New Mexico Instructional Scope

Algebra 1 Real Number System Guide

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

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Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Extend the properties of exponents to rational exponents.
  - HSN.RN.A.1
  - HSN.RN.A.2
- Use properties of rational and irrational numbers.
  - HSN.RN.B.3

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Grade	*CCSS Domain*	*CCSS Cluster*
A1	**The Real Number System**	**Extend the properties of exponents to rational exponents**

Cluster Standard: HSN.RN.A.1

Standard	**Standards for Mathematical Practice**
Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)^3}$ to hold, so $(5^{1/3})^3$ must equal 5.*	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Exponent notation is a remarkable success story in the expansion of mathematical ideas. It is not obvious at first that a number such as sqrt(2) can be represented as a power of 2. But reflecting that $(\text{sqrt}(2))^2=2$ and thinking about the properties of exponents, it is natural to define $2^{(1/2)}=\text{sqrt}(2)$ since if we follow the rule $(a^b)^c=a^{(bc)}$ then $(2^{(1/2)})^2=2^{((1/2)*2)}=2^1=2$.	• Explain how integer exponent properties apply to rational exponent properties. • Show how a rational exponent (whose numerator is not one) can be expanded as a whole number multiplied by a fraction. • Justify that raising the base to a power and then taking the root is equivalent to taking the root and then raising the base to a power.
DOK	**Blooms**
2	Apply, Analyze

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Grade	*CCSS Domain*	*CCSS Cluster*
A1	**The Real Number System**	**Extend the properties of exponents to rational exponents**

Cluster Standard: HSN.RN.A.2

Standard	**Standards for Mathematical Practice**
Rewrite expressions involving radicals and rational exponents using the properties of exponents.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 3**: Construct viable arguments and critique the reasoning of others. • **SMP 7**: Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Because rational exponents have been introduced in such a way as to preserve the laws of exponents, students can now use those laws in a wider variety of situations. For example, they can rewrite the formula for the volume of a sphere of radius r, $V=(4/3)(\text{pi})(r^3)$ to express the radius in terms of the volume, $r=((3/4)(V/\text{pi}))^{(1/3)}$.	• Apply properties of exponents to simplify algebraic expressions with fractional exponents. • Apply power of zero, negative exponent, product, quotient, power to a power, product to a power, and quotient rules of exponents to simplify or write equivalent expressions. • Convert radical expression to expressions with rational exponents and vice versa. • Identify the exponent property used when rewriting expressions and recognize when laws of exponents cannot be used to rewrite an expression.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- Struggle to connect rational exponents to its radical form. Students tend to multiply the number by the exponent.
- When using the Power of a Power Property some students may forget to multiply the entire quantity by the exponent and only multiply the variable.

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rade	*CCSS Domain*	*CCSS Cluster*
A1	**The Real Number System**	**Use properties of rational and irrational numbers**

Cluster Standard: HSN.RN.B.3

Standard	Standards for Mathematical Practice
Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• An important difference between rational and irrational numbers is that rational numbers form a number system. If you add, subtract, multiply, or divide two rational numbers, you get another rational number (provided the divisor is not 0 in the last case). The same is not true of irrational numbers. Although in applications of mathematics the distinction between rational and irrational numbers is irrelevant, since we always deal with finite decimal approximations (and therefore with rational numbers), thinking about the properties of rational and irrational numbers is good practice for mathematical reasoning habits such as constructing viable arguments and attending to precision. (SMP3, SMP6).	• Identify the difference between a rational and an irrational number. • Perform operations on rational and irrational numbers. • Explain that the sum and product of two rational numbers is rational. • Explain that the sum and product of a rational number and a nonzero irrational number are irrational.
DOK	**Blooms**
2	Apply, Analyze

Common Misconceptions

- Students may think that the quotient of two rational numbers isn't always rational because some quotients do not appear to terminate or repeat.
- Students may wrongly believe that a single explanation is an explanation or proof of a property.

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Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **The Real Number System**	Strand: **Extend the properties of exponents to rational exponents**

Suggested Student Discourse Questions

- Explain why the terms "base" and "power" represent things that have a close physical resemblance to their definitions. Can you think of other terms about exponents that have the same property?
- Share the method you used to simplify the rational exponent. Give feedback about their method to another student.
- How many steps are needed to simplify this rational exponent? Are there other ways to simplify it using the same number of steps?
- Look at measurements of various objects provided by NASA. How do the exponents used in the scientific notation representation of those measurements demonstrate the size of the objects?

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ASSESSMENT GUIDE

- Extend the properties of exponents to rational exponents.
- Use properties of rational and irrational numbers.

Grade	*CCSS Domain*	*CCSS Strand*
A1	**The Real Number System**	**Extend the properties of exponents to rational exponents**

Sample Task #1 (Constructed Response)

What is an equivalent expression to $\sqrt{(a+c)^3} \cdot \sqrt{a+c}$?
SAT, #4383735 (Modified)

Sample Task #2 (Multiple Choice)

Which of the following expressions is equivalent to $\sqrt[3]{b} \cdot b \cdot \sqrt[5]{b^2}$ for $b > 0$?

A. $b^{\frac{2}{15}}$

B. $b^{\frac{6}{15}}$

C. $b^{\frac{11}{15}}$

D. $b^{\frac{26}{15}}$

MLSS AND CLR GUIDE

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- Extend the properties of exponents to rational exponents.
- Use properties of rational and irrational numbers.

CCSS Domain	*CCSS Cluster*
The Real Number System	**Extend the properties of exponents to rational exponents**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on: (1) how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, and (2) allowing for a notation for radicals in terms of rational exponents and rewriting expressions involving radicals and rational exponents using the properties of exponents, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, learning about what systems are in place at home to organize and simplify the household can provide students with a powerful connection on why we simplify expressions.	
Cross-Curricular Connections	Science: Rational exponents can be applied to scientific notation. Consider providing a connection for students to explore rational exponents in this context, such as to determine the maximum distance of each planet from the sun. Music: The frequencies in the musical range of various instruments can be modeled using rational exponents. Consider providing a connection for students to find the highest and lowest frequencies.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures*	• Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, “it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time.” For example, rewriting expressions involving radicals and rational exponents using the properties of exponents elicits and uses student thinking, which is critical, because students need to experience mathematics that allow students to use different

	and languages? • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	approaches to find the same end result. When rewriting expressions with rational exponents in order to simplify expressions, students will be provided the opportunity to see the order in which you perform the root and the power does not make a difference with the end result. Also, by using student thinking in regards to simplifying exponential expressions using the exponent properties, students are provided the opportunity to reason, communicate their reasoning and justify their solution. Students will have the opportunity to build on the knowledge that mathematics is a powerful tool and all approaches should be validated.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to using square root and cube root symbols. **(8.EE.2)** • Connect to understanding and applying the properties of integer exponents. **(8.EE.1)**	• Connect to applying the properties of exponents to rewrite exponential functions. **(HSA.SSE.3)**	• Connect to solving equations using rational exponents and radical operations. **(HSA.REI.2)** • Connect to continuing to use exponent properties as they arise in various situations**. (HSF.IF.8, HSF.LE)**

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this*	Some learners may benefit from targeted pre-teaching that focuses on the rewriting of expressions involving radicals and rational exponents by applying the properties of exponents. Students may have unfinished

	cluster within your HQIM?	learning in regards to simplifying expressions with exponents or using the exponent properties and would benefit from the access of that prior learning. Re-visiting expanded form and connecting to exponent properties as well as anchor charts would be beneficial to provide students with access to this content.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.EE.A.1 This standard provides a foundation for work with extending exponent properties to rewriting expressions with radicals and rational exponents because this was the first-time students were introduced to applying the exponent properties in simplifying and generating equivalent expressions. Students will benefit from time to access and apply this prior knowledge. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework

A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• An expression with a rational exponent can be rewritten as a radical expression.
• The denominator of the rational exponent is the index (root) of the radical expression and the numerator of the rational exponent is the exponent of the radical expression.
• How to select appropriate quantities and/or create appropriate labels for quantities for a real-world context.
• The appropriate levels of measurement precision when using digital and concrete
	• Translate fluently between expressions with rational exponents and radical expressions.
	• Simplify expressions with rational exponents and radical expressions using the properties of exponents.
	• Determine the correct units in multi-step and real-world problems.
	• Choose the appropriate level
		• Build on students' experience with the following skills:
		  ○ Connect to using square root and cube root symbols . (8.EE.2)
		  ○ Connect to understanding and applying the properties of integer exponents. (8.EE.1)
		  ○ Writing and solving one-step and two-step equations
		
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of representation for mathematical ideas
		
		• Encourage students to use alternative tools to better access the grade level content. Examples

tools, such as calculators, rulers, and protractors.	of precision to report based on the meaning of the quantities in a problem.	include: ○ **Desmos calculator** ○ **Square calculator**

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on rewriting radical and rational exponent expressions by clarifying mathematical ideas and/or concepts through a short mini-lesson because students must build a conceptual understanding of the meaning of a rational exponent by decomposing the exponent into parts. Students must understand the parts of the rational exponent ⅔; ⅔ can be rewritten as 2 times ⅓; the numerator of 2 is the power and the denominator 3 of (⅓) is the cube root.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit of rewriting radical expressions and expressions with rational exponent by offering opportunities to understand and explore different strategies because students must make the connection that a rational exponent can be broken down to the root and the power. Students need time to explore performing different operations first to understand the mathematical relationships between inverse operations. <i.e. (5 ^3)^(⅓) is 5>

Extension

Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the application of and development of abstract thinking skills when rewriting radical expressions and expressions with rational exponents by using the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay.

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CCSS Domain	*CCSS Cluster*
The Real Number System	**Use properties of rational and irrational numbers**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on (1) why the sum or product of two rational numbers is rational; (2) that the sum of a rational number and an irrational number is irrational; and (3) that the product of a nonzero rational number and an irrational number is irrational, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, learning about student interests, how they are able to explore their interests and make connections that provide meaning to their interests can help students understand that learning occurs from the ability to make connections and make sense of how it connects to their world.	
Cross-Curricular Connections	Science: Two irrational numbers that are of great importance in physics are e and π. Consider providing a connection for students to explore irrational numbers in this context, and the fact that whenever we compute a number answer we must use rational numbers to do it, most generally a finite-precision decimal representation. Social Studies: In high school the New Mexico Social Studies Standards state students should explain and analyze "tension and cooperation between religion and new scientific discoveries". Consider providing a connection for students to learn about Hippias's who was rumored to have been murdered for divulging the existence of irrational numbers.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize*	• Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and

	the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice leads to more students viewing themselves as capable mathematicians. The types of mathematical tasks are critical because students must develop conceptual understanding of this concept making conjectures regarding the sum and products of rational and irrational numbers, conduct investigations by exploring many cases, providing counter examples, if possible, to refute the conjecture, and justifying their claims through verbal and written communication. Students who are given rules, do not remember them unless they make a personal connection to the rule. Discovery is the connection students need to truly understand and remember the outcomes of sums and products of rational and irrational numbers.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to identifying and comparing rational and irrational numbers. **(8.NS.2)** • Connect to computing rational and irrational values when working with volume, surface area, and circles. **(7.G.4,6)**	• Connect to using the same strategies as classifying one number as rational or irrational to classify sums and products.	• Connect to rationalizing denominators using an understanding of products of irrational numbers. **(HSN.CN.5)** • Connect to working with irrational numbers when solving equations. **(HSN.CN.7)** • Connect to simplifying radicals using an understanding of irrational numbers. **(HSA.REI.2)**

		• Connect to calculating and interpreting measurements using irrational numbers. **(HSF.TF.1-3)**

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying properties of rational and irrational numbers because some students have either unfinished learning or misconceptions such as confusing repeating with non-terminating regarding rational and irrational numbers. Assessing students' prior learning of rational and irrational numbers and addressing student misconceptions is imperative to avoid further misconceptions on classifying sums and products of rational and irrational numbers.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.NS.A.1 This standard provides a foundation for work with the classification of sums and products of rational and irrational numbers because this standard introduces the concept that a number can't be both rational and irrational simultaneously. In prior grades, students were presented with only the rational number system. Students must understand the difference between a rational and irrational number before they can classify expressions. Allowing time for those discussions and addressing misconceptions regarding the real number system will diminish further misconceptions from developing. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework

A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• An expression with a rational exponent can be rewritten as a radical expression.
• The denominator of the rational exponent is the index (root) of the radical expression and the numerator of the rational exponent is the exponent of the radical expression.
• How to select appropriate quantities and/or create appropriate labels for quantities for a real-world context.
• The appropriate levels of measurement precision when using digital and concrete tools, such as calculators, rulers, and protractors.
	• Translate fluently between expressions with rational exponents and radical expressions.
	• Simplify expressions with rational exponents and radical expressions using the properties of exponents.
	• Determine the correct units in multi-step and real-world problems.
	• Choose the appropriate level of precision to report based on the meaning of the quantities in a problem.
		• Build on students' experience with the following skills:
		  ○ Connect to using square root and cube root symbols . (8.EE.2)
		  ○ Connect to understanding and applying the properties of integer exponents. (8.EE.1)
		  ○ Writing and solving one-step and two-step equations
		
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of representation for mathematical ideas
		
		• Encourage students to use alternative tools to better access the grade level content. Examples include:
		  ⊄ Desmos calculator
		  ⊄ Square calculator

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on Classifying Expressions as Rational or Irrational by revisiting student thinking through a short mini-lesson because students cannot classify expressions as rational or irrational before they have developed conceptual understanding of rational and irrational numbers. Revisiting student thinking before the presentation of this concept will show student's unfinished learning, student misconceptions, and students' level of reasoning. Understanding student thinking is essential to present this concept in a way that students can extend their thinking.

Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit Classifying Sums and Products as Rational or Irrational by helping students move from specific answers to generalizations for certain types of problems because students need time to reason and apply their thinking using generalizations to develop conceptual understanding. For example, students should be given time and tools (calculators) to investigate whether the sum of two irrational numbers are always, sometimes, or never rational. Students can explore sums of different irrational numbers to determine if they are always irrational or can a counterexample be found. Students then should be allowed time to communicate their thinking verbally and in writing to write a general statement regarding the posed question.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension addressing the classification of sums and products of rational and irrational numbers because students use reasoning skills to make conjectures and provide counterexamples to disprove conjectures and develop deep understanding of the concept. For example, "Does the product of a rational and irrational number always produce an irrational product? If not, can you provide a case where it does NOT hold true."