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New Mexico Instructional Scope

Algebra 2 Building Functions Guide

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

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Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Build a function that models a relationship between two quantities.
  - HSF.BF.A.1
- Build new functions from existing functions.
  - HSF.BF.B.3
  - HSF.BF.B.4.A

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Grade	*CCSS Domain*	*CCSS Cluster*
A2	**Interpreting Functions**	**Build a function that models a relationship between two quantities**

Cluster Standard: HSF.BF.A.1

Standard	**Standards for Mathematical Practice**
Write a function that describes a relationship between two quantities.* • HSF.BF.A.1.B Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model*.	• **SMP2:** Reason abstractly and quantitatively. • **SMP4**: Model with mathematics. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should write **functions** for given **relationships** between **quantities**. Students can use functions to **model real-life situations** and make **predictions**. Students should be able to use functions to describe relationships between two quantities, usually x and $f(x)$, where $f(x)$ is some **output** value that depends on the **input** value x. Within a context, students should be able to express a given relationship as a function.	• Build a function using different functions and arithmetic operations in context.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- Students may want to try to use a linear function, specifically the slope-intercept form for every situation.
- Students may tend to focus on the symbolic form of a function and may need additional support in working with other forms.

Grade	*CCSS Domain*	*CCSS Cluster*
A2	**Interpreting Functions**	**Build new functions from existing functions**

Cluster Standard: HSF.BF.B.3

Standard	Standards for Mathematical Practice
Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	• **SMP5**: Use appropriate tools strategically. • **SMP8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should describe the effect of **stretches, shrinkages**, **vertical and horizontal transformations** on **functions**. They should be able to find the value of the **transformation** when given a **graph** and be able to explain effects of transformations using technology. Students should know that adding a **constant** k to a function will change the graph of the function depending not only on the value of the constant, but on where it is inserted as well. If $y = f(x)$ is changed to $y = f(x) + k$, the curve will **shift** vertically (up for $k > 0$, down if $k < 0$). Adding k to x such that $y = f(x + k)$ will shift the curve horizontally (left for $k > 0$, right for $k < 0$). Multiplying $f(x)$ by a constant k stretches ($k > 1$) or **squishes** ($0 < k < 1$) the graph vertically. If $k < 0$, the graph is also **flipped** over the ***x*-axis**.	• Identify vertical transformations from a function or a graph. • Identify horizontal transformations from a function or a graph. • Identify a shrink or a stretch from a function or a graph. • Write the results from such transformations. • Recognize odd and even functions. • Identify transformations of a function on a graph. • Describe the effects of transformations on parent functions.

Multiplying x by k stretches ($k > 0$) or squishes ($k < 0$) the graph horizontally.	
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	*CCSS Domain*	*CCSS Cluster*
A2	**Interpreting Functions**	**Build new functions from existing functions**

Cluster Standard: HSF.BF.B.4.A

Standard	**Standards for Mathematical Practice**
Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.*	• **SMP 6:** Attend to precision. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should be able to find the **inverse** of **functions** and recognize that other functions may not have an inverse unless there are **restrictions** placed on the **domain**. If $f(x) = y$ is a function, the inverse function can be found by switching the place of x and y (f(y) = x), and then solving for y so that f-1(x) = y. For instance, if the function $f(x)$ is $y = 2x^3$, then the inverse function $f^{-1}(x)$ consists of switching the places of x and y ($x = 2y^3$) and then solving for y.	• Write the inverse of a function. • Determine restrictions on the domain to allow for an inverse to exist. • Relate using an inverse as an operation that undoes another operation. • Solve an equation of the form $f(x)=c$ for a function f that has an inverse and write an expression for the inverse.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- Students often have difficulty determining the direction of the horizontal shifts.
- Students often confuse the notation for the inverse and negative numbers.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Building Functions**	Strand: **Build a function that models a relationship between two quantities**

Suggested Student Discourse Questions

- In what ways are the independent and dependent variables related? How do they differ?
- Think about a relationship between two variables (time spent doing homework vs grade in class) - can you sketch a graph modeling this relationship? (I.e Which is the independent variable? Which is dependent? Why?) Now pass your sketch to your partner. Does your partner's sketched function make sense in the context they came up with? Why or why not?
- How is the rate of change of this (insert function here - linear, quadratic, exponential, cubic, logarithmic) function represented in a table? How is it represented in the function's graph?
- Can you write an equation or draw a graph to model your monthly cell phone data use? What happens if you don't have unlimited data? Can you write a second equation or draw a second graph to model what happens when you hit your data limit? How could you combine both functions (or graphs) to model your monthly cell data use both before and after it hits the limit?

ASSESSMENT GUIDE

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

Grade	*CCSS Domain*	*CCSS Strand*
A2	**Building Functions**	**Build a function that models a relationship between two quantities**
	Sample Task #1 (Constructed Response)	
	Standards Aligned Instructionally Embedded Formative Assessment Resources: Source: http://tasks.illustrativemathematics.org/content-standards/HSF/BF/A/2/tasks/1695 This type of assessment question requires students to analyze a number pattern described in context and fit both a recursive function to the pattern and use it to answer questions. Students will engage with SMP1 and SMP4 as they persevere to express the pattern mathematically and model the scenario with an equation	

Grade	*CCSS Domain*	*CCSS Strand*
A2	**Building Functions**	**Build new functions from existing functions**
	Sample Task #1 (Constructed Response)	
	Standards Aligned Instructionally Embedded Formative Assessment Resources: http://tasks.illustrativemathematics.org/content-standards/HSF/BF/B/3/tasks/742 This type of assessment question requires students to apply vertical and horizontal translations as well as a reflection to a given graph. Further, students are asked to identify the location of specific coordinates on the new graphs. This will engage students with SMP7 as they use the structure of the graph, the expression of the transformation and/or a table of values to create new graphs and identify the imaged points. https://www.engageny.org/resource/algebra-i-module-3-topic-c-lesson-17	

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MLSS AND CLR GUIDE

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

CCSS Domain	*CCSS Cluster*
Building Functions	**Build a function that models a relationship between two quantities**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on building a function that models a relationship between two quantities, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, learning about the various ways that functions relate to quantities can be a great way to connect school and home with making references to those quantities that can be encountered at home and how they relate to the tasks being created in the classroom. Making that connection allows for students to become more comfortable with learning the content and provides evidence of prior knowledge that the student can bring into the lessons.	
Cross-Curricular Connections	"https://www.nextgenscience.org/topic-arrangement/hsinheritance-and-variation-traits"Science: In high school the NGSS students should apply concepts of statistics and probability to explain the variation and distribution of expressed traits in a population. Consider providing a connection for students to examine scientific data and predict the effect of a change in one variable on another. https://www.nextgenscience.org/topic-arrangement/hsinheritance-and-variation-traits	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes*	• Task: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein

	regarding the mathematical abilities of students of marginalized cultures and languages? ● *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	2006)." For example, when building a function that models a relationship between two quantities the types of mathematical tasks are critical because when students are able to make connections, it is easier for them to learn and store information, like making a connection to background knowledge or prior learning to create an optimal environment for learning, as they bring this knowledge with them to class each day.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
● Connect to Algebra 1 work focusing on linear, exponential, and quadratic within this cluster. ● Connect to recognizing situations that grow by a constant rate or percent. **(HSF.LE.1)**	● Connect to identifying patterns in the function's rate of change, specifying intervals of increase and decrease, and graphing to model functions. **(HSF.IF.4,6)** ● Connect to discussing the relative strengths and weaknesses of each representation and which are most efficient to be able to assist them in making symbolic functions. **(HSF.IF.9)** ●	● Connect to arithmetic and geometric sequences and use them to model situations. **(HSF.BF.A.2)**

Suggested Instructional Strategies

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Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that previews new contexts for tasks within the unit (e.g., cell phone plans). When building a function that models a relationship between two quantities, the new contexts will show an alignment to new material that will be covered.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.F.B.4: This standard provides a foundation for work with building a function that makes a relationship between two quantities because prior learning on constructing a function modeling a linear relationship between two quantities is learned and will be expanded on in the current unit. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• What the domain of a function represents in terms of the function and what values are appropriate (i.e. discrete v. continuous values and other restrictions based on the type of function and the given context).
• In functions there is an underlying structure that determines the transformation of any function, regardless of its type.
	• Find and interpret key features of a graph or table of a function, including extreme values, end behavior, and intervals of increase and decrease.
	• identify and graph parent functions and their transformations (i.e. vertical translation, horizontal translation, vertical stretch/shrink,
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
		  ○ Building and solving proportional relationships (7.RP.A.2)
		  ○ Recognize and use function notation (HSF.IF.A)
		  ○ Recognizing inverse functions (HSF.BF.B.4)
		  ○ Graphing, solving and modeling quadratic, linear, exponential and absolute value functions (HSF.LE.A, HSF.LE.B)
		  ○ Recognize, build and solve trigonometric ratios (HSG.CRT.C)
		• Cognitive Strategies

• How to use trigonometric ratios, reference angles, and symmetry to find patterns on the unit circle. • Inverse trigonometric functions have restricted domains and ranges and are one to one.	reflect over x-axis, etc.). • Calculate values of sine, cosine, and tangent for given angles. • Solve trigonometric equations, including those written in quadratic form and equations containing more than one angle.	○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas • Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers ○ SOH CAH TOA

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?	For example, students may benefit from re-engaging with content during a unit on building a function that models a relationship between two quantities by revisiting student thinking through a short mini-lesson because students should be able to recall prior knowledge in the content previously learned and can use that prior knowledge to build on current content.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit building a function that models a relationship between two quantities by helping students move from specific answers to generalizations for certain types of problems because recalling prior knowledge will aid the student with current understanding and show the alignment to prior knowledge and will engage the student in the current content.

Extension

Essential Question	***Examples***

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying building a function that models a relationship between two quantities because activating prior knowledge will allow for students to take the understanding of the current content to a greater level and will allow for better understanding of the content.

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CCSS Domain	*CCSS Cluster*
Building Functions	**Build new functions from existing functions**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on building new functions from existing functions, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, learning about the characteristics of building functions from existing functions allows for making connections to building in any concept which can be done at home such as working on a home project that requires building on to something that already exists. Students can make connections to prior learning (something that already exists) and build onto that knowledge.	
Cross-Curricular Connections	Science: The equation for velocity, $M(v) = 6v^2$, is one where the variable, v, has directions. Therefore, an inverse function of $M(v)$ cannot give back both a positive and negative velocity. Consider providing a connection for students to consider how they will handle this situation.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in*	• Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when building new functions from existing functions goal setting is critical because it allows students to take ownership of the content and what the expectations for learning are as they are clearly identified while making a meaningful connection between the learning and daily lives.

	creating mathematical identities as capable mathematicians that can use mathematics within school and society?	

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to the work in Algebra 1 with linear, exponential, quadratic, and absolute value functions for this cluster. **(HSF.BF.B)**	• Connect to graphing functional relationships. **(HSF.IF.4)** • Connect to trigonometric functions. **(HS.F-TF.B)** •	• Connecting to understanding the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents. **(HS.F-BF.B.5)**

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on building new functions from existing functions because in prior lessons and grade levels, students have been introduced to many aspects and content of functions.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	HS.F-IF.A.1: This standard provides a foundation for work with building new functions from existing functions because this prerequisite has students understanding functions based on domain and range. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

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Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• What the domain of a function represents in terms of the function and what values are appropriate (i.e. discrete v. continuous values and other restrictions based on the type of function and the given context). • In functions there is an underlying structure that determines the transformation of any function, regardless of its type. • How to use trigonometric ratios, reference angles, and symmetry to find patterns on the unit circle. • Inverse trigonometric functions have restricted domains and ranges and are one to one.	• Find and interpret key features of a graph or table of a function, including extreme values, end behavior, and intervals of increase and decrease. • identify and graph parent functions and their transformations (i.e. vertical translation, horizontal translation, vertical stretch/shrink, reflect over x-axis, etc.). • Calculate values of sine, cosine, and tangent for given angles. • Solve trigonometric equations, including those written in quadratic form and equations containing more than one angle.	• Build on students' experience with the following skills: ○ Graphing on the coordinate plane (6.NS.C.8) ○ Building and solving proportional relationships (7.RP.A.2) ○ Recognize and use function notation (HSF.IF.A) ○ Recognizing inverse functions (HSF.BF.B.4) ○ Graphing, solving and modeling quadratic, linear, exponential and absolute value functions (HSF.LE.A, HSF.LE.B) ○ Recognize, build and solve trigonometric ratios (HSG.CRT.C) • Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas • Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers ○ SOH CAH TOA
Re-Teach		

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?	For example, students may benefit from re-engaging with content during a unit on building new functions from existing functions by revisiting student thinking through a short mini-lesson because students will be able to activate prior learning on functions and make the connection between the content.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit building new functions from existing functions by offering opportunities to understand and explore different strategies because providing students with various strategies allows for further depth in understanding and further delving into the depth of the content.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension such as the opportunity to explore links between various topics when building new functions from existing functions because their learning potential is increased, and prior knowledge activation is improved upon and built upon.