New Mexico Instructional Scope

Geometry Geometric Measurement and Dimension Guide

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

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Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Explain volume formulas and use them to solve problems
  - HSG.GMD.A.1
  - HSG.GMD.A.3
- Visualize relationships between two-dimensional and three-dimensional objects
  - HSG.GMD.B.4

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Grade	*CCSS Domain*	*CCSS Cluster*
G	**Geometric Measurement & Dimension**	**Explain volume formulas and use them to solve problems**

Cluster Standard: HSG.GMD.A.1

Standard	Standards for Mathematical Practice
Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	• **SMP2:** Reason abstractly and quantitatively. • **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students move from applying volume formulas to justifying them. Students will be exposed to advanced concepts in an informal setting. Learners will deconstruct complex geometric shapes into basic three-dimensional shapes to calculate their surface areas and volumes.	• Demonstrate Cavalieri's Principle concretely. • Give an informal argument for circumference and area formulas for circles. • Give an informal argument for volume formulas of cylinders, pyramids, and cones. • Construct viable arguments to validate the circumference of a circle, volume of a cylinder, volume of a pyramid, and volume of a cube by using Cavalieri's Principle.
DOK	**Blooms**
1-3	Understand, Apply, Analyze

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Grade	*CCSS Domain*	*CCSS Cluster*
G	**Geometric Measurement & Dimension**	**Explain volume formulas and use them to solve problems**

Cluster Standard: HSG.GMD.A.3

Standard	**Standards for Mathematical Practice**
Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	• **SMP1:** Make sense of problems and persevere in solving them. • **SMP4:** Model with mathematics. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students move from applying volume formulas to justifying them. Students will be exposed to advanced concepts in an informal setting. Learners will deconstruct complex geometric shapes into basic three-dimensional shapes to calculate their surface areas and volumes.	• Identify these geometric shapes: cylinders, pyramids, cones, and spheres. • Calculate volume for cylinders, pyramids, cones and spheres. • Use formulas to solve problems involving three-dimensional figures. • Apply volume to real world problems.
DOK	**Blooms**
1-2	Understand, Apply

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Grade	*CCSS Domain*	*CCSS Cluster*
G	**Geometric Measurement & Dimension**	**Visualize relationships between two-dimensional and three-dimensional objects**

Cluster Standard: HSG.GMD.B.4

Standard	Standards for Mathematical Practice
Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects	• **SMP1:** Make sense of problems and persevere in solving them.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students move from applying volume formulas to justifying them. Students will be exposed to advanced concepts in an informal setting. Learners will deconstruct complex geometric shapes into basic three-dimensional shapes to calculate their surface areas and volumes.	• Explain why a cross section is a two-dimensional representation of a slice of a three-dimensional object. • Realize that the cross section can be different depending on location and angle where the three-dimensional object is cut. • Identify possible cross sections in a given object • Identify three-dimensional objects generated by the rotations of two-dimensional objects
DOK	**Blooms**
1-2	Apply, Analyze

Common Misconceptions

• Students may mix up which formula to use for a given figure. • Students may have difficulty identifying the base of a figure. • When considering units, students may struggle to	• Students may struggle to visualize cross sections and rotations. • Students may need support with the concept of "slicing" a three-dimensional figure

interpret inches cubed or forget these units in the final answer	

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ASSESSMENT GUIDE

- Explain volume formulas and use them to solve problems
- Visualize relationships between two-dimensional and three-dimensional objects

Grade	*CCSS Domain*	*CCSS Strand*
G	**Geometric Measurement and Dimension**	**Explain volume formulas and use them to solve problems**

Sample Task #1 (Constructed Response)

Area of a circle

The goal of this task is to explain why the area enclosed by a circle C of radius r is πr^2. Recall that π is the ratio of the circumference of a circle to its diameter and that this ratio is independent of the size of the circle.

a.
Draw a picture of a regular octagon O inscribed in C. Find a formula for the area of the octagon in terms of its perimeter.

b.
Reasoning as in part (a), find a formula for the area of a regular n sided polygon, for $n \geq 3$, inscribed in C: the formula should give the area of the polygon *in terms of* its perimeter.

c. Using your formula from part (b), explain why the area of C is πr^2.

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Grade	*CCSS Domain*	*CCSS Strand*
	Geometric Measurement and Dimension	**Visualize relationships between two-dimensional and three-dimensional objects**

Sample Task #1 (Constructed Response)

Global Positioning System I

The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}.$$

a. For each of the following equations, describe the set of solutions geometrically and sketch this solution set in x-y-z coordinates:

i. $x^2 + y^2 + z^2 = 1.$

ii. $(x - 3)^2 + y^2 + z^2 = 4.$

iii. $x^2 + y^2 + \left(z - \frac{3}{2}\right)^2 = 1.$

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MLSS AND CLR GUIDE

- Explain volume formulas and use them to solve problems
- Visualize relationships between two-dimensional and three-dimensional object

CCSS Domain	*CCSS Cluster*
Geometric Measurement and Dimension	**Explain volume formulas and use them to solve problems**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on explaining volume formulas and using them to solve problems, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, learning the Cavalieri's principle is a great entry point and supports students to create arguments and use it to solve for the volume of a given geometric figure.	
Cross-Curricular Connections	Because volume can be found for any given item, this connection can be made to a variety of areas: science-beakers, social studies-coffins, art-paint bottles, music- variety of instruments.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures*	• Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice makes sense of mathematics and perseverance in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge

	and languages? • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students. For example, when studying, explaining volume formulas and use them to solve problems supporting productive struggle is critical because students may not see the connections of other formulas to volume and they need those to solve problems involving volume. However, students may see the connection by presenting the actual image which is relevant and relatable to them, such as, car tires, basketball, and any other shape that is visible to them.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In 7th grade, learners worked with area, and circumference which extended to find components needed for surface area and volume. Throughout grades 6, 7, and 8 students calculated the volumes and surface areas of prisms, cones, cylinders, and spheres which will connect to their work within this cluster.	• Students will continue to expand their work to include composite figures. They will also justify volume formulas and other constructions.	• In Calculus, students will apply Cavalieri's principle to calculate volumes for solids of rotation.

Suggested Instructional Strategies

Pre-Teach

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Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that rehearses prior learning when explaining volume formulas and using them to solve problems because students have to use different formulas that are necessary to support them on solving problems involving volume of three-dimensional figures, such as, cylinder, pyramid, and cone.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. This standard provides a foundation for work with explaining volume formulas and using them to solve problems because students need to master the different formulas and use them to solve real-world problems. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on GMD.A: Explain Volume Formulas and Use Them to Solve Problems by providing specific feedback to students on their work through a short mini-lesson because teachers need to monitor students on how they use different formulas and use them precisely to answer a given problem. Providing students specific feedback will help students to create strong justifications of their answers.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit GMD.A: Explain Volume Formulas and Use Them to Solve Problems by helping students move from specific answers to

		generalizations for certain types of problems because students need to master how they justify answers mathematically and base from specific concepts to come up with a general, accurate and precise answers.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the opportunity to explore links between various topics when explaining volume formulas and using them to solve problems because these topics are interconnected and it will help students to master the whole concepts. Students need to have a strong foundation of different formulas and let them use it to formulate and come up with justifications on volumes.

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Geometry Geometric Measurement and Dimension Guide

CCSS Domain	*CCSS Cluster*
Geometric Measurement and Dimension	**Visualize relationships between two-dimensional and three-dimensional objects**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on visualizing relationships between two-dimensional and three-dimensional objects, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, let the students identify two-dimensional and three-dimensional objects and let them discuss the difference.	
Cross-Curricular Connections	Students can produce interesting pieces of art by rotating a two-dimensional object that has been dipped in ink. This can also link to Language Arts and Science by having students verbally describe a conjecture of what they think an end result may be and then experimentally verifying and reflecting upon their initial thoughts. Stressing the importance of precise language in writing descriptions deepens the connection between ELA and math.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of*	• Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge concepts for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when visualizing relationships between two-dimensional and three-

	your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	dimensional objects the use of mathematical representations within the classroom is critical because students need to critically think about the relationship of two-dimensional and three-dimensional objects. We can help our students by presenting them the actual image and image that they can relate to, like a ball, car tires, cellphone and any other object that is visible to them.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• In 6th grade, students represented three-dimensional figures using nets made up of rectangles and triangles to calculate surface area. In 7th grade students moved on to describe the two-dimensional figures that result from slicing three-dimensional figures, focusing on right rectangular prisms and pyramids. These skills prepare students to address the content within this cluster.	• Students use cross section dimensions in volume calculations (i.e. the height of the triangle when calculating the volume of the cone)	• In later courses, students study conic sections which can be described as cross sections of a cone. Calculus concepts will build on the volume of solids of rotation.

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will*	Some learners may benefit from targeted pre-teaching

	prepare students to productively struggle with the mathematics for this cluster within your HQIM?	that uses images/resources (especially those being used the first time) when visualizing relationships between two-dimensional and three-dimensional objects because students need to have a visual representation of the object. As much as possible, use objects that are familiar to students.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	7.G.A.3: Describe the two-dimensional figures that result from slicing three- dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. This standard provides a foundation for work with visualizing relationships between two-dimensional and three-dimensional objects because students need to visualize how it looks when a three-dimensional object is being cut. What are the different products after cutting? If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• Transformations as functions and symmetry in terms of transformations.
• Two figures are congruent if there is a sequence of transformations that maps one onto another.
• Two figures are similar if they have the same shape with congruent angles and proportional side lengths.
• The formal language for relationships
	• Carry out rotations, reflections, translations, and dilations using a variety of tools and compare and contrast their effects.
	• Prove two triangles are congruent using ASA, SAS, and SSS or two triangles are similar using AA, SAS, and SSS.
	• Build formal justifications (proofs) for the
		• Build on students' experience with the following skills:
		  ○ Understand and use the coordinate axis
		  ○ Write and solve linear equations, especially proportions
		  ○ Recognize and draw geomteric shapes (square, triangle, trapezoid, etc.)
		  ○ Understand and use the formulas of geometric shapes (Perimeter, Area, Volume, etc.)
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies

between angles including vertical angles, angles created when a transversal intersects parallel lines, angles and sides of triangles, the segments and angles of parallelograms, and the trigonometric ratios sine, cosine, and tangent.	theorems about lines and angles, triangles, and parallelograms. Find unknown side lengths and angle measures of right triangles using trigonometric ratios and the Pythagorean Theorem.	○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ⊄ Desmos graphing calculator ⊄ Desmos scientific calculator ⊄ Desmos geometry tool ⊄ GeoGebra ⊄ Graphing or scientific calculator ⊄ Google Drawing ⊄ Geometric tools (ruler, protractor, compass, etc.) ⊄ Tracing paper ⊄ Graph paper and mirror/string/etc. ⊄ Craft tools (scissors, string, construction paper, etc.) ⊄ Paper folding

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on visualize relationships between two-dimensional and three-dimensional objects by providing specific feedback to students on their work through a short mini-lesson because students have to properly visualize the object, it's movement to the plane so students can accurate use the object to solve problems.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit visualizing relationships between two-dimensional and three-dimensional objects by offering opportunities to understand and explore different strategies because offering students different ways of visualizing two-dimensional and three-dimensional objects will give them the opportunity to express their thinking through illustrations and analysis.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the application of and development of abstract thinking skills when visualizing relationships between two-dimensional and three-dimensional objects because students will deepen their understanding of the topic and they can use this in the real-world. Challenge the students to do a frustum of two-dimensional and three-dimensional objects.