New Mexico Instructional Scope

Geometry Expressing Geometric Properties with Equations Guide

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

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Geometry Expressing Geometric Properties with Equations Guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Translate between the geometric description and the equation for a conic section
  - HSG.GPE.A.1
  - HSG.GPE.A.2
- Use coordinates to prove simple geometric theorems algebraically
  - HSG.GPE.B.4
  - HSG.GPE.B.5
  - HSG.GPE.B.6
  - HSG.GPE.B.7

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Grade	*CCSS Domain*	*CCSS Cluster*
G	**Expressing Geometric Properties with Equations**	**Translate between the geometric description and the equation for a conic section**

Cluster Standard: HSG.GPE.A.1

Standard	**Standards for Mathematical Practice**
Derive the equation of a circle given the center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	• **SMP2:** Reason abstractly and quantitatively. • **SMP3**: Construct viable arguments and critique the reasoning of others. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
The introduction of coordinates into geometry connects geometry and algebra, allowing algebraic proofs of geometric theorems.	• Explain how the Pythagorean Theorem can be used to derive the equation of a circle. • Write the equation of a circle, given the center and radius. • Complete the square within the equation of a circle in order to find the center and radius
DOK	**Blooms**
2-3	Apply, Analyze

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Geometry Expressing Geometric Properties with Equations Guide

Grade	*CCSS Domain*	*CCSS Cluster*
G	**Expressing Geometric Properties with Equations**	**Translate between the geometric description and the equation for a conic section**

Cluster Standard: HSG.GPE.A.2

Standard	Standards for Mathematical Practice
Derive the equation of a parabola given a focus and directrix.	• **SMP1:** Make sense of problems and persevere in solving them. • **SMP8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
The introduction of coordinates into geometry connects geometry and algebra, allowing algebraic proofs of geometric theorems.	• Describe the characteristics of a parabola given its equation. • Derive the equation for a parabola given the focus and directrix.
DOK	**Blooms**
1-3	Analyze

Common Misconceptions

- Students commonly swap h and k when working with the equations for the circle.
- Students will make similar mistakes with *h* and *k*, when finding the vertex of a parabola

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Grade	*CCSS Domain*	*CCSS Cluster*
G	**Expressing Geometric Properties with Equations**	**Use coordinates to prove simple geometric theorems algebraically**

Cluster Standard: HSG.GPE.B.4

Standard	**Standards for Mathematical Practice**
Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*	• **SMP2:** Reason abstractly and quantitatively. • **SMP6:** Attend to precision. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
The focus of this cluster is coordinate geometry. Students work with coordinates to find slope, distances, midpoints, and locations that are at a specified ratio from an endpoint. They then use this information to prove geometric relationships such as properties of quadrilaterals or location of a point on a circle. Using slope criteria for parallel and perpendicular lines, students write equations of lines. Using lengths computed from coordinates, students find perimeters and areas of polygons.	• Use the distance formula to find the distance between coordinates. • Find the slope of a line connecting two coordinates. • Determine if a point lies on a specific circle. • Use coordinates to prove that a quadrilateral is, or is not, a parallelogram, rectangle, rhombus, square, or trapezoid. • Use coordinates to prove a triangle's classification by its sides.
DOK	**Blooms**
1-2	Understand, Apply

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Grade	*CCSS Domain*	*CCSS Cluster*
G	**Expressing Geometric Properties with Equations**	**Translate between the geometric description and the equation for a conic section**

Cluster Standard: HSG.GPE.B.5

Standard	**Standards for Mathematical Practice**
Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	• **SMP2:** Reason abstractly and quantitatively. • **SMP5:** Use appropriate tools strategically. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
The focus of this cluster is coordinate geometry. Students work with coordinates to find slope, distances, midpoints, and locations that are at a specified ratio from an endpoint. They then use this information to prove geometric relationships such as properties of quadrilaterals or location of a point on a circle. Using slope criteria for parallel and perpendicular lines, students write equations of lines. Using lengths computed from coordinates, students find perimeters and areas of polygons.	• Prove how parallel lines increase at the same rate of change. • Explain that perpendicular lines intersect at a right angle. • Construct an equation of a line that is parallel or perpendicular to a given line. • Calculate slope from given ordered pairs. • Classify lines or segments as parallel or perpendicular given slopes, graphs, and/or equations of lines. • Write equations for parallel lines and perpendicular lines given a point and an equation of a line.
DOK	**Blooms**
1-3	Understand, Analyze

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Geometry Expressing Geometric Properties with Equations Guide

Grade	*CCSS Domain*	*CCSS Cluster*
G	**Expressing Geometric Properties with Equations**	**Translate between the geometric description and the equation for a conic section**

Cluster Standard: HSG.GPE.B.6

Standard	**Standards for Mathematical Practice**
Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	• **SMP1:** Make sense of problems and persevere in solving them. • **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
The focus of this cluster is coordinate geometry. Students work with coordinates to find slope, distances, midpoints, and locations that are at a specified ratio from an endpoint. They then use this information to prove geometric relationships such as properties of quadrilaterals or location of a point on a circle. Using slope criteria for parallel and perpendicular lines, students write equations of lines. Using lengths computed from coordinates, students find perimeters and areas of polygons.	• Determine the coordinates of a point of a given partition on a directed segment. • Use the midpoint formula, the section formula, and the distance formula to find the partition point of a given line segment. • Determine the ratio of a partition using the distance formula. • Given two points, find the point on a line segment between the two points that divides the segment into a given ratio
DOK	**Blooms**
1-2	Understand, Apply

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Geometry Expressing Geometric Properties with Equations Guide

Grade	*CCSS Domain*	*CCSS Cluster*
G	**Expressing Geometric Properties with Equations**	**Translate between the geometric description and the equation for a conic section**

Cluster Standard: HSG.GPE.B.7

Standard	**Standards for Mathematical Practice**
Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	• **SMP2:** Reason abstractly and quantitatively. • **SMP4:** Model with mathematics. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
The focus of this cluster is coordinate geometry. Students work with coordinates to find slope, distances, midpoints, and locations that are at a specified ratio from an endpoint. They then use this information to prove geometric relationships such as properties of quadrilaterals or location of a point on a circle. Using slope criteria for parallel and perpendicular lines, students write equations of lines. Using lengths computed from coordinates, students find perimeters and areas of polygons.	• Use the distance formula to find the length of sides of a polygon. • Choose the appropriate formula for perimeter or area of a given polygon. • Calculate areas and perimeters of polygons. • Use appropriate labels for the areas and perimeters.
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

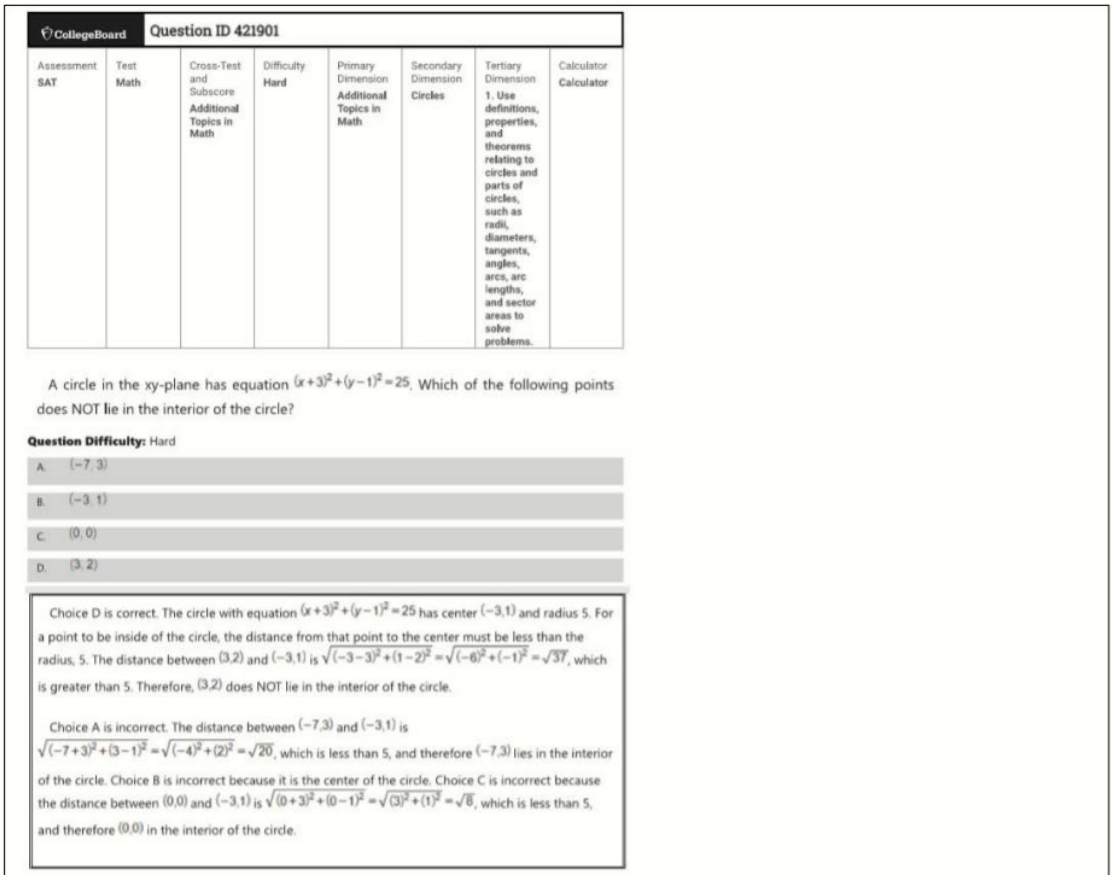
- Students may misunderstand the negative reciprocal slope with perpendicular lines.

- Students commonly forget to take the square root of the constant to find the radius in the equation of a circle.

ASSESSMENT GUIDE

- Translate between the geometric description and the equation for a conic section
- Use coordinates to prove simple geometric theorems algebraically

Grade	*CCSS Domain*	*CCSS Strand*
G	**Expressing Geometric Properties with Equations**	**Translate between the geometric description and the equation for a conic section**
	Sample Task #1 (Constructed Response)	
	SAT Item #: 421901 The linked assessment question addresses G-GPE.B., specifically the question requires students to analyze a given equation and determine if a point is within the circle.	



CollegeBoard | Question ID 421901

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Additional Topics in Math	Hard	Additional Topics in Math	Circles	1. Use definitions, properties, and theorems relating to circles and parts of circles, such as radii, diameters, tangents, angles, arcs, arc lengths, and sector areas to solve problems.	Calculator

A circle in the xy-plane has equation $(x+3)^2+(y-1)^2=25$. Which of the following points does NOT lie in the interior of the circle?

Question Difficulty: Hard

A. $(-7, 3)$

B. $(-3, 1)$

C. $(0, 0)$

D. $(3, 2)$

Choice D is correct. The circle with equation $(x+3)^2+(y-1)^2=25$ has center $(-3,1)$ and radius 5. For a point to be inside of the circle, the distance from that point to the center must be less than the radius, 5. The distance between $(3,2)$ and $(-3,1)$ is $\sqrt{(-3-3)^2+(1-2)^2}=\sqrt{(-6)^2+(-1)^2}=\sqrt{37}$, which is greater than 5. Therefore, $(3,2)$ does NOT lie in the interior of the circle.

Choice A is incorrect. The distance between $(-7,3)$ and $(-3,1)$ is $\sqrt{(-7+3)^2+(3-1)^2}=\sqrt{(-4)^2+(2)^2}=\sqrt{20}$, which is less than 5, and therefore $(-7,3)$ lies in the interior of the circle. Choice B is incorrect because it is the center of the circle. Choice C is incorrect because the distance between $(0,0)$ and $(-3,1)$ is $\sqrt{(0+3)^2+(0-1)^2}=\sqrt{(3)^2+(1)^2}=\sqrt{8}$, which is less than 5, and therefore $(0,0)$ in the interior of the circle.

Additional Assessment

http://tasks.illustrativemathematics.org/content-standards/HSG/GPE/B/4/tasks/605

The linked assessment question addresses G-GPE.B, specifically the question requires students to create their own quadrilateral and state verbally what observations they can make. Students then apply generic coordinate algebra to prove their observation. This assessment should be given to students after they've had practice applying coordinate math to prove conjectures about geometric figures. Students will engage in SMP2, SMP6 and possibly SMP3 if they are asked to share and critique others' work.

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Geometry Expressing Geometric Properties with Equations Guide

Grade	CCSS Domain	CCSS Strand
G	Expressing Geometric Properties with Equations	Use coordinates to prove simple geometric theorems algebraically

Sample Task #1 (Constructed Response)

A circle in the xy-plane has equation $(x+3)^2+(y-1)^2=25$. Which of the following points does NOT lie in the interior of the circle?

Sample Task #2 (Multiple Choice)

a.

What are the coordinates of the point on the directed line segment from $K(-5,-4)$ to $L(5,1)$ that partitions the segment into a ratio of 3 to 2?

(1) $(-3,-3)$
(2) $(-1,-2)$
(3) $(0,-\frac{3}{2})$
(4) $(1,-1)$

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Geometry Expressing Geometric Properties with Equations Guide

MLSS AND CLR GUIDE

- Translate between the geometric description and the equation for a conic section
- Use coordinates to prove simple geometric theorems algebraically

CCSS Domain	*CCSS Cluster*
Expressing Geometric Properties with Equations	**Translate between the geometric description and the equation for a conic section**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on translating between the geometric description and the equation for a conic section, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. Relating these standards to real-life context such as finding the distance from one building to another in the community will solidify the school learning to the actual application of standards/skills.	
Cross-Curricular Connections	During a unit focused on the use of coordinates to prove simple geometric theorems algebraically, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, allow students to relate this math to their home and community by plotting points to create an approximate map of a sectioned off area at home or parking lots in the community and find the amount of fencing needed.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the*	• Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics,

	negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	which can create a vicious cycle of low expectations and low achievement. For example, when studying translating between the geometric description and the equation for a conic section, goal setting is critical because students may be at varying levels of academic and language proficiency. To help students identify goals, teachers can use strategies such as writing prompts to gauge their thinking. Teachers can also provide students with a means to track their personal data (this assists in knowing where you currently are in attaining your goal). Teachers should work to build rapport, relationships and respect with their students and amongst each other to create that positive classroom culture in which students are willing to share/monitor their goals with one another without being judged.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Students have worked with coordinates, slope, and the Pythagorean Theorem in 8th grade math. This work exploring facts about right triangles connects to the foundational formulas in analytic geometry. Additionally, in Algebra I, students have been rewriting expressions in different forms (factoring and completing the square) which directly correlates to the work	• Students will connect the information in this cluster to learning later in the course by extending the precise definitions of circles and polygons to work with coordinates on the plane.	• Learners will continue with graphing quadratic functions, showing vertices, intercepts, and identifying maxima or minima in the Algebra II course.

they will complete in this cluster when creating algebraic proofs of the theorems.		

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching on translating between geometric descriptions and the equations for a conic section because students will need to know the distance formula which is learned in Grade 8. A review of distance between two points on the coordinate grid will help students in this cluster.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.G.B.8: This standard provides a foundation for work with translating between the geometric descriptions and the equation for a conic section because students have to apply the Pythagorean theorem to find the distance between two points. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on translating between the geometric description and the equation for a conic section by critiquing student approaches/solutions to make connections through a short mini-lesson because this cluster requires students to generalize patterns they see through exploration. These patterns may not be the same for every student but connecting the different patterns can reveal opportunities to deepen understanding and/or correct misunderstandings.

Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit of translating between the geometric description and the equation for a conic section by helping students move from specific answers to generalizations for certain types of problems because this cluster calls on students to recognize and generalize patterns. Students may need extra support in moving from concrete examples to generic patterns.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension such as the opportunity to explore concepts in greater depth than other students when translating between the geometric description and the equation for a conic section because some students may see generalizations easier than others. Allowing these students to move faster through the concrete examples to get to the abstract generalizations will allow them to stretch their expression of mathematical reasoning from concrete to abstract.

New Mexico Instructional Scope

Geometry Expressing Geometric Properties with Equations Guide

CCSS Domain	*CCSS Cluster*
Expressing Geometric Properties with Equations	**Use coordinates to prove simple geometric theorems algebraically**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on the use of coordinates to prove simple geometric theorems algebraically, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, allow students to relate this math to their home and community by plotting points to create an approximate map of a sectioned off area at home or parking lots in the community and find the amount of fencing needed.	
Cross-Curricular Connections	Home Economics: Connect to construction and agriculture.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and*	• Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying the use of coordinates to prove simple geometric theorems algebraically the pattern of questions within the classroom is critical because it can allow students to build upon each other's ideas. Further, teachers can tap into student's prior knowledge and use it to promote learning for all students. Teachers can

	language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	utilize strategic sequencing and questioning to encourage all students to participate in engaging with the content and seeing connections between multiple representations and solution methods. This can facilitate cross-content connections. When posing purposeful questions to the whole group, teachers should have protocols in place (classroom management) that tend to how students will respond to and discuss questions. Finally, teachers can use activities in which students are able to share (partners or groups) their thoughts and ideas in a judgement-free zone.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In 6th grade, learners find the area of polygons by composing into rectangles or decomposing into triangles and other shapes. They also draw polygons in the coordinate plane given coordinates for the vertices and find the length of horizontal and vertical sides. In 7th grade, learners solve real-world and mathematical problems involving area of triangles, quadrilaterals, and polygons. In 8th grade, learners apply the Pythagorean Theorem to find the distance between two points in a coordinate system. In Algebra I, learners write equations of lines given a slope and point.	• Learners have already had experience with properties of quadrilaterals, equations of circles, and finding area and perimeter earlier in the course. They now apply this knowledge to working with coordinates. Learners will use the concept of distance and midpoint throughout the rest of the geometry course. They apply the concepts later when calculating volumes and surface areas or when proving types of quadrilaterals given the ordered pairs of their vertices. They also use distance and midpoint when writing and deriving the equation of circles.	• Distance is an application important for many future concepts. For example, when writing equations of conic sections or converting between polar and rectangular coordinates or finding the magnitude of vectors.

Suggested Instructional Strategies

New Mexico Instructional Scope

Geometry Expressing Geometric Properties with Equations Guide

Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on the use of coordinates to prove simple geometric theorems algebraically because students will need to be familiar with discovering geometric properties before they can make sense of how to use coordinates to understand the properties.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	HS.G-GPE.A.1: This standard provides a foundation for using coordinates to prove simple geometric theorems algebraically because students should be able to work with and derive equations of geometric shapes before proving theorems. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?	For example, students may benefit from re-engaging with content during a unit on using coordinate to prove simple geometric theorems algebraically by clarifying mathematical ideas and/or concepts through a short mini-lesson because students may struggle to apply knowledge of geometric figures to coordinate algebra or vice-versa.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit of using coordinates to prove simple geometric theorems algebraically by addressing conceptual understanding because revisiting basic geometric shapes in the coordinate plane will assist in discovering geometric properties, which in turn will help them understand how coordinates can help prove theorems

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics because students will have the autonomy to make connections to personalized real-life situations in connection to this standard.