New Mexico Instructional Scope
Algebra 1 Interpreting Functions Guide

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	Conceptual Understanding	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	Application	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	Procedural Skill and Fluency	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Understand the concept of a function and use function notation.
  - HSF.IF.A.1
  - HSF.IF.A.2
  - HSF.IF.A.3
- Interpret functions that arise in applications in terms of the context.
  - HSF.IF.B.4
  - HSF.IF.B.5
  - HSF.IF.B.6
- Analyze functions using different representations.
  - HSF.IF.C.7
  - HSF.IF.C.8
  - HSF.IF.C.9

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Understand the concept of a function and use function notation

Cluster Standard: HSF.IF.A.1

Standard	Standards for Mathematical Practice
Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 4:** Model with mathematics.

Clarification Statement	Students Who Demonstrate Understanding Can...
• Students need to understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. 8.F.A are foundational standards; however, this is students' first opportunity to work with function notation as it is explicitly left out of the Grade 8 standards.	• Distinguish between functions and nonfunctions from a graph. • Distinguish between functions and nonfunctions from a table. • Distinguish between functions and nonfunctions from an equation • Identify the domain and range of a function given a graph, table, or algebraic representation. • Understand the value of a function with proper notation: f(x) = y, the y value is the value of the function at a particular value of x.

DOK	Blooms
1	Remember, Understand

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Understand the concept of a function and use function notation

Cluster Standard: HSF.IF.A.2

Standard	Standards for Mathematical Practice
Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should be able to use function notation in a flexible way such as knowing how to plug in a value and get the corresponding output. They should also be able to understand and use x and $F(x)$ interchangeably with x and y when explaining the context of a problem. Students should know that all they must do is isolate an equation for y and then replace it with $f(x)$ (read as "f of x").	• Identify mathematical relationships and convey them using proper function notation • Find the input for a given output when given in function notation. • Identify the domain and range for any given function, presented in function notation or given as a verbal description, and define a reasonable domain in terms of a context or mathematical situation.
DOK	**Blooms**
1-2	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Understand the concept of a function and use function notation

Cluster Standard: HSF.IF.A.3

Standard	Standards for Mathematical Practice
Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.	• **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. Students should see patterns emerge when comparing the *x* and *y* values to each other. Students should know that these patterns are not coincidences and, students should know that these patterns can be thought of as sequences, or a list of numbers. Sequences can be either arithmetic (where the same number is added or subtracted) or geometric (where the same number is multiplied or divided).	• Observe a sequence as a function whose domain consists of integers. • Consider various possible sequences and determine whether they can be expressed explicitly or must be written as a function of the previous terms.
DOK	**Blooms**
1	Remember, Understand

Common Misconceptions

- Students may not recognize $f(x)$ = is the same as y =. They also will often confuse $f(x)$ with the product of f and x and not recognize that it is a form of notation.
- Students often show a lack of understanding for what '*n*' represents and often struggle to understand the notation of recurrence sequences, using different values of *n* for a given term.
- Students may believe that any relationship having an input and an output are functions, and therefore misuse function notation or terminology.

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Interpret functions that arise in applications in terms of the context

Cluster Standard: HSF.IF.B.4

Standard	Standards for Mathematical Practice
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students interpret the key features of the different functions listed in the standard. When given a table or graph of a function that models a real-life situation, explain the meaning of the characteristics of the table or graph in the context of the problem. Key features of a linear function are slope and intercepts; of a quadratic function are intervals of increase/decrease, positive/negative, maximum/minimum, symmetry, and intercepts; of an exponential function include *y*-intercept and increasing/decreasing intervals; and of an absolute value include *y*-intercept, minimum or maximum, increasing or decreasing intervals, and symmetry.	• Identify intercepts of a function. • Identify intervals where the function is increasing. • Identify intervals where the function is decreasing. • Identify intervals where the function is positive. • Identify intervals where the function is negative. • Identify relative maximums of a function. • Identify relative minimums of a function. • Identify symmetries in the functions. • Identify the end behavior of the functions. • Sketch graphs given a list of key features or a verbal model. • Sketch functions that model key feature behavior. • Label intercepts and intervals of a graph. • Interpret where the function is increasing, decreasing, positive, or negative. • Interpret relative maximums and minimums. • Interpret various symmetries, end behaviors, and periodicity.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Interpret functions that arise in applications in terms of the context

Cluster Standard: HSF.IF.B.5

Standard	Standards for Mathematical Practice
Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should focus their attention on possible input and output values, framing them as the domain and range of a function. When given a description of a function that represents a situation, the students should determine reasonable domain and range. Students relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Students need to explain the reasonableness of a domain for a given context. Students should understand that the domain of a function is the set of all possible inputs and the range is the set of all possible outputs. Also looking at if a function is continuous (time, amount of liquid filling a container) or discrete (number of people or things) and connecting back to number classifications.	• Make connections between a graph of a function and its domain. • Make connections between the graph of a function and the context it describes. • Identify when the domain of a given context is discrete or continuous and explain why.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Interpret functions that arise in applications in terms of the context

Cluster Standard: HSF.IF.B.6

Standard	Standards for Mathematical Practice
Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	• **SMP 4:** Model with mathematics. • **SMP 5:** Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students will calculate and interpret the average rate of change of a linear, quadratic, piecewise linear (to include absolute value), and exponential function (presented symbolically or as a table) over a specified interval. Students will estimate the rate of change from a graph. In addition to finding average rates of change from functions given symbolically, graphically, or in a table, students may collect data from experiments or simulations (ex. falling ball, velocity of a car, etc.) and find average rates of change over various intervals.	• Calculate the average rate of change of a function over a specified interval presented symbolically. • Calculate the average rate of change of a function over a specified interval presented in a table. • Interpret the average rate of change of a function over a specified interval presented symbolically for a given context. • Interpret the average rate of change of a function over a specified interval presented in a table for a given context. • Estimate the rate of change of a function from a graph.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- Students may confuse independent and dependent variables.
- Students may believe that the domain for all functions is all real numbers.
- Students may struggle with the concepts of rate of change and slope.
- Students may focus on the *y* values of the graph instead of the *x* values of the interval, when identifying key features of a graph.

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Analyze functions using different representations

Cluster Standard: HSF.IF.C.7

Standard	Standards for Mathematical Practice
Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. • HSF.IF.C.7.A: Graph linear and quadratic functions and show intercepts, maxima, and minima. • HSF.IF.C.7.B: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. • HSF.IF.C.7.E: Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	• **SMP 4:** Model with mathematics. • **SMP 7:** Look for and make use of structure.

Clarification Statement	Students Who Demonstrate Understanding Can...
• Students should be able to describe the significant features of different functions graphically and algebraically. Students should be able to use the significant features to sketch the graph of the function. Students should graph linear and quadratic functions and show intercepts, maxima, and minima. Students should know the slope-intercept form of linear functions, $y = mx + b$, and how to extract enough information from the equation to be able to draw it. When graphing roots, remember that for $\sqrt[n]{x}$, if n is even, the domain includes all positive integers. Otherwise, negative values are included as well. When graphing roots of the for $y =$	• Graph functions expressed symbolically showing key features of the graph by hand in simple cases and with technology for more complicated cases. • Graph linear functions showing intercepts. • Graph quadratic functions showing intercepts, maxima and minima. • Graph piecewise defined functions (step functions and absolute value functions) showing intercepts, maxima, and minima. • Compare and contrast linear, quadratic and exponential functions. • Explain issues of domain, range and usefulness when examining piecewise-defined functions.

$a\sqrt{x}+b$, remember the *y*-intercept is b. Students should remember that roots are fractional exponents. Students should know to look at the highest degree of the polynomial and its coefficient, ax^n. If *n* is even, the function will extend either up or down on both ends (as *x* goes to positive or negative infinity). If *n* is odd, they'll go in opposite directions. If *a* is positive, the even powered functions will go up and the odd powered functions will start down and go up. If *a* is negative, the even powered functions will go down, and the odd powered functions will start up and go down.	
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Analyze functions using different representations

Cluster Standard: HSF.IF.C.8

Standard	Standards for Mathematical Practice
Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. • HSF.IF.C.8.A: Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. • HSF.IF.C.8.B: Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay.	• **SMP 4:** Model with mathematics. • **SMP 7:** Look for and make use of structure.

Clarification Statement	Students Who Demonstrate Understanding Can...
• Students should be able to rewrite quadratic and exponential functions in different ways to find key features of the expression and interpret those key features in terms of the context they represent. Students should be able to find the *x*-intercepts of a quadratic function using both factoring and completing the square.	• Rewrite a function to find and highlight key features. • Factor a quadratic expression to find zeros, extrema and symmetry • Interpret the meaning of zeros, extrema and symmetry within the context of a problem. • Complete the square for a quadratic function to reveal its key features. • Interpret the key features of a quadratic expression in terms of the context it represents. • Use properties of exponents to relate parts of an exponential function to its context (e.g., describe the initial value, growth/decay rate or factor and the growth period). • Identify how key features of an exponential function relate to characteristics in a real-world context. • Classify real-world problems as an exponential growth or decay.

DOK	Blooms
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Analyze functions using different representations

Cluster Standard: HSF.IF.C.9

Standard	Standards for Mathematical Practice
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	• **SMP 5:** Use appropriate tools strategically. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should be able to compare two given functions (linear, exponential, quadratic) whether that be as a function or equation, in a table, in a graph, or by verbal description. Students should start by knowing the difference between linear, quadratic and exponential functions, and be able to identify them by equation and by graph. Students should be able to compare two functions even when they're both represented differently. To do this successfully, they must be able to translate between an equation, a graph, words, and a table of values, and understand how certain aspects of one representation impact the rest.	• Make comparisons between functions in different forms using their knowledge of key features.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- Students may have difficulty identifying the key features needed to sketch the graphs or identifying those features algebraically.
- Students may have difficulty with contextualizing and decontextualizing expressions.
- Students will often confuse functions given in a table as a representation of a finite set of numbers rather than a subset of the entire function. They also may have difficulty with the abstractness of determining what is happening with a function over intervals of the domain that they cannot see.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Interpreting Functions**	Strand: **Analyze functions using different representations**

Suggested Student Discourse Questions

- Can you identify the _____________ (intercepts, zeros, rate of change, maxima, minima) on this graph? Explain what the ___________ (intercepts, zeros, rate of change, maxima, minima) mean in your own words.
- Give a linear, quadratic, exponential, or piecewise function to each pair of partners. One partner sketches a graph, the other creates a table of values. Direct both to identify the intercepts, zeros, rate of change, maxima, minima. They work together to find each algebraically using the equation. In what ways are the strategies different? How are they similar?
- In what ways are the _________ (intercepts, zeros, rate of change, maxima, minima) portrayed in this graph? How are they portrayed in the table of values? How are they portrayed in the equation?
- How do the ___________ (intercepts, zeros, rate of change, maxima, minima) relate to the real-world problem?

ASSESSMENT GUIDE

- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

Grade	*CCSS Domain*	*CCSS Cluster*
A1	**Interpreting Functions**	**Understand the concept of a function and use function notation**
	Sample Task #1 (Constructed Response)	
	An arrow is shot into the air. A function representing the relationship between the number of seconds it is in the air, t, and the height of the arrow in meters, h, is given by $h(t) = -4.9t^2 + 29.4t + 2.5.$ a. Complete the square for this function. Show all work. b. What is the maximum height of the arrow? Explain how you know. c. How long does it take the arrow to reach its maximum height? Explain how you know. Engage NY - Algebra 1 Module 4, End of Module Assessment, #3	

Sample Task #2 (Multiple Choice)

$T = 1{,}000 + 18h$

In the equation above, T represents Brittany's total take-home pay, in dollars, for her first week of work, where h represents the number of hours she worked that week and 1,000 represents a sign-on bonus. If Brittany's total take-home pay was $1,576, for how many hours was Brittany paid for her first week of work?

A. 16

B. 32

C. 55

D. 88

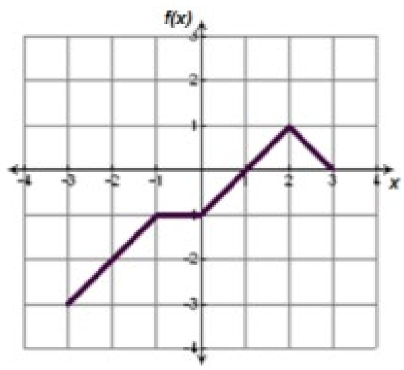
SAT, #1053407

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Interpret functions that arise in applications in terms of the context

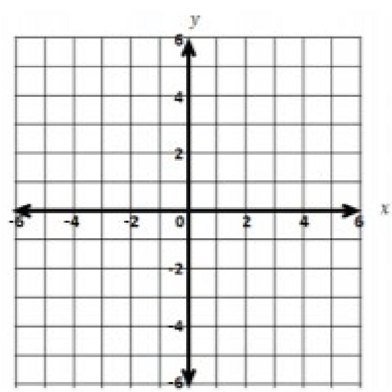
Sample Task #1 (Constructed Response)

The graph of a piecewise function f is shown to the right. The domain of f is $-3 \le x \le 3$.

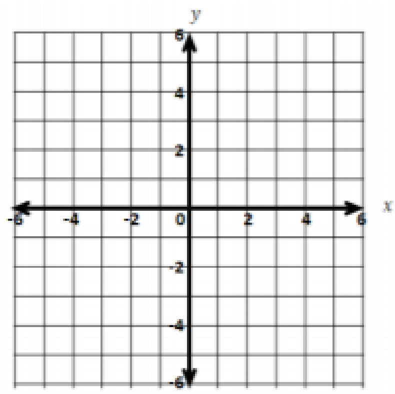
a. Create an algebraic representation for f. Assume that the graph of f is composed of straight line segments.



b. Sketch the graph of $y = 2f(x)$, and state the domain and range.



c. Sketch the graph of $y = f(2x)$, and state the domain and range.



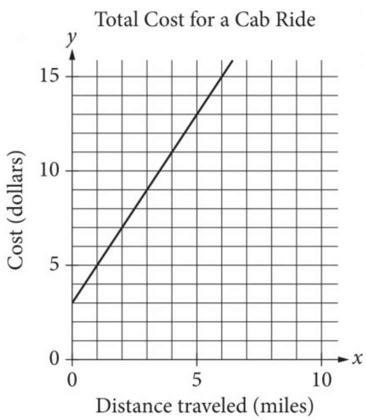
d. How does the range of $y = f(x)$ compare to the range of $y = kf(x)$, where $k > 1$?

e. How does the domain of $y = f(x)$ compare to the domain of $y = f(kx)$, where $k > 1$?

Engage NY - Algebra 1
Module 3, End of Module Assessment, #5

Sample Task #2 (Multiple Choice)

The line graphed in the *xy*-plane below models the total cost, in dollars, for a cab ride, *y*, in a certain city during non-peak hours based on the number of miles traveled, *x*.



According to the graph, what is the cost for each additional mile traveled, in dollars, of a cab ride?

A. \$2.00

B. \$2.60

C. \$3.00

D. \$5.00

Rationale

Choice A is correct. The cost of each additional mile traveled is represented by the slope of the given line. The slope of the line can be calculated by identifying two points on the line and then calculating the ratio of the change in y to the change in x between the two points. Using the points $(1,5)$ and $(2,7)$, the slope is equal to $\frac{7-5}{2-1}$, or 2. Therefore, the cost for each additional mile traveled of the cab ride is \$2.00.

Choice B is incorrect and may result from calculating the slope of the line that passes through the points $(5,13)$ and $(0,0)$. However, $(0,0)$ does not lie on the line shown. Choice C is incorrect. This is the y-coordinate of the y-intercept of the graph and represents the flat fee for a cab ride before the charge for any miles traveled is added. Choice D is incorrect. This value represents the total cost of a 1-mile cab ride.

SAT, #5209215

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Analyze functions using different representations

Sample Task #1 (Constructed Response)

Sydney was studying the following functions:

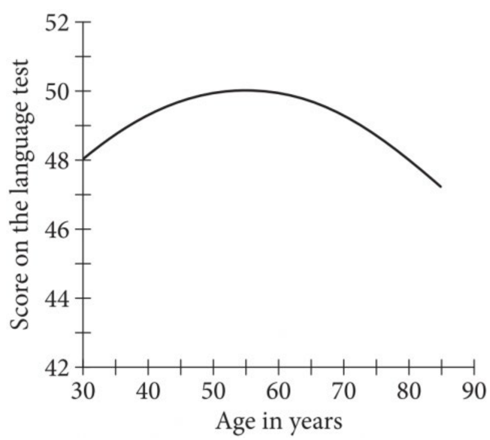
$$f(x) = 2x + 4 \text{ and } g(x) = 2(2)^x + 4$$

She said that linear functions and exponential functions are basically the same. She based her statement on plotting points at $x = 0$ and $x = 1$ and graphing the functions.

Help Sydney understand the difference between linear functions and exponential functions by comparing and contrasting f and g. Support your answer with a written explanation that includes use of the average rate of change and supporting tables and/or graphs of these functions.

Engage NY - Algebra 1
Module 3, Mid-Module Assessment, #2

Sample Task #2 (Multiple Choice)



A scientist tested a group of adults aged 30 to 85. The graph shows the quadratic function S, which models their scores on a language test as a function of their age x, in years. Which of the following could define S ?

A. $S(x) = -\frac{1}{320}(x-50)^2+55$

B. $S(x) = -\frac{1}{320}(x-55)^2+50$

C. $S(x)=\frac{1}{320}(x-50)^2+55$

D. $S(x)=\frac{1}{320}(x-55)^2+50$

Rationale

Choice B is correct. The vertex form of a quadratic function $y=f(x)$ in the xy-plane is represented by the equation $f(x)=a(x-h)^2+k$, where (h,k) represents the vertex. A positive value of a results in a vertex that's the lowest point of the graph of $y=f(x)$, and a negative value of a results in a vertex that's the highest point of the graph of $y=f(x)$. The vertex of the given graph has its highest point at approximately $(55,50)$. Therefore, a must be negative. The equation $S(x)=-\frac{1}{320}(x-55)^2+50$ represents a graph with a vertex at $(55,50)$ with a value of a that is negative.

Choice A is incorrect. The vertex of the graph is $(55,50)$, not $(50,55)$. Choice C is incorrect. The positive value of a results in the vertex being the lowest point of the graph instead of the highest. Choice D is incorrect and may result from using an incorrect vertex point and a positive value of a, which would result in the vertex being the lowest point of the graph instead of the highest.

SAT, #1054197

MLSS AND CLR GUIDE

- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

CCSS Domain	*CCSS Cluster*	
Interpreting Functions	Understand the concept of a function and use function notation	
Culturally and Linguistically Responsive Instruction		
Relevance to Families and Communities	During a unit focused on the concept of a function and function notation, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, allowing students to look at home budgets, utility bills (the cost as a function of usage etc.) or even bringing in examples of functions from various careers represented at home can help students make connections between the abstract idea of functions and how/where they exist in real life.	
Cross-Curricular Connections	Science: Radioactive decay is a function that is a sequence. Consider providing a connection where students know the half-life and starting amount of a substance and use that to define a function and determine the amount left after a certain amount of time.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of*	• Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it hinders those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying the concept of a function and function notation the types of mathematical tasks are critical because this cluster is conceptual in nature. The types of vocabulary introduced/continued within this cluster are vital to success in future mathematics especially those within the domain of interpreting functions. Students who are unfamiliar with the idea of a function or the concept of function notation will

	your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	struggle with these foundational ideas if explicit instruction is neglected.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	***Current Learning***	***Future Learning***
• Connect to analyzing proportional relationships and solving real-world math problems using numerical and algebraic expressions and equations. **(7.RPA.2-3)** • Connect to describing the functional relationship between two quantities qualitatively by analyzing a graph. **(8.F.5)** • Connect to constructing a function to model a linear relationship. **(8.F.4)**	• Connect to writing recursive and explicit formulas for arithmetic and geometric sequences. **(HSF.BF.2)** • Connect to writing functions for linear, quadratic, and exponential relationships. **(HSF.BF.1- 2)**	• Connect to use function notation with all types of functions. **(HSF.IF.2)** • **Connect to deriving the formula for a geometric series. (HSA.SSE.4)**

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on the concept of a function and function notation because the foundation for this cluster is developed in 8th grade. Students are introduced to functions as relationships having a unique output for input. Building from this idea is a crucial connection for students developing a deeper understanding of functions and function notation.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.F.A.1: This standard provides a foundation for work with the concept of a function and function notation because it is the foundational concept of the function. Understanding the definition of a function is crucial to making sense of the more complicated functions that are seen in Algebra 1. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
		Potential Scaffolds
● A function is a special relationship between two sets in which each domain value corresponds to one and only one range number. ● The similarities and differences of linear, quadratic, and exponential functions. ● That an arithmetic recursive formula is addition of a repeated constant and a	● Use multiple representations (including graphs, tables, and symbols) to determine the domain and range and describe important behaviors of functions. ● Graph linear, quadratic, and exponential by hand and using	● Build on students' experience with the following skills: ○ Graphing on the coordinate plane (6.NS.C.8) ○ Know and recognize linear functions (8.EE.C.A.7) ○ Calculate arithmetic sequence (7.EE.B.4) ○ Apply properties of exponents (8.EE.A) ● Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-

geometric recursive formula is multiplication of a repeated constant. ● Over time, a quadratic function will grow faster than a linear function, and an exponential function will grow faster than both a linear and a quadratic function.	technology and identify and label key features. ● Create and translate between recursive and explicit definitions of arithmetic and geometric sequences. ● Identify when a table, graph, equation, and/or verbal description exhibits a linear or exponential relationship.	monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Graphing calculator ○ Desmos ○ Graphic organizers ○ Sketch a graph ○ Create a table of values
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on the concept of a function and function notation by clarifying mathematical ideas and/or concepts through a short mini lesson because the cluster is conceptual in nature. Making sense of the concepts is key to analyzing them and interpreting in the next two clusters.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit the concept of a function and function notation by addressing conceptual understanding because this cluster is conceptual in nature. Anything that we do to deepen students' understanding of the concept of function and function notation will be a key to extend their understanding of functions and function notation in additional standards within this domain.

<table>
<tr><th colspan="3">Extension</th></tr>
<tr><th colspan="2">Essential Question</th><th>Examples</th></tr>
<tr><td colspan="2">What type of extension will offer additional challenges to ‘broaden’ your student’s knowledge of the mathematics developed within your HQIM?</td><td>Some learners may benefit from an extension focused on the concept of a function and function notation because students gain a deeper understanding of functions, once they see applications in real life disciplines other than mathematics. In making cross curricular links students will not only deepen their understanding of the widely applicable nature of functions but also prepare themselves for the next levels of analyzing and interpreting functions.</td></tr>
<tr><th>CCSS Domain</th><th colspan="2">CCSS Cluster</th></tr>
<tr><th>Interpreting Functions</th><th colspan="2">Interpret functions that arise in applications in terms of the context</th></tr>
<tr><th colspan="3">Culturally and Linguistically Responsive Instruction</th></tr>
<tr><td>Relevance to Families and Communities</td><td colspan="2">During a unit focused on Interpreting functions that arise in applications in terms of a context, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, allow students to look at home budgets, utility bills (the cost as a function of usage etc.) or even bringing in examples of functions from various careers represented at home can help students make connections between the abstract idea of functions and how/where they exist in real life.</td></tr>
<tr><td>Cross-Curricular Connections</td><td colspan="2">Science: Average rate of change can be modeled in contexts involving temperature, speed or height. Consider providing a connection where students collect bivariate data and then make a contextualized explanation of an average rate of change for a model they have created.</td></tr>
<tr><td>Validate/Affirm/Build/Bridge</td><td>How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the</td><td>Task: When planning with your HQIM, consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to “portray mathematics as useful and important in students’ lives and promote students’ lived experiences as important in mathematics class.” Tasks can also be designed to “promote social justice to engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006).” For example, when interpreting functions</td></tr>
</table>

	mathematical abilities of students of marginalized cultures and languages? ● *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	that arise in applications in the terms of a context the types of mathematical tasks are critical because student engagement in this area leads to greater understanding of the key features of functions and how they relate to the context. When students are beginning to make sense of the parts of a function (or its various representations), they need it to be related to an idea they already understand. In doing this we aren't trying to teach the concept in the application and the mathematics because the students already understand the context and can focus on the mathematics of the task.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	***Current Learning***	***Future Learning***
● Connect to interpreting the equation $y = mx + b$ as a linear function and using the equation to solve problems in context. **(8.F.3)** ● Connect to interpreting key features of linear equations in relation to a contextual situation. **(8.F.4)**	● Connect to discovering features of families of functions. **(HSF.IF.7)** ● Connect to distinguishing between situations modeled by linear and exponential functions. **(HSF.LE.1)**	● Connect to finding key features of the entire family of functions. **(HSF.IF.4)**

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that previews new contexts for tasks within the unit (e.g., cell phone plans) when studying the interpretations of functions that arise in applications. Understanding the key aspects of a context is the key to unlocking a problem for students.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.F.B.5: This standard provides a foundation for work with interpreting functions that arise in applications in terms of a context because the given standard is the foundational piece of interpreting linear functions. Once students can efficiently and accurately interpret linear functions, they can apply that knowledge to the more complex quadratic and exponential functions of Algebra 1. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
		Potential Scaffolds

• A function is a special relationship between two sets in which each domain value corresponds to one and only one range number.
• The similarities and differences of linear, quadratic, and exponential functions.
• That an arithmetic recursive formula is addition of a
	• Use multiple representations (including graphs, tables, and symbols) to determine the domain and range and describe important behaviors of functions.
	• Graph linear, quadratic, and exponential by hand and using
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
		  ○ Know and recognize linear functions (8.EE.C.A.7)
		  ○ Calculate arithmetic sequence (7.EE.B.4)
		  ○ Apply properties of exponents (8.EE.A)
		
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-

repeated constant and a geometric recursive formula is multiplication of a repeated constant. • Over time, a quadratic function will grow faster than a linear function, and an exponential function will grow faster than both a linear and a quadratic function.	technology and identify and label key features. • Create and translate between recursive and explicit definitions of arithmetic and geometric sequences. • Identify when a table, graph, equation, and/or verbal description exhibits a linear or exponential relationship.	monitoring strategies ○ Introduce multiple means of representation for mathematical ideas • Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Graphing calculator ○ Desmos ○ Graphic organizers ○ Sketch a graph ○ Create a table of values
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on interpreting functions that arise is applications in terms of the context by providing specific feedback to students on their work through a short mini lesson because in interpreting functions within a context providing feedback and allowing students to revise their work can be a powerful tool in deepening their understanding.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit interpreting functions that arise in applications in terms of the context by confronting student misconceptions because as in the section on re-teach targeted in this cluster students need feedback for learning. They need to see what they don't understand, celebrate their successes and revise their work to deepen their understanding of functions and interpreting functions in context.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to ‘broaden’ your student’s knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the opportunity to make connections between the abstract and isolated nature of functions in mathematics and applications in science, history, psychology, sociology and other topics that may be of greater interest to students. If they can research functions in other disciplines, they will have more “buy in” to the importance of interpreting functions.

CCSS Domain	*CCSS Cluster*
Interpreting Functions	Analyze functions using different representations

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on analyzing functions using different representations, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, allow students to look at home budgets, utility bills (the cost as a function of usage etc.) or even bring in examples of functions from various careers represented at home that can help students make connections between the abstract idea of functions and how/where they exist in real life. You can then extend these functions by having students make tables, graphs, and write functions related to what they find at home.	
Cross-Curricular Connections	Science: In high school the NGSS builds on K–8 experiences and progresses to using, synthesizing, and developing models to predict and show relationships among variables between systems and their components in the natural and designed worlds. Consider providing a connection for students to use a model based on evidence to illustrate the relationships between systems or between components of a system.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes*	• Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use

	regarding the mathematical abilities of students of marginalized cultures and languages? • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when analyzing functions using different representations the use of mathematical representations within the classroom is critical because it is the focus of this cluster. Students must be able to connect a table to the algebraic written function, and its graph (in any order). All three representations are vital to making sense in mathematics applications. Students often come to us with strengths using one or more of those representations and we can build on those strengths and extend them to the other representations. In connecting what they already know to what they need to add to their "toolbox", students build strength in mathematical representations.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	***Current Learning***	***Future Learning***
• Connect to graphing linear functions. **(8.F.5)** • Connect to comparing properties of linear functions represented in different ways. **(8.F.2)** • Connect to identifying and using key features of linear functions. **(8.F.4)** • Connect to writing linear equations. **(8.F.4)**	• Connect to writing linear, quadratic, and exponential functions to describe relationships between quantities. **(HSA.CED.1-3)** • Connect to analyzing transformations of parent functions for linear, quadratic, and exponential functions. **(HSF.BF.3)**	• Connect to graphing all parent functions by hand and using technology and identifying their key features. **(HSF.IF.7)** • Connect to factoring to complete the square with quadratic functions with complex zeros. **(HSN.CN.7)**

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when analyzing functions using different representations. It is in this cluster that students begin to broaden the scope of the functions they are working with and are specifically introduced to the ideas of quadratic, exponential, piecewise defined, and absolute value functions. In allowing them time to struggle and grapple with the mathematics we are allowing them to make sense of the functions and internalize the understanding of the functions key features when presented in various ways.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	6.EE.A.3: This standard provides a foundation for work analyzing functions using different representations because this standard lays the foundation for order of operations and understanding the idea of equivalent expressions. The ideas presented in this standard allow students to start slowly with expressions that are linear in nature leading up to the use of the distributive property as well as associative and commutative properties that are the precursors for factoring and rearranging higher order functions. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
		Potential Scaffolds
• A function is a special relationship between two sets in which each domain value corresponds to one and only one range	• Use multiple representations (including graphs, tables, and symbols) to determine the	• Build on students' experience with the following skills: ○ Graphing on the coordinate plane (6.NS.C.8) ○ Know and recognize linear functions (8.EE.C.A.7)

number. ● The similarities and differences of linear, quadratic, and exponential functions. ● That an arithmetic recursive formula is addition of a repeated constant and a geometric recursive formula is multiplication of a repeated constant. ● Over time, a quadratic function will grow faster than a linear function, and an exponential function will grow faster than both a linear and a quadratic function.	domain and range and describe important behaviors of functions. ● Graph linear, quadratic, and exponential by hand and using technology and identify and label key features. ● Create and translate between recursive and explicit definitions of arithmetic and geometric sequences. ● Identify when a table, graph, equation, and/or verbal description exhibits a linear or exponential relationship.	○ Calculate arithmetic sequence ([7.EE.B.4](7.EE.B.4)) ○ Apply properties of exponents ([8.EE.A](8.EE.A)) ● Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Graphing calculator ○ Desmos ○ Graphic organizers ○ Sketch a graph ○ Create a table of values
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?	For example, students may benefit from re-engaging with content during a unit on analyzing functions using different representations by critiquing student approaches/solutions to make connections through a short mini lesson because so much of this cluster can be learned through student choice of solution method. When we allow students to share their thinking and make connections between their work and that of others, they are encouraged to try a solution method that they hadn't tried before and might be more efficient. They can also see their errors and make revisions. Jo Boaler (youcubed.com) tells us that brain research suggests that we learn more from when we make mistakes than we do when we get things right all the time. Therefore, constructive criticism and feedback

		that is more meaningful than just a percentage, and vital for our students' success in learning mathematics.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit analyzing functions using different representations by offering opportunities to understand and explore different strategies because as stated above students will approach the problems with the method that makes the most sense to them at first even if it isn't the most efficient strategy. Looking at ideas from other students allows kids to engage in math practice 5 and perhaps make more meaning of different more efficient strategies. We know that we can pick the best strategy from the outset of the problem - but it's because we have a lot of practice and often, we can't necessarily explain why we chose a specific method. It is helpful for our students to think about why one strategy might be better than another and learn when to use specific strategies based on the problem type, they are given when analyzing functions given in different ways.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when analyzing functions given in different ways because this cluster is widely applicable to other disciplines such as science and statistics. If students can explore something of interest to them related to this cluster, they may think of ways to analyze functions that make more sense to them and their peers. Allowing them to explore the widely applicable nature of functions given in multiple representations will also allow them to become more informed citizens of our society at large.