New Mexico Instructional Scope

Algebra 2 Reasoning with Equations and Inequalities Guide

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

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Algebra 2 Reasoning with Equations and Inequalities Guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Understand solving equations as a process of reasoning and explain the reasoning.
  - REI.A.2
- Represent and solve equations and inequalities graphically.
  - HSA.REI.D.11

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Grade	*CCSS Domain*	*CCSS Cluster*
A2	**Reasoning with Equations and Inequalities**	**Understand solving equations as a process of reasoning and explain the reasoning**

Cluster Standard: REI.A.2

Standard	Standards for Mathematical Practice
Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	• **SMP 6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• This cluster builds on the framework of solving equations and extends it to rational and radical equations (and the knowledge of extraneous solutions). • Equations are solved as a process of reasoning using properties of operations and equality, which can justify each step of the process. Students solve simple rational and radical equations using a variety of methods and explain why and where in the solution process the extraneous solution arose.	• Determine the domain of a rational function. • Determine the domain of a radical function. • Solve radical equations in one variable. • Solve rational equations in one variable. • Explain and give examples how extraneous solutions may arise when solving rational and radical equations.
DOK	**Blooms**
1-3	Apply, Analyze

Common Misconceptions

- Students may struggle identifying when there is an extraneous solution.
- Struggle with expression versus equation.
- When students multiply or divide both sides of an inequality by a negative value, they forget to reverse the inequality symbol.
- Students may sometimes forget to consider the cases when the LCD is positive and negative when solving a

rational inequality algebraically,

Grade	*CCSS Domain*	*CCSS Cluster*
A2	**Reasoning with Equations and Inequalities**	**Represent and solve equations and inequalities graphically**

Cluster Standard: HSA.REI.D.11

Standard	Standards for Mathematical Practice
Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 5:** Use appropriate tools strategically. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• HSA.REI.D.11: Just as the **algebraic** work with **equations** can be reduced to a series of algebraic moves unsupported by reasoning, so can the **graphical visualization of solutions**. The simple idea that an equation $f(x) = g(x)$ can be **solved (approximately)** by graphing $y = f(x)$ and $y = g(x)$ and finding the **intersection points** involves a number of pieces of conceptual understanding. [This method] seeks to convert an equation in one variable, $f(x) = g(x)$, to a s**ystem of equations** in two **variables**, $y = f(x)$ and $y = g(x)$, by introducing a second variable y and **setting it equal** to each side of the equation. If x is a solution to the original equation, then $f(x)$ and $g(x)$ are equal, and thus **(x, y)** is a solution to the new system.	• Recognize what the solution $y = f(x)$ and $y = g(x)$ means on a graph. • Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. • Find approximate solutions for the system $y = f(x)$ and $y = g(x)$ using graphs and tables. • Find successive approximations and use them to solve the system $y = f(x)$ and $y = g(x)$. • Use paper/pencil or technology to produce a table of values. • Explain what x-coordinate of a common ordered pair represents in the context of the problem.
DOK	**Blooms**

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1-3	Understand, Apply, Analyze, Evaluate

Common Misconceptions

- Students often interpret the solutions to an equation or graphical representation of an equation as only integer values.
- Students may believe an estimate of a value between two integer points is sufficient, but the standard states that students should find successive approximations to approximate the solution.
- Students believe the graph of a function is simply a line or curve "connecting the dots," without recognizing that the graph represents all solutions to the equation.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Reasoning with Equations & Inequalities**	Strand: **Understand solving equations as a process of reasoning and explain the reasoning**

Suggested Student Discourse Questions

- Why do we see extraneous solutions to this _____ (rational / radical) equation? Explain your thinking using mathematical vocabulary.
- Turn and talk to your partner, and come up with a method to identify extraneous solutions to this _____ (rational / radical)
- How are extraneous solutions to this _____ (rational / radical) equation identified graphically? Algebraically? Using a table?
- Given the area of _____ (classroom, your bedroom, football field, grocery store,

equation. How is your method different from that of another group's? How is it similar?	lake), and one side length, why must the width be a positive measure? How is this modeled in a radical equation?

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Algebra 2 Reasoning with Equations and Inequalities Guide

Domain: **Reasoning with Equations & Inequalities**	Strand: **Represent and solve equations and inequalities graphically**
Suggested Student Discourse Questions	
• In what ways are inequalities different from equations? In what way are they the same? • Turn and talk with your partner. How did they work this problem? How is that different from your method? Which method do you prefer? Why?	• How does the graph of this _____ (equation / inequality) model the equation? • How does a linear inequality model _____? (Ex: correct combination of drugs to give to a patient, legal speed limit on a highway, data limit on cell phone plans, time needed to walk to school, maximum profit based on materials used)

NEW MEXICO
Public Education Department

ASSESSMENT GUIDE

- Understand solving equations as a process of reasoning and explain the reasoning.
- Represent and solve equations and inequalities graphically.

Grade	*CCSS Domain*	*CCSS Strand*
A2	**Reasoning with Equations and Inequalities**	**Understand solving equations as a process of reasoning and explain the reasoning**

Sample Task #1 (Constructed Response)

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: An Extraneous Solution Illustrative Mathematics

Megan is working solving the equation

$$\frac{2}{x^2-1} - \frac{1}{x-1} = \frac{1}{x+1}.$$

She says

> ***If I clear the denominators I find that the only solution is $x = 1$ but when I substitute in $x = 1$ the equation does not make any sense.***

a. Is Megan's work correct?

b. Why does Megan's method produce an x value that does not solve the equation?

IM Commentary
The goal of this task is to examine how extraneous solutions can arise when solving rational equations. The task presents an operation, "clearing denominators," which appears to lead to a contradiction. To resolve the contradiction, we examine more carefully what is happening when we clear denominators (MP6). One way to describe the process is that we find a common denominator for both sides and set the numerators equal to each other. This gives solutions to the original equation provided the solutions are in the domain of the rational functions on both sides, that is, provided they are not zeros of one or more of the denominators. In this case, the solution x=1 makes the numerators equal to one another but also makes the denominators of two of the expressions zero, and so x=1 is an extraneous solution.

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Grade	*CCSS Domain*	*CCSS Strand*
A2	**Reasoning with Equations and Inequalities**	**Represent and solve equations and inequalities graphically**

Sample Task #1 (Multiple Choice)

Source: SAT

$$x+1=\frac{2}{x+1}$$

Question 1474935 Answers

In the equation above, which of the following is a possible value of $x+1$?

- A. $1-\sqrt{2}$
- B. $\sqrt{2}$
- C. 2
- D. 4

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Algebra 2 Reasoning with Equations and Inequalities Guide

MLSS AND CLR GUIDE

- Understand solving equations as a process of reasoning and explain the reasoning.
- Represent and solve equations and inequalities graphically.

CCSS Domain	*CCSS Cluster*
Reasoning with Equations and Inequalities	**Understand solving equations as a process of reasoning and explain the reasoning**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on understanding solving equations as a process of reasoning and explaining the reasoning, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, some students learn differently when it comes to expressing their ideas, linguistically speaking they learned different languages. Learning about the different structures for the process and explaining it across the languages in the classroom can lead to a more robust understanding of solving equations for all students by making connections to the different structures of understanding solving equations in other languages.	
Cross-Curricular Connections	This lesson uses right triangle trigonometry and a rational function to explore the percent of your visual field that is occupied by the area of a television. Sofa Away From Me: A Lesson by Mathalicious	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?*	• Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their “mathematical, social, and cultural competence”. By valuing these representations and

	• *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying understanding solving equations as a process of reasoning and explaining the reasoning the use of mathematical representations within the classroom is critical because linguistically and culturally speaking, experiences of students provide different and varied types of representations for solving mathematical problems varies on different types of exposures they have in their home and communities. Students' abilities in explaining mathematical reasoning varies on their language preferences and representations.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In Algebra 1, students solved linear and quadratic equations.	• In this course, students will extend their solving skills learned in Algebra 1 to rational and radical equations. Students will also relate the solving of other nonlinear equations learned in this course to solving rational and radical equations.	• In future math classes students will solve more challenging nonlinear equations, including trigonometric equations.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this*	Some learners may benefit from targeted pre-teaching that focuses on solving equations and explaining each step because students may need to justify the inverse operation used in each step with viable arguments.

	cluster within your HQIM?	Students may practice expressing their mathematical thinking verbally and symbolically.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	6.EE.B.5: This standard provides a foundation for work with reasoning and solving one-variable equations because students need to understand each step of solving one-variable equations and explain the reason for each step. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework

A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• Different forms of an expression can be equivalent and are useful in different contexts.
• The addition, subtraction, multiplication, or division of rational expressions results in another rational expression.
• When a situation and its potential constraints will be represented by all available types of equations/inequalities, including simple root function, or a system of those equations/inequalities.
• When solving graphically/with a table is more efficient than
	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form.
	• Perform the operations of addition, subtraction, multiplication, and division with rational expressions.
	• Determine reasonable solutions based on the context of real-world problems from graphs of equations/inequalities and systems of equations/inequalities.
	• Solve systems using a graph and a table as well as rewrite an equation as two functions (and vice
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
		  ○ Solving systems of equations / inequalities (8.EE.C.8)
		  ○ Adding / subtracting / multiplying / dividing and simplify fractions
		  ○ Writing and solving one-step and two-step equations (HSA.REI.B.3, HSA.REI.B.4)
		  ○ Modeling linear, exponential, quadratic and absolute value functions (HSF.LE.A, HSF.LE.B)
		  ○ Different forms of linear (linear standard form, point-slope form, slope intercept form) and quadratic equations (quadratic standard form and vertex form) (HSF.LE.A)
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of

solving algebraically.	versa).	representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on explaining the reason of each step of solving equations by critiquing student approaches/solutions to make connections through a short mini-lesson because students need to understand why the specific inverse operation is used and develop the viable argument using properties of equality.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit explaining the steps of solving equations by offering opportunities to understand and explore different strategies because students need to understand why some steps are interchangeable when solving the equations. Students need to explain the order of applying the inverse operations and how that relates to the order of operation of the equations.

Extension

Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when solving complex equations because students may deepen

	their understanding of inverse operation (e.g. logarithm as the inverse operation of exponent). Students explore strategies of solving equations with complex operations and justify their reason in cooperative learning groups.

CCSS Domain	*CCSS Cluster*
Reasoning with Equations and Inequalities	**Represent and solve equations and inequalities graphically**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on representing and solving equations and inequalities graphically, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, bring in the different languages spoken in the home and connect it to the tools available to translate different languages, i.e. Google translate, closed captions on televisions, etc. make connections that show that in the culture of mathematics, tools are used to translate mathematics and help us make sense of what we are seeing.	
Cross-Curricular Connections	Students will model projectile motion in both function and parametric graphing. This was designed as an in-class modeling activity to be used prior to actually launching air-powered projectile rockets. A set of data is given in a spreadsheet and students create model functions using a variety of methods: vertex form (then using grab and move to fit the curve to the data points), standard form (using matrices), and quadratic regression. Rocket Simulation Activity	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures*	• Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student

	and languages? • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when representing and solving equations and inequalities graphically the use of mathematical representations within the classroom is critical. When students are given a situation in two variables, they must find the value of one variable given the value of the other, create an equation to represent the situation, use technology to create a graph, and interpret each representation. Understanding how lines and tables represent solution sets of linear relationships will help students make sense of graphs of solutions to linear inequalities, and later, to make sense of solutions to systems of linear equations in their Algebra 1 class.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to the work of Algebra 1 in this cluster around linear and exponential. **(HSA.REI.D)**	• Connect this cluster across all of Algebra 2, particularly when each new function is presented.	• Connect this cluster to future work around determining specific solutions to new functions (i.e. zeros). Also, in Calculus, when students discuss the area between two curves and volume with rotation.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this*	Some learners may benefit from targeted pre-teaching that introduces new representations (e.g., number lines) when representing and solving equations and inequalities graphically. Different representations within the same

	cluster within your HQIM?	problem allow students to make connections between the graph, table, word problem, and equations. When two expressions are equal to each other, the variable equal to a numerical value is the solution algebraically and graphically.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	6.EE.B.5: This standard provides a foundation for work to represent and solve equations and inequalities graphically because by substituting numerical values into an equation to determine if the equation is true, the student will comprehend that the answer is a solution. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework

A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• Different forms of an expression can be equivalent and are useful in different contexts.
• The addition, subtraction, multiplication, or division of rational expressions results in another rational expression.
• When a situation and its potential constraints will be represented by all available types of equations/inequalities, including simple root function, or a system of those equations/inequalities.
	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form.
	• Perform the operations of addition, subtraction, multiplication, and division with rational expressions.
	• Determine reasonable solutions based on the context of real-world problems from graphs of equations/inequalities and systems of equations/inequali
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
		  ○ Solving systems of equations / inequalities (8.EE.C.8)
		  ○ Adding / subtracting / multiplying / dividing and simplify fractions
		  ○ Writing and solving one-step and two-step equations (HSA.REI.B.3, HSA.REI.B.4)
		  ○ Modeling linear, exponential, quadratic and absolute value functions (HSF.LE.A, HSF.LE.B)
		  ○ Different forms of linear (linear standard form, point-slope form, slope intercept form) and quadratic equations (quadratic standard form and vertex form) (HSF.LE.A)
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies

● When solving graphically/with a table is more efficient than solving algebraically.	ties. ● Solve systems using a graph and a table as well as rewrite an equation as two functions (and vice versa).	○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on representing and solving equations and inequalities graphically by critiquing student approaches/solutions to make connections through a short mini-lesson because connections between solution (algebraically) and intersection (graphically) are equivalent. When students compare answers graphically and algebraically, intersections are the solution.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit to represent and solve equations and inequalities graphically by offering opportunities to understand and explore different strategies because interpreting solution using the different representations allows the students to visualize the answer that was only written as a system of equations or two expressions equal to each other. Students can check their work graphically to confirm their answer.
Extension		

Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the opportunity to explore links between various topics when representing and solving equations and inequalities graphically because different types of equations (logarithmic, exponential, trigonometric, etc.) can use the graphing method to find solutions to word problems or algebraically.