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Public Education Department

New Mexico Instructional Scope

Algebra 2 Trigonometric Functions Guide

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

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Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Extend the domain of trigonometric functions using the unit circle.
  - HSF.TF.A.1
  - HSF.TF.A.2
- Model periodic phenomena with trigonometric functions.
  - HSF.TF.B.5
- Prove and apply trigonometric identities
  - HSF.TF.C.8

Grade	*CCSS Domain*	*CCSS Cluster*
A2	**Trigonometric Functions**	**Extend the domain of trigonometric functions using the unit circle**

Cluster Standard: HSF.TF.A.1

Standard	**Standards for Mathematical Practice**
Understand the radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	• **SMP2:** Reason abstractly and quantitatively. • **SMP5:** Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students will be able to extend their knowledge of circle and trigonometric ratios (sine and cosine) to arc length, evaluating using a unit circle, and graphing trigonometric functions (sine and cosine).	• Find the measures of an angle in standard position and the reference angle. • Find arc length using radian measure on the unit circle. • Convert between degrees and radians.
DOK	**Blooms**
1-2	Understand, Apply

Grade	*CCSS Domain*	*CCSS Cluster*
A2	**Trigonometric Functions**	**Extend the domain of trigonometric functions using the unit circle**

Cluster Standard: HSF.TF.A.2

Standard	Standards for Mathematical Practice
Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	• **SMP 6:** Attend to precision. • **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students will be able to extend their knowledge of circle and trigonometric ratios (sine and cosine) to arc length, evaluating using a unit circle, and graphing trigonometric functions (sine and cosine).	• Identify the relationship between the unit circle and the coordinate plane.
DOK	**Blooms**
1-2	Understand

Common Misconceptions

- Students may confuse the direction of positive and negative angle measures, writing the wrong sign with the measure.
- Students may mix-up radian and degree measures

Grade	*CCSS Domain*	*CCSS Cluster*
A2	**Trigonometric Functions**	**Model periodic phenomena with trigonometric functions**

Cluster Standard: HSF.TF.B.5

Standard	Standards for Mathematical Practice
Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	• **SMP 4:** Model with mathematics. • **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students apply the concept of inverse functions to trigonometric functions and use that concept to solve problems.	• Graph sine and cosine functions using radian and degrees. • Identify properties of the sine function. • Model a real-world situation using trigonometric functions. Students can then use inverse trigonometric functions to find solutions.
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Students may mix-up sine and cosine.

Grade	*CCSS Domain*	*CCSS Cluster*
A2	**Trigonometric Functions**	**Prove and apply trigonometric identities**

Cluster Standard: HSF.TF.C.8

Standard	**Standards for Mathematical Practice**
Prove the Pythagorean identity $\sin^2(\vartheta) + \cos^2(\vartheta) = 1$ and use it to find $\sin(\vartheta)$, $\cos(\vartheta)$, or $\tan(\vartheta)$ given $\sin(\vartheta)$, $\cos(\vartheta)$, or $\tan(\vartheta)$ and the quadrant of the angle.	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 5:** Use appropriate tools strategically. • **SMP 6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students will make connections between their knowledge of the Pythagorean Theorem, trigonometric ratios, the unit circle and coordinate plane.	• Use the concepts from the Pythagorean identity to calculate trigonometric ratios in any quadrant on the coordinate plane
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Students may struggle to explain how the identities frame responses.

ASSESSMENT GUIDE

- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions.
- Prove and apply trigonometric identities.

Grade	*CCSS Domain*	*CCSS Strand*
A2	**Trigonometric Functions**	**Extend the domain of trigonometric functions using the unit circle**

Sample Task #1 (Constructed Response)

CollegeBoard **Question ID 423225**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Additional Topics in Math	Hard	Additional Topics in Math	Circles	6. Convert between angle measures in degrees and radians.	No Calculator

The number of radians in a 720-degree angle can be written as $a\pi$, where a is a constant. What is the value of a ?

Question Difficulty: Hard

The correct answer is 4. There are π radians in a $180°$ angle. An angle measure of $720°$ is 4 times greater than an angle measure of $180°$. Therefore, the number of radians in a $720°$ angle is 4π.

This type of assessment question requires students to translate an angle measurement from degrees to radians. Students may use a conversion factor to do so or may use the fact that pi radians is equivalent to 180 degrees,

Grade	*CCSS Domain*	*CCSS Strand*
A2	**Trigonometric Functions**	**Model periodic phenomena with trigonometric functions**

Sample Task #1 (Constructed Response)

Standards Aligned Instructionally Embedded Formative Assessment Resources:

http://tasks.illustrativemathematics.org/content-standards/HSF/TF/B/5/tasks/816

This type of assessment question requires students to model the population of two species using trigonometric functions given a table of data. Since students may use positive or negative sine and cosine functions to model the data, consider having students work in corporate groups and comparing solutions between groups.

Students will engage with SMP1, SMP4 and if comparing work with other groups, SMP3.

Grade	*CCSS Domain*	*CCSS Strand*
A2	**Trigonometric Functions**	**Prove and apply trigonometric identities**

Sample Task #1 (Constructed Response)

Standards Aligned Instructionally Embedded Formative Assessment Resources:

http://tasks.illustrativemathematics.org/content-standards/HSF/TF/C/8/tasks/1835

This type of assessment question requires students to take a given ratio for a trigonometric function and use it to exactly state the value of two other trigonometric functions. This will require knowledge of the trigonometric functions as ratios of side lengths and possibly trigonometric identities. Students will engage with SMP 7 as they use the structure of the ratio to determine the remaining trigonometric values.

Additional Assessment:

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	https://www.map.mathshell.org/lessons.php?unit=9255&collection=8&redir=1

MLSS AND CLR GUIDE

- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions.
- Prove and apply trigonometric identities.

CCSS Domain	*CCSS Cluster*
Trigonometric Functions	**Extend the domain of trigonometric functions using the unit circle**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on extending the domain of trigonometric functions using the unit circle, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, periodicity refers to a pattern that repeats over time. Consider providing examples of numeric and non-numeric patterns that exist (springs bouncing, pendulums swinging, temperature fluctuations throughout the year, hunger growing and decreasing as you approach mealtime) and then ask students to provide some examples of their own.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of*	• Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion, we position them as knowers and doers of mathematics. Using equitable talk moves students in how they talk about who is and isn't capable of mathematics. As a result, we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example,

	your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	when extending the domain of trigonometric functions using the unit circle facilitating meaningful mathematical discourse is critical because some students will naturally see and understand the patterns of periodic functions quicker than others. Eliciting peer-to-peer explanations can build a cooperative environment in the classroom. Further, by providing sentence frames for students, all students can engage in mathematical conversations, even if they are not sure what the solution is. When students speak about mathematics, they engage meaningfully in the content and deepen their understanding.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In Geometry, students learned the relationship of trigonometric ratios and special right triangles (which leads to evaluating on the unit circle). Students also learned about radian measure and how to convert to degree measure.	• Students will use this knowledge of trigonometric functions (sine and cosine) and apply transformations and identify key features of these trigonometric functions. They will also be able to evaluate trig ratios on a coordinate plane (not on the unit circle).	• In Precalculus and Calculus courses, students will connect this learning cluster to other trigonometric ratios and functions (tangent, cosecant, secant and cotangent) and inverse trigonometric functions.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when extending the domain of trigonometric functions using the unit circle because this is the first time many students will be working with a unit circle so providing that visual at the very beginning and explaining its purpose can be helpful in later parts of the lessons

Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	F.IF.A1: This standard provides a foundation for work with extending the domain of trigonometric functions using the unit circle because this is the standard where students gain conceptual understanding of domain and range as sets of inputs and outputs for a given function. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework

A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• What the domain of a function represents in terms of the function and what values are appropriate (i.e. discrete v. continuous values and other restrictions based on the type of function and the given context).
• In functions there is an underlying structure that determines the transformation of any function, regardless of its type.
• How to use trigonometric ratios, reference angles, and symmetry to find patterns on the unit circle.
• Inverse trigonometric functions have restricted domains and ranges and are one to one.
	• Find and interpret key features of a graph or table of a function, including extreme values, end behavior, and intervals of increase and decrease.
	• identify and graph parent functions and their transformations (i.e. vertical translation, horizontal translation, vertical stretch/shrink, reflect over x-axis, etc.).
	• Calculate values of sine, cosine, and tangent for given angles.
	• Solve trigonometric equations, including those written in quadratic form and equations
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
		  ○ Building and solving proportional relationships (7.RP.A.2)
		  ○ Recognize and use function notation (HSF.IF.A)
		  ○ Recognizing inverse functions (HSF.BF.B.4)
		  ○ Graphing, solving and modeling quadratic, linear, exponential and absolute value functions (HSF.LE.A, HSF.LE.B)
		  ○ Recognize, build and solve trigonometric ratios (HSG.CRT.C)
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of representation for mathematical ideas
		• Encourage students to use alternative tools to better access the grade level content. Examples include:
		  ○ Desmos.com

	containing more than one angle.	○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers ○ SOH CAH TOA

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on extending the domain of trigonometric functions using the unit circle by providing specific feedback to students on their work through a short mini-lesson because students may have a fear of trigonometry that may affect their willingness to interact with the content. Providing student specific feedback, identifying things students are doing correctly and/or effort students are putting forth can encourage students to continue engaging with the material. There are many opportunities for students to make minor errors and if this is the focus, students may shut down. Focused and encouraging feedback can help to counteract any feelings that arise as errors are made.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit extending the domain of trigonometric functions using the unit circle by addressing conceptual understanding because the goal of this cluster is understanding the basis of periodic functions, rather than procedural computations. Students may lack conceptual understanding of domain/range, radian measures of angles and/or the periodicity of functions. Each of these concepts should be explored so students can adequately express their understanding.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as extending the domain of trigonometric functions using the unit circle because once students understand the

	behavior of periodic functions, they can extend the pattern on their own with little guidance from the teacher.

CCSS Domain	*CCSS Cluster*
Trigonometric Functions	**Model periodic phenomena with trigonometric functions**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on modelling periodic phenomena with trigonometric functions, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. Have students provide examples of music they enjoy and discuss how music creates sound waves that are modeled by trigonometric functions. Using free online tools, you can create sound wave images (as a class or on an individual basis) and discuss the features of these images as they relate to periodic functions.	
Cross-Curricular Connections	Many of the Navajo rug designs you will discover by following the project will be good examples of symmetrical balance. Symmetrical balance is a type of visual balance where the overall composition is arranged to look like it is the same on both sides of the center of the design. In other words, it is a design which could be folded in half, and as the design folds, each part of the design would match up with its symmetrical counterpart on the opposite side of the center. The rug design on the right is symmetrical left-to-right. If a line was drawn vertically down the center of the rug, the arrangement of shapes and colors would appear to be exactly the opposite of each other on both sides of that line. http://web.archive.org/web/20080130134231/http:/www.mpsaz.org/arts/elements/balance/page1.html	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures*	• Task: When planning with your HQIM, consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)." For example, when studying modelling periodic phenomena with trigonometric functions the types of mathematical tasks are critical because we often experience patterns in the real-world,

	and languages? ● *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	whether it be related to science or societal issues. This is an opportunity for students to explore claims like "climate change is not real" or "the violent crime rate always rises in the warmer months" mathematically by attempting to apply features of periodic functions to describe them, or showing that these features do not exist within the data.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
● In Geometry and Algebra II, students have defined trigonometric ratios using the acute angles of right triangles	● In Algebra II, students define inverse functions	● Inverse trigonometric functions play a major role in Calculus, when using operations such as differentiation and integration

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that models periodic phenomena with trigonometric functions because there is new vocabulary introduced in this cluster that relates to the graphs of trigonometric functions. These terms will then be applied to contextual scenarios. By rehearsing how to precisely use these terms to describe graphs can support student's later work in using them to describe scenarios.

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Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	F-IF.C.7: This standard provides a foundation for work with modelling periodic phenomena with trigonometric functions because this standard is where students focus on describing key features of other function families. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework

A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• What the domain of a function represents in terms of the function and what values are appropriate (i.e. discrete v. continuous values and other restrictions based on the type of function and the given context).
• In functions there is an underlying structure that determines the transformation of any function, regardless of its type.
• How to use trigonometric ratios, reference angles, and symmetry to find patterns on the unit circle.
• Inverse trigonometric functions have restricted domains and ranges and are one to one.
	• Find and interpret key features of a graph or table of a function, including extreme values, end behavior, and intervals of increase and decrease.
	• identify and graph parent functions and their transformations (i.e. vertical translation, horizontal translation, vertical stretch/shrink, reflect over x-axis, etc.).
	• Calculate values of sine, cosine, and tangent for given angles.
	• Solve trigonometric equations, including those written in quadratic form and equations containing more
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
		  ○ Building and solving proportional relationships (7.RP.A.2)
		  ○ Recognize and use function notation (HSF.IF.A)
		  ○ Recognizing inverse functions (HSF.BF.B.4)
		  ○ Graphing, solving and modeling quadratic, linear, exponential and absolute value functions (HSF.LE.A, HSF.LE.B)
		  ○ Recognize, build and solve trigonometric ratios (HSG.CRT.C)
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of representation for mathematical ideas
		• Encourage students to use alternative tools to better access the grade level content. Examples include:
		  ○ Desmos.com
		  ○ Graphing calculator

	than one angle.	○ Sketch a graph ○ Create a table of values ○ Algebra tiles ○ Graphic organizers ○ SOH CAH TOA
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on modelling periodic phenomena with trigonometric functions by critiquing student approaches/solutions to make connections through a short mini-lesson because students may formulate solutions from multiple perspectives (table, graph, calculation, logical reasoning). Presenting multiple solution methods allows students to think from a new perspective and analyze the features of these functions in a new way. This may illuminate errors in their own work or in the work of others.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit modelling periodic phenomena with trigonometric functions by offering opportunities to understand and explore different strategies because students may, at times, have an easier time solving a problem using a table rather than a graph or equation and vice versa. Allowing students the opportunity to explore these different strategies and to discuss their usefulness will help students deepen their understanding of the concepts as well as build their skills of using different representations to solve problems.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when modelling periodic phenomena with trigonometric functions because once a student has a conceptual understanding of periodic functions, the applications are easy to see in science and career specific scenarios. Linking these can allow students to explore their

	interests beyond mathematics.

CCSS Domain	*CCSS Cluster*
Trigonometric Functions	**Prove and apply trigonometric identities**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on proving and applying trigonometric identities, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, connecting the process of proofs with the concept of replacing equivalent items can allow students to relate to the process. Students may have experienced this in recipes (replacing butter with oil), in making purchases (choosing one brand over another), etc. Use this as an opportunity to talk about equivalence. In some instances, the outcome may be changed by replacement, but in this mathematical process, equivalence is preserved.	
Cross-Curricular Connections	Economics – Substitution and utility when making purchases	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school*	• Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when proving and applying trigonometric identities, goal setting is critical because these algebraic proofs can be very challenging for students and if they do not have a productive mind frame as a starting point, they will struggle to make any progress. Encouraging students to set a goal to simply start these proofs by choosing to work from left to right or right to left or a goal to accurately rewrite an expression in terms of sin/cos can give students the support they need in engaging with difficult material. The importance is not that they can complete every proof, but rather that they have a goal they can achieve, and they work

	mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	mathematically toward that goal.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In Geometry, students learned the relationship of trigonometric ratios. In 8th grade, Algebra 1, and Geometry, students also learned the Pythagorean Theorem and how to graph on a coordinate plane	• Students will use their knowledge of the unit circle and relate that to determine trigonometric ratios not on the unit circle.	• In Precalculus and Calculus courses, students will connect this learning cluster to other trigonometric ratios (tangent, cosecant, secant and cotangent).

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when proving and applying trigonometric identities. Algebraic proofs can be very challenging for students and we want to confront that fact by providing extra supports and time for students to engage with the material, whether their work is exactly correct or not. The extra time experiencing the material will build deeper understanding.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.GB.7: This standard provides a foundation for work with proving and applying trigonometric identities because students used the visual structure of right triangles to apply the Pythagorean theorem. This process can be thought of as a concrete version of the algebraic

		process this cluster calls for. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework

A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• What the domain of a function represents in terms of the function and what values are appropriate (i.e. discrete v. continuous values and other restrictions based on the type of function and the given context).
• In functions there is an underlying structure that determines the transformation of any function, regardless of its type.
• How to use trigonometric ratios, reference angles, and symmetry to find patterns on the unit circle.
• Inverse trigonometric functions have restricted domains and ranges and are one to one.
	• Find and interpret key features of a graph or table of a function, including extreme values, end behavior, and intervals of increase and decrease.
	• identify and graph parent functions and their transformations (i.e. vertical translation, horizontal translation, vertical stretch/shrink, reflect over x-axis, etc.).
	• Calculate values of sine, cosine, and tangent for given angles.
	• Solve trigonometric equations, including those written in quadratic form and equations containing more than one angle.
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
		  ○ Building and solving proportional relationships (7.RP.A.2)
		  ○ Recognize and use function notation (HSF.IF.A)
		  ○ Recognizing inverse functions (HSF.BF.B.4)
		  ○ Graphing, solving and modeling quadratic, linear, exponential and absolute value functions (HSF.LE.A, HSF.LE.B)
		  ○ Recognize, build and solve trigonometric ratios (HSG.CRT.C)
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of representation for mathematical ideas
		• Encourage students to use alternative tools to better access the grade level content. Examples include:
		  ○ Desmos.com
		  ○ Graphing calculator
		  ○ Sketch a graph
		  ○ Create a table of values
		  ○ Algebra tiles
		  ○ Graphic organizers

		○ SOH CAH TOA
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on proving and applying trigonometric identities by critiquing student approaches/solutions to make connections through a short mini-lesson because students frequently are able to take certain steps in a proof but may find themselves feeling stuck. Whether this is because they have made an error or just cannot see the next step, showing these approaches to their peers can help students make connections between the work they did and that of others, as well as critique steps that are different.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit proving and applying trigonometric identities by confronting student misconceptions because students may expect that each proof will progress in the same format, or that identities will always appear in the same way. Every proof is different and showing students that identities can be applied in a variety of ways may help them feel freed from looking for specific instances of the identities.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students. For example, prove and apply trigonometric identities because when students see the patterns and process clearly, we should allow them to challenge themselves at their own pace to try increasingly more challenging proofs.