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New Mexico Instructional Scope

Algebra 2 Complex Number System Guide

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

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Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Perform arithmetic operations with complex numbers.
  - HSN.CN.A.1
  - HSN.CN.A.2
- Use complex numbers in polynomial identities and equations.
  - HSN.CN.C.7

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Grade	*CCSS Domain*	*CCSS Cluster*
A2	**The Complex Number System**	**Perform arithmetic operations with complex numbers**

Cluster Standard: HSN.CN.A.1

Standard	**Standards for Mathematical Practice**
Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	• **SMP1:** Make sense of problems and persevere in solving them. • **SMP2:** Reason abstractly and quantitatively. • **SMP6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Complex numbers expand the number system to include square roots of negative numbers and allow applications of complex numbers to electronics. Students use the properties of operations as it applies to complex numbers to simplify expressions and to build foundations to solve quadratic equations having complex solutions.	• Identify the real number and the imaginary number of a complex number • Define an imaginary number (i.e. $i^2 = -1$). • Define complex numbers. • Find the complex conjugate. • Describe complex numbers in terms of their real and imaginary parts.
DOK	**Blooms**
1-2	Understand

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Grade	*CCSS Domain*	*CCSS Cluster*
A2	**The Complex Number System**	**Perform arithmetic operations with complex numbers**

Cluster Standard: HSN.CN.A.2

Standard	Standards for Mathematical Practice
Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 2:** Reason abstractly and quantitatively. • **SMP 6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Complex numbers expand the number system to include square roots of negative numbers and allow applications of complex numbers to electronics. Students use the properties of operations as it applies to complex numbers to simplify expressions and to build foundations to solve quadratic equations having complex solutions.	• Recognize that $i^2 = -1$ • Use the properties of operations to add and subtract complex numbers • Use the distributive property and the relation $i^2 = -1$ to multiply complex numbers. • Apply the commutative, associative, and distributive properties to complex numbers in order to add, subtract, and multiply.
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Since most variables are letters and symbols, students may confuse *i* as a variable.
- Students may try to simplify a complex number by combining the real part and the imaginary part.

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Grade	*CCSS Domain*	*CCSS Cluster*
A2	**The Complex Number System**	**Use complex numbers in polynomial identities and equations**

Cluster Standard: HSN.CN.C.7

Standard	**Standards for Mathematical Practice**
Solve quadratic equations with real coefficients that have complex solutions.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 3**: Construct viable arguments and critique the reasoning of others.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students will be able to use multiple methods to solve quadratic equations with complex solutions.	• Determine the number and nature of quadratic solutions • Solve a quadratic equation using various methods (e.g., factoring, completing the square, quadratic formula)
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Students may confuse non-real, imaginary and irrational numbers.

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ASSESSMENT GUIDE

- Perform arithmetic operations with complex numbers.
- Use complex numbers in polynomial identities and equations.

Grade	*CCSS Domain*	*CCSS Strand*
A2	**The Complex Number System**	Perform arithmetic operations with complex numbers
	Sample Task #1 (Constructed Response)	

CollegeBoard **Question ID 5344950**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Additional Topics in Math	Hard	Additional Topics in Math	Complex numbers	1. Apply knowledge and understanding of the complex number system to add, subtract, multiply, and divide with complex numbers and solve problems.	No Calculator

$i^2+(-i)^2$

In the complex number system, what is the value of the given expression? (Note: $i=\sqrt{-1}$)

Question Difficulty: Hard

A. -2

B. 0

C. 2

D. $2i$

Choice A is correct. The power of a product property states that $(xy)^a=x^ay^a$. Using this property, the second term of the given expression can be rewritten as $(-1\times i)^2=(-1)^2i^2$, or i^2. Substituting i^2 in place of $(-i)^2$ in the given expression yields i^2+i^2, or $2i^2$. Since $i=\sqrt{-1}$, $i^2=-1$ and $2i^2=2(-1)$, or -2.

Choice B is incorrect and may result from rewriting $(-i)^2$ as $-i^2$ instead of i^2. Choice C is incorrect and may result from rewriting i^2 as 1 instead of -1. Choice D is incorrect and may result from rewriting i^2 as i instead of -1.

Grade	*CCSS Domain*	*CCSS Strand*
A2	**The Complex Number System**	**Use complex numbers in polynomial identities and equations**

Sample Task #1 (Constructed Response)

Renee reasons as follows to solve the equation $x^2+x+1=0$.

First I will rewrite this as a square plus some number.

$x^2+x+1=(x+12)^2+3/4$

	Now I can subtract 34 from both sides of the equation $(x+12)^2=-3/4$. *But I can't take the square root of a negative number so I can't solve this equation.* a. Show how Renee might have continued to find the complex solutions of $x^2+x+1=0$. b. Apply Renee's reasoning to find the solutions to $x^2+4x+6=0$. http://tasks.illustrativemathematics.org/content-standards/HSN/CN/C/7/tasks/1690

MLSS AND CLR GUIDE

- Perform arithmetic operations with complex numbers.
- Use complex numbers in polynomial identities and equations.

CCSS Domain	*CCSS Cluster*
The Complex Number System	**Perform arithmetic operations with complex numbers**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on performing arithmetic operations with complex numbers, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students, learning about the history of the complex number systems and how the complex numbers originated and used in other countries. Different families can contribute small history pieces and will eventually turn into a big presentation to the class by the student.	
Cross-Curricular Connections	**Science -** Science and Electrical Engineering use complex numbers, especially when dealing with light and radio wave. http://faculty.wcas.northwestern.edu/~infocom/Ideas/electric.html **History -** The ancient Greeks once believed that all numbers were rational numbers; that is, that every number could be expressed as the ratio of two integers, and they were very disturbed when it was demonstrated that the measure of the hypotenuse of an isosceles right triangle, having arms of unit measure, was not a rational number. http://mathforum.org/library/drmath/view/55747.html	
Validate/Affirm/Build/Bridge	• *How can you design*	• Eliciting and Using Evidence of Student Thinking:

	your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? ● *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time." For example, when performing arithmetic operations with complex numbers, eliciting and using student thinking is critical because when making mistakes and finding errors, students make adjustments and begin asking questions without any repercussions. They are comfortable and know mistakes are allowed and corrections can be made. Mistakes allow students to try instead of leaving questions blank. Challenging questions can also lead to critical thinking and when a task is complete, whether it's right or wrong, students feel the ownership of learning.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
● In Algebra 1, students solved quadratic equations using a variety of methods. Their solutions however were limited to real solutions.	● Students will learn to solve quadratic and higher-order polynomial equations that have complex answers as those found within this cluster.	● Students will relate this knowledge of complex numbers to solving rational equations, trigonometric equations and trigonometric form in subsequent math courses (Pre-Calculus, AP Calculus, College Algebra, etc).

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that introduces new representations (e.g., number lines) when studying to perform arithmetic operations with complex numbers because students no longer will be using real numbers on both axes in their graphs. The y-axis will be used for the imaginary numbers.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.NS.A.1: This standard provides a foundation for work with performing arithmetic operations with complex numbers because all numbers are classified as rational or irrational. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on performing arithmetic operations with complex numbers by revisiting student thinking through a short mini-lesson because one of the students' misconception is that the i is another variable. Check for misconceptions using aggressive monitoring.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit performing arithmetic operations with complex numbers by offering opportunities to understand and explore different strategies because students may make the connection between the properties of equations and the procedures within the complex number operations.
Extension		

Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the opportunity to explore links between various topics because students explore how the operations will be used in later lessons by watching a short video.

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CCSS Domain	*CCSS Cluster*
The Complex Number System	**Use complex numbers in polynomial identities and equations**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on using complex numbers in polynomial identities and equations, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, learning about the different complex numbers and making reference to how they could be used at home or in the community can be a great way to connect the tasks to their own personal tasks.	
Cross-Curricular Connections	During a unit focused on using complex numbers in polynomial identities and equations, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, learning about the different complex numbers and making reference to how they could be used at home or in the community can be a great way to connect the tasks to their own personal tasks.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical*	• Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when using complex numbers in polynomial identities and equations, goal setting is critical because students are able to make connections to their learning and prior knowledge can be accessed when goals are clearly identified.

	identities as capable mathematicians that can use mathematics within school and society?	

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In Algebra 1, students solved quadratic equations using a variety of methods. Their solutions were limited however to real solutions.	• Students learn to solve polynomial equations that have complex answers.	• Students will connect this knowledge of complex numbers to solving rational equations, trigonometric equations and trigonometric form in subsequent math courses (Pre-Calculus, AP Calculus, College Algebra, etc).

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that uses complex numbers in polynomial identities and equations because students will have to recall prior knowledge from previous grade levels.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	7.EE.A.1: This standard provides a foundation for work using complex numbers in polynomial identity and equations because students should be able to apply properties of operation strategies. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

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Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on using complex numbers in polynomial identities and equations by providing specific feedback to students on their work through a short mini-lesson because students who are having difficulty or who may be struggling will be able to get immediate feedback which will help them to better understand possible misconceptions.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit using complex numbers in polynomial identities and equations by offering opportunities to understand and explore different strategies. Students will be able to visualize different perspectives with the different strategies and may get a better understanding of the content being presented.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when using complex numbers in polynomial identities and equations because students will be able to direct their studying to the specific areas that they need further clarification in.