NEW MEXICO
Public Education Department

New Mexico Instructional S
Geometry Circles Gu

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

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Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Understand and apply theorems about circles
  - HSG.C.A.1
  - HSG.C.A.2
  - HSG.C.A.3
- Find arc lengths and areas of sectors of circles
  - HSG.C.B.5

Grade	*CCSS Domain*	*CCSS Cluster*
G	Circles	Understand and apply theorems about circle

Cluster Standard: HSG.C.A.1

Standard	Standards for Mathematical Practice
Prove that all circles are similar.	• **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP5:**Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Learners will apply concepts of similarity to circles and their related components, explore inscribed and circumscribed circles and their associated polygons through constructions. This cluster builds many of the basic properties for angles, lines, and segments related to circles. Learners explore those properties and form conjectures. They should then be encouraged to create justifications regarding why their conjectures are correct. Learners refer back to their work with transformations to better understand these relationships.	• Show that all circles are similar by proving that the ratio of a circle's circumference to its diameter for different sized circles is a constant • Calculate the circumference of a circle, given the diameter or radius. • Calculate angles inside and outside of a circle. • Prove that circles are similar. • Compare the ratios of the radius and circumference of multiple circles to determine similarity.
DOK	**Blooms**
3-4	Analyze, Evaluate

Grade	CCSS Domain	CCSS Cluster
G	Circles	Understand and apply theorems about circle

Cluster Standard: HSG.C.A.2

Standard	Standards for Mathematical Practice
Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	• **SMP1:** Make sense of problems and persevere in solving them. • **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP5:** Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Learners will apply concepts of similarity to circles and their related components, explore inscribed and circumscribed circles and their associated polygons through constructions. This cluster builds many of the basic properties for angles, lines, and segments related to circles. Learners explore those properties and form conjectures. They should then be encouraged to create justifications regarding why their conjectures are correct. Learners refer back to their work with transformations to better understand these relationships.	• Identify central angles, inscribed angles, circumscribed angles, tangent line and chords on a circle from a drawing. • Construct and explain examples of central angles, inscribed angles, circumscribed angles, tangent line and chords on a circle. • Describe the relationship between central angles, inscribed angles, circumscribed angles, tangent lines, and chords.
DOK	**Blooms**
1-2	Understand, Apply

Grade	*CCSS Domain*	*CCSS Cluster*
G	Circles	Understand and apply theorems about circle

Cluster Standard: HSG.C.A.3

Standard	Standards for Mathematical Practice
Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	• **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP5:** Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Learners will apply concepts of similarity to circles and their related components, explore inscribed and circumscribed circles and their associated polygons through constructions. This cluster builds many of the basic properties for angles, lines, and segments related to circles. Learners explore those properties and form conjectures. They should then be encouraged to create justifications regarding why their conjectures are correct. Learners refer back to their work with transformations to better understand these relationships.	• Construct angle bisectors, perpendicular bisectors, inscribed circle of a triangle and circumscribed circle about a triangle. • Prove that opposite angles of an inscribed quadrilateral are supplementary.
DOK	**Blooms**
1-3	Apply, Analyze

Common Misconceptions

- Students may try to solve by sketching the circles instead of ensuring precision by using a compass or appropriate tool to construct an accurate circle.
- Students might confuse the relationships of central angle, inscribed angles, circumscribe angles, as well as tangent line and chords of a circle.

Grade	*CCSS Domain*	*CCSS Cluster*
G	Circles	Find arc lengths and areas of sectors of circl

Cluster Standard: HSG.C.B.5

Standard	Standards for Mathematical Practice
Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	• **SMP2:** Reason abstractly and quantitatively. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• This cluster explores the relationship between the length of an arc and the measure of a central angle. Learners develop a definition for the radian measure of an angle and apply radians to find the area of sectors.	• Calculate the length of an intercepted arc. • Demonstrate that the constant of proportionality between arc length and the radius of the circle is the radian measure of the central angle. • Derive the formula for the area of a sector using similarity. • Calculate the area of a sector.
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Students often struggle with precision while working within this cluster. Small errors in constructions will lead to results that do not work.

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thi critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful imple classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse they find various path and reveal knowledge or misunderstandings to educators. The process also allows educators to know students culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mec developing conceptual understanding and meaningful learning of mathematics (Michaels, O'C Resnick, 2008)

Domain: Circles	Strand: Understand and apply theorems about c

Suggested Student Discourse Questions

- Share the process you used to prove the ci similar. What feedback do you have for the
- How do you know that distances on a globe represent distances in real life?
- Which technique for determining similarity between circles is easier for you: rigid transformations on the plane or manipulatio equations?
- How are the radius and diameter of similar related to each other?

ASSESSMENT GUIDE

- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circle

Grade	*CCSS Domain*	*CCSS Strand*
G	Circles	Understand and apply theorems about circ

Sample Task #1 (Constructed Response)

Standards Aligned Instructionally Embedded Formative Assessment Resources:

SAT Item # 422459: The linked assessment question addresses G-C.A., specifically the question requires students to use knowledge of inscribed and central angles.

CollegeBoard **Question ID 422459**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Additional Topics in Math	Hard	Additional Topics in Math	Circles	1. Use definitions, properties, and theorems relating to circles and parts of circles, such as radii, diameters, tangents, angles, arcs, arc lengths, and sector areas to solve problems.	Calculator

Point P is the center of the circle in the figure above. What is the value of x ?

Question Difficulty: Hard

The correct answer is 80. If points A and P are joined, then the triangles that will be formed, APB and APC, are isosceles because $PA = PB = PC$. It follows that the base angles on both triangles each measure $20°$. Angle BAC consists of two base angles; therefore, the measure of angle BAC = $40°$. Since the measure of an angle inscribed in a circle is half the measure of the central angle that intercepts the same arc, it follows that the value of x is $80°$.

Additional Assessment:

	Additional Assessment: http://tasks.illustrativemathematics.org/content-standards/HSG/C/A/1/tasks/1368 The linked assessment question addresses G-C.A, specifically the question requires students to make use of visualizing transformations as well as knowledge of equations for a circle. This assessment should be given to students after they've been introduced to these concepts. Students will engage in SMP1 and SMP8.

Grade	*CCSS Domain*	*CCSS Strand*
G	Circles	Find arc lengths and areas of sectors of ci
	Sample Task #1 (Constructed Response)	
	Standards Aligned Instructionally Embedded Formative Assessment Resources: SAT Item # 5209208: The linked assessment question addresses G-C.B., specifically the question requires students to use knowledge of arc length.	

CollegeBoard **Question ID 5209208**

SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Circles	1. Use definitions, properties, and theorems relating to circles and parts of circles, such as radii, diameters, tangents, angles, arcs, arc lengths, and sector areas to solve problems.	No Calculator

The circle above has center O, the length of arc $\overset{\frown}{ADC}$ is 5π, and $x = 100$. What is the length of arc $\overset{\frown}{ABC}$?

Question Difficulty: Medium

A. 9π

B. 13π

C. 18π

D. $\frac{13}{2}\pi$

Choice B is correct. The ratio of the lengths of two arcs of a circle is equal to the ratio of the measures of the central angles that subtend the arcs. It's given that arc $\overset{\frown}{ADC}$ is subtended by a central angle with measure 100°. Since the sum of the measures of the angles about a point is 360°, it follows that arc $\overset{\frown}{ABC}$ is subtended by a central angle with measure $360° - 100° = 260°$. If s is the length of arc $\overset{\frown}{ABC}$, then s must satisfy the ratio $\frac{s}{5\pi} = \frac{260}{100}$. Reducing the fraction $\frac{260}{100}$ to its simplest form gives $\frac{13}{5}$. Therefore, $\frac{s}{5\pi} = \frac{13}{5}$. Multiplying both sides of $\frac{s}{5\pi} = \frac{13}{5}$ by 5π yields $s = 13\pi$.

Choice A is incorrect. This is the length of an arc consisting of exactly half of the circle, but arc $\overset{\frown}{ABC}$ is greater than half of the circle. Choice C is incorrect. This is the total circumference of the circle. Choice D is incorrect. This is half the length of arc $\overset{\frown}{ABC}$, not its full length.

MLSS AND CLR GUIDE

- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circle

CCSS Domain	*CCSS Cluster*
Circles	Understand and apply theorems about circles

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on understanding and applying theorems about circles, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, provide a task that requires students to calculate the dimensions needed for a new restaurant to build a triangular deck (with one side being the restaurant building) will relate school learning to community/home application	
Cross-Curricular Connections	Art: Consider discussing how inscribed and circumscribed angles may be used in calculating specific designs in landscape, apparel, etc. Designers are often given constraints in which to create an image and may use knowledge of these angles to help design an appropriate image.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between*	• Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice leads to more students viewing themselves as capable mathematicians. For example, when understanding and applying theorems about circles the types of mathematical tasks are critical because they will help students make connections to how the math in this cluster is applicable to real-life context. In some regards, tasks can be designed to highlight various cultures which in turn allows f

	the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	• or students to learn about the diversity amongst their peers' cultures. Tasks should be cognizant of culturally responsive academic vocabulary, language, and literacy.

Planning for Multilayered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In 8th grade, learners have worked with two-dimensional figures and verified their properties. These skills from 8.G.A. will support students in their ability to work with circles while working within this standard.	• Previous work with similarity will be applied to circles. Construction adds to learning from previous clusters by increasing skills with formal construction, building on angle congruence, perpendicular lines/segments, and properties of polygons. This will lead to work with arcs and areas of sectors, as well as prepare learners for future work in the Geometry course with 3-dimensional geometry and cross-sections.	• Unit circles, their central angles, and reference angles will build on foundational skills learned in this cluster.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*

Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas because new concepts (inscribed, circumscribed and central angles and tangent lines) are connected with prior held concepts (parallel, perpendicular, radii, etc). Students may need extra time to wrestle with the differences between these types of angles/lines and the work they have already mastered.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	SRTA2/CO.C.9: This standard provides a foundation for understanding and applying theorems about circles because these lay the groundwork for understanding similarity and congruence, both of which are the foundational concepts of this cluster. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on understand and apply theorems about circles by critiquing student approaches/solutions to make connections through a short mini-lesson because proofs and constructions can be seen from a variety of perspectives, some correct and others incorrect but will offer insight into common misconceptions all students may have.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit to understand and apply theorems about circles by confronting student misconceptions because when students do not have a firm grasp of congruence and similarity, progressing forward and applying those to circles and proofs will be impossible.
Extension		
Essential Question		***Examples***

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension understanding and applying theorems about circles because proofs with circles can be challenging and require students to pull facts from a variety of places, some implied and others explicitly stated. This develops and pushes students' abstract thinking skills.

CCSS Domain	*CCSS Cluster*
Circles	Find arc lengths and areas of sectors of circles

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on arc lengths and areas of sectors of circles, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, use every day circular items to show how sectors are part of the entire circle.	
Cross-Curricular Connections	Economics: Connect to a variety of circular foods, talking about maximizing crust or finding the largest slice.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical*	• Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it hinders students with strong prior familiarity from learning more methods for solving tasks that occur outside of school mathematics. For example, when studying arc lengths and areas of sectors of circles the types of mathematical tasks are critical because the struggle to be all-inclusive can be an issue. Where students with strong procedural knowledge will easily follow a process, some students will struggle and need adaptation and accommodations. Some ways to address this would be to adapt procedures into the students spoken language, apply terminology and problems from the student's daily life, use hands-on demonstrations, and use bi-lingual grouping of students

	identities as capable mathematicians that can use mathematics within school and society?	

Planning for Multilayered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In 7th grade, the formulas for the area and circumference of a circle are learned and then applied to solve problems. They give an informal derivation of the relationship between the circumference and area of a circle.	• Later in the Geometry course when calculating geometric probabilities, students will need to know how to calculate the area of a sector which is taught within this cluster.	• In future courses, students expand on their basic understanding of the radian measure of an angle. They apply radian measures when discovering relationships within the unit circle and while learning trigonometric relationships.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on arc lengths and areas of sectors of circles because understanding here depends on how deeply a student understands earlier concepts such as area of a circle and the terminology involved.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	7.GB.4 and 6.RP.A2: : This standard provides a foundation for work with arc lengths and areas of sectors of circles because it deals with the concept of area of circles and the concept of proportionality. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure

		students are ready to access grade level instruction and assignments.
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	Students may benefit from re-engaging with content during a unit on arc lengths and areas of sectors of circles by examining tasks from a different perspective through a short mini-lesson because we are welding concepts together to form a new concept, this process is not automatic and by backing up and looking at the problem from a different point of view.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on arc length and areas of sectors of circles by offering opportunities to understand and explore different strategies because what works for one student may not work for another.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to ‘broaden’ your student’s knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension when studying arc lengths and areas of sectors of circles because linking the concept to something that a student will experience in their own lives will add depth to their experience regarding this concept.