes, your age increases by 1. It can be interpreted as constantly adding 1 to the age you were before. In other words, write your age as $f(x) = f(x - 1) + 1$ starting with $f(1) = 1$. Students should know how to recognize that arithmetic functions that take the explicit form $A(n) = A(1) + (n - 1)d$ have the recursive form $A(n) = A(n - 1) + d$ and geometric functions with the form $G(n) = G(1) \times r^{n-1}$ have the recursive form $G(n) = G_{n-1} \times r$.	• Identify arithmetic and geometric patterns in given sequences. • Generate arithmetic or geometric sequences from recursive and explicit formulas. • Justify the translation of given and constructed arithmetic and geometric sequences between recursive and explicit formulas.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- Students may believe that arithmetic and geometric sequences are the same and may need more experience with both types to be able to recognize the difference and develop formulas to describe the patterns.
- Some students may interchange the input and the output values, which can lead to confusion about domain and range, or determining if a relation is a function. This can also affect student understanding and application of inverse functions.
- Students may want to try to use a linear function, specifically the slope-intercept form, for every situation.
- Students may tend to focus on the symbolic form of a function and may need additional support in working with other forms.

Grade	*CCSS Domain*	*CCSS Cluster*
A1	**Building Functions**	**Build new functions from existing functions**

Cluster Standard: HSF.BF.B.3

Standard	**Standards for Mathematical Practice**
Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	• **SMP 5:** Use appropriate tools strategically. • **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students should describe the effect of stretches, shrinkages, vertical and horizontal transformations of linear, quadratic and exponential functions. They should be able to find the value of the transformation when given a graph and be able to explain the effects of transformations using technology. Students should know that adding a constant *k* to a function will change the graph of the function depending not only on the value of the constant, but on where it is inserted as well. If $y = f(x)$ is changed to $y = f(x) + k$, the curve will shift vertically (up for $k > 0$, down if $k < 0$). Adding *k* to *x* such that $y = f(x + k)$ will shift the curve horizontally (left for $k > 0$, right for $k < 0$). Multiplying $f(x)$ by a constant *k* stretches ($k > 1$) or squishes ($0 < k < 1$) the graph vertically. If $k < 0$, the graph	• Describe the effect of a single transformation on graphs of functions. • Find the value of k using the graphs of a parent function, f(x), and the transformed function: f(x)+k, kf(x), f(kx), or f(x+k)

is also flipped over the *x*-axis. Multiplying *x* by *k* stretches (*k* > 0) or squishes (k < 0) the graph horizontally.	
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	*CCSS Domain*	*CCSS Cluster*
A1	**Building Functions**	**Build new functions from existing functions**

Cluster Standard: HSF.BF.A.4

Standard	**Standards for Mathematical Practice**
HSF.BF.B.4 Find inverse functions. HSF.BF.B.4.A • Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.	• **SMP 6:** Attend to precision. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should be able to find the inverse of simple linear functions and recognize that other functions may not have an inverse unless there are restrictions placed on the domain. If $f(x) = y$ is a function, the inverse function can be found by switching the place of *x* and *y* ($f(y) = x$), and then solving for *y* so that $f^{-1}(x) = y$. For instance, if the function $f(x)$ is $y = 2x^3$, then the inverse function $f^{-1}(x)$ consists of switching the places of *x* and *y* ($x = 2y^3$) and then solving for *y*.	• Describe how to determine the input of a function when the output is known, using the idea of going backwards. • Determine restrictions on the domain of a function that are required for an inverse of that function to exist.
DOK	**Blooms**

1-2	Understand, Apply, Analyze

Common Misconceptions

- Students often have difficulty determining the direction of the horizontal or vertical shifts, as well as understanding the difference between shrink and stretch
- Students often confuse the notation for the inverse and negative numbers.
- Students can easily get confused from traditional algorithms, such as "switch *x* and *y*", as this can become problematic when *x* and *y* are representing real-world quantities.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Building Functions**	Strand: **Build a function that models a relationship between two quantities**

Suggested Student Discourse Questions

- What are the differences between a recursive and explicit formula? What are the similarities? Explain using mathematical vocabulary.
- Explain your thinking in writing the recursive formula for this ________ (arithmetic,
- Explain your algebraic process in converting the recursive formula to an explicit one.
- Imagine you are __________ (laying tile on a floor, creating a pattern for a beaded necklace). Would a recursive formula or an explicit formula better help you know how

geometric) sequence.	many (tiles of a certain color, beads of a certain color) to purchase for a small room / necklace? Which formula would better help you determine how many to purchase if you wanted to tile a very large room / make a very long necklace?

ASSESSMENT GUIDE

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

Grade	*CCSS Domain*	*CCSS Strand*
A1	**Building Functions**	**Build a function that models a relationship between two quantities**
	Sample Task #1 (Constructed Response)	
	The population of a town is currently 50,000, and the population is estimated to increase each year by 3% from the previous year. Write an equation that can be used to estimate the number of years, t, it will take for the population of the town to reach 60,000. SAT, #4383286 (Modified)	
	Sample Task #2 (Multiple Choice)	
	A new savings account was opened with an initial deposit of $1,000. Each year, the account earns 2% interest on the amount of money in the account the previous year, and this interest is added to the account. If no additional deposits or withdrawals are made, which of the following functions gives the	

account value $A(t)$, in dollars, after t years?

A.
$A(t) = 1{,}000(1 + 0.02t)$

B.
$A(t) = 1{,}000(1 + 1.02t)$

C.
$A(t) = 1{,}000(0.02)^t$

D.
$A(t) = 1{,}000(1.02)^t$

Rationale

Choice D is correct. A model for a quantity that increases by a certain percentage per time period t is an exponential function in the form $A(t) = I\left(1 + \frac{r}{100}\right)^t$, where I is the initial value at time $t = 0$ for r% annual interest. It's given that $A(t)$ is the account value, in dollars, and t is the number of years after the account was opened. There was an initial deposit of $1,000 at time $t = 0$, so $I = 1{,}000$. This number increases 2% per year after year $t = 0$, so $r = 2$. Substituting these values into the function equation produces $A(t) = 1{,}000(1.02)^t$.

Choices A and B are incorrect and may result from setting up a linear function rather than an exponential function. Choice C is incorrect and may result from representing the exponential function as a decreasing function instead of an increasing function.

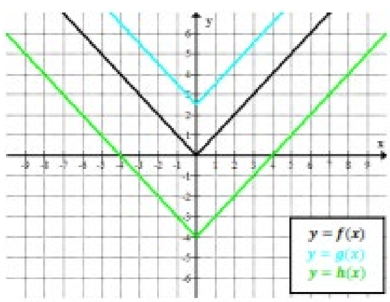
SAT, #4789285

Grade	CCSS Domain	CCSS Strand
A1	**Building Functions**	**Build new functions from existing functions**

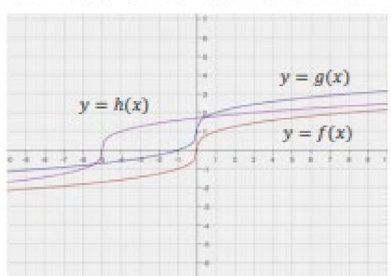
Sample Task #1 (Constructed Response)

For each of the following graphs, use the formula for the parent function f to write the formula of the translated function.

a.



b.



Sample Task #2 (Multiple Choice)

Write a function, g ,in terms of another function, f, such that the graph of g is a vertical shrink of the graph f by a factor of 0.75.

*Convert to Multiple-Choice

MLSS AND CLR GUIDE

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

CCSS Domain	CCSS Cluster
Building Functions	**Build a function that models a relationship between two quantities**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on building a function that models a relationship between two quantities, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, find something that you do in your family and create a function model to show someone can create a strong connection between your school tasks and your life tasks.	
Cross-Curricular Connections	Science: In high school the NGSS students should apply concepts of statistics and probability to explain the variation and distribution of expressed traits in a population. Consider providing a connection for students to examine scientific data and predict the effect of a change in one variable on another. https://www.nextgenscience.org/topic-arrangement/hsinheritance-and-variation-traits	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?*	• Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a

	• *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when building a function that models a relationship between two quantities the pattern of questions within the classroom is critical because by posing purposeful questions you will be able to scaffold the activity to provide multiple entry points meeting students where they are at.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to interpreting the equation $y = mx + b$ as defining a linear function. **(8.F.3)** • Connect to comparing properties of two functions, each represented in a different way. **(8.F.2)**	• Connect to identifying patterns in the function's rate of change, specifying intervals of increase and decrease, and graphing to model functions. **(HSF.IF.4,6)** • Connect to discussing the relative strengths and weaknesses of each representation and which are most efficient to be able to assist them in making symbolic functions. **(HSF.IF.9)** • Connect to recognizing situations that grow by a constant rate or percent. **(HSF.LE.1)**	• Connect to continuing to write arithmetic and geometric sequences. **(HSF.LE.2)** • Connect to using geometric series to find the sum. **(HSA.SSE.4)** • Connect to performing operations with all parent functions and composing functions. **(HSF.BF.1)**

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*

Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that previews new contexts for tasks within the unit (e.g., cell phone plans) when building a function that models a relationship between two quantities and discussing possible strategies and viable solutions.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.F.A.3 This standard provides a foundation for work with building a function that models a relationship between two quantities because students identify the type of relationship the two quantities have (linear, non-linear, exponential). If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
		Potential Scaffolds

• A function is a special relationship between two sets in which each domain value corresponds to one and only one range number.
• The similarities and differences of linear, quadratic, and exponential functions.
• That an arithmetic recursive formula is addition of a repeated constant and a geometric recursive formula is multiplication of a repeated constant.
• Over time, a quadratic function will grow faster than a linear
	• Use multiple representations (including graphs, tables, and symbols) to determine the domain and range and describe important behaviors of functions.
	• Graph linear, quadratic, and exponential by hand and using technology and identify and label key features.
	• Create and translate between recursive and explicit definitions of arithmetic and geometric sequences.
	• Identify when a table, graph,
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
		  ○ Know and recognize linear functions (8.EE.C.A.7)
		  ○ Calculate arithmetic sequence (7.EE.B.4)
		  ○ Apply properties of exponents (8.EE.A)
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of representation for mathematical ideas
		• Encourage students to use alternative tools to better access the grade level content. Examples include:
		  ○ Graphing calculator
		  ○ Desmos

function, and an exponential function will grow faster than both a linear and a quadratic function.	equation, and/or verbal description exhibits a linear or exponential relationship.	○ Graphic organizers ○ Sketch a graph ○ Create a table of values

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on building a function that models a relationship between two quantities by revisiting student thinking through a short mini lesson because some students have trouble writing their thinking and they just need more time to explain what they are thinking.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on building a function that models a relationship between two quantities by helping students move from specific answers to generalizations for certain types of problems because students need to understand that the content used in this unit is not only useful for one relationship between two quantities. It is a concept that they will continue to use every time that they have a relationship between two quantities.

Extension

Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as an open-ended task linking multiple disciplines. With the problem being open-ended it allows the students to view multiple perspectives, explain and describe their thinking without feeling pressured to one specific answer.

CCSS Domain	*CCSS Cluster*
Building Functions	**Build new functions from existing functions**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on building new functions from existing functions, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, compare functions that represent your community that you can find online. This can create a strong connection between your school tasks and your community.	
Cross-Curricular Connections	Science: The equation for velocity, $M(v) = 6v^2$, is one where the variable, v, has directions. Therefore, an inverse function of $M(v)$ cannot give back both a positive and negative velocity. Consider providing a connection for students to consider how they will handle this situation.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	• Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when building new functions from existing functions, the use of mathematical representations within the classroom is critical because students will need different representations when creating new functions from existing functions. Students will need to make connections to their previous "mathematical and cultural "knowledge.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	***Current Learning***	***Future Learning***
• Connect to recognizing and understanding that all linear functions can be written in the form $y = mx + b$. **(8.F.3)** • Connect to graphing linear relationships. **(8.F.5)**	• Connect to graphing linear, quadratic, and exponential relationships. **(HSF.IF.4)**	• Connect to extending transformation patterns to all functions. **(HSF.BF.3)** • Connect to graph transformations and compositions of transformations on a coordinate plane. **(HSF.BF.1)**

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that analyzes common misconceptions when building new functions from existing functions because students will need to make connections to the previous standard. If they still have misconceptions it is better to address before the new standard is introduced to reduce the amount of future confusion.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.F.A.3: This standard provides a foundation for work with building new functions from existing functions because students identify the type of relationship the two quantities have (linear, non-linear, exponential) and they can create new functions. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework

		Potential Scaffolds

- A function is a special relationship between two sets in which each domain value corresponds to one and only one range number. - The similarities and differences of linear, quadratic, and exponential functions. - That an arithmetic recursive formula is addition of a repeated constant and a geometric recursive formula is multiplication of a repeated constant. - Over time, a quadratic function will grow faster than a linear function, and an exponential function will grow faster than both a linear and a quadratic function.	- Use multiple representations (including graphs, tables, and symbols) to determine the domain and range and describe important behaviors of functions. - Graph linear, quadratic, and exponential by hand and using technology and identify and label key features. - Create and translate between recursive and explicit definitions of arithmetic and geometric sequences. - Identify when a table, graph, equation, and/or verbal description exhibits a linear or exponential relationship.	- Build on students' experience with the following skills: - Graphing on the coordinate plane (6.NS.C.8) - Know and recognize linear functions (8.EE.C.A.7) - Calculate arithmetic sequence (7.EE.B.4) - Apply properties of exponents (8.EE.A) - Cognitive Strategies - Repeatedly model the strategies - Monitor the students' use of the strategies - Provide feedback to students - Teach self-questioning and self-monitoring strategies - Introduce multiple means of representation for mathematical ideas - Encourage students to use alternative tools to better access the grade level content. Examples include: - Graphing calculator - Desmos - Graphic organizers - Sketch a graph - Create a table of values
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on Build new functions from existing functions by critiquing student approaches/solutions to make connections through a short mini-lesson because by having students critiquing their work or others they are able to make connections which they can use to help them build new functions.

Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after the unit building new functions from existing functions by addressing conceptual understanding because this will inform the teacher what the student understands and why it is important to understand why building new functions from existing functions is useful.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the opportunity to build new functions from existing functions because some students can do the assignments but sometimes do not fully understand the concept. This will allow them to focus on the concept in greater depth and not just on finishing the problems.