

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A [breakdown](#) of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned [assessment](#) items
- [Suggested Student Discourse Guide](#)
- A [multilayered system of supports \(MLSS\) and culturally and linguistically responsive instruction \(CLR\) guide](#)

Key		
	<i>Priority Standard</i>	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	<i>Conceptual Understanding</i>	Conceptual Understanding standards help students build a deep understanding of the <b>how</b> and <b>why</b> of mathematics.
	<i>Application</i>	Application standards help students identify the appropriate concepts and skills to tackle <b>novel real-world problems</b> .
	<i>Procedural Skill and Fluency</i>	Procedural standards help students develop <b>efficiency</b> and <b>accuracy</b> in computations.

## Standards Breakdown

- Experiment with transformations in the plane
  - [G.CO.A.1](#)
  - [G.CO.A.2](#)
  - [G.CO.A.3](#)
  - [G.CO.A.4](#)
  - [G.CO.A.5](#)
- Understand congruence in terms of rigid motions
  - [HSG.CO.B.6](#)
  - [HSG.CO.B.7](#)
  - [HSG.CO.B.8](#)
- Prove geometric theorems
  - [HSG.CO.C.9](#)
  - [HSG.CO.C.10](#)
  - [HSG.CO.C.11](#)
- Make geometric constructions
  - [HSG.CO.D.12](#)
  - [HSG.CO.D.13](#)

Grade	CCSS Domain	CCSS Cluster
<b>G</b>	<b>Congruence</b>	<b>Experiment with transformations in the plane</b>
<b>Cluster Standard: G.CO.A.1</b>		
<b>Standard</b>		<b>Standards for Mathematical Practice</b>
<p>State and apply precise definitions of angle, circle, perpendicular, parallel, ray, line segment, and distance based on the undefined notions of point, line, and plane.</p>		<ul style="list-style-type: none"> <li>● <b>SMP1:</b> Make sense of problems and persevere in solving them.</li> <li>● <b>SMP7:</b> Look for and make use of structure.</li> </ul>
<b>Clarification Statement</b>		<b>Students Who Demonstrate Understanding Can...</b>
<ul style="list-style-type: none"> <li>● Students formalize their transformation language by building precise definitions based on properties.</li> </ul>		<ul style="list-style-type: none"> <li>● Demonstrate the knowledge of precise definitions of angles, line, point, plane, circles, perpendicular and parallel lines, and line segments</li> <li>● Calculate the linear distance and arc length</li> <li>● Demonstrate the use of proper notation</li> <li>● Make connections with rigid motions in relation to the definitions of words above</li> </ul>
<b>DOK</b>		<b>Blooms</b>
2-3		Understand, Apply

Grade	CCSS Domain	CCSS Cluster
<b>G</b>	<b>Congruence</b>	<b>Experiment with transformations in the plane</b>
<b>Cluster Standard: G.CO.A.2</b>		
<b>Standard</b>		<b>Standards for Mathematical Practice</b>
<p>Represent transformations in the plane. (e.g., using transparencies and/or geometry software) <i>a.</i> Describe transformations as functions that take points in the plane as inputs and give other points as outputs. <i>b.</i> Compare transformations that preserve distance and angle to those that do not (e.g., translation versus dilation).</p>		<ul style="list-style-type: none"> <li>● <b>SMP 1:</b> Make sense of problems and persevere in solving them.</li> <li>● <b>SMP 3:</b> Construct viable arguments and critique the reasoning of others.</li> <li>● <b>SMP 6:</b> Attend to precision.</li> </ul>
<b>Clarification Statement</b>		<b>Students Who Demonstrate Understanding Can...</b>
<ul style="list-style-type: none"> <li>● They use formal notation and precise descriptions of transformations and sequences of transformations.</li> </ul>		<ul style="list-style-type: none"> <li>● Write a function that maps a preimage to its image or a description of a transformation.</li> <li>● Understand which transformations result in figures with congruent sides and angles and which do not</li> <li>● Represent transformations in the plane</li> <li>● Compare rigid motions that preserve distance and angle measure (translations, reflections, rotations) to transformations that do not preserve both distance and angle measure (e.g. dilations)</li> <li>● Understand that rigid motions produce congruent figures while dilations produce similar figures</li> </ul>
<b>DOK</b>		<b>Blooms</b>
2-3		Understand, Apply

Grade	CCSS Domain	CCSS Cluster
<b>G</b>	<b>Congruence</b>	<b>Experiment with transformations in the plane</b>
<b>Cluster Standard: G.CO.A.3</b>		
<b>Standard</b>		<b>Standards for Mathematical Practice</b>
<p>Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and/or reflections that map the figure onto itself.</p>		<ul style="list-style-type: none"> <li>● <b>SMP1:</b> Make sense of problems and persevere in solving them.</li> <li>● <b>SMP4:</b> Use appropriate tools strategically.</li> <li>● <b>SMP5:</b> Use appropriate tools strategically.</li> <li>● <b>SMP7:</b> Look for and make use of structure.</li> </ul>
<b>Clarification Statement</b>		<b>Students Who Demonstrate Understanding Can...</b>
<p>Rotational and reflection symmetry are identified with a specific degree of rotation or line(s) of symmetry.</p>		<ul style="list-style-type: none"> <li>● Identify and describe the different symmetries (line symmetry, rotational symmetry, point symmetry) of a figure</li> <li>● Determine the maximum possible lines of symmetries that exist for a given polygon</li> <li>● Determine the order and angle of a rotational symmetry</li> <li>● Determine the symmetries of a parallelogram, rectangle, rhombus, square, trapezoid and regular polygon</li> <li>● Understand symmetry in terms of transformations" (The Common Core Mathematics Companion)</li> <li>● Explore which shapes are symmetric and what symmetries they will have" (The Common Core Mathematics Companion)</li> <li>● Develop generalizations for the symmetries held by various geometric shapes" (The Common Core Mathematics Companion)</li> <li>● Determine the properties of a shape based on its symmetries" (The Common Core Mathematics Companion)</li> </ul>
<b>DOK</b>		<b>Blooms</b>
2-3		Understand, Apply

Grade	CCSS Domain	CCSS Cluster
<b>G</b>	<b>Congruence</b>	<b>Experiment with transformations in the plane</b>
<b>Cluster Standard: G.CO.A.4</b>		
<b>Standard</b>		<b>Standards for Mathematical Practice</b>
Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.		<ul style="list-style-type: none"> <li>● <b>SMP3:</b> Construct viable arguments and critique the reasoning of others.</li> <li>● <b>SMP7:</b> Look for and make use of structure.</li> </ul>
<b>Clarification Statement</b>		<b>Students Who Demonstrate Understanding Can...</b>
They use formal notation and precise descriptions of transformations and sequences of transformations. Rotational and reflection symmetry are identified with a specific degree of rotation or line(s) of symmetry.		<ul style="list-style-type: none"> <li>● Describe rotations, reflections and translations</li> <li>● Determine and apply the properties of the isometric transformations</li> <li>● Identify which transformation has taken place based on the properties found between the preimage and image.</li> <li>● Identify the orientation relationship between the preimage and image.</li> <li>● Explore properties of transformations using common geometric relationships (e.g., parallel, perpendicular, and congruence)" (The Common Core Mathematics Companion).</li> <li>● Develop definitions of the transformations in terms of their properties" (The Common Core Mathematics Companion).</li> </ul>
<b>DOK</b>		<b>Blooms</b>
2-3		Understand, Apply

Grade	CCSS Domain	CCSS Cluster
<b>G</b>	<b>Congruence</b>	<b>Experiment with transformations in the plane</b>
<b>Cluster Standard: G.CO.A.5</b>		
<b>Standard</b>		<b>Standards for Mathematical Practice</b>
Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure.		<ul style="list-style-type: none"> <li>● <b>SMP1:</b> Make sense of problems and persevere in solving them.</li> <li>● <b>SMP3:</b> Construct viable arguments and critique the reasoning of others.</li> <li>● <b>SMP5:</b> Use appropriate tools strategically.</li> </ul>
<b>Clarification Statement</b>		<b>Students Who Demonstrate Understanding Can...</b>
They use formal notation and precise descriptions of transformations and sequences of transformations. Rotational and reflection symmetry are identified with a specific degree of rotation or line(s) of symmetry.		<ul style="list-style-type: none"> <li>● Model a sequence of transformations including reflections, rotations, and translations with a geometric figure.</li> <li>● Determine the sequence of transformations performed between a given preimage and image.</li> <li>● Describe which single transformation is the result of two reflections over parallel lines.</li> <li>● Describe which single transformation is the result of two reflections over intersecting lines.</li> <li>● Identify a transformation by its coordinate rule and then apply it to transform the shape.</li> <li>● Demonstrate how some composite transformations are not commutative.</li> </ul>
<b>DOK</b>		<b>Blooms</b>
1-2		Understand, Apply

### Common Misconceptions

- Students may confuse transformation and translation and/or not know how to express the differences between the two terms.

- Students may confuse the connection between the terms (angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.) in the standard G.CO.A.1 and their application in this standard.

Grade	CCSS Domain	CCSS Cluster
<b>G</b>	<b>Congruence</b>	<b>Understand congruence in terms of rigid motions</b>
<b>Cluster Standard: HSG.CO.B.6</b>		
<b>Standard</b>		<b>Standards for Mathematical Practice</b>
Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.		<ul style="list-style-type: none"> <li><b>SMP3:</b> Construct viable arguments and critique the reasoning of others.</li> <li><b>SMP6:</b> Attend to precision.</li> <li><b>SMP7:</b> Look for and make use of structure.</li> </ul>
<b>Clarification Statement</b>		<b>Students Who Demonstrate Understanding Can...</b>
Students create a definition of triangle congruence in terms of rigid motions. They work to develop a set of criteria for triangle congruence and build a foundation for geometric proofs.		<ul style="list-style-type: none"> <li>Demonstrate that two figures are congruent if there is a sequence of rigid motions that map one figure to another.</li> <li>Express verbally and in writing that two figures are congruent if and only if they have the same shape and size.</li> <li>Model composite transformations to map one figure onto another.</li> <li>Recognize and explain the effects of rigid motion on orientation and location of a figure.</li> <li>Define congruence as a test to see if two figures are congruent.</li> </ul>
<b>DOK</b>		<b>Blooms</b>
1-2		Understand, Apply

Grade	CCSS Domain	CCSS Cluster
<b>G</b>	<b>Congruence</b>	<b>Understand congruence in terms of rigid motions</b>
<b>Cluster Standard: HSG.CO.B.7</b>		
<b>Standard</b>		<b>Standards for Mathematical Practice</b>
<p>Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.</p>		<ul style="list-style-type: none"> <li>● <b>SMP3:</b> Construct viable arguments and critique the reasoning of others.</li> <li>● <b>SMP6:</b> Attend to precision.</li> <li>● <b>SMP7:</b> Look for and make use of structure.</li> </ul>
<b>Clarification Statement</b>		<b>Students Who Demonstrate Understanding Can...</b>
<ul style="list-style-type: none"> <li>● Students create a definition of triangle congruence in terms of rigid motions. They work to develop a set of criteria for triangle congruence and build a foundation for geometric proofs.</li> </ul>		<ul style="list-style-type: none"> <li>● Identify corresponding angles and sides based on congruence statements.</li> <li>● Develop and write congruence statements for two congruent triangles.</li> <li>● Determine if two triangles are congruent based on their corresponding parts.</li> <li>● Compare given figures to determine congruence and indicate whether the figure went through a rigid transformation.</li> <li>● Explain, using rigid motions, why in congruent triangles, corresponding parts must be congruent.</li> </ul>
<b>DOK</b>		<b>Blooms</b>
1-2		Understand, Apply

Grade	CCSS Domain	CCSS Cluster
<b>G</b>	<b>Congruence</b>	<b>Understand congruence in terms of rigid motions</b>
<b>Cluster Standard: HSG.CO.B.8</b>		
<b>Standard</b>		<b>Standards for Mathematical Practice</b>
Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.		<ul style="list-style-type: none"> <li>● <b>SMP3:</b> Construct viable arguments and critique the reasoning of others.</li> <li>● <b>SMP6:</b> Attend to precision.</li> <li>● <b>SMP7:</b> Look for and make use of structure.</li> </ul>
<b>Clarification Statement</b>		<b>Students Who Demonstrate Understanding Can...</b>
<ul style="list-style-type: none"> <li>● Students create a definition of triangle congruence in terms of rigid motions. They work to develop a set of criteria for triangle congruence and build a foundation for geometric proofs.</li> </ul>		<ul style="list-style-type: none"> <li>● Create a method to determine unknown measurements of congruent triangles.</li> <li>● Explain the approach that was used to determine the congruency of two triangles given limited parts of triangles.</li> <li>● Explain the approach that was used to determine the congruence of two triangles.</li> <li>● Apply the criteria of SSS, SAS, ASA to prove triangle congruency.</li> </ul>
<b>DOK</b>		<b>Blooms</b>
1-2		Understand, Apply

### Common Misconceptions

- Combinations such as SSA or AAA are also a congruence criterion for triangles. All transformations, including dilations, are rigid motions. Any two figures that have the same area represent a rigid transformation. Students should recognize that the areas remain the same, but preservation of side and angle lengths determine that the transformation is rigid.

Grade	CCSS Domain	CCSS Cluster
<b>G</b>	<b>Congruence</b>	<b>Prove geometric theorems</b>
<b>Cluster Standard: HSG.CO.C.9</b>		
<b>Standard</b>		<b>Standards for Mathematical Practice</b>
<p>Prove theorems about lines and angles. <b>Theorems</b> include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.</p>		<ul style="list-style-type: none"> <li>● <b>SMP2:</b> Reason abstractly and quantitatively.</li> <li>● <b>SMP5:</b> Use appropriate tools strategically.</li> <li>● <b>SMP7:</b> Look for and make use of structure.</li> <li>● <b>SMP8:</b> Look for and express regularity in repeated reasoning.</li> </ul>
<b>Clarification Statement</b>		<b>Students Who Demonstrate Understanding Can...</b>
<ul style="list-style-type: none"> <li>● Students focus on formalizing geometric proof structure and language. They write formal proofs focusing on angle relationships, triangle segment and angle relationships, and parallelogram properties.</li> </ul>		<ul style="list-style-type: none"> <li>● Prove the vertical angle theorem and alternate interior angle theorem.</li> <li>● Prove corresponding angles are congruent.</li> <li>● Prove the converse of the alternate interior angle theorem and the corresponding angle theorem and use it to show that two lines are parallel.</li> <li>● Use perpendicular bisectors to locate the circumcenter of a triangle and to find the center of a circle given three points on the circle.</li> <li>● Express proofs both in writing and orally by using precise mathematical language</li> <li>● Examine and critique proofs produced by other students as well as their own</li> </ul>
<b>DOK</b>		<b>Blooms</b>
2-3		Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
<b>G</b>	<b>Congruence</b>	<b>Prove geometric theorems</b>
<b>Cluster Standard: HSG.CO.C.10</b>		
<b>Standard</b>		<b>Standards for Mathematical Practice</b>
<p>Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to <math>180^\circ</math>; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.</p>		<ul style="list-style-type: none"> <li>● <b>SMP2:</b> Reason abstractly and quantitatively.</li> <li>● <b>SMP5:</b> Use appropriate tools strategically.</li> <li>● <b>SMP7:</b> Look for and make use of structure.</li> <li>● <b>SMP8:</b> Look for and express regularity in repeated reasoning.</li> </ul>
<b>Clarification Statement</b>		<b>Students Who Demonstrate Understanding Can...</b>
<p>Students focus on formalizing geometric proof structure and language. They write formal proofs focusing on angle relationships, triangle segment and angle relationships, and parallelogram properties.</p>		<ul style="list-style-type: none"> <li>● Prove and apply that the sum of the interior angles of a triangle is <math>180^\circ</math>.</li> <li>● Prove and apply that the base angles of an isosceles triangle are congruent.</li> <li>● Prove and apply the midsegment (midline) of triangle theorem.</li> <li>● Prove that the medians of a triangle meet at a point, a point of concurrency.</li> <li>● Prove and apply the exterior angle theorem.</li> <li>● Determine the conditions for forming a triangle, when given three lengths.</li> </ul>
<b>DOK</b>		<b>Blooms</b>
1-2		Understand, Apply

Grade	CCSS Domain	CCSS Cluster
<b>G</b>	<b>Congruence</b>	<b>Prove geometric theorems</b>
<b>Cluster Standard: HSG.CO.C.11</b>		
<b>Standard</b>		<b>Standards for Mathematical Practice</b>
Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.		<ul style="list-style-type: none"> <li>● <b>SMP2:</b> Reason abstractly and quantitatively.</li> <li>● <b>SMP5:</b> Use appropriate tools strategically.</li> <li>● <b>SMP7:</b> Look for and make use of structure.</li> <li>● <b>SMP8:</b> Look for and express regularity in repeated reasoning.</li> </ul>
<b>Clarification Statement</b>		<b>Students Who Demonstrate Understanding Can...</b>
Students focus on formalizing geometric proof structure and language. They write formal proofs focusing on angle relationships, triangle segment and angle relationships, and parallelogram properties.		<ul style="list-style-type: none"> <li>● Prove properties of parallelograms and then apply them.</li> <li>● Prove the properties of rectangles and then apply them.</li> <li>● Prove the properties of rhombi and then apply them.</li> <li>● Prove the properties of squares and then apply them.</li> <li>● Classify a quadrilateral by its properties.</li> <li>● Identify the conditions necessary to prove that a quadrilateral is a parallelogram.</li> </ul>
<b>DOK</b>		<b>Blooms</b>
1-2		Understand, Apply

### Common Misconceptions

- Students may have a hard time generalizing and spend unnecessary time looking for multiple counterexamples to prove or disprove a proof, or, they may assume a conjecture is always true because it worked in all examples that were explored. Additionally, they may assume the converse of a statement is always true.

Grade	CCSS Domain	CCSS Cluster
<b>G</b>	<b>Congruence</b>	<b>Make geometric constructions</b>
<b>Cluster Standard: HSG.CO.D.12</b>		
<b>Standard</b>		<b>Standards for Mathematical Practice</b>
<p>Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.): <i>Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.</i></p>		<ul style="list-style-type: none"> <li>● <b>SMP1:</b> Make sense of problems and persevere in solving them.</li> <li>● <b>SMP4:</b> Model with mathematics.</li> <li>● <b>SMP5:</b> Use appropriate tools strategically.</li> <li>● <b>SMP6:</b> Attend to precision.</li> </ul>
<b>Clarification Statement</b>		<b>Students Who Demonstrate Understanding Can...</b>
<p>This cluster focuses on hands-on basic constructions. Students use geometric tools (compass, straightedge, software, etc.) to generate foundational pieces of geometry.</p>		<ul style="list-style-type: none"> <li>● Use construction techniques (compass, straight edge, software, etc.) to create figures.</li> <li>● Perform constructions including: copy a segment, copy an angle, bisect segments and angles, construct perpendicular lines/segments, construct parallel lines.</li> </ul>
<b>DOK</b>		<b>Blooms</b>
1-2		Understand, Apply

Grade	CCSS Domain	CCSS Cluster
<b>G</b>	<b>Congruence</b>	<b>Make geometric constructions</b>
<b>Cluster Standard: HSG.CO.D.13</b>		
<b>Standard</b>		<b>Standards for Mathematical Practice</b>
Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.		<ul style="list-style-type: none"> <li>● <b>SMP1:</b> Make sense of problems and persevere in solving them.</li> <li>● <b>SMP4:</b> Model with mathematics.</li> <li>● <b>SMP5:</b> Use appropriate tools strategically.</li> <li>● <b>SMP6:</b> Attend to precision.</li> </ul>
<b>Clarification Statement</b>		<b>Students Who Demonstrate Understanding Can...</b>
This cluster focuses on hands-on basic constructions. Students use geometric tools (compass, straightedge, software, etc.) to generate foundational pieces of geometry.		<ul style="list-style-type: none"> <li>● Construct an equilateral triangle inscribed within a circle using construction techniques</li> <li>● Construct a square inscribed within a circle using construction techniques</li> <li>● Construct a regular hexagon inscribed within a circle using construction techniques.</li> </ul>
<b>DOK</b>		<b>Blooms</b>
3-4		Apply, Create

### Common Misconceptions

- Some students may believe that a construction is the same as a sketch or drawing. Emphasize the need for precision and accuracy when doing constructions. Stress the idea that a compass and straightedge are identical to a protractor and ruler. Explain the difference between measurement and construction.

### Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)

Domain: **Congruence**

Strand: **Experiment with transformations in the plane**

#### Suggested Student Discourse Questions

- |  |  |
|--|--|
| <ul style="list-style-type: none"> <li>● Share how you used transformations to find lines of symmetry- give feedback to your partner about their strategy.</li> <li>● How can you determine the size of a field irrigated with a single length of pipe anchored in the center and being rotated around the field?</li> </ul> | <ul style="list-style-type: none"> <li>● Is it possible to use the x-y axis to perform rotations without a protractor or compass? How do you know?</li> <li>● Is there a relationship between the coordinates of corresponding points in a transformation? Is the relationship always the same? Why or why not?</li> </ul> |
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Domain: **Congruence**

Strand: **Understand congruence in terms of rigid motions**

#### Suggested Student Discourse Questions

- |  |  |
|--|--|
| <ul style="list-style-type: none"> <li>● Share with your group/partner the process you used to determine if one shape maps to another. What feedback can you give to fellow students?</li> </ul> | <ul style="list-style-type: none"> <li>● Did you follow the same process in making rigid motions as your fellow students? If not, what were the differences?</li> <li>● How do rigid motions in transformations</li> </ul> |
|--|--|

- How could you use rigid transformations to determine if the walls of a room in a house could be the same height?

preserve congruence between corresponding sides and corresponding angles?

Domain: <b>Congruence</b>	Strand: <b>Prove geometric theorems</b>
<b>Suggested Student Discourse Questions</b>	
<ul style="list-style-type: none"><li>● Look at the way fellow students wrote their proofs. Did they follow the same steps as you? What differences and similarities do you see?</li><li>● If you knew the length of one object, could you use it to estimate the length of another object based on what you see around them? Why or why not?</li></ul>	<ul style="list-style-type: none"><li>● Think of the easiest way to prove one segment is parallel to another segment. Then, change the steps to make the proof different. Do this again as many times as you can.</li><li>● Could it be proven that two lines are perpendicular to the segments crossing between them? If not, what else would you need to know in order to write such a proof?</li></ul>

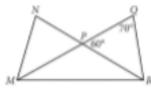
## ASSESSMENT GUIDE

- [Experiment with transformations in the plane](#)
- [Understand congruence in terms of rigid motions](#)
- [Prove geometric theorems](#)
- [Make geometric constructions](#)

Grade	CCSS Domain	CCSS Strand
<b>G</b>	<b>Congruence</b>	<b>Experiment with transformations in the plane</b>
	<b>Sample Task #1 (Constructed Response)</b>	
	<p>Source: <a href="http://tasks.illustrativemathematics.org/content-standards/HSG/CO/A/tasks/1468">http://tasks.illustrativemathematics.org/content-standards/HSG/CO/A/tasks/1468</a></p> <p>The linked assessment question addresses G-CO.A, specifically the question requires students to look at lines of symmetry using reflections. Two different arguments are presented using triangle congruence and another which uses rotations and reflections. This assessment should be given to students after they've been introduced to the formal definition of reflections. Students will engage in SMP1, SMP7, and potentially SMP3 depending on if students work in groups to share their solutions.</p>	

Grade	CCSS Domain	CCSS Strand
<b>G</b>	<b>Congruence</b>	<b>Understand congruence in terms of rigid motions</b>
	<b>Sample Task #1 (Constructed Response)</b>	
	<p>SAT Item #: 422659: The linked assessment question addresses G-CO.B., specifically the question requires students to identify and use congruent angles to find the angle measure in a triangle.</p>	

CollegeBoard Question ID 422659							
SAT	Math	Additional Topics in Math	Medium	Additional Topics in Math	Lines, angles, and triangles	1. Use concepts and theorems relating to congruence and similarity of triangles to solve problems.	No Calculator



In the figure above,  $\overline{MO}$  and  $\overline{NR}$  intersect at point P,  $NP = QP$ , and  $MP = PR$ . What is the measure, in degrees, of  $\angle QMR$ ? (Disregard the degree symbol when gridding your answer.)

**Question Difficulty:** Medium

The correct answer is 30. It is given that the measure of  $\angle QPR$  is  $60^\circ$ . Angle MPR and  $\angle QPR$  are collinear and therefore are supplementary angles. This means that the sum of the two angle measures is  $180^\circ$ , and so the measure of  $\angle MPR$  is  $120^\circ$ . The sum of the angles in a triangle is  $180^\circ$ . Subtracting the measure of  $\angle MPR$  from  $180^\circ$  yields the sum of the other angles in the triangle MPR. Since  $180 - 120 = 60$ , the sum of the measures of  $\angle QMR$  and  $\angle NRM$  is  $60^\circ$ . It is given that  $MP = PR$ , so it follows that triangle MPR is isosceles. Therefore  $\angle QMR$  and  $\angle NRM$  must be congruent. Since the sum of the measure of these two angles is  $60^\circ$ , it follows that the measure of each angle is  $30^\circ$ .

An alternate approach would be to use the exterior angle theorem, noting that the measure of  $\angle QPR$  is equal to the sum of the measures of  $\angle QMR$  and  $\angle NRM$ . Since both angles are equal, each of them has a measure of  $30^\circ$ .

Additional Assessment:

Properties of Congruent Triangles: <https://tasks.illustrativemathematics.org/content-standards/HSG/CO/B/7/tasks/1637>

The linked assessment question addresses G-CO.B, specifically the question requires students to look at two triangles and connect the concept of congruence of corresponding parts to congruence of shape in terms of rigid motion. Two different approaches are prompted: one assuming the triangles can be mapped on each other and asking students to explain which parts are congruent, and another assuming the triangles are congruent and asking students to create the sequence of transformations to map one onto the other. This assessment should be given to students after they've had time to work with rigid motion and have a firm grasp of naming parts of triangles. Students will engage in SMP3, SMP6 and SMP7.

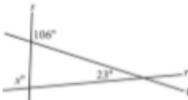
Grade	CCSS Domain	CCSS Strand
<b>G</b>	<b>Congruence</b>	Prove geometric theorems

**Sample Task #1 (Constructed Response)**

SAT Item #: 422005 The linked assessment question addresses G-CO.C., specifically the question requires students to know and apply theorems about exterior and interior angles.

CollegeBoard		Question ID 422005					
Assessment SAT	Test Math	Cross-Test and Subscore Additional Topics in Math	Difficulty Hard	Primary Dimension Additional Topics in Math	Secondary Dimension Lines, angles, and triangles	Tertiary Dimension 4. Know and directly apply relevant theorems such as b. triangle similarity and congruence criteria;	Calculator No Calculator

Intersecting lines  $r$ ,  $s$ , and  $t$  are shown below.



What is the value of  $x$  ?

**Question Difficulty:** Hard

The correct answer is 97. The intersecting lines form a triangle, and the angle with measure of  $x^\circ$  is an exterior angle of this triangle. The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles of the triangle. One of these angles has measure of  $23^\circ$  and the other, which is supplementary to the angle with measure  $106^\circ$ , has measure of  $180^\circ - 106^\circ = 74^\circ$ . Therefore, the value of  $x$  is  $23 + 74 = 97$ .

Additional Assessment:

Midpoints of the Sides of a Parallelogram: <https://tasks.illustrativemathematics.org/content-standards/HSG/CO/C/11/tasks/35>

The linked assessment question addresses G-CO.C, specifically the question requires students to prove segments of a parallelogram are congruent. Students use knowledge of corresponding parts of congruent triangles are congruent to form arguments. This assessment should be given to students after they've worked with corresponding parts of triangles. Students will engage in SMP 2 and SMP 7.

<i>Grade</i>	<i>CCSS Domain</i>	<i>CCSS Strand</i>
<b>G</b>	<b>Congruence</b>	Make geometric constructions
<b>Sample Task #1 (Constructed Response)</b>		
<p>Standards Aligned Instructionally Embedded Formative Assessment Resources:            Source: <a href="https://tasks.illustrativemathematics.org/content-standards/HSG/CO/D/12/tasks/966">https://tasks.illustrativemathematics.org/content-standards/HSG/CO/D/12/tasks/966</a></p> <p>The linked assessment question addresses G-CO.D, specifically the question requires students to prove that a specific segment is a perpendicular bisector to another segment. Students may use knowledge of corresponding parts of congruent triangles are congruent to form arguments, or may work through explanation using congruence as it follows from rigid motion transformations. This assessment should be given to students after they've been introduced to geometric construction tools and have a firm grasp of congruence. Students will engage in SMP1, SMP5 and SMP6.</p>		

## MLSS AND CLR GUIDE

- [Experiment with transformations in the plane](#)
- [Understand congruence in terms of rigid motions](#)
- [Prove geometric theorems](#)
- [Make geometric constructions](#)

CCSS Domain		CCSS Cluster	
Congruence		Experiment with transformations in the plane	
<b>Culturally and Linguistically Responsive Instruction</b>			
<b>Relevance to Families and Communities</b>	<p>During a unit focused on Exploring Transformations in the Plane, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, viewing artwork from a variety of cultures or having students select and bring in artwork that shows specific transformations can build the bridge between things they may have seen at home to what they are seeing in school. These cultural connections can be made explicitly or can be incorporated into word problems and other abstract application style questions.</p>		
<b>Cross-Curricular Connections</b>	<p>Art: Support students in making connections between geometric transformation and art, especially in relation to the idea of repetition and perspective in artwork.<sup>2</sup></p>		
<b>Validate/Affirm/Build/Bridge</b>	<ul style="list-style-type: none"> <li>• <i>How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?</i></li> <li>• <i>How can you create connections between</i></li> </ul>	<ul style="list-style-type: none"> <li>• <b>Equity Based Practices (Tasks):</b> The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying Exploring Transformations in the Plane the types of mathematical tasks are critical because the purpose of the standard is for students</li> </ul>	

	<p><i>the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?</i></p>	<p>to explore and experience the transformations for themselves. This requires teachers to select tasks and activities that provide for students to experiment with the transformations in the plane rather than explicitly tell students what steps to take.</p>
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## Planning for Multi-Layered System of Supports

### Vertical Alignment

<i>Previous Learning</i>	<i>Current Learning</i>	<i>Future Learning</i>
<ul style="list-style-type: none"> <li>This cluster aligns directly to the learning in 8th grade, when students were introduced to congruence and similarity using physical models, transparencies, or geometry software. They have described sequences of rigid motions informally and in terms of coordinates. Learners have verified experimentally the properties of transformations, and describe their effects on two-dimensional figures using coordinates.</li> </ul>	<ul style="list-style-type: none"> <li>Transformation definitions will serve as the basis for theorems that will be proven later in the year.</li> </ul>	<ul style="list-style-type: none"> <li>In Algebra II, students will connect their knowledge of transformations to functions. They will use their knowledge of transformation language to compare a function to its parent function, identify lines of symmetry, and other characteristics and behavior of functions.</li> </ul>

### Suggested Instructional Strategies

#### Pre-Teach

<i>Level of Intensity</i>	<i>Essential Question</i>	<i>Examples</i>
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Targeted	<i>What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?</i>	For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying transformations in the plane because students are experimenting with transformations and precise language will be important.
Intensive	<i>What critical understandings will prepare students to access the mathematics for this cluster?</i>	4.GA.1: This standard provides a foundation for work with transformations in the plane because students should have already built a firm grasp of key vocabulary such as angle, point, line, parallel, perpendicular, etc. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

**Universal Support Framework**

A student should know/understand...	A student should be able to do...	<i>Potential Scaffolds</i>
<ul style="list-style-type: none"> <li>Transformations as functions and symmetry in terms of transformations.</li> <li>Two figures are congruent if there is a sequence of transformations that maps one onto another.</li> <li>Two figures are similar if they have the same shape with congruent angles and proportional side lengths.</li> <li>The formal language for relationships between angles including vertical angles, angles created when a transversal</li> </ul>	<ul style="list-style-type: none"> <li>Carry out rotations, reflections, translations, and dilations using a variety of tools and compare and contrast their effects.</li> <li>Prove two triangles are congruent using ASA, SAS, and SSS or two triangles are similar using AA, SAS, and SSS.</li> <li>Build formal justifications (proofs) for the theorems about lines and angles, triangles, and</li> </ul>	<ul style="list-style-type: none"> <li>Build on students' experience with the following skills:               <ul style="list-style-type: none"> <li>Understand and use the coordinate axis</li> <li>Write and solve linear equations, especially proportions</li> <li>Recognize and draw geometric shapes (square, triangle, trapezoid, etc.)</li> <li>Understand and use the formulas of geometric shapes (Perimeter, Area, Volume, etc.)</li> </ul> </li> <li>Cognitive Strategies               <ul style="list-style-type: none"> <li>Repeatedly model the strategies</li> <li>Monitor the students' use of the strategies</li> <li>Provide feedback to students</li> <li>Teach self-questioning and self-monitoring strategies</li> <li>Introduce multiple means of representation for mathematical ideas</li> </ul> </li> </ul>

<p>intersects parallel lines, angles and sides of triangles, the segments and angles of parallelograms, and the trigonometric ratios sine, cosine, and tangent.</p>	<p>parallelograms.</p> <ul style="list-style-type: none"> <li>Find unknown side lengths and angle measures of right triangles using trigonometric ratios and the Pythagorean Theorem.</li> </ul>	<ul style="list-style-type: none"> <li>Encourage students to use alternative tools to better access the grade level content. Examples include: <ul style="list-style-type: none"> <li>Desmos graphing calculator</li> <li>Desmos scientific calculator</li> <li>Desmos geometry tool</li> <li>GeoGebra</li> <li>Graphing or scientific calculator</li> <li>Google Drawing</li> <li>Geometric tools (ruler, protractor, compass, etc.)</li> <li>Tracing paper</li> <li>Graph paper and mirror/string/etc.</li> <li>Craft tools (scissors, string, construction paper, etc.)</li> <li>Paper folding</li> </ul> </li> </ul>
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**Re-Teach**

<i>Level of Intensity</i>	<i>Essential Question</i>	<i>Examples</i>
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on transformations in the plane by clarifying mathematical ideas and/or concepts through a short mini lesson because students may struggle with rotations or reflections about a point/line that is not at the origin.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on transformations in the plane by offering opportunities to understand and explore different strategies because there may be several approaches to map one object onto another.

**Extension**

<i>Essential Question</i>	<i>Examples</i>
What type of extension will offer additional challenges to	Some learners may benefit from an extension such as the application and development of abstract thinking skills

<p>'broaden' your student's knowledge of the mathematics developed within your HQIM?</p>	<p>when studying transformations in the plane. Geometric software can allow students to experiment with combinations of transformations and using such software can allow for abstract application of knowledge of transformations.</p>
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<i>CCSS Domain</i>		<i>CCSS Cluster</i>	
<b>Congruence</b>		<b>Understand congruence in terms of rigid motions</b>	
<b>Culturally and Linguistically Responsive Instruction</b>			
<b>Relevance to Families and Communities</b>	<p>During a unit focused on Congruence in terms of Rigid Motion, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, teachers can create connections from native language to English by focusing on cognates (Words that sound the same in two different languages). Incorporating the usage of cognates throughout a unit validates and affirms all languages and can encourage students to explore these terms in language other than their native language.</p>		
<b>Cross-Curricular Connections</b>	Computer Science: program to create visual demo of transformations		
<b>Validate/Affirm/Build/Bridge</b>	<ul style="list-style-type: none"> <li><i>How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?</i></li> <li><i>How can you create connections between the cultural and</i></li> </ul>	<ul style="list-style-type: none"> <li><b>Building Procedural Fluency from Conceptual Understanding:</b> Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying Understanding Congruence in terms of Rigid Motion the types of mathematical tasks are critical because students need the time and experience in connecting prior knowledge of congruence to rigid motions in the plane. Tasks should activate knowledge of both congruence and rigid motions, and build the bridge between them.</li> </ul>	

	<p><i>linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?</i></p>	
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## Planning for Multi-Layered System of Supports

### Vertical Alignment

<i>Previous Learning</i>	<i>Current Learning</i>	<i>Future Learning</i>
<ul style="list-style-type: none"> <li>In 7th grade, students focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. In 8th grade, students develop understanding of congruence using physical models, transparencies, or geometry software. Students also understand that figures are congruent if the second can be obtained from the first by a sequence of rotations, reflections, and/or translations. These foundational skills are applied within this standard to continue to explore congruence and the impact of rigid motions on geometric shapes.</li> </ul>	<ul style="list-style-type: none"> <li>Students will use triangle congruence concepts to develop future postulates and theorems. Concepts of triangle congruence serve to build a foundation for work with triangle proofs in future clusters.</li> </ul>	<ul style="list-style-type: none"> <li>In later courses, students consider triangle congruence and the ambiguous case when working with the Law of Sines and Law of Cosines.</li> </ul>

Suggested Instructional Strategies		
Pre-Teach		
<i>Level of Intensity</i>	<i>Essential Question</i>	<i>Examples</i>
Targeted	<i>What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?</i>	For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying understanding congruence in terms of rigid motion because students may incorrectly interchange congruence and similarity.
Intensive	<i>What critical understandings will prepare students to access the mathematics for this cluster?</i>	8.GA.2: This standard provides a foundation for work with understanding congruence in terms of rigid motion because this is where the concept of congruence is solidified in terms of one object being able to be moved directly on top of another and match perfectly. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
A student should know/understand...	A student should be able to do...	<i>Potential Scaffolds</i>
<ul style="list-style-type: none"> <li>● Transformations as functions and symmetry in terms of transformations.</li> <li>● Two figures are congruent if there is a sequence of transformations that maps one onto another.</li> <li>● Two figures are similar if they have the same shape with congruent angles and</li> </ul>	<ul style="list-style-type: none"> <li>● Carry out rotations, reflections, translations, and dilations using a variety of tools and compare and contrast their effects.</li> <li>● Prove two triangles are congruent using ASA, SAS, and SSS or two triangles are similar using AA, SAS, and SSS.</li> </ul>	<ul style="list-style-type: none"> <li>● Build on students' experience with the following skills:               <ul style="list-style-type: none"> <li>○ Understand and use the coordinate axis</li> <li>○ Write and solve linear equations, especially proportions</li> <li>○ Recognize and draw geometric shapes (square, triangle, trapezoid, etc.)</li> <li>○ Understand and use the formulas of geometric shapes (Perimeter, Area, Volume, etc.)</li> </ul> </li> <li>● Cognitive Strategies               <ul style="list-style-type: none"> <li>○ Repeatedly model the strategies</li> <li>○ Monitor the students' use of the</li> </ul> </li> </ul>

<p>proportional side lengths.</p> <ul style="list-style-type: none"> <li>The formal language for relationships between angles including vertical angles, angles created when a transversal intersects parallel lines, angles and sides of triangles, the segments and angles of parallelograms, and the trigonometric ratios sine, cosine, and tangent.</li> </ul>	<ul style="list-style-type: none"> <li>Build formal justifications (proofs) for the theorems about lines and angles, triangles, and parallelograms.</li> <li>Find unknown side lengths and angle measures of right triangles using trigonometric ratios and the Pythagorean Theorem.</li> </ul>	<p>strategies</p> <ul style="list-style-type: none"> <li>Provide feedback to students</li> <li>Teach self-questioning and self-monitoring strategies</li> <li>Introduce multiple means of representation for mathematical ideas</li> </ul> <ul style="list-style-type: none"> <li>Encourage students to use alternative tools to better access the grade level content. Examples include: <ul style="list-style-type: none"> <li>Desmos graphing calculator</li> <li>Desmos scientific calculator</li> <li>Desmos geometry tool</li> <li>GeoGebra</li> <li>Graphing or scientific calculator</li> <li>Google Drawing</li> <li>Geometric tools (ruler, protractor, compass, etc.)</li> <li>Tracing paper</li> <li>Graph paper and mirror/string/etc.</li> <li>Craft tools (scissors, string, construction paper, etc.)</li> <li>Paper folding</li> </ul> </li> </ul>
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**Re-Teach**

<i>Level of Intensity</i>	<i>Essential Question</i>	<i>Examples</i>
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?	For example, students may benefit from re-engaging with content during a unit on understanding congruence in terms of rigid motion by clarifying mathematical ideas and/or concepts through a short mini-lesson because students may incorrectly apply the triangle congruence theorems (ASA, SSS, etc) and could benefit from clarifying these and connecting them to the concept of congruence.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit understanding congruence in terms of rigid motion by helping students move from specific answers to generalizations for certain types of problems because this cluster requires students

		to prove, generally speaking, if two triangles are congruent by applying knowledge of congruence rather than using specific triangles with concrete measurements.
<b>Extension</b>		
<i>Essential Question</i>		<i>Examples</i>
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension focused on the application of and development of abstract thinking skills when understanding congruence in terms of rigid motion. Their established knowledge base of congruence could allow them to generate examples and non-examples of congruent shapes.

<i>CCSS Domain</i>		<i>CCSS Cluster</i>	
<b>Congruence</b>		<b>Prove geometric theorems</b>	
<b>Culturally and Linguistically Responsive Instruction</b>			
<b>Relevance to Families and Communities</b>	During a unit focused on proving geometric theorems, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, the history of geometry spans across cultures. This history can be incorporated as students learn how to build and create formal proofs from informal diagrams, experimentation and/or oral arguments.		
<b>Cross-Curricular Connections</b>	Links to history can be made by exploring how geometric proofs developed over time in different cultures. Further, links to computer science can be made by discussing and displaying how coding reflects a variety of steps to get from one point to another, as is mirrored in proofs.		
<b>Validate/Affirm/Build/Bridge</b>	<ul style="list-style-type: none"> <li>How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students</li> </ul>	<ul style="list-style-type: none"> <li>Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion, we position them as knowers and doers of mathematics. Using equitable</li> </ul>	

	<p><i>and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?</i></p> <ul style="list-style-type: none"> <li>• <i>How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?</i></li> </ul>	<p>talk moves students and the ways students talk about who is and isn't capable of mathematics. As a result, we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying Proving Geometric Theorems facilitating meaningful mathematical discourse is critical because proofs of theorems can frequently be seen from multiple perspectives. Purposefully sequencing, discussing and validating these different perspectives can help students internalize their worth in the classroom. It can also show students that knowing a piece of the larger puzzle often "unlocks" the door to the next step in a proof. The key is in selecting and sequencing a variety of perspectives to share out and focus on how it helps the process, rather than whether it is complete and correct.</p>
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## Planning for Multi-Layered System of Supports

### Vertical Alignment

<i>Previous Learning</i>	<i>Current Learning</i>	<i>Future Learning</i>
<ul style="list-style-type: none"> <li>• In 7th grade, students use facts about supplementary, complementary, vertical, and adjacent angles. In 8th grade, learners use informal arguments to establish facts about the angle sum and exterior angle of triangles, and about the angles created when parallel lines are cut by a transversal. These angle facts connect this cluster as students apply them when</li> </ul>	<ul style="list-style-type: none"> <li>• The formalized theorems within this cluster will be used to build theorems and proofs for concepts in future clusters within the Geometry course.</li> </ul>	<ul style="list-style-type: none"> <li>• Understanding the logical flow of developing a proof will be used in future courses such as when proving trigonometric identities</li> </ul>

creating proofs.		
<b>Suggested Instructional Strategies</b>		
<b>Pre-Teach</b>		
<b>Level of Intensity</b>	<b>Essential Question</b>	<b>Examples</b>
Targeted	<i>What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?</i>	For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., structured proofs) when studying proving geometric theorems because students may be unfamiliar with traditionally structured mathematical proofs.
Intensive	<i>What critical understandings will prepare students to access the mathematics for this cluster?</i>	G.CO.B6: This standard provides a foundation for work with proving geometric theorems because this is where students have formalized an understanding of congruence in terms of rigid motion, which is required for every aspect of this cluster. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
<b>Universal Support Framework</b>		
A student should know/understand...	A student should be able to do...	<b>Potential Scaffolds</b>
<ul style="list-style-type: none"> <li>● Transformations as functions and symmetry in terms of transformations.</li> <li>● Two figures are congruent if there is a sequence of transformations that maps one onto another.</li> <li>● Two figures are similar if they have</li> </ul>	<ul style="list-style-type: none"> <li>● Carry out rotations, reflections, translations, and dilations using a variety of tools and compare and contrast their effects.</li> <li>● Prove two triangles are congruent using ASA, SAS, and SSS or two triangles</li> </ul>	<ul style="list-style-type: none"> <li>● Build on students' experience with the following skills:             <ul style="list-style-type: none"> <li>○ Understand and use the coordinate axis</li> <li>○ Write and solve linear equations, especially proportions</li> <li>○ Recognize and draw geometric shapes (square, triangle, trapezoid, etc.)</li> <li>○ Understand and use the formulas of geometric shapes (Perimeter, Area, Volume, etc.)</li> </ul> </li> <li>● Cognitive Strategies</li> </ul>

<p>the same shape with congruent angles and proportional side lengths.</p> <ul style="list-style-type: none"> <li>• The formal language for relationships between angles including vertical angles, angles created when a transversal intersects parallel lines, angles and sides of triangles, the segments and angles of parallelograms, and the trigonometric ratios sine, cosine, and tangent.</li> </ul>	<p>are similar using AA, SAS, and SSS.</p> <ul style="list-style-type: none"> <li>• Build formal justifications (proofs) for the theorems about lines and angles, triangles, and parallelograms.</li> <li>• Find unknown side lengths and angle measures of right triangles using trigonometric ratios and the Pythagorean Theorem.</li> </ul>	<ul style="list-style-type: none"> <li>○ Repeatedly model the strategies</li> <li>○ Monitor the students' use of the strategies</li> <li>○ Provide feedback to students</li> <li>○ Teach self-questioning and self-monitoring strategies</li> <li>○ Introduce multiple means of representation for mathematical ideas</li> </ul> <ul style="list-style-type: none"> <li>• Encourage students to use alternative tools to better access the grade level content. Examples include: <ul style="list-style-type: none"> <li>☞ Desmos graphing calculator</li> <li>☞ Desmos scientific calculator</li> <li>☞ Desmos geometry tool</li> <li>☞ GeoGebra</li> <li>☞ Graphing or scientific calculator</li> <li>☞ Google Drawing</li> <li>☞ Geometric tools (ruler, protractor, compass, etc.)</li> <li>☞ Tracing paper</li> <li>☞ Graph paper and mirror/string/etc.</li> <li>☞ Craft tools (scissors, string, construction paper, etc.)</li> <li>☞ Paper folding</li> </ul> </li> </ul>
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**Re-Teach**

<i>Level of Intensity</i>	<i>Essential Question</i>	<i>Examples</i>
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?	For example, students may benefit from re-engaging with content during a unit on proving geometric theorems by critiquing student approaches/solutions to make connections through a short mini-lesson because students will often see different methods of progressing through a proof which can benefit all students to see. Further, students may make specific claims or statements that are unsupported or incorrect, and critiquing this reasoning will strengthen all students' understanding of the content.

Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit about proving geometric theorems by offering opportunities to understand and explore different strategies because students may struggle with writing their own proofs, particularly in the beginning. It may be helpful to provide many examples and ask students to analyze each statement as to whether it can be mathematically supported or not. This could also be an opportunity to allow students to show their thinking with pictures rather than a formal proof structure.
<b>Extension</b>		
<i>Essential Question</i>		<i>Examples</i>
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying proving geometric theorems because giving students the opportunity to select certain theorems to prove can be both challenging and engaging for those with a firm grasp of the process.

<i>CCSS Domain</i>	<i>CCSS Cluster</i>
<b>Congruence</b>	<b>Make geometric constructions</b>
<b>Culturally and Linguistically Responsive Instruction</b>	
<b>Relevance to Families and Communities</b>	During a unit focused on Making Geometric Constructions, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, architects and other technical careers make regular use of the same tools used in geometric constructions. Connecting the use of mathematical tools with the real world can solidify the importance and relevance of material being learned, as well as encourage students to goal-set for future careers and interests.

<b>Cross-Curricular Connections</b>	Art: drafting, geometric shape work	
<b>Validate/Affirm/Build/Bridge</b>	<ul style="list-style-type: none"> <li><i>How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?</i></li> <li><i>How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?</i></li> </ul>	<ul style="list-style-type: none"> <li>Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge concepts for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their “mathematical, social, and cultural competence”. By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying Making Geometric Constructions the use of mathematical representations within the classroom is critical because students may have a varied background in reading/writing technical written directions and/or using rulers, straightedges, compasses, geometric software, etc. This background could be established in prior schooling or in specific cultural/home usage. Connecting tools they are familiar with to tools that may be new or uncomfortable to them shows the value of their current knowledge at the same time as expanding that knowledge base.</li> </ul>

## Planning for Multi-Layered System of Supports

### Vertical Alignment

<i>Previous Learning</i>	<i>Current Learning</i>	<i>Future Learning</i>
<ul style="list-style-type: none"> <li>This cluster connects to students previously taught skills from 7<sup>th</sup> grade when they constructed</li> </ul>	<ul style="list-style-type: none"> <li>Construction connects and adds on to learning from previous clusters within the Geometry course by</li> </ul>	<ul style="list-style-type: none"> <li>Construction techniques could be applied to unit circle, and conic sections in future courses.</li> </ul>

<p>geometric shapes when given certain conditions in the 7.G.A cluster. Additionally, in 8th grade within the 8.G.A cluster, students worked with two-dimensional figures and verified their properties.</p>	<p>building on triangle congruence theorems (SSS, SAS), properties of parallel and perpendicular lines, and polygons and their properties.</p>	
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**Suggested Instructional Strategies**

**Pre-Teach**

<i>Level of Intensity</i>	<i>Essential Question</i>	<i>Examples</i>
<p>Targeted</p>	<p><i>What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?</i></p>	<p>Some learners may benefit from targeted pre-teaching that rehearses prior learning experiences working with geometric construction tools (straight edge, ruler, compass, etc) because students may not use them consistently, correctly, or may need a refresher.</p>
<p>Intensive</p>	<p><i>What critical understandings will prepare students to access the mathematics for this cluster?</i></p>	<p>7.GA.2: This standard provides a foundation for work with making geometric constructions because this standard called for students to use a variety of tools to draw shapes that fit given constraints. This is where they should have mastered use of ruler/straight edge and compass. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.</p>

**Universal Support Framework**

<p>A student should know/understand...</p>	<p>A student should be able to do...</p>	<p><i>Potential Scaffolds</i></p>
<ul style="list-style-type: none"> <li>● Transformations as functions and symmetry in terms of transformations.</li> <li>● Two figures are congruent if there is a</li> </ul>	<ul style="list-style-type: none"> <li>● Carry out rotations, reflections, translations, and dilations using a variety of tools and compare and</li> </ul>	<ul style="list-style-type: none"> <li>● Build on students' experience with the following skills:               <ul style="list-style-type: none"> <li>○ Understand and use the coordinate axis</li> <li>○ Write and solve linear equations, especially proportions</li> <li>○ Recognize and draw geometric shapes</li> </ul> </li> </ul>

<p>sequence of transformations that maps one onto another.</p> <ul style="list-style-type: none"> <li>Two figures are similar if they have the same shape with congruent angles and proportional side lengths.</li> <li>The formal language for relationships between angles including vertical angles, angles created when a transversal intersects parallel lines, angles and sides of triangles, the segments and angles of parallelograms, and the trigonometric ratios sine, cosine, and tangent.</li> </ul>	<p>contrast their effects.</p> <ul style="list-style-type: none"> <li>Prove two triangles are congruent using ASA, SAS, and SSS or two triangles are similar using AA, SAS, and SSS.</li> <li>Build formal justifications (proofs) for the theorems about lines and angles, triangles, and parallelograms.</li> <li>Find unknown side lengths and angle measures of right triangles using trigonometric ratios and the Pythagorean Theorem.</li> </ul>	<p>(square, triangle, trapezoid, etc.)</p> <ul style="list-style-type: none"> <li>Understand and use the formulas of geometric shapes (Perimeter, Area, Volume, etc.)</li> <li>Cognitive Strategies           <ul style="list-style-type: none"> <li>Repeatedly model the strategies</li> <li>Monitor the students' use of the strategies</li> <li>Provide feedback to students</li> <li>Teach self-questioning and self-monitoring strategies</li> <li>Introduce multiple means of representation for mathematical ideas</li> </ul> </li> <li>Encourage students to use alternative tools to better access the grade level content. Examples include:           <ul style="list-style-type: none"> <li>Desmos graphing calculator</li> <li>Desmos scientific calculator</li> <li>Desmos geometry tool</li> <li>GeoGebra</li> <li>Graphing or scientific calculator</li> <li>Google Drawing</li> <li>Geometric tools (ruler, protractor, compass, etc.)</li> <li>Tracing paper</li> <li>Graph paper and mirror/string/etc.</li> <li>Craft tools (scissors, string, construction paper, etc.)</li> <li>Paper folding</li> </ul> </li> </ul>
<b>Re-Teach</b>		
<i>Level of Intensity</i>	<i>Essential Question</i>	<i>Examples</i>
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?	For example, students may benefit from re-engaging with content during a unit on making geometric constructions by clarifying mathematical ideas and/or concepts through a short mini-lesson because students may have simple errors (e.g. copying a segment without measurement, etc.) that require short and direct

		instruction.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on making geometric constructions by offering opportunities to understand and explore different strategies> because some students may work more efficiently with one consistent tool, while others may benefit from having a variety of tools at their disposal. Further, some students benefit from using geometric software to visualize their constructions rather than pencil and paper.
<b>Extension</b>		
<i><b>Essential Question</b></i>		<i><b>Examples</b></i>
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying making geometric constructions because this can challenge students to construct any shape, explaining or showing their methods for each.