



## New Mexico Measures of Student Success and Achievement Assessments Performance Level Descriptors: Mathematics

This document contains the Performance Level Descriptors (PLDs) for the New Mexico Measures of Student Success and Achievement (NM-MSSA) Assessment. These PLDs represent intended interpretations of solid student achievement on the assessment for intended uses of test scores.

### Principles that Guided the Development of PLDs for the NM-MSSA Assessment

Development of the PLDs began with the assumption that the grade-level content standards represent what students should know and be able to do at the end of a given grade level. Prior research on learning, cognition, and development in the subject areas, a variety of resources (e.g., *Living Word*, *Reading Teacher's Book of Lists*), and teaching experiences of content experts informed the development of definitions for solid achievement at each level.

For example, consider the following grade 5 standards for Operations and Algebraic Thinking as an example of the process of starting from the standard and moving toward defining student achievement.

#### **5.OA.1 Write and interpret numerical expressions.**

5.OA.1.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

5.OA.1.2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

#### **5.OA.2 Analyze patterns and relationships.**

5.OA.2.3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.

Students' understanding and skill in each of these mathematics standards could range from rote memorization to deep understanding. The PLDs in the table below illustrate ranges of understanding and use of the sub-objectives of these standards across the Nearing Proficient, Proficient, and Advanced performance levels.

Nearing Proficient	Proficient	Advanced
Evaluate and write simple numerical expressions that record calculations with numbers using one of the four operations, including expressions set in parentheses.	Evaluate and write simple numerical expressions that record calculations with numbers using two of the four operations and multiple sets of parentheses, brackets, and braces.  Identify an accurate interpretation of a simple numerical expression without evaluating it.	Evaluate and write simple numerical expressions that record calculations with numbers using the four operations and multiple sets of parentheses, brackets, and braces.  Interpret numerical expressions without evaluating them.
Generate two number patterns that follow the same rule.	Generate two number patterns that follow two given rules.  Form ordered pairs consisting of corresponding terms from two patterns and graph the ordered pairs on a coordinate grid.	Generate two number patterns that follow two given rules, and identify apparent relationships between corresponding terms from the two patterns.  Form ordered pairs consisting of corresponding terms from two patterns that follow two rules, graph the ordered pairs on a coordinate grid, and explain the apparent relationship between the two patterns.

In the PLDs above, verbs, adverbs, and adjectives are used to define item demands and levels of student knowledge and skill for the Nearing Proficient, Proficient, and Advanced performance levels because the level of demand in a test item depends upon the way understanding and skills are assessed. For example, it is easier for students to identify a correct response than to produce it.

In some cases, a content standard might not be represented in all three performance levels. These instances are purposeful and represent cases where components of a standard do not lend themselves to all three levels.

A final consideration was whether the entire breadth of a content standard represents an on-grade-level expectation or whether certain components represent different levels of performance. For example, consider the following grade 8 standard from the *Functions* domain:

8.F.1.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, and by verbal descriptions.)

For this standard, students are expected to compare the properties of two functions that are represented in different ways. A prerequisite skill for this standard is to understand how a single function is represented in one way, which is an expectation at the Nearing Proficient level. Research suggests that understanding functions in multiple ways is more challenging than understanding single solutions to equations. Therefore, the expectation at the Proficient level is to represent a single function in two ways and that the ability to compare two functions, each represented in a different way, is an expectation at the Advanced level, as indicated below.

Nearing Proficient	Proficient	Advanced
Represent a function in one way (algebraically, graphically, or numerically in tables)	Represent a function in two different ways (algebraically, graphically, or numerically in tables)	Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions)

## Organization of the PLDs

The NM-MSSA PLDs for each grade level are organized into two parts. The first part is a descriptive summary of the overall achievement of students at a given performance level. The second part lists the more specific definitions of the sub-objectives in the overall descriptions.

Due to the scaffolded nature of the performance levels, a student with a level of Nearing Proficient or above is presumed to also have knowledge and skills addressed in lower levels. For example, a student at the Proficient level most likely also possesses the knowledge and skills outlined in the Nearing Proficient level.

The Performance Level Descriptors for each grade and content area stem from these policy PLDs. Policy PLDs define the knowledge and skill level expectations for all grades and content areas for the NM-MSSA and NM-ASR.

- **Advanced.** Students demonstrate evidence of **thorough** understanding and use of college and career readiness knowledge, skills, and abilities.
- **Proficient.** Students demonstrate evidence of **satisfactory** understanding and use of college and career readiness knowledge, skills, and abilities.
- **Nearing Proficiency.** Students demonstrate evidence of **partial** understanding and use of college and career readiness knowledge, skills, and abilities.
- **Novice.** Students demonstrate evidence of **emerging** understanding and use of college and career readiness knowledge, skills, and abilities.

# Performance Level Descriptors

# Grade 3 Mathematics Performance Level Descriptors

## Advanced

By the end of the year, third graders at the **Advanced** level can solve two-step mathematical problems using whole numbers; multiply and divide whole numbers within 100; represent and generate equivalent fractions; compare fractions with the same numerator or same denominator, justifying the comparison; reason with shapes and their attributes; work within measurement systems to solve problems involving time, volume, and mass; solve problems involving area and perimeter, including representing whole-number products as rectangular areas in mathematical reasoning; solve two-step problems involving data from scaled bar graphs; create and use models to represent and solve problems; analyze others' arguments and identify flaws in arguments if appropriate; and identify, define, and explain numeric patterns.

## Proficient

By the end of the year, third graders at the **Proficient** level can solve two-step mathematical problems using whole numbers; multiply and divide whole numbers within 50; represent and generate equivalent fractions; compare fractions with the same numerator or same denominator; reason with shapes and their attributes; work within measurement systems to solve problems involving time, volume, and mass; solve problems involving area and perimeter; solve one-step problems involving data from scaled bar graphs; create and use models to represent and solve problems; analyze others' arguments and identify flaws in arguments if appropriate; and identify, define, and explain numeric patterns.

## Nearing Proficient

By the end of the year, third graders at the **Nearing Proficient** level can solve simple mathematical problems using addition and subtraction facts; understand what a fraction represents; and identify, describe, and create simple predictable patterns.

## Novice

By the end of the year, third graders at the **Novice** level demonstrate evidence of emerging understanding and use of mathematical concepts and practices, procedural skill and fluency, and the application of mathematics to solve problems.

**Mathematics Grade 3—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Operations and Algebraic Thinking</b>		<ul style="list-style-type: none"> <li>• Interpret products and quotients of whole numbers.</li> </ul>	<ul style="list-style-type: none"> <li>• Describe contexts in which a total number of objects or a number of shares/groups can be expressed as a product or quotient.</li> </ul>
	<ul style="list-style-type: none"> <li>• Use multiplication and division within 5 groups of 5 to solve word problems.</li> </ul>	<ul style="list-style-type: none"> <li>• Use multiplication and division with products or dividends less than or equal to 100 to solve word problems.</li> </ul>	<ul style="list-style-type: none"> <li>• Create models to represent multiplication and division equations in word problems.</li> </ul>
	<ul style="list-style-type: none"> <li>• Determine an unknown product or quotient in a multiplication or division equation with products or dividends less than or equal to 50.</li> </ul>	<ul style="list-style-type: none"> <li>• Create models to represent multiplication equations in word problems.</li> </ul>	
	<ul style="list-style-type: none"> <li>• Determine an unknown product or quotient in a multiplication or division equation with products or dividends less than or equal to 50.</li> </ul>	<ul style="list-style-type: none"> <li>• Determine an unknown whole number in a multiplication or division equation with products or dividends greater than 50 and less than or equal to 100.</li> </ul>	
	<ul style="list-style-type: none"> <li>• Apply the commutative property as a strategy to multiply.</li> </ul>	<ul style="list-style-type: none"> <li>• Apply the commutative and associative properties as strategies to multiply.</li> </ul>	<ul style="list-style-type: none"> <li>• Apply the commutative, associative, and distributive properties as strategies to multiply and divide.</li> </ul>
	<ul style="list-style-type: none"> <li>• Apply the commutative property as a strategy to multiply.</li> </ul>	<ul style="list-style-type: none"> <li>• Understand division as an unknown factor problem.</li> </ul>	
	<ul style="list-style-type: none"> <li>• Fluently multiply and divide with products or dividends less than or equal to 25.</li> </ul>	<ul style="list-style-type: none"> <li>• Fluently multiply and divide with products or dividends greater than 25 and less than or equal to 50.</li> </ul>	<ul style="list-style-type: none"> <li>• Apply the commutative, associative, and distributive properties as strategies to multiply and divide.</li> </ul>
	<ul style="list-style-type: none"> <li>• Fluently multiply and divide with products or dividends less than or equal to 25.</li> </ul>	<ul style="list-style-type: none"> <li>• Fluently multiply and divide with products or dividends greater than 25 and less than or equal to 50.</li> </ul>	<ul style="list-style-type: none"> <li>• Fluently multiply and divide with products or dividends greater than 50 and less than or equal to 100.</li> </ul>
	<ul style="list-style-type: none"> <li>• Identify arithmetic patterns.</li> </ul>		<ul style="list-style-type: none"> <li>• Fluently multiply and divide with products or dividends greater than 50 and less than or equal to 100.</li> </ul>
			<ul style="list-style-type: none"> <li>• Identify arithmetic patterns and explain them using the properties of operations.</li> </ul>
			<ul style="list-style-type: none"> <li>• Identify arithmetic patterns and explain them using the properties of operations.</li> </ul>

## Mathematics Grade 3—Concepts & Procedures

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Number and Operations in Base 10</b>	<ul style="list-style-type: none"> <li>Round whole numbers to the nearest 10s place.</li> </ul>	<ul style="list-style-type: none"> <li>Round whole numbers to the nearest 10s and 100s place.</li> <li>Fluently add and subtract whole numbers within 1,000 with or without grouping, using the standard algorithm and strategies based on place value, properties of operations, and the relationship between addition and subtraction.</li> </ul>	
	<ul style="list-style-type: none"> <li>Multiply one-digit whole numbers by 10.</li> </ul>	<ul style="list-style-type: none"> <li>Multiply one-digit whole numbers by multiples of 10 in the range 10–90.</li> </ul>	
<b>Number and Operations – Fractions</b>	<ul style="list-style-type: none"> <li>Understand a fraction <math>1/b</math> as the quantity formed by 1 part when a whole is partitioned into <math>b</math> equal parts.</li> </ul>	<ul style="list-style-type: none"> <li>Understand a fraction <math>a/b</math> as the quantity formed by <math>a</math> parts of size <math>1/b</math>.</li> </ul>	
	<ul style="list-style-type: none"> <li>Represent a fraction <math>1/b</math> on a number line.</li> </ul>	<ul style="list-style-type: none"> <li>Represent a fraction <math>a/b</math> on a number line.</li> </ul>	
		<ul style="list-style-type: none"> <li>Understand two fractions as equivalent when they are the same size or the same point on a number line.</li> </ul>	
		<ul style="list-style-type: none"> <li>Recognize and generate simple equivalent fractions (denominators of 2, 3, 4, 6, and 8).</li> </ul>	<ul style="list-style-type: none"> <li>Recognize and generate simple equivalent fractions (denominators 2, 3, 4, 6, and 8), and explain why the fractions are equivalent.</li> </ul>
		<ul style="list-style-type: none"> <li>Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.</li> </ul>	
	<ul style="list-style-type: none"> <li>Compare two fractions with the same numerator or the same denominator referring to the same whole. Record the comparisons with the symbols <math>&gt;</math>, <math>=</math>, or <math>&lt;</math>.</li> </ul>	<ul style="list-style-type: none"> <li>Justify the comparison of two fractions referring to the same whole (e.g., by using a visual fraction model).</li> </ul>	

## Mathematics Grade 3—Concepts & Procedures

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Measurement and Data</b>	<ul style="list-style-type: none"> <li>Tell time and write time to the nearest 5 minutes.</li> </ul>	<ul style="list-style-type: none"> <li>Tell time and write time to the nearest minute.</li> </ul>	
		<ul style="list-style-type: none"> <li>Measure time intervals in minutes within the same hour.</li> </ul>	<ul style="list-style-type: none"> <li>Measure time intervals in minutes.</li> </ul>
		<ul style="list-style-type: none"> <li>Solve word problems involving addition and subtraction of time intervals in minutes within the same hour.</li> </ul>	<ul style="list-style-type: none"> <li>Solve word problems involving addition and subtraction of time intervals in minutes.</li> </ul>
	<ul style="list-style-type: none"> <li>Measure liquid volumes using liters.</li> </ul>	<ul style="list-style-type: none"> <li>Measure and estimate liquid volumes using liters, and masses using grams and kilograms.</li> </ul>	<ul style="list-style-type: none"> <li>Measure and estimate liquid volumes and masses using metric measures beyond standard units (grams, kilograms, and liters).</li> </ul>
		<ul style="list-style-type: none"> <li>Use the four operations to solve one-step word problems involving masses or volumes that are given in the same units.</li> </ul>	
	<ul style="list-style-type: none"> <li>Draw a picture graph and a bar graph, scaled by 1s and 2s, to represent a data set with four or fewer categories.</li> </ul>	<ul style="list-style-type: none"> <li>Draw a picture graph and a bar graph, both scaled by 1s, 2s, or 5s, to represent a data set with several categories.</li> </ul>	<ul style="list-style-type: none"> <li>Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories.</li> </ul>
		<ul style="list-style-type: none"> <li>Solve one-step how many more/less problems using scaled bar graphs.</li> </ul>	<ul style="list-style-type: none"> <li>Solve one- and two-step how many more/less problems using scaled bar graphs.</li> </ul>
	<ul style="list-style-type: none"> <li>Measure lengths using rulers marked with halves of an inch (U.S.).</li> </ul>	<ul style="list-style-type: none"> <li>Measure lengths using rulers marked with halves and fourths of an inch (U.S.).</li> </ul>	<ul style="list-style-type: none"> <li>Measure lengths using rulers marked with halves, fourths, and eighths of an inch (U.S.).</li> </ul>
	<ul style="list-style-type: none"> <li>Make a line plot to represent measurement data, using a scale of whole numbers, halves, or quarters.</li> </ul>		
<ul style="list-style-type: none"> <li>Recognize area as an attribute of squares and rectangles.</li> </ul>	<ul style="list-style-type: none"> <li>Recognize area as an attribute of any plane figure.</li> </ul>		
	<ul style="list-style-type: none"> <li>Know that a square with a side length of 1 unit has an area of 1 square unit and can be used to measure area.</li> </ul>		

## Mathematics Grade 3—Concepts & Procedures

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Measurement and Data</b>	<ul style="list-style-type: none"> <li>Find the area of a rectangle with whole-number side lengths by tiling it.</li> </ul>	<ul style="list-style-type: none"> <li>Show that the area of a rectangle with whole-number side lengths can be found both by tiling it and by multiplying the side lengths.</li> </ul>	
		<ul style="list-style-type: none"> <li>Multiply side lengths to find areas of rectangles with whole-number side lengths to solve mathematical problems.</li> </ul>	<ul style="list-style-type: none"> <li>Use tiling to show that the area of a rectangle with whole-number side lengths <math>a</math> and <math>b + c</math> is the sum of <math>a \times b</math> and <math>a \times c</math>. Use area models to represent the distributive property in mathematical reasoning.</li> </ul>
		<ul style="list-style-type: none"> <li>Find areas of rectilinear figures (figures formed by straight lines) by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts.</li> </ul>	<ul style="list-style-type: none"> <li>Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.</li> </ul>
	<ul style="list-style-type: none"> <li>Solve real-world and mathematical problems by finding the perimeter, given side lengths in pictures of rectangular plane figures.</li> </ul>	<ul style="list-style-type: none"> <li>Solve real-world and mathematical problems by using the perimeter to find an unknown side length.</li> </ul>	<ul style="list-style-type: none"> <li>Solve real-world and mathematical problems that involve rectangles with the same perimeter and different areas or the same area and different perimeters.</li> </ul>
<b>Geometry</b>	<ul style="list-style-type: none"> <li>Recognize that figures with the same number of sides belong to the same category.</li> </ul>	<ul style="list-style-type: none"> <li>Recognize that figures with the same attribute belong to the same category and name the category.</li> </ul>	
	<ul style="list-style-type: none"> <li>Partition shapes into parts with equal areas.</li> </ul>	<ul style="list-style-type: none"> <li>Partition shapes into parts with equal areas, based on a unit fraction of the whole.</li> </ul>	<ul style="list-style-type: none"> <li>Partitions shapes into parts with equal areas and express the area of each part as a unit fraction of the whole.</li> </ul>
<b>Problem-Solving</b>	<ul style="list-style-type: none"> <li>Discuss how to solve a problem.</li> </ul>	<ul style="list-style-type: none"> <li>Explain the meaning of a problem and look for ways to solve it.</li> </ul>	<ul style="list-style-type: none"> <li>Listen to the strategies others used to solve a problem and differentiate approaches.</li> </ul>
		<ul style="list-style-type: none"> <li>Check the solution to a problem by asking, "Does this make sense?"</li> </ul>	<ul style="list-style-type: none"> <li>Use another method to check the answer to a problem.</li> </ul>
	<ul style="list-style-type: none"> <li>Use concrete objects to help in solving problems.</li> </ul>	<ul style="list-style-type: none"> <li>Use concrete objects and pictures to help in solving problems.</li> </ul>	

**Mathematics Grade 3—Mathematical Practices**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Quantitative Reasoning</b>		<ul style="list-style-type: none"> <li>Recognize that a number represents a specific quantity.</li> </ul>	
<b>Logical Reasoning</b>	<ul style="list-style-type: none"> <li>Construct an argument using concrete objects.</li> </ul>	<ul style="list-style-type: none"> <li>Create a representation of the quantitative components of a given problem.</li> <li>Construct an argument using concrete objects, pictures, and drawings.</li> <li>Explain their thinking to others.</li> </ul>	<ul style="list-style-type: none"> <li>Create a representation of the quantitative components and relationships in a given problem.</li> <li>Construct an argument using concrete objects, pictures, drawings, and mathematical expressions.</li> <li>Respond to others' thinking.</li> <li>Ask questions designed to refine others' thinking.</li> </ul>
<b>Modeling</b>	<ul style="list-style-type: none"> <li>Represent problem situations with concrete objects.</li> </ul>	<ul style="list-style-type: none"> <li>Represent problem situations with concrete objects, numbers, pictures, lists, charts, and graphs.</li> </ul>	<ul style="list-style-type: none"> <li>Represent problem situations with concrete objects, pictures, lists, charts, graphs, words (mathematical language), actions, and mathematical expressions.</li> </ul>
<b>Patterns and Structures</b>	<ul style="list-style-type: none"> <li>Identify a pattern or mathematical structure in a situation.</li> </ul>	<ul style="list-style-type: none"> <li>Use common mathematical properties (commutative and distributive properties) to solve problems.</li> </ul>	

# Grade 4 Mathematics Performance Level Descriptors

## Advanced

By the end of the year, fourth graders at the **Advanced** level can solve multistep mathematical problems using multi-digit whole numbers with or without regrouping; solve addition and subtraction problems using fractions and/or mixed numbers with or without like denominators; multiply one-digit whole numbers times multi-digit whole numbers with or without regrouping; multiply one-digit whole numbers times unit fractions; represent fractions as decimal numbers; compare fractions with unlike denominators, equivalent fractions with like and unlike denominators, and decimal numbers; identify, describe, and compare two-dimensional figures based on their geometric properties; solve problems requiring conversions within measurement systems; solve nonstandard problems; generate and explain figural and numeric patterns; construct models to represent and solve problems; develop and communicate arguments supported by data; and analyze complex mathematical representations.

## Proficient

By the end of the year, fourth graders at the **Proficient** level can solve multistep mathematical problems using multi-digit whole numbers and fractions with like denominators; multiply one-digit whole numbers by multi-digit whole numbers and unit fractions; represent and compare fractions, equivalent fractions, and decimal numbers; identify and describe the geometric properties of geometric figures; work within measurement systems to solve problems; use models to represent and solve nonstandard problems; analyze others' arguments and identify flaws in arguments if appropriate; and generate, define, and explain figural and numeric patterns.

## Nearing Proficient

By the end of the year, fourth graders at the **Nearing Proficient** level can solve simple mathematical problems using math facts and unit fractions; and identify, describe, and create simple predictable patterns.

## Novice

By the end of the year, fourth graders at the **Novice** level demonstrate evidence of emerging understanding and use of mathematical concepts and practices, procedural skill and fluency, and the application of mathematics to solve problems.

**Mathematics Grade 4—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Operations and Algebraic Thinking</b>	<ul style="list-style-type: none"> <li>Represent verbal statements of multiplicative comparisons as multiplication equations.</li> </ul>	<ul style="list-style-type: none"> <li>Interpret a multiplication equation as a comparison.</li> <li>Represent verbal statements of multiplicative equations that involve a letter for an unknown.</li> </ul>	<ul style="list-style-type: none"> <li>Explain why multiplication can be interpreted as a comparison.</li> </ul>
		<ul style="list-style-type: none"> <li>Multiply or divide to solve word problems involving multiplicative comparison.</li> </ul>	<ul style="list-style-type: none"> <li>Distinguish multiplicative comparison from additive comparison.</li> </ul>
		<ul style="list-style-type: none"> <li>Solve multistep word problems posed with whole numbers and having whole-number answers.</li> </ul>	<ul style="list-style-type: none"> <li>Solve multistep word problems posed with whole numbers and having whole-number answers, including problems in which remainders must be interpreted. Assess the reasonableness of answers using mental computation and estimation strategies (including rounding).</li> </ul>
	<ul style="list-style-type: none"> <li>Find one or two factor pairs for a whole number less than or equal to 50.</li> </ul>	<ul style="list-style-type: none"> <li>Find all factor pairs for multiples of 2 and 5 for a whole number less than or equal to 100.</li> </ul>	<ul style="list-style-type: none"> <li>Find all factor pairs for a whole number less than or equal to 100.</li> </ul>
		<ul style="list-style-type: none"> <li>Recognize that a whole number is a multiple of each of its factors.</li> </ul>	
	<ul style="list-style-type: none"> <li>Determine whether a whole number between 1 and 50 is a prime or composite number.</li> </ul>	<ul style="list-style-type: none"> <li>Determine whether a whole number between 1 and 100 is a prime or composite number.</li> </ul>	
		<ul style="list-style-type: none"> <li>Identify multiples of a one-digit number.</li> </ul>	
	<ul style="list-style-type: none"> <li>Generate a shape pattern that follows a given rule.</li> </ul>	<ul style="list-style-type: none"> <li>Generate a number pattern that follows a given rule.</li> </ul>	<ul style="list-style-type: none"> <li>Identify features of a shape or number pattern that were not explicit in the rule itself.</li> </ul>

**Mathematics Grade 4—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Number and Operations – Base 10</b>		<ul style="list-style-type: none"> <li>Recognize that in a multi-digit whole number, a digit in one place represents ten times what it would represent in the place to its right.</li> </ul>	
	<ul style="list-style-type: none"> <li>Write numeric representations of number names less than or equal to 999.</li> </ul>	<ul style="list-style-type: none"> <li>Read and write multi-digit whole numbers using number names.</li> </ul>	<ul style="list-style-type: none"> <li>Read and write multi-digit whole numbers using expanded form.</li> </ul>
		<ul style="list-style-type: none"> <li>Use <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record whole number comparisons.</li> </ul>	
	<ul style="list-style-type: none"> <li>Round whole numbers less than 1,000 to any place value.</li> </ul>	<ul style="list-style-type: none"> <li>Round whole numbers between 1,000 and 10,000 to any place value.</li> </ul>	<ul style="list-style-type: none"> <li>Round whole numbers between 10,000 and 1,000,000 to any place value.</li> </ul>
	<ul style="list-style-type: none"> <li>Fluently add and subtract multi-digit whole numbers without regrouping using the standard algorithm.</li> </ul>	<ul style="list-style-type: none"> <li>Fluently add and subtract multi-digit whole numbers with regrouping using the standard algorithm.</li> </ul>	
		<ul style="list-style-type: none"> <li>Multiply a whole number of up to four digits by a one-digit whole number using strategies based on place value and the properties of operations. Illustrate the calculation by using area models and/or rectangular arrays.</li> </ul>	<ul style="list-style-type: none"> <li>Multiply two two-digit numbers using strategies based on place value and the properties of operations. Illustrate the calculation by using area models, rectangular arrays, and equations.</li> </ul>
<ul style="list-style-type: none"> <li>Find quotients of two-digit numbers divided by one-digit numbers without remainders.</li> </ul>	<ul style="list-style-type: none"> <li>Find whole-number quotients and remainders with up to two-digit dividends and one-digit divisors. Illustrate the calculation by using areas models and/or rectangular arrays.</li> </ul>	<ul style="list-style-type: none"> <li>Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors. Illustrate the calculation by using equations.</li> </ul>	

**Mathematics Grade 4—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Number and Operations – Fractions</b>	<ul style="list-style-type: none"> <li>Identify fractions that are equivalent to unit fractions with denominators 2, 3, 4, 6, 8, or 10.</li> </ul>	<ul style="list-style-type: none"> <li>Generate equivalent fractions and explain why two fractions are equivalent using visual fraction models.</li> <li>Compare two fractions with unlike numerators and like denominators. Record the results with symbols <math>&gt;</math>, <math>=</math>, or <math>&lt;</math>. Justify the conclusion.</li> </ul>	<ul style="list-style-type: none"> <li>Generate equivalent fractions and explain why two fractions are equivalent using visual fraction models and common denominators.</li> </ul>
		<ul style="list-style-type: none"> <li>Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.</li> </ul>	
		<ul style="list-style-type: none"> <li>Decompose fractions into unit fractions with the same denominator.</li> </ul>	<ul style="list-style-type: none"> <li>Use different methods to decompose fractions with the same denominator. Justify decompositions.</li> </ul>
	<ul style="list-style-type: none"> <li>Add fractions to whole numbers to create mixed numbers.</li> </ul>	<ul style="list-style-type: none"> <li>Add and subtract mixed numbers with like denominators.</li> </ul>	
	<ul style="list-style-type: none"> <li>Solve word problems involving addition of fractions referring to the same whole and having like denominators.</li> </ul>	<ul style="list-style-type: none"> <li>Solve problems involving addition and subtraction of fractions referring to the same whole and having like denominators.</li> </ul>	<ul style="list-style-type: none"> <li>Solve problems involving addition and subtraction of fractions referring to the same whole and having unlike denominators.</li> </ul>
	<ul style="list-style-type: none"> <li>Understand a fraction <math>a/b</math> as a multiple of <math>1/b</math>.</li> </ul>	<ul style="list-style-type: none"> <li>Multiply unit fractions by whole numbers.</li> </ul>	<ul style="list-style-type: none"> <li>Solve word problems involving multiplication of a fraction by a whole number. Represent such problems using visual fraction models and equations.</li> </ul>
	<ul style="list-style-type: none"> <li>Express fraction with denominator 10 as an equivalent fraction with denominator 100. Use this technique to add two fractions with respective denominators 10 and 100.</li> </ul>	<ul style="list-style-type: none"> <li>Use decimal notation for fractions with denominators 10 or 100.</li> </ul>	
		<ul style="list-style-type: none"> <li>Compare two decimals to hundredths. Record the results with the symbols <math>&gt;</math>, <math>=</math>, or <math>&lt;</math>. Justify the conclusion.</li> </ul>	<ul style="list-style-type: none"> <li>Compare two decimals to thousandths. Record the results with the symbols <math>&gt;</math>, <math>=</math>, or <math>&lt;</math>. Justify the conclusion.</li> </ul>

## Mathematics Grade 4—Concepts & Procedures

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Measurement and Data</b>	<ul style="list-style-type: none"> <li>Identify relative sizes of units within the same system.</li> <li>Record measurement equivalents in a two-column table.</li> </ul>	<ul style="list-style-type: none"> <li>Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit within the same system (e.g., km→m→cm; kg→g; lb→oz (U.S.); l→ml; hr.→min.→sec.).</li> </ul>	
	<ul style="list-style-type: none"> <li>Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money with whole numbers and simple fractions.</li> </ul>	<ul style="list-style-type: none"> <li>Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money that involve whole numbers, simple fractions, and simple decimals, and require expressing measurements given in a larger unit in terms of a smaller unit.</li> </ul>	
		<ul style="list-style-type: none"> <li>Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.</li> </ul>	
	<ul style="list-style-type: none"> <li>Apply perimeter formulas for rectangles in real-world and mathematical problems.</li> </ul>	<ul style="list-style-type: none"> <li>Apply area formulas in real-world and mathematical problems.</li> </ul>	
	<ul style="list-style-type: none"> <li>Make a line plot to display a data set of measurements in <math>\frac{1}{2}</math> fractions of a unit.</li> </ul>	<ul style="list-style-type: none"> <li>Make a line plot to display a data set of measurements in <math>\frac{1}{4}</math> or <math>\frac{1}{8}</math> fractions of a unit.</li> </ul>	<ul style="list-style-type: none"> <li>Make a line plot to display a data set of measurements in fractions of a unit appropriate for a fourth grader (denominators of 3, 5, 6, 8, and 10).</li> </ul>
		<ul style="list-style-type: none"> <li>Solve problems involving addition and subtraction by using information presented in line plots.</li> </ul>	
<ul style="list-style-type: none"> <li>Recognize angles as geometric shapes that are formed whenever two rays share a common endpoint.</li> </ul>	<ul style="list-style-type: none"> <li>Understand the following concepts of angle measurement: An angle is measured with reference to a circle with its center at the common endpoint of the rays. An angle that turns through <math>\frac{1}{360}</math> of a circle is called a “one-degree angle” and can be used to measure angles. An angle that turns through <math>n</math> one-degree angles is said to have an angle measure of <math>n</math> degrees.</li> </ul>		

**Mathematics Grade 4—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Measurement and Data</b>	<ul style="list-style-type: none"> <li>Read the measure of an angle from a protractor that has been placed on the angle.</li> </ul>	<ul style="list-style-type: none"> <li>Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.</li> </ul>	<ul style="list-style-type: none"> <li>Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.</li> <li>Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical contexts.</li> </ul>
<b>Geometry</b>	<ul style="list-style-type: none"> <li>Identify points, lines, line segments, and rays. Identify these in two-dimensional figures.</li> <li>Identify parallel lines in shapes.</li> <li>Distinguish between right triangles and non-right triangles.</li> <li>Identify or draw one line of symmetry for a two-dimensional shape.</li> </ul>	<ul style="list-style-type: none"> <li>Identify points, lines, line segments, rays, angles (right, acute, obtuse), perpendicular lines, and parallel lines. Identify these in two-dimensional figures.</li> <li>Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of right angles.</li> <li>Identify or draw all of the lines of symmetry for a two-dimensional shape.</li> </ul>	<ul style="list-style-type: none"> <li>Draw points, lines, line segments, rays, angles (right, acute, obtuse), perpendicular lines, and parallel lines. Draw two-dimensional figures with these attributes.</li> <li>Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of specific angles.</li> <li>Identify right triangles in polygons.</li> </ul>

## Mathematics Grade 4—Mathematical Practices

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Problem-Solving</b>	<ul style="list-style-type: none"> <li>Discuss how to solve a problem.</li> </ul>	<ul style="list-style-type: none"> <li>Check their thinking about how to solve a problem by asking, “Does this make sense?”</li> </ul>	<ul style="list-style-type: none"> <li>Listen to the strategies others used to solve a problem and differentiate approaches.</li> </ul>
	<ul style="list-style-type: none"> <li>Explain the meaning of a problem and look for ways to solve it.</li> </ul>	<ul style="list-style-type: none"> <li>Compare two solutions to a problem.</li> </ul>	<ul style="list-style-type: none"> <li>Use another method to check their answer to a problem.</li> </ul>
	<ul style="list-style-type: none"> <li>Use concrete objects and pictures to help in solving problems.</li> </ul>	<ul style="list-style-type: none"> <li>Use expressions and equations to help in solving problems.</li> </ul>	<ul style="list-style-type: none"> <li>Transform representations to get the needed information.</li> </ul>
<b>Quantitative Reasoning</b>	<ul style="list-style-type: none"> <li>Recognize that a number represents a specific quantity.</li> </ul>	<ul style="list-style-type: none"> <li>Extend concepts of quantity from whole numbers to fractions.</li> </ul>	<ul style="list-style-type: none"> <li>Extend concepts of quantity from whole numbers and fractions to decimals.</li> </ul>
	<ul style="list-style-type: none"> <li>Record calculations with numbers.</li> </ul>	<ul style="list-style-type: none"> <li>Create a representation of the quantitative components of a given problem, considering the appropriate units involved and the meaning of quantities.</li> </ul>	
		<ul style="list-style-type: none"> <li>Identify important quantities in a practical situation.</li> </ul>	
	<ul style="list-style-type: none"> <li>Round numbers in a problem situation by using place value concepts and understand how the rounded numbers relate to the original quantity.</li> </ul>	<ul style="list-style-type: none"> <li>Decontextualize a problem by writing simple numerical expressions.</li> </ul>	
<b>Logical Reasoning</b>	<ul style="list-style-type: none"> <li>Construct an argument using concrete objects, pictures, and drawings.</li> </ul>	<ul style="list-style-type: none"> <li>Explain their thinking and make connections between models and equations.</li> </ul>	
		<ul style="list-style-type: none"> <li>Explain their thinking to others and respond to others’ thinking.</li> </ul>	<ul style="list-style-type: none"> <li>Ask and answer questions designed to refine a group’s thinking about a problem.</li> </ul>

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**Mathematics Grade 4—Mathematical Practices**

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**Modeling**

- Represent problem situations with concrete objects, numbers, pictures, lists, charts, and graphs.
- Represent problem situations with equations.
- Explain the connections between two or three representations (concrete objects, numbers, pictures, lists, charts, graphs, and equations).
- Evaluate results in the context of the situation and reflect on whether the results make sense.
- Use multiple representations as needed to solve a problem.

**Patterns and Structures**

- Use the structure of tree diagrams and arrays to describe the multiplication principle of counting.
  - Use properties of operations to explain calculations (partial product model).
  - Identify attributes for presorted groups of shapes.
  - Generate number or shape patterns that follow a given rule.
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# Grade 5 Mathematics Performance Level Descriptors

## Advanced

By the end of the year, fifth graders at the **Advanced** level can solve multistep mathematical problems using multi-digit numbers, fractions, and mixed numbers; divide a whole number with up to four digits by a two-digit whole number; represent, compare, and compute decimal numbers to the hundredths place; use exponents to denote powers of 10; solve problems involving the volume of right rectangular prisms; identify and describe the geometric properties of geometric figures and use them to classify figures; work within measurement systems to solve problems; use models to represent and solve nonstandard problems; analyze others' arguments and identify flaws in arguments if appropriate; and identify, define, and explain numeric patterns.

## Proficient

By the end of the year, fifth graders at the **Proficient** level can solve multistep mathematical problems using multi-digit numbers and fractions; divide a whole number with up to four digits by a two-digit whole number; represent, compare, and compute decimal numbers to the tenths place; find the volume of right rectangular prisms; identify and describe the geometric properties of geometric figures; work within measurement systems to solve problems; use models to represent and solve nonstandard problems; analyze others' arguments and identify flaws in arguments if appropriate; and identify, define, and explain numeric patterns.

## Nearing Proficient

By the end of the year, fifth graders at the **Nearing Proficient** level can add and subtract fractions with unlike denominators; solve mathematical problems using whole numbers and fractions with like denominators; work within measurement systems to solve problems; and identify, describe, and create patterns.

## Novice

By the end of the year, fifth graders at the **Novice** level demonstrate evidence of emerging understanding and use of mathematical concepts and practices, procedural skill and fluency, and the application of mathematics to solve problems.

## Mathematics Grade 5—Concepts & Procedures

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Operations and Algebraic Thinking</b>	<ul style="list-style-type: none"> <li>Evaluate numerical expressions that use parentheses.</li> </ul>	<ul style="list-style-type: none"> <li>Evaluate numerical expressions that use parentheses, brackets, and/or braces.</li> </ul>	
	<ul style="list-style-type: none"> <li>Generate two number patterns that follow two given rules.</li> </ul>	<ul style="list-style-type: none"> <li>Write and interpret simple numerical expressions that record calculations with numbers using two of the four operations.</li> <li>Form ordered pairs consisting of corresponding terms from two number patterns that follow two given rules. Graph the ordered pairs on a coordinate grid.</li> </ul>	<ul style="list-style-type: none"> <li>Write and interpret simple numerical expressions that record calculations with numbers using the four operations.</li> <li>Identify apparent relationships between corresponding terms from two number patterns with different rules.</li> </ul>
<b>Number and Operations – Base 10</b>	<ul style="list-style-type: none"> <li>Recognize that in a multi-digit whole number, a digit in one place represents 10 times what it represents in the place to its right.</li> </ul>	<ul style="list-style-type: none"> <li>Recognize that in a multi-digit number, a digit in one place is 1/10 of what it represents in the place to its left.</li> </ul>	<ul style="list-style-type: none"> <li>Recognize that in a multi-digit number, a digit in one place represents one hundred times what it represents two places to its right and 1/100 of what it represents in two places to its left.</li> </ul>
	<ul style="list-style-type: none"> <li>Explain patterns in the number of zeros of the product when multiplying a number by a power of 10.</li> </ul>	<ul style="list-style-type: none"> <li>Use exponents to denote powers of 10.</li> </ul>	<ul style="list-style-type: none"> <li>Explain what a numeral to the 10th power means.</li> </ul>
	<ul style="list-style-type: none"> <li>Read and write multi-digit numbers to the tenths place, using base-ten numerals, number names, and expanded form.</li> </ul>	<ul style="list-style-type: none"> <li>Read and write multi-digit decimal numbers to the hundredths place, using base-ten numerals, number names, and expanded form.</li> </ul>	<ul style="list-style-type: none"> <li>Read and write multi-digit decimal numbers to the thousandths place, using base-ten numerals, number names, and expanded form.</li> </ul>
	<ul style="list-style-type: none"> <li>Use <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record decimal number comparisons to the tenths place.</li> </ul>	<ul style="list-style-type: none"> <li>Use <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record decimal number comparisons to the thousandths place.</li> </ul>	<ul style="list-style-type: none"> <li>Use <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record decimal number comparisons to the ten thousandths place.</li> </ul>
		<ul style="list-style-type: none"> <li>Round decimal numbers to any whole-number place and to the hundredths place.</li> </ul>	<ul style="list-style-type: none"> <li>Round decimal numbers to any whole-number place and to any decimal place.</li> </ul>
	<ul style="list-style-type: none"> <li>Fluently multiply up to a four-digit whole number by a one-digit whole number using the standard algorithm.</li> </ul>	<ul style="list-style-type: none"> <li>Fluently multiply multi-digit whole numbers using the standard algorithm.</li> </ul>	

**Mathematics Grade 5—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Number and Operations – Base 10</b>	<ul style="list-style-type: none"> <li>Find whole-number quotients with up to two-digit dividends and two-digit divisors.</li> </ul>	<ul style="list-style-type: none"> <li>Find whole-number quotients with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation using rectangular arrays and/or area models.</li> </ul>	<ul style="list-style-type: none"> <li>Find whole-number quotients with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation using equations, rectangular arrays, and/or area models.</li> </ul>
	<ul style="list-style-type: none"> <li>Add and subtract multi-digit decimals to the tenths place.</li> </ul>	<ul style="list-style-type: none"> <li>Add, subtract, multiply, and divide decimals to the hundredths place, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</li> </ul>	<ul style="list-style-type: none"> <li>Relate the strategy used to add, subtract, multiply, and divide decimals to the hundredths place by relating the strategy to a written method. Explain the reasoning used.</li> </ul>
<b>Number and Operations – Fractions</b>	<ul style="list-style-type: none"> <li>Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.</li> </ul>	<ul style="list-style-type: none"> <li>Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.</li> </ul>	
		<ul style="list-style-type: none"> <li>Interpret a fraction as division of the numerator by the denominator.</li> </ul>	<ul style="list-style-type: none"> <li>Solve word problems involving division of whole numbers that result in fractions or mixed numbers.</li> </ul>
		<ul style="list-style-type: none"> <li>Interpret the product of a whole number and a fraction in terms of partitioning a whole into parts defined by the denominator.</li> </ul>	
	<ul style="list-style-type: none"> <li>Find the area of a rectangle that has one dimension as a unit fraction and the other dimension as a whole number by tiling it using unit squares of the unit fraction side length.</li> </ul>	<ul style="list-style-type: none"> <li>Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths. Show that the area is the same as would be found by multiplying the side lengths.</li> </ul>	<ul style="list-style-type: none"> <li>Represent fractions as the product of two fractions that can be lengths of a rectangle.</li> </ul>

**Mathematics Grade 5—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Number and Operations – Fractions</b>		<ul style="list-style-type: none"> <li>• Multiply fractional side lengths to find areas of rectangles.</li> </ul>	<ul style="list-style-type: none"> <li>• Interpret the product of fractions and whole numbers as compared to the value of one of the factors based on the value of the other factor.</li> </ul>
		<ul style="list-style-type: none"> <li>• Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number. Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number.</li> </ul>	<ul style="list-style-type: none"> <li>• Relate the principle of fraction equivalence <math>a/b = (nxa)/(nxb)</math> to the effect of multiplying <math>a/b</math> by 1.</li> </ul>
	<ul style="list-style-type: none"> <li>• Solve real-world problems involving multiplication of fractions.</li> </ul>	<ul style="list-style-type: none"> <li>• Solve real-world problems involving multiplication of fractions and mixed numbers.</li> </ul>	
	<ul style="list-style-type: none"> <li>• Divide a unit fraction by a non-zero whole number using a visual fraction model.</li> </ul>	<ul style="list-style-type: none"> <li>• Divide a unit fraction by a non-zero whole number, including solving word problems that result in division of a unit fraction by a non-zero whole number.</li> </ul>	<ul style="list-style-type: none"> <li>• Divide a unit fraction by a fraction, including solving word problems that result in division of a unit fraction by a unit fraction.</li> </ul>
		<ul style="list-style-type: none"> <li>• Create a story context that results in the division of a unit fraction by a non-zero whole number.</li> </ul>	<ul style="list-style-type: none"> <li>• Create a story context that results in the division of a unit fraction by a unit fraction.</li> </ul>
<b>Measurement and Data</b>	<ul style="list-style-type: none"> <li>• Convert among standard measurement units within the same system, using whole numbers.</li> </ul>	<ul style="list-style-type: none"> <li>• Solve one-step word problems involving conversions of standard measurement units within the same system.</li> </ul>	<ul style="list-style-type: none"> <li>• Solve multistep word problems involving conversions of standard measurement units within the same system.</li> </ul>
	<ul style="list-style-type: none"> <li>• Make a line plot to display a data set of measurements in fractions of a unit (<math>1/2</math>, <math>1/4</math>, <math>1/8</math>).</li> </ul>	<ul style="list-style-type: none"> <li>• Use operations on fractions for this grade to solve a one-step problem involving information presented in line plots.</li> </ul>	<ul style="list-style-type: none"> <li>• Use operations on fractions for this grade to solve multistep problems involving information presented in line plots.</li> </ul>
	<ul style="list-style-type: none"> <li>• Measure volume of a right rectangular prism by packing it with and counting unit cubes.</li> </ul>	<ul style="list-style-type: none"> <li>• Solve mathematical or word problems involving volume of a right rectangular prism by packing it with and counting unit cubes and comparing the result with applying the formulas <math>V = l \times w \times h</math> or <math>V = B \times h</math>.</li> </ul>	<ul style="list-style-type: none"> <li>• Solve word problems involving volume of a solid figure composed of non-overlapping right rectangular prisms.</li> </ul>

**Mathematics Grade 5—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Geometry</b>	<ul style="list-style-type: none"> <li>Graph points (both terms are whole numbers) in the first quadrant of the coordinate plane.</li> <li>Identify properties (e.g., number of sides and angles, types of angles, parallel sides) of two-dimensional figures.</li> </ul>	<ul style="list-style-type: none"> <li>Explain how a coordinate grid represents information.</li> <li>Represent problems by graphing points (one term is a fraction with a denominator of 2 or 4) in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.</li> <li>Classify two-dimensional figures based on their properties.</li> </ul>	<ul style="list-style-type: none"> <li>Construct coordinate grids appropriate for a problem situation.</li> <li>Represent problems by graphing points (one or both terms are fractions with a denominator of 2, 4, or 10) in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.</li> <li>Classify two-dimensional figures in hierarchy based on their properties.</li> </ul>

**Mathematics Grade 5—Concepts & Procedures****Mathematics Grade 5—Mathematical Practices**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Problem-Solving</b>	<ul style="list-style-type: none"><li>• Explain the meaning of a problem.</li></ul>	<ul style="list-style-type: none"><li>• Plan a solution pathway in order to solve a problem.</li></ul>	<ul style="list-style-type: none"><li>• Identify an efficient means to solve a problem.</li></ul>
		<ul style="list-style-type: none"><li>• Identify givens and constraints.</li></ul>	<ul style="list-style-type: none"><li>• Identify givens, constraints, relationships, and goals.</li></ul>
	<ul style="list-style-type: none"><li>• Draw diagrams of important features and relationships.</li></ul>	<ul style="list-style-type: none"><li>• Transform representations to get the needed information.</li></ul>	<ul style="list-style-type: none"><li>• Explain relationships between the representations used to solve a problem.</li></ul>
		<ul style="list-style-type: none"><li>• Solve problems by applying understandings of operations with whole numbers, decimals, and fractions including mixed numbers.</li></ul>	
		<ul style="list-style-type: none"><li>• Solve problems related to volume and measurement conversions.</li></ul>	
	<ul style="list-style-type: none"><li>• Check thinking by asking follow-up questions such as, “Does this make sense?”</li></ul>	<ul style="list-style-type: none"><li>• Check thinking by asking follow-up questions such as, “Does this make sense?” and “Can I solve the problem in a different way?”</li></ul>	<ul style="list-style-type: none"><li>• Check thinking by asking follow-up questions such as, “Does this make sense?”, “Can I solve this problem in a different way?”, and “What is the most efficient way to solve the problem?”</li></ul>
<b>Quantitative Reasoning</b>	<ul style="list-style-type: none"><li>• Recognize that a number represents a specific quantity.</li></ul>		<ul style="list-style-type: none"><li>• Link abstract representations to a concrete situation.</li></ul>
		<ul style="list-style-type: none"><li>• Connect quantities to written symbols. Extend the creation of such representations from whole numbers to fractions and decimals.</li></ul>	<ul style="list-style-type: none"><li>• Write simple expressions to express a quantity as the result of combining other quantities using the four operations. Extend the creation of expressions from whole numbers to fractions and decimals.</li></ul>
	<ul style="list-style-type: none"><li>• Consider both the appropriate units involved and the meaning of the quantities presented in a problem.</li></ul>		

## Mathematics Grade 5—Mathematical Practices

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Logical Reasoning</b>	<ul style="list-style-type: none"> <li>Make an argument using concrete referents, such as objects, pictures, and drawings.</li> </ul>	<ul style="list-style-type: none"> <li>Explain calculations based upon models and properties of operations and rules that generate patterns.</li> </ul>	<ul style="list-style-type: none"> <li>Demonstrate and explain the relationship between volume and multiplication.</li> </ul>
	<ul style="list-style-type: none"> <li>Identify information needed to clarify other's arguments.</li> </ul>	<ul style="list-style-type: none"> <li>Identify reasons in an argument.</li> </ul>	<ul style="list-style-type: none"> <li>Identify reasons and assumptions in an argument.</li> </ul>
		<ul style="list-style-type: none"> <li>Compare the reasonableness of two plausible arguments.</li> </ul>	<ul style="list-style-type: none"> <li>Compare the effectiveness (efficiency of process used to arrive at solution) of two plausible arguments.</li> </ul>
		<ul style="list-style-type: none"> <li>Identify relevant questions to clarify arguments.</li> </ul>	<ul style="list-style-type: none"> <li>Ask relevant questions to clarify arguments.</li> </ul>
<b>Modeling</b>	<ul style="list-style-type: none"> <li>Create and use models to describe phenomena.</li> </ul>	<ul style="list-style-type: none"> <li>Create and use models to solve real-world problems and describe phenomena.</li> </ul>	
		<ul style="list-style-type: none"> <li>Explain the connections between different representations when modeling a problem.</li> </ul>	
		<ul style="list-style-type: none"> <li>Evaluate results in the context of the situation and determine whether the results make sense.</li> </ul>	<ul style="list-style-type: none"> <li>Evaluate the utility of models to determine which models are most useful and efficient to solve problems.</li> </ul>
<b>Patterns and Structures</b>	<ul style="list-style-type: none"> <li>Identify a pattern or mathematical structure in a situation.</li> </ul>		
		<ul style="list-style-type: none"> <li>Use properties of operations as strategies to add, subtract, multiply, and divide with whole numbers, fractions, and decimals.</li> </ul>	
		<ul style="list-style-type: none"> <li>Examine numerical patterns and relate them to a rule or a graphical representation.</li> </ul>	<ul style="list-style-type: none"> <li>Use repeated reasoning to understand algorithms and make generalizations about patterns.</li> </ul>

# Grade 6 Mathematics Performance Level Descriptors

## Advanced

By the end of the year, sixth graders at the **Advanced** level can solve single- step mathematical equations; use ratio and rate reasoning to solve real-world and mathematical problems; compute with decimal numbers; find and position rational numbers on a number line or plot points of rational numbers on a coordinate plane; write and evaluate numerical and algebraic expressions, including those with whole-number exponents; find the area of polygons, volumes of right rectangular prisms, and surface area of three-dimensional figures made up of rectangles and triangles; plot numerical data on a dot plot, histogram, or box plot; summarize numerical data; construct and manipulate models to represent and solve nonstandard problems; analyze others' arguments and identify flaws in arguments if appropriate; and identify, define, and explain numeric patterns and determine nets of three- dimensional figures.

## Proficient

By the end of the year, sixth graders at the **Proficient** level can solve single- step mathematical equations; use ratio and rate reasoning to solve real-world and mathematical problems; compute with decimal numbers; find and position rational numbers on a number line or plot points of rational numbers on a coordinate plane; write and evaluate numerical and algebraic expressions, including those with exponents to 4; find the area of polygons, volumes of right rectangular prisms, and surface area of three-dimensional figures made up of rectangles and triangles; plot numerical data on a dot plot, histogram, or box plot; use models to represent and solve nonstandard problems; analyze others' arguments and identify flaws in arguments if appropriate; and identify, define, and explain numeric patterns and determine nets of three- dimensional figures.

## Nearing Proficient

By the end of the year, sixth graders at the **Nearing Proficient** level can solve simple mathematical equations by replacing the variable with a probable correct answer; describe relationships between quantities using ratio language; write and evaluate numerical and algebraic expressions; find the area of polygons and volumes of right rectangular prisms; plot numerical data on a dot plot; and identify, describe, and develop patterns in computations and relationships between quantities.

## Novice

By the end of the year, sixth graders at the **Novice** level demonstrate evidence of emerging understanding and use of mathematical concepts and practices, procedural skill and fluency, and the application of mathematics to solve problems.

**Mathematics Grade 6—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
<p>At these performance levels, students can:</p> <p><b>Ratios and Proportional Relationships</b></p>	<ul style="list-style-type: none"> <li>Understand the concept of a ratio.</li> </ul>	<ul style="list-style-type: none"> <li>Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.</li> </ul>	
	<ul style="list-style-type: none"> <li>Understand the concept of unit rate <math>a/b</math> associated with a ratio <math>a : b</math> with <math>b \neq 0</math>. Find unit rates when given whole number quantities that divide without a remainder.</li> </ul>	<ul style="list-style-type: none"> <li>Understand the concept of unit rate <math>a/b</math> associated with a ratio <math>a : b</math> with <math>b \neq 0</math>. Find unit rates when given whole number quantities that divide without a remainder. Use rate language in the context of a ratio relationship.</li> </ul>	
		<ul style="list-style-type: none"> <li>Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, or double number line diagrams.</li> </ul>	<ul style="list-style-type: none"> <li>Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by using equations.</li> </ul>
		<ul style="list-style-type: none"> <li>Make tables of equivalent ratios relating quantities in whole number measurements and plot the pairs of values on the coordinate plane.</li> </ul>	<ul style="list-style-type: none"> <li>Make tables of equivalent ratios relating quantities in whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</li> </ul>
		<ul style="list-style-type: none"> <li>Solve unit rate problems including those involving unit pricing and constant speed.</li> </ul>	
		<ul style="list-style-type: none"> <li>Find the whole in a percent of a quantity given the part and the percent.</li> </ul>	
			<ul style="list-style-type: none"> <li>Use ratio reasoning to convert measurement units, manipulating and transforming units appropriately when multiplying or dividing quantities.</li> </ul>

**Mathematics Grade 6—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Expressions and Equations</b>	<ul style="list-style-type: none"> <li>Write and evaluate numerical expressions that contain any combination of operations and grouping symbols but do not include whole-number exponents.</li> </ul>	<ul style="list-style-type: none"> <li>Write numerical expressions involving whole-number exponents. Evaluate numerical expressions involving whole-number exponents up to 3.</li> </ul>	<ul style="list-style-type: none"> <li>Write and evaluate numerical expressions involving whole-number exponents.</li> </ul>
		<ul style="list-style-type: none"> <li>Write expressions that record operations with numbers and with letters standing for numbers.</li> </ul>	
		<ul style="list-style-type: none"> <li>Identify parts of an expression using mathematical terms (e.g., sum, difference, product, quotient, term, factor).</li> </ul>	<ul style="list-style-type: none"> <li>Identify parts of an expression using mathematical terms (e.g., sum, difference, product, quotient, term, factor, coefficient, constant, variable).</li> </ul>
	<ul style="list-style-type: none"> <li>Evaluate expressions arising from formulas used in real-world problems.</li> </ul>	<ul style="list-style-type: none"> <li>Evaluate expressions in the conventional order when there are no grouping symbols to specify a particular order of operations.</li> </ul>	
		<ul style="list-style-type: none"> <li>Identify simple equivalent expressions.</li> </ul>	<ul style="list-style-type: none"> <li>Apply the properties of operations to generate equivalent expressions.</li> </ul>
	<ul style="list-style-type: none"> <li>Use substitution to determine whether a given number in a specified set makes a one-variable equation true where the variable term appears only on one side of the equation.</li> </ul>	<ul style="list-style-type: none"> <li>Use substitution to determine whether a given number in a specified set makes a one-variable equation true where the variable term appears on either or both sides of the equation.</li> </ul>	<ul style="list-style-type: none"> <li>Use substitution to determine whether a given number in a specified set makes a one-variable equation or inequality true.</li> </ul>
		<ul style="list-style-type: none"> <li>Use variables to represent unknown values and write expressions to represent real world and mathematical problems.</li> </ul>	
	<ul style="list-style-type: none"> <li>Solve equations of the form <math>x + p = q</math> and <math>px = q</math> for cases in which <math>p</math>, <math>q</math>, and <math>x</math> are all nonnegative whole numbers.</li> </ul>	<ul style="list-style-type: none"> <li>Solve mathematical problems by writing and solving equations of the form <math>x + p = q</math> and <math>px = q</math> for cases in which <math>p</math>, <math>q</math>, and <math>x</math> are all nonnegative rational numbers.</li> </ul>	

**Mathematics Grade 6—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Expressions and Equations</b>		<ul style="list-style-type: none"> <li>Write an inequality of the form <math>x &gt; c</math> or <math>x &lt; c</math> to represent a constraint or condition in a real- world or mathematical problem.</li> </ul>	<ul style="list-style-type: none"> <li>Write an inequality of the form <math>x &gt; c</math> or <math>x &lt; c</math> to represent a constraint or condition in a real- world or mathematical problem. State that an inequality given in this form has infinitely many solutions.</li> </ul>
		<ul style="list-style-type: none"> <li>Represent on a number line an inequality of the form <math>x &gt; c</math> or <math>x &lt; c</math> for cases in which <math>c</math> and <math>x</math> are nonnegative whole numbers.</li> </ul>	<ul style="list-style-type: none"> <li>Represent on a number line an inequality of the form <math>x &gt; c</math> or <math>x &lt; c</math> for cases in which <math>c</math> and <math>x</math> are nonnegative rational numbers.</li> </ul>
		<ul style="list-style-type: none"> <li>Use variables to represent two quantities in a real-world problem that change in relationship to one another.</li> </ul>	<ul style="list-style-type: none"> <li>Use variables to represent two quantities in a real-world or mathematical problem that change in relationship to one another.</li> </ul>
		<ul style="list-style-type: none"> <li>Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable.</li> </ul>	<ul style="list-style-type: none"> <li>Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.</li> </ul>
<b>The Number System</b>	<ul style="list-style-type: none"> <li>Divide a fraction by a unit fraction.</li> </ul>	<ul style="list-style-type: none"> <li>Divide a fraction by a fraction.</li> </ul>	<ul style="list-style-type: none"> <li>Use multiplication to interpret the quotient when a fraction is divided by another fraction.</li> </ul>
		<ul style="list-style-type: none"> <li>Solve word problems that involve division of a fraction by a fraction.</li> </ul>	<ul style="list-style-type: none"> <li>Create a context that involves dividing a fraction by a fraction and solve for a given quotient of two fractions.</li> </ul>
	<ul style="list-style-type: none"> <li>Fluently divide multi-digit whole numbers by two-digit divisors using the standard algorithm.</li> </ul>	<ul style="list-style-type: none"> <li>Fluently divide multi-digit whole numbers using the standard algorithm.</li> </ul>	
	<ul style="list-style-type: none"> <li>Fluently add and subtract multi-digit decimals.</li> </ul>	<ul style="list-style-type: none"> <li>Fluently add, subtract, and multiply multi-digit decimals using the standard algorithm.</li> </ul>	<ul style="list-style-type: none"> <li>Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm.</li> </ul>
	<ul style="list-style-type: none"> <li>Find the greatest common factor of two numbers less than or equal to 20 and the least common multiple of two numbers less than or equal to 5.</li> </ul>	<ul style="list-style-type: none"> <li>Find the greatest common factor of two numbers less than or equal to 100 and the least common multiple of two numbers less than or equal to 12.</li> </ul>	<ul style="list-style-type: none"> <li>Find the greatest common factor and/ or the least common multiple of any two whole numbers.</li> </ul>

**Mathematics Grade 6—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>The Number System</b>		<ul style="list-style-type: none"> <li>Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.</li> </ul>	<ul style="list-style-type: none"> <li>Use the distributive property to express a sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factor.</li> </ul>
	<ul style="list-style-type: none"> <li>Use positive and negative numbers to represent quantities in real-world contexts.</li> </ul>	<ul style="list-style-type: none"> <li>Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in the context.</li> </ul>	
		<ul style="list-style-type: none"> <li>Locate numbers on opposite sides of 0 on a number line.</li> </ul>	<ul style="list-style-type: none"> <li>Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on a number line. Recognize that the opposite of the opposite of a number is the number itself.</li> </ul>
		<ul style="list-style-type: none"> <li>Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane.</li> </ul>	<ul style="list-style-type: none"> <li>Recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.</li> </ul>
	<ul style="list-style-type: none"> <li>Find or position integers on a horizontal or vertical number line.</li> </ul>	<ul style="list-style-type: none"> <li>Find and position integers and rational numbers on a horizontal or vertical number line.</li> </ul>	
	<ul style="list-style-type: none"> <li>Find and position pairs of integers on a coordinate plane.</li> </ul>	<ul style="list-style-type: none"> <li>Find and position pairs of integers and familiar rational numbers on a coordinate plane.</li> </ul>	
		<ul style="list-style-type: none"> <li>Interpret statements of inequality as statements about the relative position of two numbers on a number line.</li> </ul>	
	<ul style="list-style-type: none"> <li>Identify correct statements of order for rational numbers in real-world contexts.</li> </ul>	<ul style="list-style-type: none"> <li>Write and explain statements of order for rational numbers in real-world contexts.</li> </ul>	<ul style="list-style-type: none"> <li>Interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.</li> </ul>
<ul style="list-style-type: none"> <li>Understand the absolute value of a rational number as its distance from 0 on the number line.</li> </ul>		<ul style="list-style-type: none"> <li>Distinguish comparisons of absolute value from statements about order.</li> </ul>	

**Mathematics Grade 6—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>The Number System</b>	<ul style="list-style-type: none"> <li>Solve real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane.</li> <li>Use coordinates and absolute value to find distances between points in the first quadrant with the same first coordinate or the same second coordinate.</li> </ul>	<ul style="list-style-type: none"> <li>Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane.</li> <li>Use coordinates and absolute value to find distances between points on a coordinate plane with the same first coordinate or the same second coordinate.</li> </ul>	<ul style="list-style-type: none"> <li>Use coordinates and absolute value to find distances between points on a coordinate plane with the same first coordinate or the same second coordinate. Explain the distance in the context of the problem situation.</li> </ul>
<b>Statistics and Probability</b>	<ul style="list-style-type: none"> <li>Recognize a statistical question as one that anticipates variability in the data related to the question.</li> </ul>	<ul style="list-style-type: none"> <li>Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for variability in the answers to the question.</li> <li>Pose statistical questions that can be answered by categorical data.</li> <li>Understand that a set of data collected to answer a statistical question has a distribution that can be described by its center and spread.</li> <li>Recognize that a measure of center for a numerical data set summarizes all of its values with a single number.</li> </ul>	<ul style="list-style-type: none"> <li>Pose statistical questions that can be answered by categorical or continuous data.</li> <li>Understand that a set of data collected to answer a statistical question has a distribution that can be described by its center, spread, and overall shape.</li> <li>Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.</li> </ul>
	<ul style="list-style-type: none"> <li>Display numerical data on a dot plot.</li> </ul>	<ul style="list-style-type: none"> <li>Display numerical data on a dot plot, histogram, or box plot.</li> </ul>	

**Mathematics Grade 6—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Statistics and Probability</b>	<ul style="list-style-type: none"> <li>Summarize numerical data sets by: reporting the number and range of observations and giving quantitative measures of center (median and/or mean).</li> </ul>	<ul style="list-style-type: none"> <li>Summarize numerical data sets by: reporting the number of observations, describing how an attribute of the set was measured, what its units of measure are, and by giving a quantitative measure of center (median and/or mean) and variability (interquartile range).</li> </ul>	<ul style="list-style-type: none"> <li>Summarize numerical data sets by: reporting the number of observations, describing how an attribute of the set was measured and what its units of measure are, giving quantitative measure of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.</li> <li>Predict effects on mean and median given a change in data points.</li> </ul>
<b>Geometry</b>	<ul style="list-style-type: none"> <li>Find the volume of a right rectangular prism that has one dimension with fractional edge length and the other two dimensions with whole number edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths.</li> </ul>	<ul style="list-style-type: none"> <li>Solve real-world and mathematical problems involving finding the area of right triangles, other triangles, special quadrilaterals, and polygons by decomposing into triangles and rectangles.</li> <li>Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction lengths.</li> <li>Apply the formulas for finding the volume of right rectangular prisms with fraction edge lengths to solving real-world and mathematical problems.</li> </ul>	<ul style="list-style-type: none"> <li>Solve real-world and mathematical problems involving finding the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles and decomposing into triangles, rectangles, and other shapes.</li> <li>Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism.</li> </ul>

**Mathematics Grade 6—Concepts & Procedures**

<b>Focus</b>	<b>Nearing Proficient</b>	<b>Proficient</b>	<b>Advanced</b>
At these performance levels, students can:			
<b>Geometry</b>	<ul style="list-style-type: none"><li>• Draw right triangles, squares, and rectangles in the first quadrant given the coordinates for the vertices.</li><li>• Find the side lengths of polygons in the first quadrant with the same first coordinate or the same second coordinate.</li></ul>	<ul style="list-style-type: none"><li>• Draw polygons in the coordinate plane given the coordinates for the vertices.</li><li>• Solve real world and mathematical problems involving the side lengths of polygons in the coordinate plane with the same first coordinate or the same second coordinate.</li><li>• Solve real-world or mathematical problems involving the surface area of three-dimensional figures made up of rectangles and triangles, using the nets of the figures.</li></ul>	<ul style="list-style-type: none"><li>• Solve real world and mathematical problems involving polygons in the coordinate plane.</li><li>• Solve real-world or mathematical problems involving the surface area of three-dimensional figures made up of rectangles and triangles.</li></ul>

## Mathematics Grade 6—Mathematical Practices

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Problem-Solving</b>	<ul style="list-style-type: none"> <li>Solve real-world and mathematical problems by applying understandings of operations with whole numbers, decimals, and fractions, including mixed numbers.</li> </ul>	<ul style="list-style-type: none"> <li>Solve real-world and mathematical problems by applying concepts related to ratios and discuss the process used to solve them.</li> </ul>	
	<ul style="list-style-type: none"> <li>Solve problems related to area, volume, and measurement conversions.</li> </ul>	<ul style="list-style-type: none"> <li>Solve real-world and mathematical problems by applying algebraic and geometric concepts and discuss the process used to solve them.</li> </ul>	
		<ul style="list-style-type: none"> <li>Seek the meaning of a problem, develop a representation for the problem, and then derive the solution.</li> </ul>	<ul style="list-style-type: none"> <li>Seek the meaning of a problem, develop a representation that facilitates an efficient process to find the solution, and then derive the solution.</li> </ul>
	<ul style="list-style-type: none"> <li>Check thinking by asking questions like, “Does this make sense?”</li> </ul>	<ul style="list-style-type: none"> <li>Check thinking by asking questions like, “Does this make sense?” and “Can I solve the problem in a different way?”</li> </ul>	<ul style="list-style-type: none"> <li>Check thinking by asking questions like, “Does this make sense?,” “Can I solve the problem in a different way?,” and “What is the most efficient way to solve this problem?”</li> </ul>
<b>Quantitative Reasoning</b>	<ul style="list-style-type: none"> <li>Represent familiar contexts through the use of real numbers and variables in mathematical expressions.</li> </ul>	<ul style="list-style-type: none"> <li>Represent familiar contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities.</li> </ul>	<ul style="list-style-type: none"> <li>Represent a wide variety of real-world contexts through the efficient use of real numbers and variables in mathematical expressions, equations, and inequalities.</li> </ul>
		<ul style="list-style-type: none"> <li>Contextualize to understand the meaning of the number as related to the problem.</li> </ul>	<ul style="list-style-type: none"> <li>Contextualize to understand the meaning of the number or variable as related to the problem.</li> </ul>
	<ul style="list-style-type: none"> <li>Decontextualize to manipulate numeric representations of expressions by applying properties of operations.</li> </ul>	<ul style="list-style-type: none"> <li>Decontextualize to manipulate symbolic representations by applying properties of operations.</li> </ul>	

## Mathematics Grade 6—Mathematical Practices

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Logical Reasoning</b>	<ul style="list-style-type: none"> <li>Construct arguments using drawings, models, and numeric expressions.</li> </ul>	<ul style="list-style-type: none"> <li>Construct arguments using drawings, models, numeric and algebraic expressions, equations, tables, and graphs.</li> </ul>	<ul style="list-style-type: none"> <li>Construct arguments using drawings, models, numeric and algebraic expressions, equations, inequalities, tables, and graphs.</li> </ul>
	<ul style="list-style-type: none"> <li>Identify information needed to clarify other’s arguments.</li> </ul>	<ul style="list-style-type: none"> <li>Identify reasons and assumptions in an argument.</li> </ul>	
		<ul style="list-style-type: none"> <li>Compare the reasonableness of two plausible arguments.</li> <li>Identify relevant questions to clarify arguments.</li> </ul>	<ul style="list-style-type: none"> <li>Compare the effectiveness of two plausible arguments.</li> <li>Ask relevant questions to clarify arguments.</li> </ul>
<b>Modeling</b>	<ul style="list-style-type: none"> <li>Model problem situations with concrete models and numeric expressions.</li> </ul>	<ul style="list-style-type: none"> <li>Model problem situations symbolically, graphically, in tabular form, and contextually.</li> </ul>	
	<ul style="list-style-type: none"> <li>Form numeric expressions from real-world and mathematical contexts.</li> </ul>	<ul style="list-style-type: none"> <li>Form expression, equations, or inequalities from real-world and mathematical contexts. Connect symbolic and graphical representations.</li> </ul>	
	<ul style="list-style-type: none"> <li>Use number lines to compare numbers.</li> </ul>	<ul style="list-style-type: none"> <li>Use number lines to compare numbers and represent inequalities.</li> <li>Use all representations as appropriate to a problem context.</li> </ul>	<ul style="list-style-type: none"> <li>Use all representations to efficiently and appropriately solve a problem.</li> </ul>
<b>Patterns and Structures</b>	<ul style="list-style-type: none"> <li>Identify a pattern or structure in a situation.</li> </ul>	<ul style="list-style-type: none"> <li>Identify a pattern or structure in a real-world or mathematical situation. Break down complicated phenomena into simpler parts.</li> </ul>	<ul style="list-style-type: none"> <li>Identify a pattern or structure in a real-world or mathematical situation. Break down complicated phenomena into simpler parts. Build a more complex representation from simpler parts.</li> </ul>
	<ul style="list-style-type: none"> <li>Generate patterns from rules.</li> </ul>	<ul style="list-style-type: none"> <li>Identify patterns in ratio tables and in the points when plotting quantities from common ratios on a coordinate plane. Identify patterns in computations.</li> </ul>	<ul style="list-style-type: none"> <li>Identify patterns in ratio tables and in the points when plotting quantities from common ratios on a coordinate plane. Identify patterns in computations. Identify and describe patterns in data.</li> </ul>
	<ul style="list-style-type: none"> <li>Compose and decompose two-dimensional figures to solve real-world problems involving area.</li> </ul>	<ul style="list-style-type: none"> <li>Compose and decompose two- and three-dimensional figures to solve real-world problems involving area and volume.</li> </ul>	

# Grade 7 Mathematics Performance Level Descriptors

## Advanced

By the end of the year, seventh graders at the **Advanced** level can solve problems with rational numbers of any form; solve two-step equations and inequalities, and graph the solutions of inequalities; compute unit rates and use proportional relationships to solve multi- step ratio and percent problems; add, subtract, factor, and expand linear expressions with rational coefficients; solve problems involving scale drawings, including computing actual lengths and areas from a scale drawing; solve problems involving area and circumference of a circle; solve problems involving angle measures in figures; understand that the probability of a chance event is a number between 0 and 1; develop uniform and non- uniform probability models; construct and manipulate models to represent and solve nonstandard problems; analyze others' arguments and identify flaws in arguments if appropriate; identify, define, and explain numeric patterns and patterns in tables or graphs.

## Proficient

By the end of the year, seventh graders at the **Proficient** level can solve problems with rational numbers of any form; solve two-step equations and inequalities; compute unit rates and use proportional relationships to solve multi- step ratio and percent problems; add, subtract, and expand linear expressions with rational coefficients; solve problems involving scale drawings, including computing actual lengths and areas from a scale drawing; solve problems involving angle measures in figures; understand that the probability of a chance event is a number between 0 and 1; develop uniform probability models; use models to represent and solve nonstandard problems; analyze others' arguments and identify flaws in arguments if appropriate; identify, define, and explain numeric patterns and patterns in tables or graphs.

## Nearing Proficient

By the end of the year, seventh graders at the **Nearing Proficient** level can solve addition and multiplication problems with rational numbers, subtraction problems with positive rational numbers, and division problems with rational numbers divided by non-zero integers; compute unit rates; add and subtract simple linear expressions; solve problems involving scale drawings with whole number scales; solve problems involving area and circumference of a circle; understand that the probability of a chance event is a number between 0 and 1; identify, describe, and develop patterns in computations and relationships between quantities.

## Novice

By the end of the year, seventh graders at the **Novice** level demonstrate evidence of emerging understanding and use of mathematical concepts and practices, procedural skill and fluency, and the application of mathematics to solve problems.

**Mathematics Grade 7—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Ratios and Proportional Relationships</b>	<ul style="list-style-type: none"> <li>• Compute unit rates associated with ratios of fractions <math>a/b</math>, where <math>b</math> is a unit fraction.</li> </ul>	<ul style="list-style-type: none"> <li>• Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like units.</li> </ul>	<ul style="list-style-type: none"> <li>• Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.</li> </ul>
		<ul style="list-style-type: none"> <li>• Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table.</li> </ul>	<ul style="list-style-type: none"> <li>• Decide whether two or more quantities are in a proportional relationship, e.g., by graphing on a coordinate plane and observing whether the graph is a straight line through the origin.</li> </ul>
	<ul style="list-style-type: none"> <li>• Identify the unit rate of a ratio given in a table or graph.</li> </ul>	<ul style="list-style-type: none"> <li>• Identify the unit rate of a ratio given in a table, graph, or equation.</li> </ul>	<ul style="list-style-type: none"> <li>• Identify the unit rate of a ratio given in a table, graph, equation, diagram, or verbal description.</li> </ul>
		<ul style="list-style-type: none"> <li>• Represent proportional relationships by equations.</li> </ul>	
		<ul style="list-style-type: none"> <li>• Explain what a point <math>(x, y)</math> on the graph of a proportional relationship means in terms of the situation.</li> </ul>	<ul style="list-style-type: none"> <li>• Explain what a point <math>(x, y)</math> on the graph of a proportional relationship means in terms of the situation, with special attention to the points <math>(0, 0)</math> and <math>(1, r)</math> where <math>r</math> is the unit rate.</li> </ul>
		<ul style="list-style-type: none"> <li>• Use proportional relationships to solve multi-step ratio problems.</li> </ul>	<ul style="list-style-type: none"> <li>• Use proportional relationships to solve multi-step ratio and percent problems.</li> </ul>

**Mathematics Grade 7—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Expressions and Equations</b>	<ul style="list-style-type: none"> <li>Apply properties of operations to add and subtract linear expressions with rational coefficients.</li> </ul>	<ul style="list-style-type: none"> <li>Apply properties of operations to add, subtract, and expand linear expressions with rational coefficients.</li> </ul>	<ul style="list-style-type: none"> <li>Apply properties of operations to add, subtract, factor, and expand linear expressions with integer or rational coefficients.</li> </ul>
		<ul style="list-style-type: none"> <li>Rewrite an expression in different forms in a problem context.</li> </ul>	<ul style="list-style-type: none"> <li>Rewrite an expression in different forms in a problem context and explain how the quantities in it are related.</li> </ul>
	<ul style="list-style-type: none"> <li>Solve multi-step real world and mathematical problems involving addition and multiplication with rational numbers of the same form. Solve real world and mathematical problems involving subtraction of positive rational numbers of the same form such that the minuend is greater than the subtrahend. Solve real world and mathematical problems involving division of a rational number by a non-zero integer.</li> </ul>	<ul style="list-style-type: none"> <li>Solve multi-step real world and mathematical problems involving rational numbers of any form. Convert between forms as appropriate to solve the problem. Assess the reasonableness of answers using mental computation and estimation strategies.</li> </ul>	
	<ul style="list-style-type: none"> <li>Solve equations of the form <math>px + q = r</math> and <math>p(x + q) = r</math> where <math>p</math>, <math>q</math>, and <math>r</math> are specific integers.</li> </ul>	<ul style="list-style-type: none"> <li>Solve equations of the form <math>px + q = r</math> and <math>p(x + q) = r</math> where <math>p</math>, <math>q</math>, and <math>r</math> are specific rational numbers.</li> </ul>	
		<ul style="list-style-type: none"> <li>Solve word problems leading to equations of the form <math>px + q = r</math> and <math>p(x + q) = r</math>.</li> </ul>	<ul style="list-style-type: none"> <li>Solve word problems leading to inequalities of the form <math>px + q &gt; r</math> or <math>px + q &lt; r</math>, where <math>p</math>, <math>q</math>, and <math>r</math> are specific rational numbers.</li> </ul>
		<ul style="list-style-type: none"> <li>Solve word problems leading to inequalities of the form <math>px + q &gt; r</math> or <math>px + q &lt; r</math>, where <math>p</math>, <math>q</math>, and <math>r</math> are specific integers.</li> </ul>	
		<ul style="list-style-type: none"> <li>Compare an algebraic solution of an equation to an arithmetic solution.</li> </ul>	<ul style="list-style-type: none"> <li>Compare an algebraic solution of an equation to an arithmetic solution, identifying the sequence of operations used in each approach.</li> </ul>
		<ul style="list-style-type: none"> <li>Graph the solution set of an inequality.</li> </ul>	<ul style="list-style-type: none"> <li>Graph the solution set of an inequality and interpret it in the context of the problem.</li> </ul>

## Mathematics Grade 7—Concepts & Procedures

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>The Number System</b>	<ul style="list-style-type: none"> <li>Describe situations in which opposite quantities combine to make 0.</li> </ul>	<ul style="list-style-type: none"> <li>Understand <math>p + q</math> as the number located a distance <math> q </math> from <math>p</math>, in the positive or negative direction depending on whether <math>q</math> is positive or negative.</li> </ul>	<ul style="list-style-type: none"> <li>Explain why, in <math>p + q</math> the number located a distance <math> q </math> from <math>p</math>, is in the positive or negative direction depending on whether <math>q</math> is positive or negative.</li> </ul>
		<ul style="list-style-type: none"> <li>Interpret sums of rational numbers in real world contexts.</li> </ul>	<ul style="list-style-type: none"> <li>Demonstrate sums of rational numbers by describing real world contexts.</li> </ul>
		<ul style="list-style-type: none"> <li>Understand subtraction of rational numbers as adding the additive inverse, <math>p - q = p + (-q)</math>.</li> </ul>	<ul style="list-style-type: none"> <li>Explain why subtraction of rational numbers is the same as adding the additive inverse, <math>p - q = p + (-q)</math>.</li> </ul>
		<ul style="list-style-type: none"> <li>Show that the distance between two rational numbers on the number line is the absolute value of their difference.</li> </ul>	<ul style="list-style-type: none"> <li>Show that the distance between two rational numbers on the number line is the absolute value of their difference in real world contexts.</li> </ul>
	<ul style="list-style-type: none"> <li>Apply properties of operations as strategies to add positive and negative rational numbers and to subtract two positive rational numbers where the minuend is greater than the subtrahend.</li> </ul>	<ul style="list-style-type: none"> <li>Apply properties of operations as strategies to add and subtract rational numbers.</li> </ul>	<ul style="list-style-type: none"> <li>Apply properties of operations as strategies to add, subtract, multiply, and divide signed rational numbers.</li> </ul>
	<ul style="list-style-type: none"> <li>Apply properties of operations as strategies to multiply two integers.</li> </ul>	<ul style="list-style-type: none"> <li>Compute products of rational numbers in real world contexts.</li> </ul>	
	<ul style="list-style-type: none"> <li>Understand that integers can be divided, provided that the divisor is not zero.</li> </ul>	<ul style="list-style-type: none"> <li>Explain the rules for dividing signed numbers. Compute quotients of rational numbers in real world contexts.</li> </ul>	
	<ul style="list-style-type: none"> <li>Convert between familiar fractions and decimals.</li> </ul>	<ul style="list-style-type: none"> <li>Convert common fractions and fractions with denominators that are a factor of a power of 10 to decimals.</li> </ul>	<ul style="list-style-type: none"> <li>Convert a rational number to a decimal using long division.</li> </ul>
<ul style="list-style-type: none"> <li>Solve real world and mathematical problems involving addition and multiplication with rational numbers, subtraction of positive rational numbers such that the minuend is greater than the subtrahend, and division of a rational number by a non-zero integer.</li> </ul>	<ul style="list-style-type: none"> <li>Solve real world and mathematical problems involving the four operations with rational numbers (including complex fractions).</li> </ul>		

**Mathematics Grade 7—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Statistics and Probability</b>	<ul style="list-style-type: none"> <li>Determine whether a sample is representative of a population.</li> </ul>	<ul style="list-style-type: none"> <li>Identify a sample that is representative of a population.</li> </ul>	<ul style="list-style-type: none"> <li>Explain why a sample is or is not representative of a population.</li> </ul>
	<ul style="list-style-type: none"> <li>Use data from a random sample to draw simple inferences about a population.</li> </ul>	<ul style="list-style-type: none"> <li>Generate multiple samples of the same size to gauge the variation in estimates or predictions.</li> </ul>	<ul style="list-style-type: none"> <li>Explain conditions under which we can use data from a random sample to draw inferences about a population.</li> </ul>
	<ul style="list-style-type: none"> <li>Informally assess the degree of visual overlap of two numerical data distributions with similar variability.</li> </ul>	<ul style="list-style-type: none"> <li>Informally assess the degree of visual overlap of two numerical data distributions with similar variability and measure the difference between the centers.</li> </ul>	<ul style="list-style-type: none"> <li>Informally assess the degree of visual overlap of two numerical data distributions with similar variability and measure the difference between the centers by expressing it as a multiple of a measure of variability.</li> </ul>
	<ul style="list-style-type: none"> <li>Use measures of center for numerical data from random samples to draw informal comparative inferences about two populations.</li> </ul>	<ul style="list-style-type: none"> <li>Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.</li> </ul>	
	<ul style="list-style-type: none"> <li>Understand that the probability of a chance event is a number between 0 and 1 indicating the likelihood of the event occurring.</li> </ul>	<ul style="list-style-type: none"> <li>Understand that a probability near 0 indicates an unlikely event and a probability near 1 indicates a likely event.</li> </ul>	<ul style="list-style-type: none"> <li>Understand that a probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither likely or unlikely, and a probability near 1 indicates a likely event.</li> </ul>
		<ul style="list-style-type: none"> <li>Approximate the probability of a chance event by collecting sample data on the chance process that produced it and computing the probability for a larger sample.</li> </ul>	<ul style="list-style-type: none"> <li>Approximate the probability of a chance event by collecting data on the chance process that produced it and observing its long-run relative frequency.</li> </ul>
		<ul style="list-style-type: none"> <li>Develop a uniform probability model by assigning equal probabilities to all outcomes.</li> </ul>	<ul style="list-style-type: none"> <li>Develop a uniform probability model by assigning equal probabilities to all outcomes and use the model to determine the probabilities of events.</li> </ul>
		<ul style="list-style-type: none"> <li>Evaluate whether frequencies in data generated from a chance process reflect a uniform model.</li> </ul>	<ul style="list-style-type: none"> <li>Predict the approximate relative frequency given the probability.</li> </ul>

**Mathematics Grade 7—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Statistics and Probability</b>			<ul style="list-style-type: none"> <li>• Develop a probability model (uniform or not) by observing frequencies in data generated from a chance process.</li> </ul>
			<ul style="list-style-type: none"> <li>• Compare probabilities from a model to observed frequencies and explain possible sources of discrepancies between the two.</li> </ul>
		<ul style="list-style-type: none"> <li>• Understand that the probability of a compound event is the fraction of the outcomes in the sample space for which the compound event occurs.</li> </ul>	
		<ul style="list-style-type: none"> <li>• Represent sample spaces for compound events using methods such as organized lists and tree diagrams.</li> </ul>	<ul style="list-style-type: none"> <li>• Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams.</li> </ul>
<b>Geometry</b>	<ul style="list-style-type: none"> <li>• Solve problems involving scale drawings of geometric figures, including computing actual lengths from a scale drawing.</li> </ul>	<ul style="list-style-type: none"> <li>• Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing.</li> </ul>	<ul style="list-style-type: none"> <li>• Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.</li> </ul>
	<ul style="list-style-type: none"> <li>• Draw geometric shapes with given conditions (e.g., number of sides, types of angles, parallel sides, lengths of sides, . . .).</li> </ul>	<ul style="list-style-type: none"> <li>• Draw geometric shapes with given conditions, focusing on triangles from three measures of angles or sides.</li> </ul>	<ul style="list-style-type: none"> <li>• Draw geometric shapes with given conditions, focusing on triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.</li> </ul>
		<ul style="list-style-type: none"> <li>• Describe the two-dimensional figures that result from slicing right rectangular prisms and right rectangular pyramids.</li> </ul>	<ul style="list-style-type: none"> <li>• Describe the two-dimensional figures that result from slicing right rectangular prisms, right rectangular pyramids, spheres, and cones.</li> </ul>
		<ul style="list-style-type: none"> <li>• Solve problems involving the area and circumference of a circle.</li> </ul>	<ul style="list-style-type: none"> <li>• Explain the relationship between circumference and area of circles in real world problems.</li> </ul>

**Mathematics Grade 7—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<p><b>Geometry</b></p>	<ul style="list-style-type: none"> <li>Use facts about supplementary and complementary angles to solve problems.</li> <li>Solve real world and mathematical problems involving area and volume of two- and three- dimensional objects composed of triangles, rectangles, squares, cubes, and right prisms.</li> </ul>	<ul style="list-style-type: none"> <li>Use facts about supplementary, complementary, vertical, and adjacent angles in a one-step problem to solve for unknown angles. (The problem may be presented as a one-step equation).</li> <li>Solve real world and mathematical problems involving area, volume, and surface area of two- and three- dimensional objects composed of triangles, quadrilaterals, cubes, and right prisms.</li> </ul>	<ul style="list-style-type: none"> <li>Give an informal derivation of the relationship between the circumference and area of a circle.</li> <li>Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. (The problem may be presented as a multi-step equation.)</li> <li>Solve real world and mathematical problems involving area, volume, and surface area of two- and three- dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</li> </ul>

## Mathematics Grade 7—Mathematical Practices

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Problem-Solving</b>	<ul style="list-style-type: none"> <li>Solve problems involving ratios and discuss solution strategy.</li> </ul>	<ul style="list-style-type: none"> <li>Solve problems involving ratios and rates and discuss solution strategy.</li> </ul>	
	<ul style="list-style-type: none"> <li>Solve problems through the application of algebraic and geometric concepts.</li> </ul>		
	<ul style="list-style-type: none"> <li>Seek the meaning of a problem and look for a solution pathway.</li> </ul>	<ul style="list-style-type: none"> <li>Seek the meaning of a problem and look for efficient ways to represent and solve it.</li> </ul>	<ul style="list-style-type: none"> <li>Identify simpler forms of the original problem in order to gain insight into its solution and solve such problems.</li> </ul>
	<ul style="list-style-type: none"> <li>Check thinking by asking questions like, “Does this make sense?” and “Can I solve the problem in a different way?”</li> </ul>	<ul style="list-style-type: none"> <li>Check thinking by asking questions like, “Does this make sense?”, “Can I solve the problem in a different way?”, and “What is the most efficient way to solve the problem?”</li> </ul>	<ul style="list-style-type: none"> <li>Identify the more efficient and effective of two solutions.</li> </ul>
<b>Quantitative Reasoning</b>	<ul style="list-style-type: none"> <li>Represent familiar contexts through the use of real numbers and variables in mathematics expressions and equations.</li> </ul>	<ul style="list-style-type: none"> <li>Represent familiar contexts through the use of real numbers and variables in mathematics expressions, equations, and inequalities.</li> </ul>	<ul style="list-style-type: none"> <li>Represent a wide variety of real-world contexts through the efficient use of real numbers and variables in mathematics expressions, equations, and inequalities.</li> </ul>
		<ul style="list-style-type: none"> <li>Contextualize to understand the meaning of the number or variable as related to the problem.</li> </ul>	
	<ul style="list-style-type: none"> <li>Decontextualize to manipulate symbolic representations in expressions and equations by applying properties of operations.</li> </ul>	<ul style="list-style-type: none"> <li>Decontextualize to manipulate symbolic representations in expressions, equations, and inequalities by applying properties of operations.</li> </ul>	

## Mathematics Grade 7—Mathematical Practices

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Logical Reasoning</b>	<ul style="list-style-type: none"> <li>Construct arguments using drawings, models, expressions, and equations.</li> </ul>	<ul style="list-style-type: none"> <li>Construct arguments using drawings, models, expressions, equations, inequalities, tables, and graphs.</li> </ul>	
	<ul style="list-style-type: none"> <li>Identify reasons and assumptions in an argument.</li> </ul>		
	<ul style="list-style-type: none"> <li>Compare the reasonableness of two plausible arguments.</li> </ul>	<ul style="list-style-type: none"> <li>Compare the effectiveness of two plausible arguments.</li> </ul>	
	<ul style="list-style-type: none"> <li>Identify relevant questions to clarify arguments.</li> </ul>	<ul style="list-style-type: none"> <li>Asks relevant questions to clarify arguments.</li> </ul>	<ul style="list-style-type: none"> <li>Explains how their thinking is an efficient and effective argument.</li> </ul>
<b>Modeling</b>	<ul style="list-style-type: none"> <li>Model familiar problem situations symbolically, graphically, and contextually.</li> </ul>	<ul style="list-style-type: none"> <li>Model problem situations symbolically, graphically, tabularly, and contextually.</li> </ul>	
	<ul style="list-style-type: none"> <li>Form expressions and equations from real world and mathematical contexts.</li> </ul>	<ul style="list-style-type: none"> <li>Form expressions, equations, or inequalities from real world and mathematical contexts. Connect symbolic and graphical representations.</li> </ul>	
		<ul style="list-style-type: none"> <li>Represent two quantities simultaneously.</li> </ul>	<ul style="list-style-type: none"> <li>Explore covariance and represent two quantities simultaneously.</li> </ul>
		<ul style="list-style-type: none"> <li>Use measures of center and variability and data displays to draw inferences, make comparisons, and formulate predictions.</li> </ul>	<ul style="list-style-type: none"> <li>Use measures of center and data displays to draw inferences, make comparisons, and formulate predictions.</li> </ul>
		<ul style="list-style-type: none"> <li>Create probability models from data sets.</li> </ul>	<ul style="list-style-type: none"> <li>Use experiments or simulations to generate data sets and create probability models.</li> </ul>
	<ul style="list-style-type: none"> <li>Use all representations as appropriate to a problem context.</li> </ul>	<ul style="list-style-type: none"> <li>Use all representations to efficiently and appropriately solve a problem.</li> </ul>	

**Mathematics Grade 7—Mathematical Practices**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Patterns and Structures</b>	<ul style="list-style-type: none"> <li>Identify a pattern in a situation. Break down complicated phenomena into simpler parts.</li> </ul>	<ul style="list-style-type: none"> <li>Identify a pattern or structure in a situation. Break down complicated phenomena into simpler parts. Build a more complex representation from simpler parts.</li> </ul>	
	<ul style="list-style-type: none"> <li>Identify patterns in ratio tables.</li> </ul>	<ul style="list-style-type: none"> <li>Make the connection between the constant of proportionality in a ratio table with the slope of a graph (without defining slope).</li> </ul>	
	<ul style="list-style-type: none"> <li>Compose and decompose two- and three-dimensional figures to solve real world problems involving area and volume.</li> </ul>		
		<ul style="list-style-type: none"> <li>Examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities.</li> </ul>	
		<ul style="list-style-type: none"> <li>Create, explain, evaluate, and modify probability models to describe simple events.</li> </ul>	<ul style="list-style-type: none"> <li>Create, explain, evaluate, and modify probability models to describe simple and compound events.</li> </ul>
		<ul style="list-style-type: none"> <li>Make connections between covariance, rates, and representations showing the relationship between quantities.</li> </ul>	

# Grade 8 Mathematics Performance Level Descriptors

## Advanced

By the end of the year, eighth graders at the **Advanced** level can identify rational and irrational numbers and give rational approximates of irrational numbers and estimate values of expressions that include irrational numbers; solve linear equations in one variable and systems of linear equations; compare proportional relationships and properties of linear functions represented in different ways; sketch graphs that exhibit features of a function described verbally; solve problems with numbers expressed in scientific notation; translate, dilate, rotate, and reflect figures and use these transformations to determine whether figures are congruent and/or similar; apply the Pythagorean theorem to solve real world and mathematical problems, including those on a coordinate plane; construct and interpret scatterplots of bivariate data and two-way tables of categorical data; solve problems involving the volumes of cylinders, cones, and spheres; construct and manipulate models to represent and solve nonstandard problems; analyze others' arguments and identify flaws in arguments if appropriate; identify, define, and explain numeric patterns and patterns in tables or graphs.

## Proficient

By the end of the year, eighth graders at the **Proficient** level can identify rational and irrational numbers and give rational approximates of irrational numbers; solve linear equations in one variable and systems of linear equations; compare proportional relationships and properties of linear functions represented in different ways; solve addition, subtraction, and multiplication problems with numbers expressed in scientific notation; translate, dilate, rotate, and reflect figures and use these transformations to determine whether figures are congruent and/or similar; apply the Pythagorean theorem to solve real world and mathematical problems, including those on a coordinate plane; construct and interpret scatterplots of bivariate data and two-way tables of categorical data; solve problems involving the volumes of cylinders and cones; use models to represent and solve nonstandard problems; analyze others' arguments and identify flaws in arguments if appropriate; identify, define, and explain numeric patterns and patterns in tables or graphs.

## Nearing Proficient

By the end of the year, eighth graders at the **Nearing Proficient** level can identify rational and irrational numbers; solve simple linear equations in one variable; graph proportional relationships; identify relationships that are functions; use properties of positive exponents to generate equivalent expressions and write numbers in scientific notation; translate and reflect figures; construct scatterplots of bivariate data; find the volume of cylinders; identify, describe, and develop patterns in computations, relationships between quantities, and bivariate data.

## Novice

By the end of the year, eighth graders at the **Novice** level demonstrate evidence of emerging understanding and use of mathematical concepts and practices, procedural skill and fluency, and the application of mathematics to solve problems.

**Mathematics Grade 8—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
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At these performance levels, students can:

**Functions**

- |   |   |  |
|---|---|--|
| <ul style="list-style-type: none"> <li>Understand that a function is a rule that assigns to each input exactly one output.</li> </ul>                       |   |  |
|   | <ul style="list-style-type: none"> <li>Construct a graph of a function.</li> </ul>  | <ul style="list-style-type: none"> <li>Explain how a graph of a function shows the relationship between an independent and a dependent variable.</li> </ul>  |
| <ul style="list-style-type: none"> <li>Compare two functions represented in the same way (algebraically, graphically, or numerically in tables).</li> </ul> | <ul style="list-style-type: none"> <li>Compare two functions represented in different ways (algebraically, graphically, or numerically in tables).</li> </ul>   |  |
|   | <ul style="list-style-type: none"> <li>Interpret the equation <math>y = mx + b</math> as defining a linear function whose graph is a straight line.</li> </ul>  | <ul style="list-style-type: none"> <li>Explain the meaning of the variables in a <math>y = mx + b</math> function in a real-world situation.</li> </ul>  |
|   | <ul style="list-style-type: none"> <li>Give examples of functions that are not linear.</li> </ul>   |  |
|   | <ul style="list-style-type: none"> <li>Construct a function to model a linear relationship between two quantities.</li> </ul>   |  |
|   | <ul style="list-style-type: none"> <li>Determine the rate of change and initial value of the function from two <math>(x, y)</math> values, including reading these from a table or from a graph.</li> </ul>             | <ul style="list-style-type: none"> <li>Explain rate of change and initial value of the function from a description of a relationship in a real-world situation, including reading these from a table or from a graph.</li> </ul> |
|   | <ul style="list-style-type: none"> <li>Describe qualitatively (e.g., where the function is increasing/decreasing, linear/nonlinear) the functional relationship between two quantities by analyzing a graph.</li> </ul> | <ul style="list-style-type: none"> <li>Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</li> </ul>  |

**Mathematics Grade 8—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Expressions and Equations</b>	<ul style="list-style-type: none"> <li>Apply the properties of positive integer exponents to the power of 3 to generate equivalent numerical expressions.</li> </ul>	<ul style="list-style-type: none"> <li>Apply the properties of integer exponents to generate equivalent numerical expressions.</li> </ul>	
	<ul style="list-style-type: none"> <li>Use square root symbols to represent solutions to equations of the form <math>x^2 = p</math>, where <math>p</math> is a positive rational number.</li> </ul>	<ul style="list-style-type: none"> <li>Use square root and cube root symbols to represent solutions to equations of the form <math>x^2 = p</math> and <math>x^3 = p</math>, where <math>p</math> is a positive rational number.</li> </ul>	<ul style="list-style-type: none"> <li>Explain the relationship between square root and cube root solutions and equations of the form <math>x^2 = p</math> and <math>x^3 = p</math>, where <math>p</math> is a positive rational number.</li> </ul>
	<ul style="list-style-type: none"> <li>Evaluate square roots of familiar perfect squares.</li> </ul>	<ul style="list-style-type: none"> <li>Evaluate cube roots of familiar perfect cubes.</li> </ul>	
	<ul style="list-style-type: none"> <li>Know that <math>\sqrt{2}</math> is irrational.</li> </ul>		<ul style="list-style-type: none"> <li>Know why square roots may be irrational.</li> </ul>
	<ul style="list-style-type: none"> <li>Use numbers in the form of a single digit times a positive integer power of 10 to estimate numbers to 100,000.</li> </ul>	<ul style="list-style-type: none"> <li>Use numbers in the form of a single digit times an integer power of 10 to estimate very large or very small quantities.</li> </ul>	
		<ul style="list-style-type: none"> <li>Express how many times as much one number in the form of a single digit times an integer power of 10 is compared to another of the same form.</li> </ul>	
	<ul style="list-style-type: none"> <li>Perform addition with numbers expressed in scientific notation with positive integer powers of 10.</li> </ul>	<ul style="list-style-type: none"> <li>Perform addition and subtraction with numbers expressed in scientific notation with integer powers of 10, including problems where both decimal and scientific notation are used.</li> </ul>	<ul style="list-style-type: none"> <li>Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities as appropriate for a real-world situation.</li> </ul>
		<ul style="list-style-type: none"> <li>Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities.</li> </ul>	<ul style="list-style-type: none"> <li>Interpret scientific notation in the context of a situation.</li> </ul>
	<ul style="list-style-type: none"> <li>Interpret the slope of a graph as the unit rate.</li> </ul>	<ul style="list-style-type: none"> <li>Graph proportional relationships, interpreting the unit rate as the slope of the graph.</li> </ul>	<ul style="list-style-type: none"> <li>Graph proportional relationships, interpreting the unit rate in the context of the situation.</li> </ul>
			<ul style="list-style-type: none"> <li>Compare two different proportional relationships represented in different ways.</li> </ul>

**Mathematics Grade 8—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Expressions and Equations</b>	<ul style="list-style-type: none"> <li>Draw a line to represent equations of the form <math>y = mx</math> and <math>y = mx + b</math>.</li> </ul>	<ul style="list-style-type: none"> <li>Represent a proportional relationship in two ways.</li> </ul>	<ul style="list-style-type: none"> <li>Explain why the slope <math>m</math> is the same between any two distinct points on a non-vertical line in the coordinate plane.</li> </ul>
		<ul style="list-style-type: none"> <li>Use similar triangles to explain why the slope <math>m</math> is the same between any two distinct points on a non-vertical line in the coordinate plane.</li> </ul>	<ul style="list-style-type: none"> <li>Explain the relationship between <math>y = mx</math> and <math>y = mx + b</math>.</li> </ul>
		<ul style="list-style-type: none"> <li>Derive the equation <math>y = mx</math> for a line through the origin and <math>y = mx + b</math> for a line intercepting the vertical axis at <math>b</math>.</li> </ul>	<ul style="list-style-type: none"> <li>Give examples of and solve linear equations in one variable with rational coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms, with one solution, infinitely many solutions, or no solution.</li> </ul>
	<ul style="list-style-type: none"> <li>Solve linear equations in one variable with integer coefficients with one solution.</li> </ul>	<ul style="list-style-type: none"> <li>Solve linear equations in one variable with rational coefficients with one solution, infinitely many solutions, or no solution.</li> </ul>	
	<ul style="list-style-type: none"> <li>Solve systems of two linear equations in two variables given in slope-intercept form algebraically and estimate solutions by graphing the equations.</li> </ul>	<ul style="list-style-type: none"> <li>Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</li> </ul>	<ul style="list-style-type: none"> <li>Explain the solutions to a system of two linear equations in two variables in the context of a real-world situation.</li> </ul>
		<ul style="list-style-type: none"> <li>Solve systems of two linear equations algebraically and estimate solutions by graphing the equations.</li> </ul>	
		<ul style="list-style-type: none"> <li>Solve simple systems of two linear equations in two variables by inspection.</li> </ul>	<ul style="list-style-type: none"> <li>Solve real world and mathematical problems involving two linear equations in two variables.</li> </ul>
		<ul style="list-style-type: none"> <li>Represent real world and mathematical problems leading to two linear equations in two variables.</li> </ul>	

**Mathematics Grade 8—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Number System</b>	<ul style="list-style-type: none"> <li>Know whether a number is rational or irrational.</li> <li>Understand informally that every number has a decimal expansion.</li> </ul>	<ul style="list-style-type: none"> <li>Show that the decimal expansion for a rational number eventually terminates or repeats.</li> <li>Identify the rational number for a repeating decimal expansion.</li> <li>Use rational approximations of irrational numbers to compare the size of irrational numbers and locate them approximately on a number line.</li> </ul>	<ul style="list-style-type: none"> <li>Provide an example that shows that the decimal expansion for an irrational number does not eventually repeat .</li> <li>Convert a decimal expansion that repeats eventually into a rational number.</li> <li>Use rational approximations of irrational numbers, locate them approximately on a number line, and estimate the value of expressions.</li> </ul>
<b>Statistics and Probability</b>	<ul style="list-style-type: none"> <li>Construct scatterplots for bivariate measurement data.</li> <li>Informally fit a straight line to model a relationship between two quantitative variables on a scatterplot that suggests a linear association and explain why the line fits the data.</li> </ul>	<ul style="list-style-type: none"> <li>Describe patterns of association between two quantities represented on a scatter plot (positive/negative association, linear/nonlinear association).</li> <li>Informally fit a straight line to model a relationship between two quantitative variables on a scatterplot that suggests a linear association and assess the model fit by judging the closeness of the data points to the line.</li> </ul>	<ul style="list-style-type: none"> <li>Describe patterns of association between two quantities represented on a scatter plot (clustering, outliers, positive/negative association, linear/nonlinear association).</li> <li>Informally fit a straight line to model a relationship between two quantitative variables on a scatterplot that suggests a linear association, write a linear equation to represent the straight line, and informally assess the model fit by judging the closeness of the data points to the line.</li> </ul>

**Mathematics Grade 8—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Statistics and Probability</b>	<ul style="list-style-type: none"> <li>Identify patterns of association in categorical data based on frequencies in a two-way table.</li> </ul>	<ul style="list-style-type: none"> <li>Use the equation of a linear model to solve problems in the context of bivariate measurement data, identifying the slope and intercept.</li> </ul>	<ul style="list-style-type: none"> <li>Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept in the context of the problem situation.</li> </ul>
		<ul style="list-style-type: none"> <li>Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.</li> </ul>	<ul style="list-style-type: none"> <li>Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects and explain the patterns of association between the variables.</li> </ul>
			<ul style="list-style-type: none"> <li>Use relative frequencies calculated for rows and columns to describe possible association between two variables.</li> </ul>
<b>Geometry</b>	<ul style="list-style-type: none"> <li>Identify accurate reflections and translations.</li> </ul>	<ul style="list-style-type: none"> <li>Verify experimentally that for reflections, rotations, and translations lines are taken to lines, line segments are taken to line segments of the same length, angles are taken to angles of the same measure, and parallel lines are taken to parallel lines.</li> </ul>	
	<ul style="list-style-type: none"> <li>Understand that two regular polygons are congruent if they have exactly the same side lengths and angles.</li> </ul>	<ul style="list-style-type: none"> <li>Determine whether a two-dimensional figure is the result of taking an initial figure through a sequence of reflections, rotations, and translations. Identify such figures as congruent to the initial figure.</li> </ul>	
		<ul style="list-style-type: none"> <li>Given two congruent figures, describe two transformations that exhibit the congruence between them.</li> </ul>	<ul style="list-style-type: none"> <li>Given two congruent figures, describe a sequence of transformations that exhibits the congruence between them.</li> </ul>
		<ul style="list-style-type: none"> <li>Apply translations, rotations, dilations, and reflections on two-dimensional figures using coordinates.</li> </ul>	

**Mathematics Grade 8—Concepts & Procedures**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<p><b>Geometry</b></p>	<ul style="list-style-type: none"> <li>• Describe the effect of translations, rotations, dilations, and reflections on two-dimensional figures using coordinates.</li> <li>• Determine whether a two-dimensional figure is similar to another using a sequence of rotations, reflections, translations, and dilations.</li> <li>• Given two similar two-dimensional figures, describe a sequence of up to 3 transformations that exhibit the similarity between them.</li> <li>• Apply the Pythagorean Theorem to determine the unknown whole number hypotenuse length in right triangles mathematical problems in two dimensions (the side lengths are whole numbers).</li> <li>• Apply the Pythagorean Theorem to find the whole- number distance between two points within the same quadrant in a coordinate system.</li> </ul>	<ul style="list-style-type: none"> <li>• Describe the effect of translations, rotations, dilations, and reflections on two-dimensional figures using coordinates.</li> <li>• Determine whether a two-dimensional figure is similar to another using a sequence of rotations, reflections, translations, and dilations.</li> <li>• Given two similar two-dimensional figures, describe a sequence of up to 3 transformations that exhibit the similarity between them.</li> <li>• Use informal arguments to establish facts about the angle sum and exterior angle of triangles and about the angles created when parallel lines are cut by a transversal.</li> <li>• Apply the Pythagorean Theorem to determine the unknown whole number side lengths in right triangles in real world and mathematical problems in two and three dimensions (the side lengths are whole numbers).</li> <li>• Apply the Pythagorean Theorem to find the whole- number distance between two points in a coordinate system.</li> </ul>	<ul style="list-style-type: none"> <li>• Given two similar two-dimensional figures, describe a sequence of transformations that exhibit the similarity between them.</li> <li>• Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.</li> <li>• Apply the Pythagorean theorem to determine the unknown side lengths in right triangles in real world and mathematical problems in two and three dimensions (the side lengths are rational numbers).</li> <li>• Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</li> <li>• Solve problems involving the volumes of cylinders, cones, and spheres.</li> </ul>
<ul style="list-style-type: none"> <li>• Find the volume of a cylinder.</li> </ul>		<ul style="list-style-type: none"> <li>• Solve problems involving the volumes of cylinders and cones.</li> </ul>	<ul style="list-style-type: none"> <li>• Solve problems involving the volumes of cylinders, cones, and spheres.</li> </ul>

**Mathematics Grade 8—Mathematical Practices**

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Problem-Solving</b>	<ul style="list-style-type: none"> <li>Solve real world problems through the application of algebraic and geometric concepts.</li> </ul>	<ul style="list-style-type: none"> <li>Seek the meaning of a problem and look for efficient ways to represent and solve it.</li> <li>Check thinking by asking questions like, “Does this make sense?”, “Can I solve the problem in a different way?”, and “What is the most efficient way to solve the problem?”</li> </ul>	<ul style="list-style-type: none"> <li>Monitor progress while solving an unfamiliar problem and, if necessary, devise another solution strategy.</li> </ul>
<b>Quantitative Reasoning</b>	<ul style="list-style-type: none"> <li>Represent familiar real world contexts through the use of real numbers and variables in mathematics expressions, equations, and inequalities.</li> <li>Decontextualize to manipulate symbolic representations by applying properties of operations.</li> </ul>	<ul style="list-style-type: none"> <li>Represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities.</li> <li>Examine patterns in data and assess the degree of linearity of functions.</li> <li>Contextualize to understand the meaning of the number or variable as related to the problem.</li> </ul>	<ul style="list-style-type: none"> <li>Informally describe the behavior of non-linear functions.</li> </ul>
<b>Logical Reasoning</b>	<ul style="list-style-type: none"> <li>Construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, graphs, tables, and other data displays.</li> </ul>	<ul style="list-style-type: none"> <li>Explain their thinking to others and respond to others’ thinking.</li> </ul>	<ul style="list-style-type: none"> <li>Compare the effectiveness of two plausible arguments and distinguish correct logic or reasoning from that which is flawed.</li> <li>Determine the appropriate domain to which an argument applies.</li> </ul>

## Mathematics Grade 8—Mathematical Practices

Focus	Nearing Proficient	Proficient	Advanced
At these performance levels, students can:			
<b>Modeling</b>	<ul style="list-style-type: none"> <li>Model problem situations symbolically, graphically, in tabular form, and contextually.</li> </ul>		
	<ul style="list-style-type: none"> <li>Form expressions, equations, or inequalities from real world contexts. Connect symbolic and graphical representations.</li> </ul>	<ul style="list-style-type: none"> <li>Solve systems of linear equations presented in standard forms.</li> </ul>	<ul style="list-style-type: none"> <li>Solve systems of linear equations.</li> </ul>
		<ul style="list-style-type: none"> <li>Compare properties of linear functions provided in different forms</li> </ul>	<ul style="list-style-type: none"> <li>Compare properties of functions provided in different forms.</li> </ul>
	<ul style="list-style-type: none"> <li>Represent data in scatterplots.</li> </ul>	<ul style="list-style-type: none"> <li>Use scatterplots to represent data and describe associations between variables.</li> </ul>	
	<ul style="list-style-type: none"> <li>Use all representations as appropriate to a problem context.</li> </ul>	<ul style="list-style-type: none"> <li>Use all representations to efficiently and appropriately solve a problem.</li> </ul>	
<b>Patterns and Structures</b>	<ul style="list-style-type: none"> <li>Identify a pattern or structure in a situation. Break down complicated phenomena into simpler parts. Build a more complex representation from simpler parts.</li> </ul>	<ul style="list-style-type: none"> <li>Identify patterns and/or structures, model these patterns and/or structures, and use them to solve problems.</li> </ul>	
	<ul style="list-style-type: none"> <li>Examine patterns in tables and graphs. Describe relationships.</li> </ul>	<ul style="list-style-type: none"> <li>Examine patterns in tables and graphs. Describe relationships. Generate equations.</li> </ul>	<ul style="list-style-type: none"> <li>Examine patterns in tables and graphs. Describe relationships. Generate equations, inequalities, and functions.</li> </ul>
		<ul style="list-style-type: none"> <li>Solve and model problems. Identify the equivalence between the slope of a line and the rate of change in the problem.</li> </ul>	
		<ul style="list-style-type: none"> <li>Use iterative processes to determine more precise rational approximations of irrational numbers.</li> </ul>	
	<ul style="list-style-type: none"> <li>Make connections between covariance, rates, and representations showing the relationship between quantities.</li> </ul>		



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