New Mexico Instructional Scope

Algebra 2 Arithmetic with Polynomials and Rational Expressions Guide

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards of Mathematical Practice
    - Common Misconceptions
    - Identification of Priority Standards, as identified by NMPED.
    - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

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Algebra 2 Arithmetic with Polynomials and Rational Expressions Guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	Conceptual Understanding	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	Application	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	Procedural Skill and Fluency	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Perform arithmetic operations on polynomials.
  - HSA.APR.A.1
- Understand the relationship between zeros and factors of polynomials.
  - HSA.APR.B.2
  - HSA.APR.B.3
- Use polynomial identities to solve problems.
  - HSA.APR.C.4
- Rewrite rational expressions.
  - HSA.APR.D.6

Grade	CCSS Domain	CCSS Cluster
A2	Arithmetic with Polynomials & Rational Expressions	Perform arithmetic operations on polynomials

Cluster Standard: HSA.APR.A.1

Standard	Standards for Mathematical Practice
Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	• **SMP2:** Reason abstractly and quantitatively.

Clarification Statement	Students Who Demonstrate Understanding Can...
• The development of **polynomials** and **rational expressions** in high school parallels the development of numbers in elementary and middle grades. In elementary school, students might initially see expressions for the same numbers 8 + 3 and 11, or 3/4 and 0.75, as referring to different entities: 8 + 3 might be seen as describing a calculation and 11 is its answer; 3/4 is a fraction and 0.75 is a decimal. They come to understand that these different expressions are different names for the same numbers, that properties of operations allow numbers to be written in different but **equivalent forms**, and that all of these numbers can be represented as points on the number line. In middle grades, they come to see numbers as forming a unified system, the number system, still represented by points on the number line. The whole numbers expand to the integers—with extensions of addition, subtraction, multiplication, and division, and their properties. Fractions expand to the rational numbers—and the four operations and their properties are extended. A similar evolution takes place in algebra. At first, algebraic expressions are simply numbers in which one or	• Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; • Describe the similarities between the set of integers and the system of polynomials. • Add, subtract, and multiply polynomials. • Determine whether a set or system is closed under a given operation

more letters are used to stand for a number which is either unspecified or unknown. Students learn to use the properties of operations to write expressions in different but equivalent forms. At some point they see equivalent expressions, particularly polynomial and rational expressions, as naming some underlying thing. As they see polynomial expressions as quantities rather than operations to be performed, they can perform operations such as adding, subtracting and multiplying two polynomials and identify that these operations will yield another polynomial, thus making the system of polynomials closed.	
DOK	**Blooms**
1	Remember, Understand

Common Misconceptions

- Students might think polynomials are only monomial, binomial, or trinomial.
- Students may not confuse the impact of adding and subtracting polynomials on the degree of the variable.
- Students may not fully distribute the multiplication of polynomials and only multiply like terms.
- When adding and multiplying like terms students may initially confuse $x + x$ as x^2 instead of $2x$.
- Students may not think $x^2 \bullet x = x^3$ is not an example of closure for polynomial multiplication since the result has a different exponent than the factors.

Grade	CCSS Domain	CCSS Cluster
A2	**Arithmetic with Polynomials & Rational Expressions**	**Understand the relationship between zeros and factors of polynomials**

Cluster Standard: HSA.APR.B.2

Standard	Standards for Mathematical Practice
Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	• **SMP3:** Construct viable arguments and critique the reasoning of others.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• The zeros of a polynomial are turned into linear factors and can be used to factor polynomials of any power. The degree of a polynomial will indicate the maximum number of zeros of the polynomial.	• Define the Remainder Theorem. • Use the Remainder Theorem to show the relationship between a factor and a zero.
DOK	**Blooms**
1-2	Understand, Apply

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Grade	CCSS Domain	CCSS Cluster
A2	**Arithmetic with Polynomials & Rational Expressions**	**Understand the relationship between zeros and factors of polynomials**

Cluster Standard: HSA.APR.B.3

Standard	Standards for Mathematical Practice
Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	• **SMP3:** Construct viable arguments and critique the reasoning of others.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• The zeros of a polynomial are turned into linear factors and can be used to factor polynomials of any power. The degree of a polynomial will indicate the maximum number of zeros of the polynomial.	• Determine the degree of a polynomial and the number of possible zeros of that polynomial. • Simplify polynomials into factored forms. • Identify the zeros of the polynomial using the factors. • Plot the zeros of the polynomial on a graph.
DOK	**Blooms**
1-2	Apply, Analyze

Common Misconceptions

- Division problems never have a remainder; it is okay to write R-value.
- Students often forget to distribute the –1 which is equivalent to subtraction, to terms inside the parenthesis.
- Students might make errors in signs when doing synthetic division and synthetic substitution because values are added rather than subtracted as in long division. Remind them that terms are always added for synthetic substitution and synthetic division. When listing the coefficients, there may be missing degrees and students will forget to write a zero.

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Grade	CCSS Domain	CCSS Cluster
A2	**Arithmetic with Polynomials & Rational Expressions**	**Use polynomial identities to solve problems**

Cluster Standard: HSA.APR.C.4

Standard	Standards for Mathematical Practice
Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	• **SMP7:** Look for and make use of structure. • **SMP8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students make systematic lists of all arrangements and count the number of unique subgroups. Students use prior knowledge of counting techniques to calculate the number of combinations.	• Understand that polynomial identities include but are not limited to the product of the sum and difference of two terms, the difference of two squares, the sum and difference of two cubes, the square of a binomial, etc. • Prove polynomial identities by showing steps and providing reasons. • Illustrate how polynomial identities are used to determine numerical relationships
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- There are no *y*-axis zeros.
- Easily get lost with the different coefficients and degrees.
- Multiplying the degrees.
- Students might incorrectly expand binomial expressions by choosing the wrong row of coefficients in Pascal's triangle. Remind students that for an exponent of *n*, choose row *n* of Pascal's triangle. Row *n* will be the row with

the value *n* as the second entry.

Grade	CCSS Domain	CCSS Cluster
A2	**Arithmetic with Polynomials & Rational Expressions**	**Rewrite rational expressions**

Cluster Standard: HSA.APR.D.6

Standard	Standards for Mathematical Practice
Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	• **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP5:** Use appropriate tools strategically. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Rational expressions can be rewritten using properties of fractions and elementary numerical algorithms.	• Divide polynomials using long division. • Divide polynomials using synthetic division. • Relate the algorithm of dividing multi-digit integers with polynomial long division. • Perform partial fraction decomposition. • Determine the quotient and remainder of rational expressions using inspection, long division, and/or a computer algebra system
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Students may forget to write the polynomial in descending order.

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- Students may not recognize a missing term in the divisor or dividend and forget to insert a zero for the missing term.
- Students might make errors in signs when doing synthetic division and synthetic substitution because values are added rather than subtracted as in long division. Remind them that terms are always added for synthetic substitution and synthetic division.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Arithmetic with Polynomials & Rational Expressions**	Strand: **Perform arithmetic operations on polynomials**

Suggested Student Discourse Questions

- Can you identify a maximum or minimum from a polynomial?
- What can finding a max or min tell you?
- If you add or subtract two rational numbers will you always get an irrational number?

- Compare strategies for add/subtracting with mult/and dividing? Do they work the same? What are the differences? Are they the same, why or why not the same?
- Given the relationship between the length and width of a rectangular pool, the width and total area of the surrounding walkway, how would you find the dimensions of the pool?

New Mexico Instructional Scope

Algebra 2 Arithmetic with Polynomials and Rational Expressions Guide

ASSESSMENT GUIDE

- Perform arithmetic operations on polynomials.
- Understand the relationship between zeros and factors of polynomials.
- Use polynomial identities to solve problems.
- Rewrite rational expressions.

Grade	*CCSS Domain*	*CCSS Strand*
A2	**Arithmetic with Polynomials & Rational Expressions**	**Perform arithmetic operations on polynomials**

Sample Task #1 (Constructed Response)

Source: https://satsuitequestionbank.collegeboard.org/

Question ID 5094624

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math		Passport to Advanced Mathematics	Equivalent expressions	2. Fluently add, subtract, and multiply polynomials.	No Calculator

Which of the following is equivalent to the sum of $3x^4+2x^3$ and $4x^4+7x^3$?

A. $16x^{14}$

B. $7x^8+9x^6$

C. $12x^4+14x^3$

D. $7x^4+9x^3$

Rationale

Choice D is correct. Adding the two expressions yields $3x^4+2x^3+4x^4+7x^3$. Because the pair of terms $3x^4$ and $4x^4$ and the pair of terms $2x^3$ and $7x^3$ each contain the same variable raised to the same power, they are like terms and can be combined as $7x^4$ and $9x^3$, respectively. The sum of the given expressions therefore simplifies to $7x^4+9x^3$. Choice A is incorrect and may result from adding the coefficients and the exponents in the given expressions. Choice B is incorrect and may result from adding the exponents as well as the coefficients of the like terms in the given expressions. Choice C is incorrect and may result from multiplying, rather than adding, the coefficients of the like terms in the given expressions.

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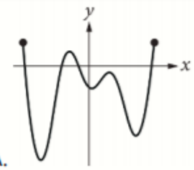
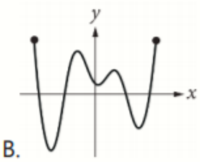
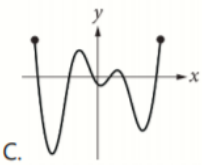
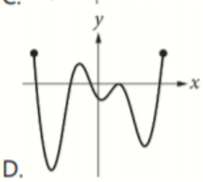
Grade	CCSS Domain	CCSS Strand
A2	**Arithmetic with Polynomials & Rational Expressions**	**Understand the relationship between zeros and factors of polynomials**
	Sample Task #1 (Multiple Choice)	

Source: SAT https://satsuitequestionbank.collegeboard.org/results

20079	■■□	Passport to Advanced Math	Passport to Advanced Mathematics	Nonlinear functions	3. For a factorable or factored polynomial or simple rational function, b. understand and use the fact that for the graph of y = f(x), the solutions to f(x) = 0 correspond to x-intercepts of the graph and f(0) corresponds to the y-intercept of the graph; interpret these key features in terms of a context;	Calculator

If the function f has five distinct zeros, which of the following could represent the complete graph of f in the xy-plane?

- A. 
- B. 
- C. 
- D. 

Rationale

- Choice D is correct. A zero of a function corresponds to an x-intercept of the graph of the function in the xy-plane. Therefore, the complete graph of the function f, which has five distinct zeros, must have five x-intercepts. Only the graph in choice D has five x-intercepts, and therefore, this is the only one of the given graphs that could be the complete graph of *f* in the xy-plane.
- Choices A, B, and C are incorrect. The number of x-intercepts of each of these graphs is not equal to five; therefore, none of these graphs could be the complete graph of f, which has five distinct zeros.

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Grade	CCSS Domain	CCSS Strand
A2	**Arithmetic with Polynomials & Rational Expressions**	**Use polynomial identities to solve problems**
	Sample Task #1 (Constructed Response)	

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Illustrative Mathematics

Felicia notices what appears to be an interesting pattern between powers of 11 and powers of x+1:

$$11^0 = 1 \qquad (x+1)^0 = 1$$
$$11^1 = 11 \qquad (x+1)^1 = x+1$$
$$11^2 = 121 \qquad (x+1)^2 = x^2+2x+1$$

- The digits of the number 11n are the same as the coefficients of the polynomial $(x+1)^n$. Is this always true?
- Does this pattern continue for n=3 and n=4?
- What is the answer to Felicia's question?

IM Commentary
This task has students combine polynomial arithmetic with pattern-matching. Students can expand powers of x+1 using either repeated multiplication (A-APR.1) or by the binomial theorem (A-APR.5), and then are asked to analyze the question of whether the similarity of coefficients with the digits of powers of 11 is a coincidence. Identifying patterns, as Felicia has done, is an important part of mathematics. In this case, there is a deep relationship between the numbers and polynomials that Felicia is investigating; on the other hand, further consideration shows that the pattern does not continue. It is important for students not only to identify patterns but also to look more deeply to understand whether or not the patterns are "generalizable" or true because of some essential mathematical structure.

Grade	CCSS Domain	CCSS Strand
A2	**Arithmetic with Polynomials & Rational Expressions**	**Rewrite rational expressions**
	Sample Task #1 (Constructed Response)	

Source: SAT

Which of the following is equivalent to $\frac{4x^2+6x}{4x+2}$?

- A. x
- B. $x+4$
- C. $x-\frac{2}{4x+2}$
- D. $x+1-\frac{2}{4x+2}$

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MLSS AND CLR GUIDE

- Perform arithmetic operations on polynomials.
- Understand the relationship between zeros and factors of polynomials.
- Use polynomial identities to solve problems.
- Rewrite rational expressions.

<table>
<tr><th>CCSS Domain</th><th colspan="2">CCSS Cluster</th></tr>
<tr><td>Arithmetic with Polynomials and Rational Expressions</td><td colspan="2">Perform arithmetic operations on polynomials</td></tr>
<tr><td colspan="3">Culturally and Linguistically Responsive Instruction</td></tr>
<tr><td>Relevance to Families and Communities</td><td colspan="2">During a unit focused on performing arithmetic operations on polynomials, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students, in becoming a critical thinker and problem solver. Students take a skill familiar to them (arithmetic operations with integers) and apply it to something new, arithmetic operations on polynomials. This unit practices learning something new from existing knowledge.</td></tr>
<tr><td>Cross-Curricular Connections</td><td colspan="2">History: The history of exponents dates back many centuries and Euclid is credited with the first known usage of exponents. He used the term ‘power’ to represent what we know today, how many times a number is multiplying by itself. The ancient Greek mathematicians used, and many other mathematicians added onto the use of exponents as they learned more about their use. Archimedes generalized the same idea of powers and later mathematicians in the Islamic golden age utilized powers of two and three in their work in algebra. In our project, you will see many other mathematicians and their contributions to the development of exponents from the 14th century up to the use of exponents today. https://www.sutori.com/story/history-of-exponents--wNbwYExXdzFNYPh1zFUYhbDc</td></tr>
<tr><td>Validate/Affirm/Build/Bridge</td><td>How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes</td><td>Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it hinders students with strong prior familiarity with school mathematics procedures for solving problems from more methods for solving tasks that occur outside of school</td></tr>
</table>

	regarding the mathematical abilities of students of marginalized cultures and languages? ● *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	mathematics. For example, when performing arithmetic operations on polynomials, the types of mathematical tasks are critical because they build on prior knowledge of arithmetic operations. Time spent on conceptual understanding of the four basic operations (addition, subtraction, multiplication, division) using integers can bridge to the conceptual understanding of those operations of polynomials. From here, time can be spent on procedural fluency of the mechanics of the operations with polynomials. Students will be expected to demonstrate proficiency on End of Course exams and the SAT in English, an opportunity presents itself to bridge home language to the language of these exams.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	***Current Learning***	***Future Learning***
● Connect to applying the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. **(8.EE.A.1)**	● Connect to using the properties of operations to write expressions in different but equivalent forms. **(HSA.SSE.A.2)**	● Connect to performing operations with rational expressions **(HSA.APR.7)** ● Connect to deriving the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems. *For example, calculate mortgage payments.* **(HS.A.SSE.B.4)**

Suggested Instructional Strategies

<table>
<tr><th colspan="3">Pre-Teach</th></tr>
<tr><th>Level of Intensity</th><th>Essential Question</th><th>Examples</th></tr>
<tr><td>Targeted</td><td>What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?</td><td>Some learners may benefit from targeted pre-teaching that focuses on arithmetic operations on polynomials because the structure of the four basic operations hold true for arithmetic operations on polynomials.</td></tr>
<tr><td>Intensive</td><td>What critical understandings will prepare students to access the mathematics for this cluster?</td><td>8.EE.A.1: This standard provides a foundation for work with arithmetic operations on polynomials because the student must know and apply the properties of integer exponents. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.</td></tr>
<tr><th colspan="3">Universal Support Framework</th></tr>
<tr><th>A student should know/understand...</th><th>A student should be able to do...</th><th>Potential Scaffolds</th></tr>
<tr><td>● Different forms of an expression can be equivalent and are useful in different contexts.
● The addition, subtraction, multiplication, or division of rational expressions results in another rational expression.
● When a situation and its potential constraints will be represented by all available types of equations/inequalities, including simple root</td><td>● Use the structure of an expression and the properties of mathematics to rewrite it in a different form.
● Perform the operations of addition, subtraction, multiplication, and division with rational expressions.
● Determine reasonable solutions based on the context of real-world</td><td>● Build on students' experience with the following skills:
○ Graphing on the coordinate plane (6.NS.C.8)
○ Solving systems of equations / inequalities (8.EE.C.8)
○ Adding / subtracting / multiplying / dividing and simplify fractions
○ Writing and solving one-step and two-step equations (HSA.REI.B.3, HSA.REI.B.4)
○ Modeling linear, exponential, quadratic and absolute value functions (HSF.LE.A, HSF.LE.B)
○ Different forms of linear (linear standard form, point-slope form, slope intercept form) and quadratic equations (quadratic standard form and vertex form) (HSF.LE.A)</td></tr>
</table>

function, or a system of those equations/inequalities. • When solving graphically/with a table is more efficient than solving algebraically.	problems from graphs of equations/inequalities and systems of equations/inequalities. • Solve systems using a graph and a table as well as rewrite an equation as two functions (and vice versa).	• Cognitive Strategies ◦ Repeatedly model the strategies ◦ Monitor the students' use of the strategies ◦ Provide feedback to students ◦ Teach self-questioning and self-monitoring strategies ◦ Introduce multiple means of representation for mathematical ideas • Encourage students to use alternative tools to better access the grade level content. Examples include: ◦ Desmos.com ◦ Graphing calculator ◦ Sketch a graph ◦ Create a table of values ◦ Algebra tiles ◦ Graphic organizers
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on arithmetic operations on polynomials by providing specific feedback to students on their work through a short mini-lesson because looking at integer rules for arithmetic operations apply directly to arithmetic operations with polynomials.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on arithmetic operations on polynomials by confronting student misconceptions because integer rules concerning positives and negatives are common errors that lead to misconceptions when performing arithmetic operations on polynomials.
Extension		

Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the opportunity to explore links between various topics when studying arithmetic operations on polynomials because addition and subtraction are inverse operations as are multiplication and division.

CCSS Domain	*CCSS Cluster*	
Arithmetic with Polynomials and Rational Expressions	**Understand the relationship between zeros and factors of polynomials**	
Culturally and Linguistically Responsive Instruction		
Relevance to Families and Communities	During a unit focused on understanding the relationship between zeros and factors of polynomials, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, students practice using mathematical tools to solve new problems. Students learn to identify what tools are at their disposal when connecting the zeros of a polynomial to its factors. This prepares them for life outside the math classroom by providing skills that are lifelong.	
Cross-Curricular Connections	Science: This approach is extended to a spherical body rolling on a curved path. Assuming that a curved path can be approximated by a sequence of many very short inclines, the problem is approached as a body rolling on this sequence of inclines, solving each with the work-energy theorem. Defining the curved path as a differentiable function, the slope of each incline is obtained through the function. Teach Engineering Roller Coaster - Spherical Body rolling Social Studies: Not much is really known about the Pythagoreans or their rather mysterious founder, Pythagoras. Several different accounts of the Pythagoreans have come down to us from antiquity. Plato and Aristotle both reference the Pythagoreans throughout their philosophical writings. Even still, the true nature of the "cult of Pythagoras" is often shrouded in mystery. Pythagorious and his followers were a mystical society that placed great importance on the mathematical relations of the universe. There is no denying that they contributed greatly to the area of mathematics and philosophy. One needs only to reflect on the Pythagorean theorem, a mathematical principle said to have been discovered by Pythagoras himself, to appreciate the profound impact they had on the development of scientific thought. https://classicalwisdom.com/philosophy/cult-of-pythagoras/	
Validate/Affirm/Build/Bridge	● *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and*	● Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural

	languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when understanding the relationship between zeros and factors of polynomials the use of mathematical representations within the classroom is critical because graphing technology and area models can be used to factor polynomials and check the zeros of those factors. Mathematics can be designed in a context to connect home culture or interests in a way that a polynomial function could represent a quantity where its solution(s) could represent a critical value within the context. For example, a cubic function could represent the profit of a fundraiser given the cost of a ticket for the fundraiser. The zero(s) of the function would represent the break-even point. The context of the fundraiser could be framed around a cultural interest. A graph could be used to show a representation of the function to support the learner in bridging different representations.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	***Current Learning***	***Future Learning***
• Connect to factoring and completing the square and using the Remainder Theorem **(standards A-APR.B.2, F-IF.C.8.a)**	• Connect to calculating the zero in the Remainder Theorem or by factoring to graph the zeros of a polynomial function **(standards A-APR.B.2, A-APR.B.3)**	• Connect to graphing key features of polynomial functions to identifying zeros and sketching a graph **(standards F-IF.C.7)**

Suggested Instructional Strategies

Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that analyzes common misconceptions regarding the relationship between zeros and factors of polynomials because when using the Remainder Theorem students must use the opposite sign of the factor in the dividend. Students must also know how to factor polynomials when the leading coefficient is equal to 1 and not equal to 1 which can lead to common misconceptions.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	A.SSE.B.3.A and A.APR.B.6: These standards provide a foundation for work with understanding the relationship between zeros and factors of polynomials because students will be producing equivalent forms of polynomials to reveal properties of the expression (in this case the factored form will reveal zeros; the Remainder Theorem can be used instead of long division to check if a factor is a zero of the expression). If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• Different forms of an expression can be equivalent and are useful in different contexts.
• The addition, subtraction, multiplication, or division of rational expressions results in another rational expression.
• When a situation and
	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form.
	• Perform the operations of addition, subtraction, multiplication, and division with rational
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
		  ○ Solving systems of equations / inequalities (8.EE.C.8)
		  ○ Adding / subtracting / multiplying / dividing and simplify fractions
		  ○ Writing and solving one-step and two-step equations (HSA.REI.B.3, HSA.REI.B.4)
		  ○ Modeling linear, exponential, quadratic and absolute value functions (HSF.LE.A, HSF.LE.B)

its potential constraints will be represented by all available types of equations/inequalities, including simple root function, or a system of those equations/inequalities. • When solving graphically/with a table is more efficient than solving algebraically.	expressions. • Determine reasonable solutions based on the context of real-world problems from graphs of equations/inequalities and systems of equations/inequalities. • Solve systems using a graph and a table as well as rewrite an equation as two functions (and vice versa).	○ Different forms of linear (linear standard form, point-slope form, slope intercept form) and quadratic equations (quadratic standard form and vertex form) (HSF.LE.A) • Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas • Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?	For example, students may benefit from re-engaging with content during a unit on understanding the relationship between zeros and factors of polynomials by critiquing student approaches/solutions to make connections through a short mini-lesson because zeros of polynomials must match the graph of the polynomial. By critiquing other students' work, the learner can immediately make connections to the correctness of the work by observing a graph.
Intensive	What assessment data will help identify content needing to be revisited for	For example, some students may benefit from intensive extra time during and after a unit on understanding the relationship between zeros and factors of polynomials by

	intensive interventions?	offering opportunities to understand and explore different strategies because investigating graphs to identify zeros and using area models to factor polynomials can offer structure to the student as a starting point.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students. because some learners may be ready to factor more difficult polynomials.

NEW MEXICO
Public Education Department

CCSS Domain	*CCSS Cluster*	
Arithmetic with Polynomials and Rational Expressions	**Use polynomial identities to solve problems**	
Culturally and Linguistically Responsive Instruction		
Relevance to Families and Communities	During a unit focused on the use of polynomial identities to solve problems, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, the student makes use of structure and decides on a method to expand binomials that is effective and efficient and makes sense to them. The Standards for Mathematical Practice come alive as they use polynomial identities to solve problems as the bridge to perseverance and making use of structure and repeated reasoning.	
Cross-Curricular Connections	In this activity, students relate the graph of a rational function to the graphs of the polynomial functions of its numerator and denominator. Students graph these polynomials one at a time and identify their y-intercepts and zeros. Asymptotes and Zeros of Rational Functions: Algebra 2	
Validate/Affirm/Build/Bridge	● *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* ● *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school*	● Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it hinders students with strong prior familiarity with school mathematics procedures for solving problems from learning more methods for solving tasks that occur outside of school mathematics. For example, when studying the use of polynomial identities to solve problems the types of mathematical tasks are critical because, for example, students' familiarity of structure can support the expansion of binomials. When squaring a binomial, students have a working knowledge of area models and the FOIL method. As the power of a binomial grows, these methods break down and become messy. The student can then decide when it would be more efficient to use the Binomial Theorem and Pascal's Triangle to expand a binomial. Conceptual understanding of expanding binomials will lead to procedural fluency as the student decides

	mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	what method works best for their learning style.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	***Current Learning***	***Future Learning***
• Students are building on their knowledge of zeros and factors of quadratics learned in Algebra 1.	• Students are learning about factoring with polynomials of degrees higher than 2 (perfect cubes, quadratics, factor by grouping, etc). Students are also understanding that not all polynomials are factorable, but still can be divided by another polynomial. Students continue to build their understanding of how factored form relates to zeros on a graph. Later in the year, these skills are used in simplifying rational expressions.	• In 4th year math (Pre-Calculus, Calculus, and college level math) students will build on their factoring skills (with rational expressions and trigonometric expressions). Students will also determine zeros of trigonometric functions in subsequent math courses.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that introduces new representations (e.g., Pascal's Triangle, the Binomial Theorem) when studying the use of polynomial identities to solve problems because there are structures that exist to make expanding binomials

<table>
<tr><td></td><td></td><td>more efficient and effective.</td></tr>
<tr><td>Intensive</td><td>What critical understandings will prepare students to access the mathematics for this cluster?</td><td>A.SSE.A.2: This standard provides a foundation for work with the use of polynomial identities to solve problems because students have looked at the structure of an expression to identify ways to rewrite it. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.</td></tr>
<tr><td colspan="3">Universal Support Framework</td></tr>
<tr><td>A student should know/understand...</td><td>A student should be able to do...</td><td>Potential Scaffolds</td></tr>
<tr><td>Different forms of an expression can be equivalent and are useful in different contexts.
The addition, subtraction, multiplication, or division of rational expressions results in another rational expression.
When a situation and its potential constraints will be represented by all available types of equations/inequalities, including simple root function, or a system of those equations/inequalities.
When solving graphically/with a table is more efficient than solving algebraically.</td><td>Use the structure of an expression and the properties of mathematics to rewrite it in a different form.
Perform the operations of addition, subtraction, multiplication, and division with rational expressions.
Determine reasonable solutions based on the context of real-world problems from graphs of equations/inequalities and systems of equations/inequalities.</td><td>Build on students' experience with the following skills:
Graphing on the coordinate plane (6.NS.C.8)
Solving systems of equations / inequalities (8.EE.C.8)
Adding / subtracting / multiplying / dividing and simplify fractions
Writing and solving one-step and two-step equations (HSA.REI.B.3, HSA.REI.B.4)
Modeling linear, exponential, quadratic and absolute value functions (HSF.LE.A, HSF.LE.B)
Different forms of linear (linear standard form, point-slope form, slope intercept form) and quadratic equations (quadratic standard form and vertex form) (HSF.LE.A)
Cognitive Strategies
Repeatedly model the strategies
Monitor the students' use of the strategies
Provide feedback to students
Teach self-questioning and self-monitoring strategies
Introduce multiple means of</td></tr>
</table>

	• Solve systems using a graph and a table as well as rewrite an equation as two functions (and vice versa).	representation for mathematical ideas • Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?	For example, students may benefit from re-engaging with content during a unit on the use of polynomial identities to solve problems by revisiting student thinking through a short mini-lesson because looking at other students' work can support all learners in understanding the structure of the Binomial Theorem. Learners listening to their peers explain their thinking can benefit all as student thinking is delivered in student friendly language.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on the use of polynomial identities to solve problems by offering opportunities to understand and explore different strategies because some students will insist on using area models or the FOIL method for expanding a binomial regardless of the power of the binomial. As the Binomial Theorem will be more effective and efficient to expand certain binomials, pockets of students are afforded the opportunity to explore and apply previous strategies.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics	Some learners may benefit from an extension to explore links between various strategies when studying the use

developed within your HQIM?	of polynomial identities to solve problems. Some learners can investigate and explain when it would be more appropriate to use an area model, the FOIL method, or the Binomial Theorem when expanding binomials.

CCSS Domain	*CCSS Cluster*
Arithmetic with Polynomials and Rational Expressions	**Rewrite rational expressions**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on rewriting rational expressions, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. Learners look for and make use of structure and make sense of problems and persevere in solving them. As students rewrite rational expressions in equivalent forms, they are building confidence in taking what they know to apply to problem solving scenarios. These Mathematical Practices skills exist outside of school as the student builds their critical thinking skills through rewriting rational expressions.	
Cross-Curricular Connections	Medicine and Analytical Chemistry: MRI and NMR Spectroscopy involves Fast Fourier Transformation that allows the creation of images from the "ringing" after the atoms are subjected to radio waves in strong magnetic fields. The Fourier series consists of terms of increasing orders. (An Algorithm for the Machine Calculation of Complex Fourier Series by James W. Cooley and John W. Tukey)	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures*	• Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school

	and languages? • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying rewriting rational expressions, the use of mathematical representations within the classroom is critical because students are in fact rewriting simple rational expressions as equivalent representations. Students are asked to draw on their mathematical competence by simplifying rational expressions previously learned within this standard domain. For example, a student might have to factor a numerator and/or denominator to simplify a rational expression. Or a student might have to perform synthetic division to rewrite a rational expression as an equivalent representation. These skills could build a bridge for students to position them as competent and capable mathematicians and leverage further study of mathematics.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	***Current Learning***	***Future Learning***
• Students are building on their knowledge of factors of quadratics learned in Algebra 1.	• Students will use the skills learned to factor and divide polynomials to simplify rational expressions.	• Students will be able to perform all operations with rational expressions.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on rewriting rational expressions because arithmetic operations with polynomials and factoring will be used when rewriting rational expressions.

Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	A.SSE.A.2: This standard provides a foundation for work with rewriting rational expressions because students use the structure of an expression to identify ways to rewrite it. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• Different forms of an expression can be equivalent and are useful in different contexts. • The addition, subtraction, multiplication, or division of rational expressions results in another rational expression. • When a situation and its potential constraints will be represented by all available types of equations/inequalities, including simple root function, or a system of those equations/inequalities. • When solving graphically/with a table is more efficient than solving algebraically.	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form. • Perform the operations of addition, subtraction, multiplication, and division with rational expressions. • Determine reasonable solutions based on the context of real-world problems from graphs of equations/inequalities and systems of equations/inequalities. • Solve systems using a graph and a table as well as rewrite an	• Build on students' experience with the following skills: ○ Graphing on the coordinate plane (6.NS.C.8) ○ Solving systems of equations / inequalities (8.EE.C.8) ○ Adding / subtracting / multiplying / dividing and simplify fractions ○ Writing and solving one-step and two-step equations (HSA.REI.B.3, HSA.REI.B.4) ○ Modeling linear, exponential, quadratic and absolute value functions (HSF.LE.A, HSF.LE.B) ○ Different forms of linear (linear standard form, point-slope form, slope intercept form) and quadratic equations (quadratic standard form and vertex form) (HSF.LE.A) • Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas • Encourage students to use alternative tools to better access the grade level content. Examples

	equation as two functions (and vice versa).	include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on rewriting rational expressions by providing specific feedback to students on their work through a short mini lesson because an expression might not be fully simplified. Students might not have applied a full set of mathematical properties to rewrite a rational expression and may benefit from focused feedback on where to go next in their work.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit rewriting rational expressions by confronting student misconceptions because the student might rewrite a rational expression incorrectly and simplify a polynomial incorrectly (e.g., (x+ y)2 = x2+ y2) or might have factored a polynomial incorrectly.

Extension

Essential Question	***Examples***
What type of extension will offer additional challenges to ‘broaden’ your student’s knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the opportunity to understand concepts in greater depth. When rewriting rational expressions some students will be ready for more complex rational expressions with more complex terms than others. Pockets of students can be paired homogeneously by ability to work on more complex rational expressions to explore in greater depth.

New Mexico Instructional Scope

Algebra 2 Arithmetic with Polynomials and Rational Expressions Guide