NEW MEXICO
Public Education Department

New Mexico Instructional Scope
Algebra 1 See Structure in Expressions Guide

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Interpret the structure of expressions.
  - HSA.SSE.A.1
  - HSA.SSE.A.2
- Interpret the structure of expressions.
  - HSA.SSE.B.3

Grade	CCSS Domain	CCSS Cluster
A1	Seeing Structure in Expressions	Interpret the structure of expressions

Cluster Standard: HSA.SSE.A.1

Standard	Standards for Mathematical Practice
Interpret expressions that represent a quantity in terms of its context. • HSA.SSE.A.1.A: Interpret parts of an expression, such as terms, factors, and coefficients. • HSA.SSE.A.1.B: Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*	• **SMP4:** Model with mathematics. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• The middle grades standards in Expressions and Equations build a ramp from arithmetic expressions in elementary school to more sophisticated work with algebraic expressions in high school. As the complexity of expressions increases, students continue to see them as being built out of basic operations; they see expressions as sums of terms and products of factors. In "Animal Populations" students compare $P + Q$ and $2P$ by seeing $2P$ as $P + P$. They distinguish between $(Q–P)/2$ and $Q – P/2$ by seeing the first as the quotient where the numerator is a difference and the second as a difference where the second term is a quotient.	• Identify parts of an expression, such as terms, factors, coefficients, exponents, etc. • Interpret simple compound expressions by viewing one or more of their parts as a single entity.
DOK	**Blooms**
1-2	Remember, Understand, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	Seeing Structure in Expressions	Interpret the structure of expressions

Cluster Standard: HSA.SSE.A.2

Standard	Standards for Mathematical Practice
Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*	• **SMP3**: Construct viable arguments and critique the reasoning of others. • **SMP8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Seeing structure in expressions entails a dynamic view of an algebraic expression, in which potential rearrangements and manipulations are ever present. An important skill for college readiness is the ability to try possible manipulations mentally without having to carry them out, and to see which ones might be fruitful and which ones might not.	• Recognize equivalent forms of expressions. • Use the structure of an expression to identify ways to rewrite it. • Make generalizations about the possible equivalent forms expressions can have (e.g., a quadratic expression can always be represented as the product of two factors containing its roots). • Rewrite expressions to identify important components, such as where zeros may occur or end behavior.
DOK	**Blooms**
1-2	Remember, Understand, Apply

Common Misconceptions

- Students may confuse the parts of an expression, such as counting variables and not terms and therefore misidentifying the number of terms an expression has.
- Students may not have a conceptual basis for patterns, such as an area model for difference of squares, and therefore struggle to recognize and apply them to new situations.

Grade	CCSS Domain	CCSS Cluster
A1	Seeing Structure in Expressions	Interpret the structure of expressions

Cluster Standard: HSA.SSE.B.3

Standard	Standards for Mathematical Practice
Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. • HSA.SSE.B.3.A: Factor a quadratic expression to reveal the zeros of the function it defines. • HSA.SSE.B.3.B: Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. • HSA.SSE.B.3.C: Use the properties of exponents to transform expressions for exponential functions. *For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%*	• **SMP3:** Construct viable arguments and critique the reasoning of others. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• The standards emphasize purposeful transformation of expressions into equivalent forms that are suitable for the purpose at hand. The standards avoid talking about simplification, because it is often not clear what the simplest form of an expression is, and even in cases where that is clear, it is not obvious that the simplest form is desirable for a given purpose. There are three commonly used forms for a quadratic expression: • Standard form, e.g., $x^2 - 2x - 3$ • Factored form, e.g., $(x + 1)(x - 3)$ • Vertex form (a square plus or minus a constant), e.g. $(x - 1)^2 - 4$ Rather than memorize the names of these forms,	• Write a quadratic expression with rational coefficients in an equivalent form by factoring and by completing the square. • Identify and use the zeros to solve or explain familiar problems. • Use properties of exponents to write equivalent forms of exponential functions with one or more variables, integer coefficients, and nonnegative rational exponents involving operations of addition, subtraction and multiplication, including distributing an exponent across terms within parentheses. • Find the maximum or minimum values of a quadratic function. • Choose an appropriate equivalent form of an expression in order to reveal a property of interest when solving problems.

students need to gain experience with them and their different uses. The traditional emphasis on simplification as an automatic procedure might lead students to automatically convert the second two forms to the first, rather than convert an expression to a form that is useful in each context. The introduction of rational exponents and systematic practice with the properties of exponents in high school widens the field of operations for manipulating expressions.	
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- When factoring a quadratic where $a > 0$, students may look at c only when determining which factors to use, rather than looking for the factors of the product a and c.
- When completing the square, students may forget to subtract the number that was added inside the parentheses.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Seeing Structure in Expressions**	Strand: **Interpret the structure of expressions**

Suggested Student Discourse Questions

- Looking at an algebraic expression, what does each symbol in the expression represent? Include any given coefficients and operations.
- Compare how you rewrote the formula with your shoulder partner. How could you improve their process?
- Try to find the most efficient way to rewrite the formula in terms of one of its variables.
- Looking at this formula, choose the variable that is most difficult to measure in real life. Rewrite the formula so that it is defined by the OTHER variables.

ASSESSMENT GUIDE

- Interpret the structure of expressions.
- Interpret the structure of expressions.

Grade	*CCSS Domain*	*CCSS Cluster*
A1	Seeing Structure in Expressions	Interpret the structure of expressions

Sample Task #1 (Constructed Response)

If $x^2 = a + b$ and $y^2 = a + c$, what is $(x^2 - y^2)^2$?

SAT, #5204412

Sample Task #2 (Multiple Choice)

Which of the following is equivalent to the expression $x^4 - x^2 - 6$?

A. $(x^2 + 1)(x^2 - 6)$

B. $(x^2 + 2)(x^2 - 3)$

C. $(x^2 + 3)(x^2 - 2)$

D. $(x^2 + 6)(x^2 - 1)$

Grade	CCSS Domain	CCSS Cluster
A1	Seeing Structure in Expressions	Interpret the structure of expressions

Sample Task #1 (Constructed Response)

An arrow is shot into the air. A function representing the relationship between the number of seconds it is in the air, t, and the height of the arrow in meters, h, is given by

$$h(t) = -4.9t^2 + 29.4t + 2.5.$$

a. Complete the square for this function. Show all work.

b. What is the maximum height of the arrow? Explain how you know.

Engage NY - Algebra 1
Module 4, End of Module Assessment, #3a/b

Sample Task #2 (Multiple Choice)

Which of the following is equivalent to the expression $x^4 - x^2 - 6$?

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B. $(x^2+2)(x^2-3)$

C. $(x^2+3)(x^2-2)$

D. $(x^2+6)(x^2-1)$

SAT, #5308651

MLSS AND CLR GUIDE

- Interpret the structure of expressions.
- Interpret the structure of expressions.

CCSS Domain	*CCSS Cluster*
See Structure in Expressions	Interpret the structure of expressions

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on interpreting the structure of expressions, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, ask them about structures that they encounter in their everyday lives that help them parse information, such as knowing which gaming platform a video game is operating on or how to substitute ingredients in a recipe based on someone's food allergies.	
Cross-Curricular Connections	Science: Many science formulas take on linear, exponential and quadratic forms. For example, $F = ma$. Consider providing a connection for students to explore these formulas and identify their structure and how knowing that structure helps them make sense of the context. Social Studies: In high school the New Mexico Social Studies Standards state students should "understand basic economic principles." Consider providing a connection for students to use expressions to model cost and revenue.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the*	• Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it hinders those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying interpreting the structure of expressions the types of mathematical tasks

	cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	are critical because the conceptual part of interpreting the structure of expressions is foundational for being able to build and understand equations later on and is not something that is going to be culturally relevant to most students' home lives.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to identifying and interpreting slope and y-intercept for linear representations. **(8.F.3-4)** • Connect to rewriting standard linear equation to slope-intercept form for systems of equations. **(8.EE.8)**	• Connect to rewriting quadratic functions to find specific key features. (HSA.SSE.B.3) • Connect to rewriting formulas to highlight quantities of interest. (HSA.CED.4)	• Connect to work with expressions of all function types. **(HSA.SSE.A.1-2 - *polynomial and rational*)**

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on interpreting the structure of expressions because when students feel comfortable with the vocabulary being used, they are more likely to use it and using the correct terminology when discussing the structure of an equation allows everyone (bother students and teachers) to communicate their ideas and understanding more clearly.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	5.OA.A.2: This standard provides a foundation for work with interpreting the structure of expressions because students write out the numerical expression without the calculation. Students become comfortable with using the vocabulary words: difference, greater than, multiple, etc. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. 6.EE.A.4: This standard provides a foundation for work with interpreting the structure of expressions because being able to tell if two expressions are equivalent is the building blocks for being able to construct and deconstruct expressions to use their structure. Being able to tell if what you have done to an expression essentially changes it or not leads to the understanding of how to use these changes to manipulate the expressions and equations to better understand their structure. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• Different forms of an expression can be equivalent and are useful in different contexts.
• The addition, subtraction, or multiplication of polynomials results in another polynomial.
• When a situation and its potential constraints will be represented by a linear or quadratic, or exponential equation/inequality or a system of those equations/inequalities.
• The relationship between solutions of equations/ inequalities and their graphical representations.
	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form.
	• Perform the operations of addition, subtraction, and multiplication with polynomials.
	• Determine reasonable solutions based on the context of real-world problems from graphs of equations/inequalities and systems of equations/inequalities.
	• Use the properties of mathematics to solve linear and quadratic equations/inequalities and systems of those equations/inequalities.
		• Build on students' experience with the following skills:
		  ○ Connect to identifying and interpreting slope and *y*-intercept for linear representations. (8.F.3-4)
		  ○ Connect to rewriting standard linear equation to slope intercept form for systems of equations. (8.EE.8)
		  ○ Connect to knowing and apply the properties of integer exponents to generate equivalent, simplified numerical expressions using the properties of exponents. (8.A.1)
		  ○ Connect to combining like terms and simplifying expressions using the distributive property (6.EE.3)
		  ○ Connect to creating and solving equations in one variable. (7.EE.4)
		  ○ Connect to reasoning with inequalities. (7.EE.4)
		  ○ Connect to solving real world problems involving two linear equations in two variables. (8.EE.8)
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of representation for mathematical ideas
		• Encourage students to use alternative tools to better access the grade level content. Examples include:
		  ○ Desmos graphing calculator
		  ○ Algebra tiles
		  ○ Graphic Organizers

		○ Sketch graph ○ Create table of values
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on interpreting the structure of expressions by clarifying mathematical ideas and/or concepts through a short mini-lesson because the structure of an expression can be looked at in many different ways and you don't students to get locked into one way of thinking about equations, like understanding that slope-intercept, point-slope, and standard form are all useful ways of looking at linear equations and can tell you different things about the equation.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, students may benefit from re-engaging with content during a unit on interpreting the structure of expressions by clarifying mathematical ideas and/or concepts through a short mini-lesson because the structure of an expression can be looked at in many different ways and you don't students to get locked into one way of thinking about equations, like understanding that slope-intercept, point-slope, and standard form are all useful ways of looking at linear equations and can tell you different things about the equation.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension interpreting the structure of expressions because looking at the structure of the different types of equations and disciplines will help reinforce concepts such as the inverse relationship between logarithmic and exponential functions.

CCSS Domain	*CCSS Cluster*
See Structure in Expressions	Interpret the structure of expressions

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on writing expressions in equivalent forms to solve problems, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, ask students to share about structures that they encounter every day that they manipulate, such as changing the formatting of a picture or video so that it is sharable on different device platforms.	
Cross-Curricular Connections	Science: Finding the zeros and maximum for a model that created a projectile motion equation may also require students to rewrite a quadratic in an equivalent form. Consider providing a connection for students to experiment with projectile motion by tossing objects themselves, possibly using technology, and then exchanging equations with another classmate or group to identify key components. Social Studies: In high school the New Mexico Social Studies Standards state students should "understand basic economic principles." Consider providing a connection for students to rewrite the model $P(1+r)^t$ for compound interest to identify the quarterly, monthly or weekly interest rate.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support*	• Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an

	students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	understanding of the culture of the students. For example, when writing expressions in equivalent forms to solve problems supporting productive struggle is critical. Students will come to this cluster with a variety of knowledge about how to manipulate equations and there are a lot of correct ways to do so; therefore, they need to be encouraged to work through the process to find the ways that are more effective on their own instead of being asked to memorize rote procedures for a given situation.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to knowing and apply the properties of integer exponents to generate equivalent, simplified numerical expressions using the properties of exponents. **(8.A.1)**	• Connect to recognizing and flexibly writing expressions (or rewriting) to use that expression and solve the problem at hand. **(HSA.SSE.A.2)**	• Connect to factoring polynomial functions of varying degrees. **(HSA.APR.3)** • Connect to completing the square to solve quadratic equations with imaginary solutions**. (HSN.CN.7)** • Connect to rewriting exponential equations as logarithmic equations**. (HSF.LE.4)**

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on writing expressions in equivalent forms to solve problems because they will be asked to combine several skills that they had previously learned independently of each other. For example, like rewriting exponents into one larger problem so reviewing these individual skills will help them be more confident in the larger problem.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	7.EEA.1: This standard provides a foundation for work with writing expressions in equivalent forms to solve problems because they need to be able to manipulate expressions in a basic sense of linear equations if they are going to be successful at manipulating more complex expressions. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• Different forms of an expression can be equivalent and are useful in different contexts. • The addition, subtraction, or multiplication of polynomials results in another polynomial. • When a situation and	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form. • Perform the operations of addition, subtraction, and multiplication with polynomials. • Determine reasonable	• Build on students' experience with the following skills: ○ Connect to identifying and interpreting slope and *y*-intercept for linear representations. (8.F.3-4) ○ Connect to rewriting standard linear equation to slope intercept form for systems of equations. (8.EE.8) ○ Connect to knowing and apply the properties of integer exponents to

its potential constraints will be represented by a linear or quadratic, or exponential equation/inequality or a system of those equations/inequalities. ● The relationship between solutions of equations/ inequalities and their graphical representations.	solutions based on the context of real-world problems from graphs of equations/ inequalities and systems of equations/ inequalities. ● Use the properties of mathematics to solve linear and quadratic equations/inequalities and systems of those equations/inequalities.	generate equivalent, simplified numerical expressions using the properties of exponents. (8.A.1) ○ Connect to combining like terms and simplifying expressions using the distributive property (6.EE.3) ○ Connect to creating and solving equations in one variable. (7.EE.4) ○ Connect to reasoning with inequalities. (7.EE.4) ○ Connect to solving real world problems involving two linear equations in two variables. (8.EE.8) ● Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos graphing calculator ○ Algebra tiles ○ Graphic Organizers ○ Sketch graph ○ Create table of values

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on writing expressions in equivalent forms to solve problems by critiquing student approaches/solutions to make connections through a short mini-lesson because there are a variety of ways to solve problems and looking at the ways that other students are solving the problems can help the students to make connections between their preferred methods and another that could help them become more efficient at solving similar problems in the future.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit writing expressions in equivalent forms to solve problems by offering opportunities to understand and explore different strategies because different strategies of looking at the equivalent forms are more efficient for certain tasks and exploring when it is most appropriate to use a particular form will help them become more flexible in their problem-solving skills.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension addressing writing expressions in equivalent forms to solve problems because some students will pick up on the technical mechanics of a particular technique quickly. Having them go more deeply into why it works will help them gain a better understanding of the overall intricacies of the method. For example, factoring using a variety of methods, like factoring by grouping and how it relates to factoring a traditional trinomial into two binomials.

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	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Create equations that describe numbers or relationships.
  - HSA.CED.A.1
  - HSA.CED.A.2
  - HSA.CED.A.3
  - HSA.CED.A.4

Grade	CCSS Domain	CCSS Cluster
A1	Creating Equations	Create equations that describe numbers or relationships

Cluster Standard: HSA.CED.A.1

Standard	Standards for Mathematical Practice
Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 4:** Reason abstractly and quantitatively.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• The repertoire of functions that is acquired during high school allows students to create more complex equations, including equations arising from linear and quadratic expressions, and simple rational and exponential expressions.	• Write a linear equation in one variable based on a given context and use their equation to solve problems. • Write quadratic equations in one variable based on a given context and explain how it can be used to solve the problem situation. • Use a created equation or inequality to solve problems.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

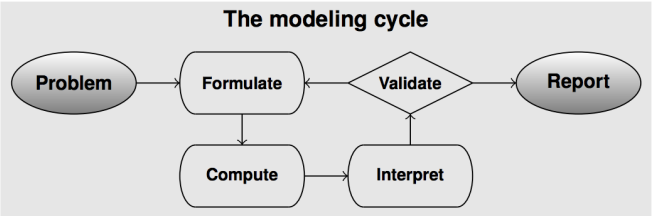
Grade	CCSS Domain	CCSS Cluster
A1	Creating Equations	Create equations that describe numbers or relationships

Cluster Standard: HSA.CED.A.2

Standard	Standards for Mathematical Practice
Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	• **SMP 1:** Make sense of problems and persevere in solving them. . • **SMP 3:** Construct viable arguments and critique the reasoning of others.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students use complex equations—including equations arising from linear and quadratic expressions, and simple rational and exponential expressions—to model relationships between quantities with equations in two variables.	• Graph equations on coordinate axes with scales clearly labeling the axes, defining what the values on the axes represent and the unit of measure. • Determine appropriate units for the labels and scale of a graph that depicts the relationship between quantities in the given context and displays adequate information about the relationship. • Analyze points on and off a graph and interpret them in context.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	Creating Equations	Create equations that describe numbers or relationships

Cluster Standard: HSA.CED.A.3

Standard	Standards for Mathematical Practice
Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• All the standards in the Creating Equations group carry a modeling star, denoting their connection with the Modeling category in high school. This connotes not only an increase in the complexity of the equations studied, but an upgrade of the student's ability in every part of the modeling cycle. 	• Identify constraints of equations, inequalities, and systems of equations and inequalities given a context • Explain why solutions of equations, inequalities, and systems of equations and inequalities are viable or non-viable given a context. • Interpret solutions analytically and graphically to answer questions about the quantities and relationships in a given context.
DOK	**Blooms**
1-3	Understand, Apply, Analyze, Evaluate

Grade	CCSS Domain	CCSS Cluster
A1	Creating Equations	Create equations that describe numbers or relationships

Cluster Standard: HSA.CED.A.4

Standard	Standards for Mathematical Practice
Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*	• **SMP 4:** Model with mathematics. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• There are situations where an equation is used to describe the relationship between a number of different quantities. For example, Ohm's Law *V = IR* relates the voltage, current, and resistance of an electrical circuit. An equation used in this way is sometimes called a formula. It is perhaps best to avoid using the terms "variable", "parameter", or "constant" when working with this formula, because there are six different ways it can be viewed as defining one quantity as a function of the other with a third held constant.	• Define a “quantity of interest” to mean any number or algebraic quantity and what it represents in terms of a given context. • Determine when it is useful to rewrite a formula by solving for one of the variables in the formula. • Make connections between solving equations and rearranging formulas. • Apply inverse operations to rearrange formulas for a specified variable.
DOK	**Blooms**
1-2	Understand, Apply

Common Misconceptions

- Choosing the correct form of the equation can often be difficult when first introducing exponential and quadratic equations to students. They will often try to make everything linear. Students may need to see or experience more linear, exponential and quadratic relationships in order to fully grasp the difference between each.
- Students tend to struggle without the benefit of having numbers involved. To support conceptual development, encourage students to start with concrete examples, or similar situations they have encountered, and work toward abstract representation of quantities and relationships.
- Students may believe that the default scales in calculators or technology are the best for every situation. Help students understand that different situations call for different scales or units.

- Students may believe that a constraint must mean an inequality, or that only *x* and *y* can represent variables.
- Support student understanding around mathematically true solutions that do not make any sense in the context of a real-world word problem, which leads to constraints for the model to realistically represent the situation.
- In formulas with subscripts, some students may believe that these can be combined by like terms. Students may need additional practice to realize that the same variables with different subscripts are actually different variables.

ASSESSMENT GUIDE

- Create equations that describe numbers or relationships

Grade	*CCSS Domain*	*CCSS Cluster*
A1	Creating Equations	Create equations that describe numbers or relationships

Sample Task #1 (Constructed Response)

A boy bought six guppies at the beginning of the month. One month later, the number of guppies in his tank had doubled. His guppy population continued to grow in this same manner. His sister bought some tetras at the same time. The table below shows the number of tetras, t, after n months have passed since they bought the fish.

n, months	0	1	2	3
t, tetras	8	16	24	32

a. Create a function g to model the growth of the boy's guppy population, where $g(n)$ is the number of guppies at the beginning of each month and n is the number of months that have passed since he bought the six guppies. What is a reasonable domain for g in this situation?

b. How many guppies will there be one year after he bought the six guppies?

c. Create an equation to determine how many months it will take to reach 100 guppies.

Engage NY - Algebra 1
Module 3, End of Module Assessment, #3a-c

Sample Task #2 (Multiple Choice)

Marisa needs to hire at least 10 staff members for an upcoming project. The staff members will be made up of junior directors, who will be paid $640 per week, and senior directors, who will be paid $880 per week. Her budget for paying the staff members is no more than $9,700 per week. She must hire at least 3 junior directors and at least 1 senior director. Which of the following systems of inequalities represents the conditions described if x is the number of junior directors and y is the number of senior directors?

A. $640x + 880y \geq 9{,}700$
$x + y \leq 10$
$x \geq 3$
$y \geq 1$

B. $640x + 880y \leq 9{,}700$
$x + y \geq 10$
$x \geq 3$
$y \geq 1$

C. $640x + 880y \geq 9{,}700$
$x + y \geq 10$
$x \leq 3$
$y \leq 1$

D. $640x + 880y \leq 9{,}700$
$x + y \leq 10$
$x \leq 3$
$y \leq 1$

Rationale

Choice B is correct. Marisa will hire x junior directors and y senior directors. Since she needs to hire at least 10 staff members, $x + y \geq 10$. Each junior director will be paid $640 per week, and each senior director will be paid $880 per week. Marisa's budget for paying the new staff is no more than $9,700 per week; in terms of x and y, this condition is $640x + 880y \leq 9{,}700$. Since Marisa must hire at least 3 junior directors and at least 1 senior director, it follows that $x \geq 3$ and $y \geq 1$. All four of these conditions are represented correctly in choice B.

Choices A and C are incorrect. For example, the first condition, $640x + 880y \geq 9{,}700$, in each of these options implies that Marisa can pay the new staff members more than her budget of $9,700. Choice D is incorrect because Marisa needs to hire at least 10 staff members, not at most 10 staff members, as the inequality $x + y \leq 10$ implies.

SAT, #422860

MLSS AND CLR GUIDE

- Create equations that describe numbers or relationships

CCSS Domain	*CCSS Cluster*
Creating Equations	**Create equations that describe numbers or relationships**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on creating equations that describe numbers or relationships, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, this could be looking at the utility bills for someone in the neighborhood and seeing how the rates are calculated using the unit rate of energy or water.	
Cross-Curricular Connections	Economics: Linear programming with a system of inequalities is often used to model the constraint of resources for production. Consider providing a connection where students are starting their own business and must maximize profit or production with the possible solutions of the system. Science: There are many formulas in science such as Ohm's Law and the Doppler formulas that may require isolating and solving for a specific variable given certain conditions. Consider providing a connection where students must rearrange the same formulas in multiple ways to highlight different quantities of interest.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and*	• Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge concepts for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as

	language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	competent and capable mathematicians. For example, when creating equations that describe numbers or relationships the use of mathematical representations within the classroom is critical. Creating equations is a skill that focuses on transforming the world around us into numbers and symbols and in doing so, care needs to be taken to emphasize that this stripping down of the world is not a discarding of the cultural aspects of the situation. Making the connections between the math that is being used and the thing it is being used to analyze is imperative in this context.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to creating and solving equations in one variable. **(7.EE.4)** • Connect to reasoning with inequalities. **(7.EE.4)** • Connect to solving real-world problems involving two linear equations in two variables. **(8.EE.8)**	• Connect to graphing equations and inequalities. **(HSF.IF.7)** • Connect to graphing systems of equations and inequalities. **(HSA.REI.7)** • Connect to solving equations in one variable including those equations with coefficients represented by variables. **(HSA.REI.3-4)** • Connect to communicating relevant domain and range for linear, exponential and quadratic functions. **(HSF.IF.4)**	• Connect to extending knowledge to include additional types of functions such as trigonometric, rational, and polynomial. **(HSA.CED.1-4)** • Connect to communicating relevant domain and range for all types of functions. **(HSF.IF.4)**

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on creating equations that describe numbers or relationships. Doing so allows students to better understand that writing expressions, equations, or inequalities to represent data is a highly useful tool that can be used in a variety of different scenarios and when wielded by them, will allow for a broader application of the concepts they are learning.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	6.EE.A.2: This standard provides a foundation for work with creating equations that describe numbers or relationships because this is the first time that students are being asked to read, write, and evaluate expressions in which letters stand for numbers and many students have trouble making this transition. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• Different forms of an expression can be equivalent and are useful in different contexts. • The addition, subtraction, or multiplication of polynomials results in another polynomial. • When a situation and its potential	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form. • Perform the operations of addition, subtraction, and multiplication with polynomials. • Determine reasonable solutions based on the	• Build on students' experience with the following skills: ○ Connect to identifying and interpreting slope and *y*-intercept for linear representations. (8.F.3-4) ○ Connect to rewriting standard linear equation to slope intercept form for systems of equations. (8.EE.8) ○ Connect to knowing and apply the properties of integer exponents to generate equivalent, simplified numerical expressions using the

constraints will be represented by a linear or quadratic, or exponential equation/inequality or a system of those equations/ inequalities. ● The relationship between solutions of equations/ inequalities and their graphical representations.	context of real-world problems from graphs of equations/inequalities and systems of equations/inequalities. ● Use the properties of mathematics to solve linear and quadratic equations/inequalities and systems of those equations/inequalities.	properties of exponents. (8.A.1) ○ Connect to combining like terms and simplifying expressions using the distributive property (6.EE.3) ○ Connect to creating and solving equations in one variable. (7.EE.4) ○ Connect to reasoning with inequalities. (7.EE.4) ○ Connect to solving real world problems involving two linear equations in two variables. (8.EE.8) ● Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos graphing calculator ○ Algebra tiles ○ Graphic Organizers ○ Sketch graph ○ Create table of values

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on creating equations that describe numbers or relationships by clarifying mathematical ideas and/or concepts through a short mini-lesson because creating equations has such a broad level of application, from linear and proportional to exponential, quadratic, logarithmic, and trigonometric meaning that this has the opportunity to be studied from many different perspectives and the better that is understood about one type of problem, the better it will be understood for the others.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit creating equations that describe numbers or relationships by addressing conceptual understanding because creating equations is best done in the context of a real-world problem and understanding the underlying relationships of why a particular equation is preferred over another will allow students to more readily choose the appropriate type of equation in the future.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension that focuses on creating equations that describe numbers or relationships because it would allow them to build the context of why they are building equations and the purposes of what using the equations would allow them to do. For example, they could explore the link between how building an equation to model the cost of a project based on the material costs and size constraints can help when calculating costs in manufacturing and construction.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Summarize, represent, and interpret data on a single count or measurement variable
  - HSS.ID. A.1
  - HSS.ID. A.2
  - HSS.ID. A.3
- Summarize, represent, and interpret data on two categorical and quantitative variables
  - HSS.ID.B.5
  - HSS.ID.B.6
- Interpret linear models
  - HSS.ID.C.7
  - HSS.ID.C.8
  - HSS.ID.C.9

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Categorical & Quantitative Data	Summarize, represent, and interpret data on a single count or measurement variable

Cluster Standard: HSS.ID. A.1

Standard	Standards for Mathematical Practice
Represent data with plots on the real number line (dot plots, histograms, and box plots).	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 3:** Construct viable arguments and critique the reasoning of others.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should not only be able to construct each of these plot types but be able to do so in a way that shows the data in a meaningful way considering things like spread and center. For example, making bins of appropriate width when making a histogram or appropriate spacing on a number line for a dot plot or a box plot. Students should know that a dot plot is a diagram that represents a data set using dots over the number line. A histogram is a diagram that shows a data set as a series of rectangles that shows how often data occur within a given interval. A box plot, also called a box and whisker plot, is a diagram that shows a data set as a distribution along the number line, divided into four equal parts using the median (the middle data value) and the upper and lower quartiles (median of upper and lower half of data, respectively).	• Summarize data using a dot plot. • Summarize data using a histogram. • Summarize data using a box plot. • Know when each of these is appropriate to be used
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Categorical & Quantitative Data	Summarize, represent, and interpret data on a single count or measurement variable

Cluster Standard: HSS.ID.A.2

Standard	Standards for Mathematical Practice
Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 3:** Construct viable arguments and critique the reasoning of others.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should use statistics appropriately to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Students should know that the center of data can give us a good sense of the data set overall. The center of the data is exactly what it sounds like: a representation of the middle of the data, or a typical value. It gives us a good first guess as to where on the number line the data will fall. Students should know the two types of centers of data: mean and median. The mean, or average, is the sum of all the data points divided by the number of data points, while the median is the value that splits the data into two intervals.	• Calculate the median, mean, interquartile range, and standard deviation of a set of data. • Identify and describe differences in two sets of data based on these calculations. • Identify and describe the shape of a set of data (skewness, symmetric, bimodal, normal).
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Categorical & Quantitative Data	Summarize, represent, and interpret data on a single count or measurement variable

Cluster Standard: HSS.ID.A.3

Standard	Standards for Mathematical Practice
Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students understand and use the context of the data to explain why its distribution takes on a shape (e.g., Is the data skewed? Are there outliers?). Students understand that the higher the value of a measure of variability, the more spread out the data set is. Measures of variability are range (100% of data), standard deviation (68-95-99.7% of data), and interquartile range (50% of data). Students explain the effect of any outliers on the shape, center, and spread of the data sets.	• Describe how the presence or removal of an outlier changes the shape, center and spread of a data set. • Explain why some data sets will tend towards skewness (e.g., tests scores with an upper limit that students do well on tend to be left skewed while heights tend to be more normally distributed). • Recognize that the shape of the data is usually connected to the relative positions of the mean and median (e.g., left skewed data has a mean that is lower than the median).
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- Students may forget to arrange the data in numerical order before finding key numbers needed for creating a box plot.
- When doing normal distribution calculations, students often report the area to the left of a boundary when they are asked about the area to the right of the boundary.

Grade	*CCSS Domain*	*CCSS Cluster*
A1	Interpreting Categorical & Quantitative Data	Summarize, represent, and interpret data on two categorical and quantitative variables

Cluster Standard: HSS.ID.B.5

Standard	Standards for Mathematical Practice
Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
	• Summarize data by creating a two-way frequency table for two categories of data. • Calculate relative frequencies in the context of data. • Identify and describe correlations in relative frequencies that could signify possible causation.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Categorical & Quantitative Data	Summarize, represent, and interpret data on two categorical and quantitative variables

Cluster Standard: HSS.ID.B.6

Standard	Standards for Mathematical Practice
Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. • HSS.ID.B.6.A: Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. • HSS.ID.B.6.B: Informally assess the fit of a function by plotting and analyzing residuals. • HSS.ID. B.6.C: Fit a linear function for a scatter plot that suggests a linear association.	• **SMP 4:** Model with mathematics. • **SMP 5:** Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students represent data on two quantitative variables on a scatter plot and describe how the variables are related. Students will fit a function to the data, use functions fitted to data to solve problems in the context of the data, use given functions or choose a function suggested by the context. (Emphasis is on linear and quadratic models.) Students will also informally assess the fit of a function by plotting and analyzing residuals and fit a linear function for a scatter plot that suggests a linear association.	• Summarize data by creating a scatter plot for two quantitative variables. • Identify and describe how these two variables are related (e.g., positive, negative or no correlation). • Explain how strongly or negatively correlated two variables are. • Determine what type of curve is most appropriate to represent a given set of data. • Create a residual graph. • Determine if a curve is an appropriate model based on the residual graph. • Estimate a line of best fit for a scatterplot of data that is linearly related.

DOK	Blooms
1-2	Understand, Apply, Analyze

Common Misconceptions

- Students may not consider outliers when analyzing data and determining the best fit of a function.
- Students often have difficulty separating causation and association with contextual data sets.

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Categorical & Quantitative Data	Interpret linear models

Cluster Standard: HSS.ID.C.7

Standard	Standards for Mathematical Practice
Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students interpret the slope (rate of change) and the *y*-intercept (constant term) of a linear model in the context of the data. Students may use graphing calculators or software to create representations of data sets, create linear models, and to assist them in interpreting the data. Students should know that all linear models take the form $y = mx + b$ where m is the slope and b is the *y*-intercept.	• Explain the slope and intercept of a linear model in the context of data from a visual model. • Explain the slope and intercept of a linear model in the context of data from written notation.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Categorical & Quantitative Data	Interpret linear models

Cluster Standard: HSS.ID.C.8

Standard	Standards for Mathematical Practice
Compute (using technology) and interpret the correlation coefficient of a linear fit.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 5:** Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should compute (using technology) and interpret the correlation coefficient of a linear fit and use it to understand the strength of a linear relationship. Students should match the correlation coefficient to its appropriate scatter plot and linear model. Students should use the correlation coefficient to determine the goodness of fit for a linear model. Students should know that it has the symbol *r* and that it ranges from -1 to 1. A coefficient equal to 1.0 suggests a positive correlation between the data. This means that as the independent variable (*x*) increases so does the dependent variable (*y*). A correlation coefficient equal to -1.0 suggests a negative correlation between the data, or as the independent variable (*x*) increases, the dependent variable decreases. If the coefficient equals 0, there is no linear correlation. However, just because the linear correlation coefficient equals 0 doesn't mean there is not another type of correlation between the data. Students should also know that in addition to being positive or negative, the correlation coefficient can be	• Find the correlation coefficient using technology. • Describe the meaning of the correlation coefficient of a given set of data in the context of the problem

weak or strong. The closer the correlation is to -1 or 1, the stronger the correlation.	
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	*CCSS Domain*	*CCSS Cluster*
A1	**Interpreting Categorical & Quantitative Data**	**Interpret linear models**

Cluster Standard: HSS.ID.C.9

Standard	**Standards for Mathematical Practice**
Distinguish between correlation and causation.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 3:** Construct viable arguments and critique the reasoning of others.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should be able to do more than just give the definition of correlation and causation. This should be developed as a skill of critical thinking where students are expected to first look at every set of data to determine if it is appropriate to be making comparisons between them. Students need to remember that correlation does not imply causation. For example, let's say we find that there's a strong positive linear correlation between the age of a tree and how many apples it produces. In fact, this correlation is so strong that $r = 0.99$. Does that mean the age of the tree causes more apples to grow? Can there be other factors? (e.g., What about rainfall? Did the farmer use fertilizer? Did he prune the trees? What were	• Explain the difference between correlation and causation. • Give examples of variables that are correlated, but have no logical causal connection (e.g., number of bee stings and ice cream sales both go up in the summer but there isn't a causal link between the two). • Give examples of variables that have both correlation and a high likelihood of causation (e.g., the amount of time spent studying for an exam and the score on the exam). • Give examples of variables that are neither correlated nor have a causal link (e.g., the number of shoes you own and how many students are in your third period class).

the summer and winter temperatures? Any one of these factors may have influenced the number of apples.)	
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- Students do not always know that slope, rate of change, and steepness are interchangeable.
- Students may try to determine the appropriateness of a line of best fit based only on the value of *r*.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Interpreting Categorical & Quantitative Data**	Strand: **Summarize, represent, and interpret data on a single count or measurement variable**

Suggested Student Discourse Questions

- In what ways do _____ (mean, median, mode) compare with (interquartile range, standard distribution) ?
- Turn and talk - which strategy to identify outliers in the data works best for you? Which one works best for your partner?
- How can a graph be used to identify outliers in the data? A table of values? An equation? How are these strategies similar? How do they differ?
- Identify the outliers in this data (population over time, home runs in a season, cost of milk vs cost of gas over time, childcare cost vs region of NM, rainfall in different NM regions). What would be an explanation of why the outlier(s) occur? Do you agree or disagree with your peers' explanations?

Domain: **Interpreting Categorical & Quantitative Data**	Strand: **Summarize, represent, and interpret data on two categorical and quantitative variables**

Suggested Student Discourse Questions

- How does the residual data give you information on the line of best fit?
- How is your line of best fit different from your partner's? How is it the same? Which line of best fit best represents the data? Why?
- Which strategy (graph, equation, table of values) is the most effective way to display the data?
- How would you use this scatter plot (hours of sleep needed vs age, a person's weight vs their height, time spent working out vs performance on the field, hours spent at work vs money in the bank, age started vaping vs life expectancy) to make personal choices for your own life?

Domain: **Interpreting Categorical & Quantitative Data**	Strand: **Interpret linear models**
Suggested Student Discourse Questions	
• How is _____ (rate of change, intercept, correlation coefficient, line of best fit) of the data modeled on this graph? • Turn and talk. Which technology (Desmos or graphing calculator) is most efficient for you to find the correlation coefficient? How is this different from your partner's strategy?	• How can you determine graphically, algebraically, and using a table, the slope (rate of change) of the line of best fit? Which strategy works best for you? • What are some examples of when there is a correlation between two sets of data, but not necessarily a causation between the two sets?

NEW MEXICO
Public Education Department

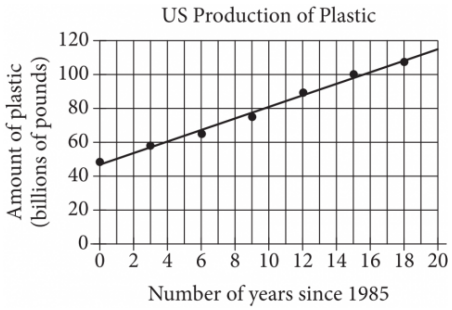
ASSESSMENT GUIDE

- Summarize, represent, and interpret data on a single count or measurement variable
- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models

Grade	*CCSS Domain*	*CCSS Cluster*
A1	Interpreting Categorical & Quantitative Data	Summarize, represent, and interpret data on a single count or measurement variable

Sample Task #1 (Constructed Response)

Between 1985 and 2003, data were collected every three years on the amount of plastic produced annually in the United States, in billions of pounds. The graph below shows the data and a line of best fit. The equation of the line of best fit is $y = 3.39x + 46.89$, where x is the number of years since 1985 and y is the amount of plastic produced annually, in billions of pounds.



What is the best interpretation of the number 3.39 in the context of the problem?

***Convert to Constructed Response**

Rationale

Choice D is correct. The number 3.39 in the equation $y = 3.39x + 46.89$ is the slope, which is the change in y per unit change in x. Because y represents the amount of plastic produced annually, in billions of pounds, and x represents the number of years since 1985, the number 3.39 represents the rate of change of the amount of plastic produced with respect to time, in units of billions of pounds per year. The change is an increase since 3.39 is positive, and it is described as an average change because the data show increases

that are sometimes more and sometimes less than 3.39.

Choice A is incorrect. It is the interpretation of the number 46.89 in the line of best fit equation, $y = 3.39x + 46.89$. Choices B and C are incorrect because they are expressed in the wrong units. The number 3.39 has units of billions of pounds per year, but choice B has units of years and choice C has units of billions of pounds.

SAT, #1474174

Sample Task #2 (Multiple Choice)

Kathy is a repair technician for a phone company. Each week, she receives a batch of phones that need repairs. The number of phones that she has left to fix at the end of each day can be estimated with the equation $P = 108 - 23d$, where P is the number of phones left and d is the number of days she has worked that week. What is the meaning of the value 108 in this equation?

A. Kathy will complete the repairs within 108 days.
B. Kathy starts each week with 108 phones to fix.
C. Kathy repairs phones at a rate of 108 per hour.
D. Kathy repairs phones at a rate of 108 per day

Rationale

Choice B is correct. The value 108 in the equation is the value of $P = 108 - 23d$ when $d = 0$. When $d = 0$ Kathy has worked 0 days that week. In other words, 108 is the number of phones left before Kathy has started work for the week. Therefore, the meaning of the value 108 in the equation is that Kathy starts each week with 108 phones to fix.

Choice A is incorrect because Kathy will complete the repairs when $P = 0$. Since $P = 108 - 23d$, this will occur when $0 = 108 - 23d$ or when $d = \frac{108}{23}$, not when $d = 108$. Therefore, the value 108 in the equation does not represent the number of days it will take Kathy to complete the repairs. Choices C and D are incorrect because the number 23 in $P = 108 - 23d$ indicates that the number of phones left will

decrease by 23 for each increase in the value of d by 1; in other words, Kathy is repairing phones at a rate of 23 per day, not 108 per hour (choice C) or 108 per day (choice D).

SAT, #19106

Grade	*CCSS Domain*	*CCSS Cluster*
A1	**Interpreting Categorical & Quantitative Data**	**Summarize, represent, and interpret data on two categorical and quantitative variables**

Sample Task #1 (Constructed Response)

Several students at Rufus King High School were debating whether males or females were more involved in after-school activities. There are three organized activities in the after-school program—intramural basketball, chess club, and jazz band. Due to budget constraints, a student can only select one of these activities. The students were not able to ask every student in the school whether they participated in the after-school program or what activity they selected if they were involved.

1. Write questions that could be included in the survey to investigate the question the students are debating. Questions that could be used for this study include the following:

2. Rufus King High School has approximately 1,500 students. Sam suggested that the first 100 students entering the cafeteria for lunch would provide a random sample to analyze. Janet suggested that they pick 100 students based on a school identification number. Who has a better strategy for selecting a random sample? How do you think 100 students could be randomly selected to complete the survey?

3. Consider the following results from 100 randomly selected students:
   - Of the 60 female students selected, 20 of them played intramural basketball, 10 played chess, and 10 were in the jazz bland. The rest of them did not participate in the after-school program.
   - Of the male students, 10 did not participate in the after-school program, 20 played intramural basketball, 8 played in the jazz band, and the rest played chess.

   A two-way frequency table to summarize the survey data was started. Indicate what label is needed in the table cell identified with a ???.

	Intramural Basketball	Chess Club	Jazz Band	???	Total
Female					
Male					
Total					

4. Complete the above table for the 100 students who were surveyed.

5. The table shows the responses to the after-school activity question for males and females. Do you think there is a difference in the responses of males and females? Explain your answer.

Engage NY - Algebra 1
Module 2, Student, Lesson #9

Sample Task #2 (Multiple Choice)

	To Fly	Freeze Time	Invisibility	Super Strength	Telepathy	Total
Females	49	60	48	1	70	228
Males	51	71	27	25	48	222
Total	100	131	75	26	118	450

Use the frequency counts from the above table to calculate the missing row of conditional relative frequencies. Round the answers to the nearest thousandth.

	To Fly	Freeze Time	Invisibility	Super Strength	Telepathy	Total
Females			$\frac{48}{228} \approx 0.211$			
Males	$\frac{51}{222} \approx 0.230$					$\frac{222}{222} = 1.000$
Total						

Suppose that a student is selected at random from those who completed the survey. If the selected student is female, what do you think was her response to the selection of a favorite superpower?

***Convert to Multiple-Choice**
Engage NY- Algebra 1
Module 2, Student, Lesson 11

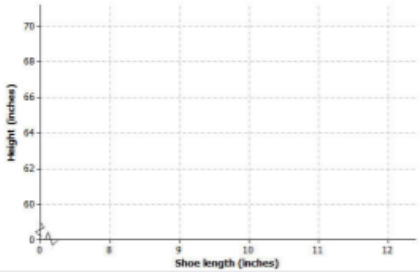
Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Categorical & Quantitative Data	Interpret linear models

Sample Task #1 (Constructed Response)

Kendra wondered if the relationship between shoe length and height might be different for men and women. To investigate, she also collected data on shoe length (in inches) and height (in inches) for 12 women.

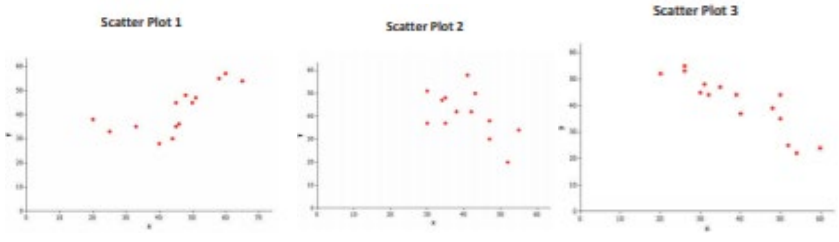
x (Shoe Length of Women)	y (Height of Women)
8.9	61
9.6	61
9.8	66
10.0	64
10.2	64
10.4	65
10.6	65
10.6	67
10.5	66
10.8	67
11.0	67
11.8	70

1. Construct a scatter plot of these data.
2. Is there a relationship between shoe length and height for these 12 women?
3. Find the equation of the least squares line. (Round values to the nearest hundredth.)



Sample Task #2 (Multiple Choice)

Which of the three scatter plots below shows the strongest linear relationship? Which shows the weakest linear relationship?



***Convert to Multiple-Choice**
Engage NY- Algebra 1
Module 2, Student, Lesson 20

NEW MEXICO Public Education Department

MLSS AND CLR GUIDE

- Summarize, represent, and interpret data on a single count or measurement variable
- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models

CCSS Domain	*CCSS Cluster*
Interpreting Categorical and Quantitative Data	**Summarize, represent, and interpret data on a single count or measurement variable**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on summarizing, representing and interpreting data in a single variable, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, using data that is relevant to the community and the summary of the data provides useful information to students regarding their families, culture and community.	
Cross-Curricular Connections	Social Studies: In high school the New Mexico Social Studies Standards state students should "explain how to use technological tools to research data, verify facts and information, and communicate findings." Consider providing a connection for students to write a report describing and analyzing a specific set of data.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between*	• Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying data on a single variable the pattern of questions within the

	the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	classroom is critical because teachers use open-ended questions to scaffold the information that students can summarize and interpret from the data set. Students explore different perspectives of interpreting the data and its implication.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	***Current Learning***	***Future Learning***
• Connect to plotting points on a coordinate grid. **(5.G.1-2)** • Connect to plotting data on dot plots and boxplot. **(6.SP.4)** • Connect to describe center and spread in a data distribution. **(6.SP.5)**	• Connect to how outliers can affect data and skew data. • Connect to classroom test scores or heights of students (something relevant to them)	• Connect to using a standard deviation to make conclusions about a set of data. **(HSS.IS.4)** • Investigate normal distributions within a context. **(HSS.IS.4)** • Calculate confidence intervals based on a normal curve, mean and standard deviation. **(HSS.IC.4)**

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on the center, spread and overall shape of the data sets because students need to understand the conceptual knowledge of center and spread in the context of the problems. Students need to interpret the information numerically and graphically.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	6.SP.A.2 and 6.SP.B.5: This standard provides a foundation for work with interpreting the center and spread of the data sets because students use their prior knowledge to compare the center and spread of different data sets and make implication in the context of the problems. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
		Potential Scaffolds
• How to choose the appropriate plot, given a set of data, based on what one wants to take away from the data. • Symmetric data is best described by mean and standard deviation and non-symmetric data is best described by the median and	• Plot data using dot plots, histograms, and box plots by hand and using technology. • Describe data sets in terms of shape, center, and spread. • Interpret the shape, center, and spread of a data set in the context of a situation.	• Build on students' experience with the following skills: ○ Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation) (6.SP.B.5.C) ○ Graphing points on the coordinate plane (6.NS.C.8) ○ Construct and interpret data plot (8.SP.A.1) ○ Construct a function to model a linear relationship (8.F.B.4) ○ Understanding, constructing,

interquartile range. ● How to find an outlier, what it represents in a data set, and how it affects the mean in a data set.	● Explain the impact of an outlier on the shape, center, and spread of a data set.	interpreting and using patterns of association in bivariate categorical data by displaying frequencies in a two-way table. (8.SP.A.4) ● Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Graphic organizers

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on interpretation of the center and the spread of the data sets by clarifying mathematical ideas and/or concepts through a short mini-lesson because students use different measures of center and spread to explain the meaning of center and spread in the context of the data set. Students use the interpretation of center and spread to compare two different data sets.
Intensive	What assessment data will help identify content needing to be revisited for	For example, some students may benefit from intensive extra time during and after a unit interpreting the center and spread of the data sets graphically and numerically

	intensive interventions?	by addressing conceptual understanding because students need to interpret the center and spread of the data sets graphically and numerically. Students interpret and compare the center and spread of 2 data sets graphically.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the application of and development of abstract thinking skills when interpreting and predicting shape, center and spread of data sets. Students may justify possible bias of the sampling method by comparing the shape, center and spread of the data.

CCSS Domain	*CCSS Cluster*
Interpreting Categorical and Quantitative Data	**Summarize, represent, and interpret data on two categorical and quantitative variables**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on summarizing, representing and interpreting data on two variables, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, using data that is relevant to the community and the summary of the data provides useful information to students regarding their families, culture and community.	
Cross-Curricular Connections	Social Studies: In high school the New Mexico Social Studies Standards state students should "explain how to use technological tools to research data, verify facts and information, and communicate findings." Consider providing a connection for students to determine the best fit of a function for a set of data and explain their choice.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to*	• Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, make sense of mathematics and persevere

	intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	in solving them, is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students. For example, when studying data on two variables supporting productive struggle is critical because students may explore multiple ways of representing and interpreting two variable data sets. Students explore different functions to model the data set and defend their choice. When the function is not the best fit for the domain, students need to develop flexible solutions and define underlying assumptions about the math model used.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to constructing and interpreting two-way tables using frequencies and relative frequencies. **(8.SP.4)** • Connect to using relative frequencies to describe a possible association between two variables. **(8.SP.4)**	• Connect to writing functions for linear, quadratic, and exponential representations. **(HSA.CED.1-3)** • Connect to identifying representations as linear, quadratic and exponential models. **(HSF.B.4)**	• Connect to constructing and interpreting two-way frequency tables to determine independence. **(HSS.CP.4)** • Connect to constructing and interpreting two-way frequency tables to calculate

• Connect to constructing and interpreting scatterplots. **(8.SP.1)** • Connect to constructing an equation or a function to model a linear relationship and determine/interpret the slope and *y*-intercept. **(8.F.4)**	• Connect to constructing and comparing linear and exponential functions to solve problems. **(HSF.LE.1)**	conditional probabilities. **(HSS.CP.4)**

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that describes possible associations between bi-variate data. Students may benefit from a mini-lesson on the types of outcomes and examples of data (hot chocolate sales and temperature). Students may also benefit from a mini lesson on how to construct and analyze a two-way frequency table and how the frequency table can help determine possible associations.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.SP.4 provides a foundation for work with understanding patterns of association of bivariate data by displaying frequencies and relative frequencies in a two-way table. If students have unfinished learning with this standard, providing opportunities for students to analyze and discuss possible associations will provide them with the opportunity for on grade level learning. Also discussing the difference between causation and correlation will benefit students before attempting grade level work for this standard.

Universal Support Framework

		Potential Scaffolds
• How to choose the appropriate plot, given a set of data,	• Plot data using dot plots, histograms, and box plots by	• Build on students' experience with the following skills: ○ Giving quantitative measures of center

based on what one wants to take away from the data. ● Symmetric data is best described by mean and standard deviation and non-symmetric data is best described by the median and interquartile range. ● How to find an outlier, what it represents in a data set, and how it affects the mean in a data set.	hand and using technology. ● Describe data sets in terms of shape, center, and spread. ● Interpret the shape, center, and spread of a data set in the context of a situation. ● Explain the impact of an outlier on the shape, center, and spread of a data set.	(median and/or mean) and variability (interquartile range and/or mean absolute deviation) (6.SP.B.5.C) ○ Graphing points on the coordinate plane (6.NS.C.8) ○ Construct and interpret data plot (8.SP.A.1) ○ Construct a function to model a linear relationship (8.F.B.4) ○ Understanding, constructing, interpreting and using patterns of association in bivariate categorical data by displaying frequencies in a two-way table. (8.SP.A.4) ● Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos.com ○ Graphing calculator ○ Sketch a graph ○ Create a table of values ○ Graphic organizers

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets,	For example, students may benefit from re-engaging with content during a unit on modeling the relation of the 2 variables with the appropriate mathematical model by

	observations) will help identify content needing to be revisited during a unit?	revisiting student thinking through a short mini-lesson because students need to explain the features of the data sets or the graphs/plots. Students connect those features to the features of different functions.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit choosing the appropriate functions to model the 2 quantitative variables of the data by confronting student misconceptions because students need to use the functions model to make prediction and implication of the data. Students need to justify if the prediction and the implication make sense in the context of the data.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension when representing the data with a function model because students explain the interval(s) when the function model fits the scatter plot. Students describe the possible situation when the function model does not fit the scatter plot and possible explanation.

CCSS Domain	*CCSS Cluster*
Interpreting Categorical and Quantitative Data	**Interpret linear models**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on interpreting linear models of the data, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, relate the mathematical models used to real-life data to interpret and predict the trend of data in order to provide useful information for decision-making in the community.	
Cross-Curricular Connections	Science: In high school the NMSS state students should "use mathematical and/or computational representations to support explanations of factors that affect carrying capacity of ecosystems at different scales." Consider providing a connection where students must analyze data and consider the relationship among various factors including boundaries, resources, climate, and competition. https://www.nextgenscience.org/topic-arrangement/hsinterdependent-relationships-ecosystems	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home*	• Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion, we position them as knowers and doers of mathematics. Using equitable talk moves students and the ways students talk about who is and isn't capable of mathematics. As a result, we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when interpreting

	culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	linear models for data, facilitating meaningful mathematical discourse is critical because students might use different linear models to represent the data sets. Students interpret and summarize the data differently using their linear models. Students compare the conclusion and defend the solution by constructing viable arguments.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to plotting points in a coordinate grid and constructing an equation or a function to model the linear relationship. **(5.G.1-2)** • Connect to constructing and interpreting scatterplots. **(8.SP.1)** • Connect to constructing an equation or a function to model a linear relationship and determining/ interpreting the slope and *y*-intercept. **(8.F.4)**	• Connect to explaining the slope and *y*-intercept as they relate to the context of the original problem. **(HSF.IF.4)** • Connect to creating linear functions and using them to solve problems. **(HSA.CED.1-3)** • Connect to interpreting key features of graphs and functions. **(HSF.IF.4)**	• Connect to determining which function fits the data. (Linear, exponential, quadratic).

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching on representing linear functions because students need to understand the features of the linear model and the meaning of the features. Students need to rehearse different ways of writing the linear model.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.F.8.4: This standard provides a foundation for work with constructing a function to model a linear relationship between 2 quantities because students need to know the information needed to model the linear function. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
		Potential Scaffolds
• How to choose the appropriate plot, given a set of data, based on what one wants to take away from the data. • Symmetric data is best described by mean and standard deviation and non-symmetric data is best described by the median and interquartile range. • How to find an outlier, what it represents in a data set, and how it	• Plot data using dot plots, histograms, and box plots by hand and using technology. • Describe data sets in terms of shape, center, and spread. • Interpret the shape, center, and spread of a data set in the context of a situation.	• Build on students' experience with the following skills: ○ Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation) (6.SP.B.5.C) ○ Graphing points on the coordinate plane (6.NS.C.8) ○ Construct and interpret data plot (8.SP.A.1) ○ Construct a function to model a linear relationship (8.F.B.4) ○ Understanding, constructing, interpreting and using patterns of association in bivariate categorical data by displaying frequencies in a two-way

affects the mean in a data set.	• Explain the impact of an outlier on the shape, center, and spread of a data set.	table. (8.SP.A.4)
		
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of representation for mathematical ideas
		
		• Encourage students to use alternative tools to better access the grade level content. Examples include:
		  ○ Desmos.com
		  ○ Graphing calculator
		  ○ Sketch a graph
		  ○ Create a table of values
		  ○ Graphic organizers

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on modeling the scatter plot with a linear function by clarifying mathematical ideas and/or concepts through a short mini-lesson because students need to use the appropriate information to model the scatter plot with linear function. Students understand the connection of rate of change and intercept of the linear function in context of the data.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit representing the scatter plot with linear function by addressing conceptual understanding because students explain the implication of rate of change and intercepts of the linear function in the context of the data. Students use the rate of change and intercepts to predict the trend of the

		behavior and describe the correlation.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension focused on modeling with linear functions and calculating the correlation coefficient because students will explain the similarities and differences of correlation and causation. Students describe and compare real-life data that has a correlation and real-life data that has causation.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards of Mathematical Practice
    - Common Misconceptions
    - Identification of Priority Standards, as identified by NMPED.
    - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Understand the concept of a function and use function notation.
  - HSF.IF.A.1
  - HSF.IF.A.2
  - HSF.IF.A.3
- Interpret functions that arise in applications in terms of the context.
  - HSF.IF.B.4
  - HSF.IF.B.5
  - HSF.IF.B.6
- Analyze functions using different representations.
  - HSF.IF.C.7
  - HSF.IF.C.8
  - HSF.IF.C.9

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Understand the concept of a function and use function notation

Cluster Standard: HSF.IF.A.1

Standard	Standards for Mathematical Practice
Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students need to understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. 8.F.A are foundational standards; however, this is students' first opportunity to work with function notation as it is explicitly left out of the Grade 8 standards.	• Distinguish between functions and nonfunctions from a graph. • Distinguish between functions and nonfunctions from a table. • Distinguish between functions and nonfunctions from an equation • Identify the domain and range of a function given a graph, table, or algebraic representation. • Understand the value of a function with proper notation: f(x) = y, the y value is the value of the function at a particular value of x.
DOK	**Blooms**
1	Remember, Understand

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Understand the concept of a function and use function notation

Cluster Standard: HSF.IF.A.2

Standard	Standards for Mathematical Practice
Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should be able to use function notation in a flexible way such as knowing how to plug in a value and get the corresponding output. They should also be able to understand and use *x* and *F*(*x*) interchangeably with *x* and *y* when explaining the context of a problem. Students should know that all they must do is isolate an equation for *y* and then replace it with *f*(*x*) (read as "*f* of *x*").	• Identify mathematical relationships and convey them using proper function notation • Find the input for a given output when given in function notation. • Identify the domain and range for any given function, presented in function notation or given as a verbal description, and define a reasonable domain in terms of a context or mathematical situation.
DOK	**Blooms**
1-2	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Understand the concept of a function and use function notation

Cluster Standard: HSF.IF.A.3

Standard	Standards for Mathematical Practice
Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*	• **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. Students should see patterns emerge when comparing the *x* and *y* values to each other. Students should know that these patterns are not coincidences and, students should know that these patterns can be thought of as sequences, or a list of numbers. Sequences can be either arithmetic (where the same number is added or subtracted) or geometric (where the same number is multiplied or divided).	• Observe a sequence as a function whose domain consists of integers. • Consider various possible sequences and determine whether they can be expressed explicitly or must be written as a function of the previous terms.
DOK	**Blooms**
1	Remember, Understand

Common Misconceptions

- Students may not recognize $f(x)$ = is the same as y =. They also will often confuse $f(x)$ with the product of f and x and not recognize that it is a form of notation.
- Students often show a lack of understanding for what 'n' represents and often struggle to understand the notation of recurrence sequences, using different values of n for a given term.
- Students may believe that any relationship having an input and an output are functions, and therefore misuse function notation or terminology.

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Interpret functions that arise in applications in terms of the context

Cluster Standard: HSF.IF.B.4

Standard	Standards for Mathematical Practice
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students interpret the key features of the different functions listed in the standard. When given a table or graph of a function that models a real-life situation, explain the meaning of the characteristics of the table or graph in the context of the problem. Key features of a linear function are slope and intercepts; of a quadratic function are intervals of increase/decrease, positive/negative, maximum/minimum, symmetry, and intercepts; of an exponential function include *y*-intercept and increasing/decreasing intervals; and of an absolute value include *y*-intercept, minimum or maximum, increasing or decreasing intervals, and symmetry.	• Identify intercepts of a function. • Identify intervals where the function is increasing. • Identify intervals where the function is decreasing. • Identify intervals where the function is positive. • Identify intervals where the function is negative. • Identify relative maximums of a function. • Identify relative minimums of a function. • Identify symmetries in the functions. • Identify the end behavior of the functions. • Sketch graphs given a list of key features or a verbal model. • Sketch functions that model key feature behavior. • Label intercepts and intervals of a graph. • Interpret where the function is increasing, decreasing, positive, or negative. • Interpret relative maximums and minimums. • Interpret various symmetries, end behaviors, and periodicity.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Interpret functions that arise in applications in terms of the context

Cluster Standard: HSF.IF.B.5

Standard	Standards for Mathematical Practice
Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should focus their attention on possible input and output values, framing them as the domain and range of a function. When given a description of a function that represents a situation, the students should determine reasonable domain and range. Students relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Students need to explain the reasonableness of a domain for a given context. Students should understand that the domain of a function is the set of all possible inputs and the range is the set of all possible outputs. Also looking at if a function is continuous (time, amount of liquid filling a container) or discrete (number of people or things) and connecting back to number classifications.	• Make connections between a graph of a function and its domain. • Make connections between the graph of a function and the context it describes. • Identify when the domain of a given context is discrete or continuous and explain why.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Interpret functions that arise in applications in terms of the context

Cluster Standard: HSF.IF.B.6

Standard	Standards for Mathematical Practice
Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	• **SMP 4:** Model with mathematics. • **SMP 5:** Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students will calculate and interpret the average rate of change of a linear, quadratic, piecewise linear (to include absolute value), and exponential function (presented symbolically or as a table) over a specified interval. Students will estimate the rate of change from a graph. In addition to finding average rates of change from functions given symbolically, graphically, or in a table, students may collect data from experiments or simulations (ex. falling ball, velocity of a car, etc.) and find average rates of change over various intervals.	• Calculate the average rate of change of a function over a specified interval presented symbolically. • Calculate the average rate of change of a function over a specified interval presented in a table. • Interpret the average rate of change of a function over a specified interval presented symbolically for a given context. • Interpret the average rate of change of a function over a specified interval presented in a table for a given context. • Estimate the rate of change of a function from a graph.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- Students may confuse independent and dependent variables.
- Students may believe that the domain for all functions is all real numbers.
- Students may struggle with the concepts of rate of change and slope.
- Students may focus on the *y* values of the graph instead of the *x* values of the interval, when identifying key features of a graph.

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Analyze functions using different representations

Cluster Standard: HSF.IF.C.7

Standard	Standards for Mathematical Practice
Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. • HSF.IF.C.7.A: Graph linear and quadratic functions and show intercepts, maxima, and minima. • HSF.IF.C.7.B: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. • HSF.IF.C.7.E: Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	• **SMP 4:** Model with mathematics. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should be able to describe the significant features of different functions graphically and algebraically. Students should be able to use the significant features to sketch the graph of the function. Students should graph linear and quadratic functions and show intercepts, maxima, and minima. Students should know the slope-intercept form of linear functions, $y = mx + b$, and how to extract enough information from the equation to be able to draw it. When graphing roots, remember that for $\sqrt[n]{x}$, if n is even, the domain includes all positive integers. Otherwise, negative values are included as well. When graphing roots of the for $y =$	• Graph functions expressed symbolically showing key features of the graph by hand in simple cases and with technology for more complicated cases. • Graph linear functions showing intercepts. • Graph quadratic functions showing intercepts, maxima and minima. • Graph piecewise defined functions (step functions and absolute value functions) showing intercepts, maxima, and minima. • Compare and contrast linear, quadratic and exponential functions. • Explain issues of domain, range and usefulness when examining piecewise-defined functions.

$a\sqrt{x}+b$, remember the *y*-intercept is b. Students should remember that roots are fractional exponents. Students should know to look at the highest degree of the polynomial and its coefficient, ax^n. If *n* is even, the function will extend either up or down on both ends (as *x* goes to positive or negative infinity). If *n* is odd, they'll go in opposite directions. If *a* is positive, the even powered functions will go up and the odd powered functions will start down and go up. If *a* is negative, the even powered functions will go down, and the odd powered functions will start up and go down.	
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Analyze functions using different representations

Cluster Standard: HSF.IF.C.8

Standard	Standards for Mathematical Practice
Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. • HSF.IF.C.8.A: Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. • HSF.IF.C.8.B: Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay.	• **SMP 4:** Model with mathematics. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should be able to rewrite quadratic and exponential functions in different ways to find key features of the expression and interpret those key features in terms of the context they represent. Students should be able to find the *x*-intercepts of a quadratic function using both factoring and completing the square.	• Rewrite a function to find and highlight key features. • Factor a quadratic expression to find zeros, extrema and symmetry • Interpret the meaning of zeros, extrema and symmetry within the context of a problem. • Complete the square for a quadratic function to reveal its key features. • Interpret the key features of a quadratic expression in terms of the context it represents. • Use properties of exponents to relate parts of an exponential function to its context (e.g., describe the initial value, growth/decay rate or factor and the growth period). • Identify how key features of an exponential function relate to characteristics in a real-world context. • Classify real-world problems as an exponential growth or decay.

DOK	Blooms
1-2	Understand, Apply, Analyze

Grade	*CCSS Domain*	*CCSS Cluster*
A1	Interpreting Functions	Analyze functions using different representations

Cluster Standard: HSF.IF.C.9

Standard	Standards for Mathematical Practice
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	• **SMP 5:** Use appropriate tools strategically. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should be able to compare two given functions (linear, exponential, quadratic) whether that be as a function or equation, in a table, in a graph, or by verbal description. Students should start by knowing the difference between linear, quadratic and exponential functions, and be able to identify them by equation and by graph. Students should be able to compare two functions even when they're both represented differently. To do this successfully, they must be able to translate between an equation, a graph, words, and a table of values, and understand how certain aspects of one representation impact the rest.	• Make comparisons between functions in different forms using their knowledge of key features.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- Students may have difficulty identifying the key features needed to sketch the graphs or identifying those features algebraically.
- Students may have difficulty with contextualizing and decontextualizing expressions.
- Students will often confuse functions given in a table as a representation of a finite set of numbers rather than a subset of the entire function. They also may have difficulty with the abstractness of determining what is happening with a function over intervals of the domain that they cannot see.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Interpreting Functions**	Strand: **Analyze functions using different representations**

Suggested Student Discourse Questions

- Can you identify the _____ (intercepts, zeros, rate of change, maxima, minima) on this graph? Explain what the _____ (intercepts, zeros, rate of change, maxima, minima) mean in your own words.
- Give a linear, quadratic, exponential, or piecewise function to each pair of partners. One partner sketches a graph, the other creates a table of values. Direct both to identify the intercepts, zeros, rate of change, maxima, minima. They work together to find each algebraically using the equation. In what ways are the strategies different? How are they similar?
- In what ways are the _____ (intercepts, zeros, rate of change, maxima, minima) portrayed in this graph? How are they portrayed in the table of values? How are they portrayed in the equation?
- How do the _____ (intercepts, zeros, rate of change, maxima, minima) relate to the real world problem?

ASSESSMENT GUIDE

- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

Grade	*CCSS Domain*	*CCSS Cluster*
A1	Interpreting Functions	Understand the concept of a function and use function notation

Sample Task #1 (Constructed Response)

An arrow is shot into the air. A function representing the relationship between the number of seconds it is in the air, t, and the height of the arrow in meters, h, is given by

$$h(t) = -4.9t^2 + 29.4t + 2.5.$$

a. Complete the square for this function. Show all work.

b. What is the maximum height of the arrow? Explain how you know.

c. How long does it take the arrow to reach its maximum height? Explain how you know.

Engage NY - Algebra 1
Module 4, End of Module Assessment, #3

Sample Task #2 (Multiple Choice)

$T = 1{,}000 + 18h$

In the equation above, T represents Brittany's total take-home pay, in dollars, for her first week of work, where h represents the number of hours she worked that week and 1,000 represents a sign-on bonus. If Brittany's total take-home pay was $1,576, for how many hours was Brittany paid for her first week of work?

A. 16

B. 32

C. 55

D. 88

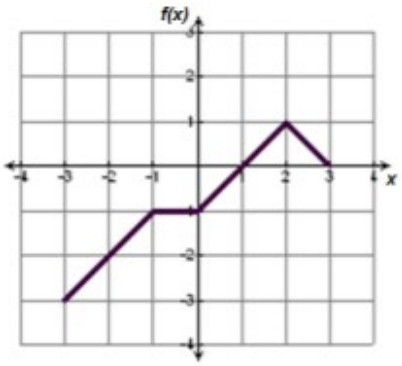
SAT, #1053407

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Interpret functions that arise in applications in terms of the context

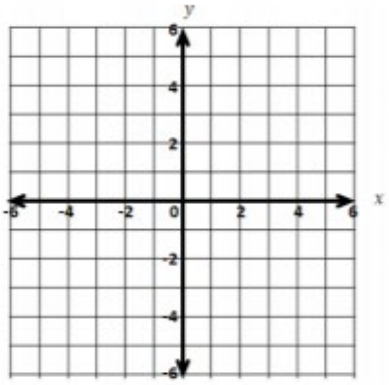
Sample Task #1 (Constructed Response)

The graph of a piecewise function f is shown to the right. The domain of f is $-3 \leq x \leq 3$.

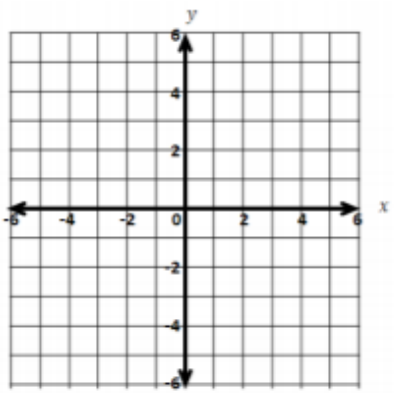
a. Create an algebraic representation for f. Assume that the graph of f is composed of straight line segments.



b. Sketch the graph of $y = 2f(x)$, and state the domain and range.



c. Sketch the graph of $y = f(2x)$, and state the domain and range.



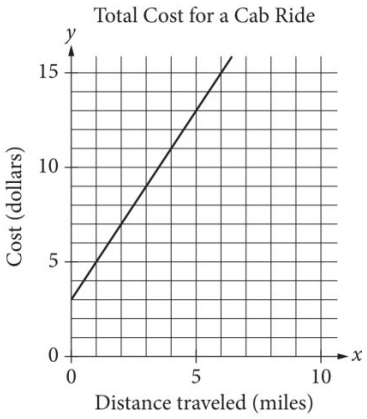
d. How does the range of $y = f(x)$ compare to the range of $y = kf(x)$, where $k > 1$?

e. How does the domain of $y = f(x)$ compare to the domain of $y = f(kx)$, where $k > 1$?

Engage NY - Algebra 1
Module 3, End of Module Assessment, #5

Sample Task #2 (Multiple Choice)

The line graphed in the *xy*-plane below models the total cost, in dollars, for a cab ride, *y*, in a certain city during non-peak hours based on the number of miles traveled, *x*.



According to the graph, what is the cost for each additional mile traveled, in dollars, of a cab ride?

A. $2.00

B. $2.60

C. $3.00

D. $5.00

Rationale

Choice A is correct. The cost of each additional mile traveled is represented by the slope of the given line. The slope of the line can be calculated by identifying two points on the line and then calculating the ratio of the change in y to the change in x between the two points. Using the points $(1,5)$ and $(2,7)$, the slope is equal to $\frac{7-5}{2-1}$, or 2. Therefore, the cost for each additional mile traveled of the cab ride is $2.00.

Choice B is incorrect and may result from calculating the slope of the line that passes through the points $(5,13)$ and $(0,0)$. However, $(0,0)$ does not lie on the line shown. Choice C is incorrect. This is the y-coordinate of the y-intercept of the graph and represents the flat fee for a cab ride before the charge for any miles traveled is added. Choice D is incorrect. This value represents the total cost of a 1-mile cab ride.

SAT, #5209215

Grade	CCSS Domain	CCSS Cluster
A1	Interpreting Functions	Analyze functions using different representations

Sample Task #1 (Constructed Response)

Sydney was studying the following functions:

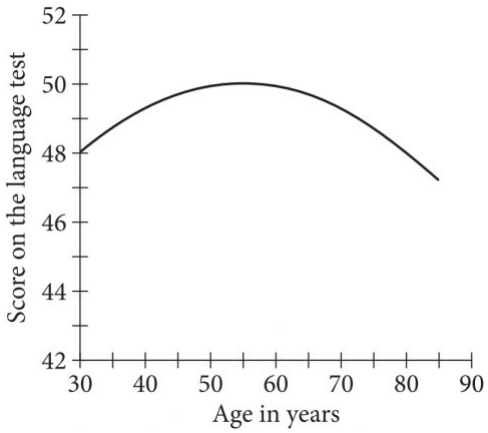
$$f(x) = 2x + 4 \text{ and } g(x) = 2(2)^x + 4$$

She said that linear functions and exponential functions are basically the same. She based her statement on plotting points at $x = 0$ and $x = 1$ and graphing the functions.

Help Sydney understand the difference between linear functions and exponential functions by comparing and contrasting f and g. Support your answer with a written explanation that includes use of the average rate of change and supporting tables and/or graphs of these functions.

Engage NY - Algebra 1
Module 3, Mid-Module Assessment, #2

Sample Task #2 (Multiple Choice)



A scientist tested a group of adults aged 30 to 85. The graph shows the quadratic function S, which models their scores on a language test as a function of their age x, in years. Which of the following could define S ?

A. $S(x) = -\frac{1}{320}(x-50)^2 + 55$

B. $S(x)=-\frac{1}{320}(x-55)^2+50$

C. $S(x)=\frac{1}{320}(x-50)^2+55$

D. $S(x)=\frac{1}{320}(x-55)^2+50$

Rationale

Choice B is correct. The vertex form of a quadratic function $y=f(x)$ in the xy-plane is represented by the equation $f(x)=a(x-h)^2+k$, where (h,k) represents the vertex. A positive value of a results in a vertex that's the lowest point of the graph of $y=f(x)$, and a negative value of a results in a vertex that's the highest point of the graph of $y=f(x)$. The vertex of the given graph has its highest point at approximately $(55,50)$. Therefore, a must be negative. The equation $S(x)=-\frac{1}{320}(x-55)^2+50$ represents a graph with a vertex at $(55,50)$ with a value of a that is negative.

Choice A is incorrect. The vertex of the graph is $(55,50)$, not $(50,55)$. Choice C is incorrect. The positive value of a results in the vertex being the lowest point of the graph instead of the highest. Choice D is incorrect and may result from using an incorrect vertex point and a positive value of a, which would result in the vertex being the lowest point of the graph instead of the highest.

SAT, #1054197

MLSS AND CLR GUIDE

- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

CCSS Domain	*CCSS Cluster*
Interpreting Functions	Understand the concept of a function and use function notation

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on the concept of a function and function notation, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, allowing students to look at home budgets, utility bills (the cost as a function of usage etc.) or even bringing in examples of functions from various careers represented at home can help students make connections between the abstract idea of functions and how/where they exist in real life.	
Cross-Curricular Connections	Science: Radioactive decay is a function that is a sequence. Consider providing a connection where students know the half-life and starting amount of a substance and use that to define a function and determine the amount left after a certain amount of time.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of*	• Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it hinders those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying the concept of a function and function notation the types of mathematical tasks are critical because this cluster is conceptual in nature. The types of vocabulary introduced/continued within this cluster are vital to success in future mathematics especially those within the domain of interpreting functions. Students who are unfamiliar with the idea of a function or the concept of function notation will

	your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	struggle with these foundational ideas if explicit instruction is neglected.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to analyzing proportional relationships and solving real-world math problems using numerical and algebraic expressions and equations. **(7.RPA.2-3)** • Connect to describing the functional relationship between two quantities qualitatively by analyzing a graph. **(8.F.5)** • Connect to constructing a function to model a linear relationship. **(8.F.4)**	• Connect to writing recursive and explicit formulas for arithmetic and geometric sequences. **(HSF.BF.2)** • Connect to writing functions for linear, quadratic, and exponential relationships. **(HSF.BF.1- 2)**	• Connect to use function notation with all types of functions. **(HSF.IF.2)** • **Connect to deriving the formula for a geometric series. (HSA.SSE.4)**

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on the concept of a function and function notation because the foundation for this cluster is developed in 8th grade. Students are introduced to functions as relationships having a unique output for input. Building from this idea is a crucial connection for students developing a deeper understanding of functions and function notation.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.F.A.1: This standard provides a foundation for work with the concept of a function and function notation because it is the foundational concept of the function. Understanding the definition of a function is crucial to making sense of the more complicated functions that are seen in Algebra 1. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
		Potential Scaffolds

• A function is a special relationship between two sets in which each domain value corresponds to one and only one range number.
• The similarities and differences of linear, quadratic, and exponential functions.
• That an arithmetic recursive formula is addition of a repeated constant and a
	• Use multiple representations (including graphs, tables, and symbols) to determine the domain and range and describe important behaviors of functions.
	• Graph linear, quadratic, and exponential by hand and using
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
		  ○ Know and recognize linear functions (8.EE.C.A.7)
		  ○ Calculate arithmetic sequence (7.EE.B.4)
		  ○ Apply properties of exponents (8.EE.A)
		
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-

geometric recursive formula is multiplication of a repeated constant. ● Over time, a quadratic function will grow faster than a linear function, and an exponential function will grow faster than both a linear and a quadratic function.	technology and identify and label key features. ● Create and translate between recursive and explicit definitions of arithmetic and geometric sequences. ● Identify when a table, graph, equation, and/or verbal description exhibits a linear or exponential relationship.	monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Graphing calculator ○ Desmos ○ Graphic organizers ○ Sketch a graph ○ Create a table of values

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on the concept of a function and function notation by clarifying mathematical ideas and/or concepts through a short mini lesson because the cluster is conceptual in nature. Making sense of the concepts is key to analyzing them and interpreting in the next two clusters.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit the concept of a function and function notation by addressing conceptual understanding because this cluster is conceptual in nature. Anything that we do to deepen students' understanding of the concept of function and function notation will be a key to extend their understanding of functions and function notation in additional standards within this domain.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension focused on the concept of a function and function notation because students gain a deeper understanding of functions, once they see applications in real life disciplines other than mathematics. In making cross curricular links students will not only deepen their understanding of the widely applicable nature of functions but also prepare themselves for the next levels of analyzing and interpreting functions.

CCSS Domain	*CCSS Cluster*
Interpreting Functions	Interpret functions that arise in applications in terms of the context

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on Interpreting functions that arise in applications in terms of a context, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, allow students to look at home budgets, utility bills (the cost as a function of usage etc.) or even bringing in examples of functions from various careers represented at home can help students make connections between the abstract idea of functions and how/where they exist in real life.	
Cross-Curricular Connections	Science: Average rate of change can be modeled in contexts involving temperature, speed or height. Consider providing a connection where students collect bivariate data and then make a contextualized explanation of an average rate of change for a model they have created.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the*	• Task: When planning with your HQIM, consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice to engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)." For example, when interpreting functions

	mathematical abilities of students of marginalized cultures and languages? • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	that arise in applications in the terms of a context the types of mathematical tasks are critical because student engagement in this area leads to greater understanding of the key features of functions and how they relate to the context. When students are beginning to make sense of the parts of a function (or its various representations), they need it to be related to an idea they already understand. In doing this we aren't trying to teach the concept in the application and the mathematics because the students already understand the context and can focus on the mathematics of the task.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to interpreting the equation $y = mx + b$ as a linear function and using the equation to solve problems in context. **(8.F.3)** • Connect to interpreting key features of linear equations in relation to a contextual situation. **(8.F.4)**	• Connect to discovering features of families of functions. **(HSF.IF.7)** • Connect to distinguishing between situations modeled by linear and exponential functions. **(HSF.LE.1)**	• Connect to finding key features of the entire family of functions. **(HSF.IF.4)**

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*

Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that previews new contexts for tasks within the unit (e.g., cell phone plans) when studying the interpretations of functions that arise in applications. Understanding the key aspects of a context is the key to unlocking a problem for students.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.F.B.5: This standard provides a foundation for work with interpreting functions that arise in applications in terms of a context because the given standard is the foundational piece of interpreting linear functions. Once students can efficiently and accurately interpret linear functions, they can apply that knowledge to the more complex quadratic and exponential functions of Algebra 1. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework

		Potential Scaffolds

• A function is a special relationship between two sets in which each domain value corresponds to one and only one range number.
• The similarities and differences of linear, quadratic, and exponential functions.
• That an arithmetic recursive formula is addition of a repeated constant and a geometric recursive formula is multiplication of a repeated constant.
• Over time, a quadratic function will grow faster than
	• Use multiple representations (including graphs, tables, and symbols) to determine the domain and range and describe important behaviors of functions.
	• Graph linear, quadratic, and exponential by hand and using technology and identify and label key features.
	• Create and translate between recursive and explicit definitions of arithmetic and
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
		  ○ Know and recognize linear functions (8.EE.C.A.7)
		  ○ Calculate arithmetic sequence (7.EE.B.4)
		  ○ Apply properties of exponents (8.EE.A)
		
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of representation for mathematical ideas
		
		• Encourage students to use alternative tools to better access the grade level content. Examples include:
		  ○ Graphing calculator

a linear function, and an exponential function will grow faster than both a linear and a quadratic function.	geometric sequences. ● Identify when a table, graph, equation, and/or verbal description exhibits a linear or exponential relationship.	○ Desmos ○ Graphic organizers ○ Sketch a graph ○ Create a table of values

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on interpreting functions that arise is applications in terms of the context by providing specific feedback to students on their work through a short mini-lesson because in interpreting functions within a context providing feedback and allowing students to revise their work can be a powerful tool in deepening their understanding.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit interpreting functions that arise in applications in terms of the context by confronting student misconceptions because as in the section on re-teach targeted in this cluster students need feedback for learning. They need to see what they don't understand, celebrate their successes and revise their work to deepen their understanding of functions and interpreting functions in context.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the opportunity to make connections between the abstract and isolated nature of functions in mathematics and applications in science, history, psychology, sociology and other topics that may be of greater interest to students. If they can research functions in other disciplines, they will have more "buy in" to the importance of interpreting functions.

CCSS Domain	*CCSS Cluster*
Interpreting Functions	Analyze functions using different representations

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on analyzing functions using different representations, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, allow students to look at home budgets, utility bills (the cost as a function of usage etc.) or even bring in examples of functions from various careers represented at home that can help students make connections between the abstract idea of functions and how/where they exist in real life. You can then extend these functions by having students make tables, graphs, and write functions related to what they find at home.	
Cross-Curricular Connections	Science: In high school the NGSS builds on K–8 experiences and progresses to using, synthesizing, and developing models to predict and show relationships among variables between systems and their components in the natural and designed worlds. Consider providing a connection for students to use a model based on evidence to illustrate the relationships between systems or between components of a system.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to*	• Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when analyzing functions using different representations the use of mathematical representations within the classroom is critical because it is the focus of this cluster. Students must be able to connect a table to the algebraic written function, and its graph (in any order). All three representations are vital to making

	support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	sense in mathematics applications. Students often come to us with strengths using one or more of those representations and we can build on those strengths and extend them to the other representations. In connecting what they already know to what they need to add to their "toolbox", students build strength in mathematical representations.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to graphing linear functions. **(8.F.5)** • Connect to comparing properties of linear functions represented in different ways. **(8.F.2)** • Connect to identifying and using key features of linear functions. **(8.F.4)** • Connect to writing linear equations. **(8.F.4)**	• Connect to writing linear, quadratic, and exponential functions to describe relationships between quantities. **(HSA.CED.1-3)** • Connect to analyzing transformations of parent functions for linear, quadratic, and exponential functions. **(HSF.BF.3)**	• Connect to graphing all parent functions by hand and using technology and identifying their key features. **(HSF.IF.7)** • Connect to factoring to complete the square with quadratic functions with complex zeros. **(HSN.CN.7)**

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when analyzing functions using different representations. It is in this cluster that students begin to broaden the scope of the functions they are working with and are specifically introduced to the ideas of quadratic, exponential, piecewise defined, and absolute value functions. In allowing them time to struggle and grapple with the mathematics we are allowing them to make sense of the functions and internalize the understanding of the functions key

		features when presented in various ways.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	6.EE.A.3: This standard provides a foundation for work analyzing functions using different representations because this standard lays the foundation for order of operations and understanding the idea of equivalent expressions. The ideas presented in this standard allow students to start slowly with expressions that are linear in nature leading up to the use of the distributive property as well as associative and commutative properties that are the precursors for factoring and rearranging higher order functions. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
		Potential Scaffolds

• A function is a special relationship between two sets in which each domain value corresponds to one and only one range number.
• The similarities and differences of linear, quadratic, and exponential functions.
• That an arithmetic recursive formula is addition of a repeated constant and a geometric recursive formula is multiplication of a repeated constant.
• Over time, a quadratic function will grow faster than a linear function, and an exponential function
	• Use multiple representations (including graphs, tables, and symbols) to determine the domain and range and describe important behaviors of functions.
	• Graph linear, quadratic, and exponential by hand and using technology and identify and label key features.
	• Create and translate between recursive and explicit definitions of arithmetic and geometric
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
		  ○ Know and recognize linear functions (8.EE.C.A.7)
		  ○ Calculate arithmetic sequence (7.EE.B.4)
		  ○ Apply properties of exponents (8.EE.A)
		
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of representation for mathematical ideas
		
		• Encourage students to use alternative tools to better access the grade level content. Examples include:
		  ○ Graphing calculator
		  ○ Desmos

will grow faster than both a linear and a quadratic function.	sequences. ● Identify when a table, graph, equation, and/or verbal description exhibits a linear or exponential relationship.	○ Graphic organizers ○ Sketch a graph ○ Create a table of values

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?	For example, students may benefit from re-engaging with content during a unit on analyzing functions using different representations by critiquing student approaches/solutions to make connections through a short mini-lesson because so much of this cluster can be learned through student choice of solution method. When we allow students to share their thinking and make connections between their work and that of others, they are encouraged to try a solution method that they hadn't tried before and might be more efficient. They can also see their errors and make revisions. Jo Boaler (youcubed.com) tells us that brain research suggests that we learn more from when we make mistakes than we do when we get things right all the time. Therefore, constructive criticism and feedback that is more meaningful than just a percentage, and vital for our students' success in learning mathematics.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit analyzing functions using different representations by offering opportunities to understand and explore different strategies because as stated above students will approach the problems with the method that makes the most sense to them at first even if it isn't the most efficient strategy. Looking at ideas from other students allows kids to engage in math practice 5 and perhaps make more meaning of different more efficient strategies. We know that we can pick the best strategy from the outset of the problem - but it's because we have a lot of practice and often, we can't necessarily explain why we chose a specific method. It is helpful for our students to think about why one strategy might be better than another and learn when to use

		specific strategies based on the problem type, they are given when analyzing functions given in different ways.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when analyzing functions given in different ways because this cluster is widely applicable to other disciplines such as science and statistics. If students can explore something of interest to them related to this cluster, they may think of ways to analyze functions that make more sense to them and their peers. Allowing them to explore the widely applicable nature of functions given in multiple representations will also allow them to become more informed citizens of our society at large.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Construct and compare linear, quadratic, and exponential models and solve problems.
  - HSF.LE.A.1
  - HSF.LE.A.2
  - HSF.LE.A.3
- Interpret expressions for functions in terms of the situation they model.
  - HSF.LE.B.5

Grade	CCSS Domain	CCSS Cluster
A1	Linear, Quadratic & Exponential Models	Construct and compare linear, quadratic, and exponential models and solve problems

Cluster Standard: HSF.LE.A.1

Standard	Standards for Mathematical Practice
Distinguish between situations that can be modeled with linear functions and with exponential functions. • HSF.LE.A.1.A: Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. • HSF.LE.A.1.B: Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. • HSF.LE.A.1.C: Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should be able to differentiate between exponential and linear functions by determining whether given relationships have a common difference or a common ratio. Students have to know the differences between linear functions and exponential functions. In simplest terms, a linear function takes the form $y = mx + b$ and an exponential function is one in which $y = ax$.	• Compare linear and exponential functions in various ways. • Show that linear functions have a common difference and that exponential functions have a common ratio. • Determine when a relationship is growing by a constant difference. • Determine when a relationship grows by a common ratio.
DOK	**Blooms**
1-3	Understand, Apply, Analyze, Evaluate

Grade	CCSS Domain	CCSS Cluster
A1	Linear, Quadratic & Exponential Models	Construct and compare linear, quadratic, and exponential models and solve problems

Cluster Standard: HSF.LE.A.2

Standard	Standards for Mathematical Practice
Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 5:** Use appropriate tools strategically.

Clarification Statement	Students Who Demonstrate Understanding Can...
• When given a variety of descriptions (whether words, graphs, or tables), students should write linear and exponential functions. Students should determine and explain (orally and in writing) whether relationships—in descriptions, tables, equations, or graphs—are functions. In a table, students should recall that when the difference in interval is constant, we can presume that our equation is most likely linear. In this case it is simply a matter of $f(x) = x + 1$. Students should understand that when graphs are involved, students should plot points. That way, students can assemble a list of input and output values from the graph. As for descriptions, words to watch out for are "exponential," "linear," "multiple," "constant," and "factor."	• Write linear and exponential functions (including arithmetic and geometric sequences) based on a graph. • Write linear and exponential functions (including arithmetic and geometric sequences) based on a description of a relationship. • Write linear and exponential functions (including arithmetic and geometric sequences) based on two ordered pairs (including from a table). • Decide whether a relationship is linear, or exponential given a table, graph or verbal description.

DOK	Blooms
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	Linear, Quadratic & Exponential Models	Construct and compare linear, quadratic, and exponential models and solve problems

Cluster Standard: HSF.LE.A.3

Standard	Standards for Mathematical Practice
Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	• **SMP 5:** Use appropriate tools strategically. • **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should understand that a function growing exponentially will eventually overtake or grow faster than either a linear or quadratic function. Students should be able to compare linear and exponential relationships by performing calculations and by interpreting graphs that show two growth patterns. Students should be able to prove that eventually, as long as the functions are headed in the same direction, a quantity increasing exponentially will "beat" linear, quadratic, and polynomial functions.	• Explore rates of change of different functions using graphs or tables. • Generalize that an exponential growth function will exceed a linear or quadratic function eventually. • Identify situations where this phenomenon is occurring.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- Students may not realize when a table or set of points increases by an interval other than 1 and not take the effect of this into account when finding the common difference or ratio.
- Students may find it difficult to attend to direction and rates of change, making it hard to then compare the graphs.

Grade	CCSS Domain	CCSS Cluster
A1	Linear, Quadratic & Exponential Models	Interpret expressions for functions in terms of the situation they model

Cluster Standard: HSF.LE.B.5

Standard	Standards for Mathematical Practice
Interpret the parameters in a linear or exponential function in terms of a context.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 7:** Look for and make use of structure.

Clarification Statement	Students Who Demonstrate Understanding Can...
• Students should be able to describe parts of linear and exponential functions in terms of a context. They should be able to describe slope and *y*-intercepts or *A* and *B* (when in standard form) for a linear function and describe and/or differentiate between the initial value and growth factor for an exponential function. In more complex problems such as exponentials and polynomials, it may be useful to break down the problem so that it's clearly understood what is changing by how much and by what. Translating the equation into words or vice versa may help understand the equation in terms of the overall context. (For instance, every additional packet of gum sold, denoted by *x,* increases the revenue *y* by 0.95 dollars. That's what the equation $y = 0.95x$ ultimately means.)	• Explain the meanings of inputs and outputs of both exponential ($y=b^x+k$) and linear functions in terms of a given context. • Explain the meaning of parts of functions in terms of context (e.g., if *x* is ice cream cones $5x$ means 5 times the number of ice cream cones). • Identify the parameters of a linear or exponential equation and know the parameters may be different based on the context (parameters include initial values, rate of change or growth factor/rate, etc.).

DOK	Blooms
1-2	Understand, Apply, Analyze

Common Misconceptions

- Students often confuse decay factor with the rate of decay.
- Students may be able to identify the slope and y-intercept but not understand their meaning.

NEW MEXICO
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Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Linear, Quadratic & Exponential Models**	Strand: **Construct and compare linear, quadratic, and exponential models and solve problems**

Suggested Student Discourse Questions

- How are the graphs of a linear function and exponential function the same? How do they differ? Explain using mathematical vocabulary.
- Explain to your partner your process in graphing this _____ (linear / exponential) function. Then, listen to your partner explain their process. Which is more efficient? Which do you think you will use next time?
- In what ways do the strategies of graphing, creating a table of values, and writing an equation modeling a _____ (linear, exponential) function differ? How are they the same?
- Which function, linear or exponential, would be best to model _____? (Population growth over time, growth of a person over time, money earned at a job over time)

Domain: **Linear, Quadratic & Exponential Models**	Strand: **Interpret expressions for functions in terms of the situation they model**

Suggested Student Discourse Questions

- How would you determine the input vs the output of a _____ (linear / exponential) function? Explain your reasoning using mathematical vocabulary.
- Turn and talk: Which strategy (graphical, tabular, algebraic) do you find most effective at quickly identifying the input and output of
- How is the ____ (input / output) of this _____ (linear / exponential) function represented in this graph? How is it represented in a table of values? In the equation?
- Given this function modeling _____ (population growth over time, growth of a

a function? Is your preference different from your partner? In what ways?	person over time, money earned at a job over time), what does the initial value (y-intercept) mean in context? What does the rate of change / growth factor or rate portray in context?

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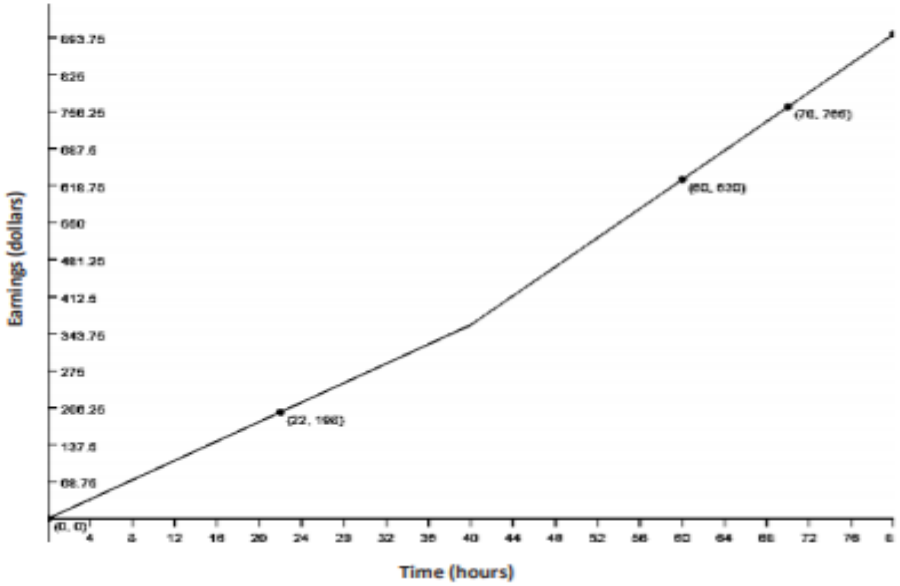
ASSESSMENT GUIDE

- Construct and compare linear, quadratic, and exponential models and solve problems
- Interpret expressions for functions in terms of the situation they model

Grade	*CCSS Domain*	*CCSS Cluster*
A1	Linear, Quadratic, & Exponential Models	Construct and compare linear, quadratic, and exponential models and solve problems

Sample Task #1 (Constructed Response)

Eduardo has a summer job that pays him a certain rate for the first 40 hours per week and time and a half for any overtime. The graph below is a representation of how much money he earns as a function of the hours he works in one week.



Eduardo's employers want to make him a salaried employee, which means he does not get overtime. If they want to pay him $480 per week but have him commit to 50 hours a week, should he agree to the salary change? Justify your answer mathematically.

a. Formulate (recall this step from Lesson 1).

   i. What type of function can be represented by a graph like this (e.g., quadratic, linear, exponential, piecewise, square root, or cube root)?

   ii. How would you describe the end behavior of the graph in the context of this problem?

Engage NY - Algebra 1
Module 5 - Student - Lesson 4

Sample Task #2 (Multiple Choice)

Match each table below to the function and the context, and explain how you made your decision.

A

x	y
1	9
2	18
3	27
4	18
5	9

B

x	y
1	12
2	24
3	36
4	48
5	60

C

x	y
0	160
1	174
2	156
3	106
4	24

D

x	y
1	2
2	4
3	8
4	16
5	32

E

x	y
2	8
3	9
4	8
5	5
6	0

Equation ____ Equation ____ Equation ____ Equation ____ Equation ____

Context ____ Context ____ Context ____ Context ____ Context ____

Equations:

$f(x) = 12x$

$h(x) = -9|x - 3| + 27$

$g(x) = -(x)(x - 6)$

$p(x) = 2^x$

$q(x) = -16x^2 + 30x + 160$

Contexts:

1. The population of bacteria doubled every month, and the total population vs. time was recorded.
2. A ball was launched upward from the top of a building, and the vertical distance of the ball from the ground vs. time was recorded.
3. The height of a certain animal's vertical leap was recorded at regular time intervals of one second; the animal returned to ground level after six seconds.
4. Melvin saves the same amount of money every month. The total amount saved after each month was recorded.
5. Chris ran at a constant rate on a straight-line path and then returned at the same rate. His distance from his starting point was recorded at regular time intervals.

Engage NY - Algebra 1
Module 5 - Student - Lesson 2
*Convert to MC

Grade	CCSS Domain	CCSS Cluster
A1	Linear, Quadratic, & Exponential Models	Interpret expressions for functions in terms of the situation they model

Sample Task #1 (Multiple Choice)

The average number of students per classroom at Central High School from 2000 to 2010 can be modeled by the equation $y = 0.56x + 27.2$, where x represents the number of years since 2000, and y represents the average number of students per classroom. Which of the following best describes the meaning of the number 0.56 in the equation?

A. The total number of students at the school in 2000
B. The average number of students per classroom in 2000
C. The estimated increase in the average number of students per classroom each year
D. The estimated difference between the average number of students per classroom in 2010 and in 2000

Rationale

Choice C is correct. In the equation $y = 0.56x + 27.2$, the value of x increases by 1 for each year that passes. Each time x increases by 1, y increases by 0.56 since 0.56 is the slope of the graph of this equation. Since y represents the average number of students per classroom in the year represented by x, it follows that, according to the model, the estimated increase each year in the average number of students per classroom at Central High School is 0.56.

Choice A is incorrect because the total number of students in the school in 2000 is the product of the average number of students per classroom and the total number of classrooms, which would appropriately be approximated by the y-intercept (27.2) times the total number of classrooms, which is not given. Choice B is incorrect because the average number of students per classroom in 2000 is given by the y-intercept of the graph of the equation, but the question is asking for the meaning of the number 0.56, which is the slope. Choice D is incorrect because 0.56 represents the estimated yearly change in the average number of students per classroom. The estimated difference between the average number of students per classroom in 2010 and 2000 is 0.56 times the number of years that have passed between 2000 and 2010, that is, $0.56 \times 10 = 5.6$. (SAT, #19789)

NEW MEXICO
Public Education Department

MLSS AND CLR GUIDE

- Construct and compare linear, quadratic, and exponential models and solve problems
- Interpret expressions for functions in terms of the situation they model

CCSS Domain	*CCSS Cluster*
Linear, Quadratic, and Exponential Models	**Construct and compare linear, quadratic, and exponential models and solve problems**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on constructing and comparing linear, quadratic, and exponential models and solving problems, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, while looking at the current events that are occurring in your community or in the world, create a model and determine the type of model it represents. This will create a strong connection on math tasks and current events that affect your life.	
Cross-Curricular Connections	Science: Exponential functions can model population growth. However, they will ultimately be limited by resource availability. Consider providing a connection where students track and/or predict when and why this will happen for a given population.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of*	• Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves as capable mathematicians. For example, when constructing and comparing linear, quadratic, and exponential models and solving problems the types of mathematical tasks are critical because the tasks need to be engaging and allow students to use multiple solution strategies which will give the students opportunities to make comparisons.

	your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to determining the growth of a linear expression by taking the ratio of rise over run for any two distinct points on the same line. **(8.EE.6)** • Connect to relating the information gathered by the ratio of rise over run to the linear equation in terms of input and output. **(8.F.4)**	• Connect to examining contextual information and distinguishing if the solution can be modeled with linear or exponential functions. **(HSA.CED.3)** • Connect to writing arithmetic and geometric sequences both recursively and with an explicit formula to model situations. **(HSF.BF.1)** • Connect to relating the knowledge of linear functions to exponential and polynomial functions and comparing their behaviors. **(HSF.If.9)**	• Connect to extending their knowledge of linear, quadratic and exponential situations to different types of functions and making comparisons. **(HSF.IF.7-9)**

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on comparing and constructing linear, quadratic, and exponential models and solving problems because this allows students to go over what they previously learned and think about the process and skills needed to construct models of functions.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.F.A.2 This standard provides a foundation for work with comparing linear, quadratic, and exponential models, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). This sets up the concept they will need for this cluster. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
		Potential Scaffolds

• A function is a special relationship between two sets in which each domain value corresponds to one and only one range number.
• The similarities and differences of linear, quadratic, and exponential functions.
• That an arithmetic recursive formula is addition of a repeated constant and a
	• Use multiple representations (including graphs, tables, and symbols) to determine the domain and range and describe important behaviors of functions.
	• Graph linear, quadratic, and exponential by hand and using
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
		  ○ Know and recognize linear functions (8.EE.C.A.7)
		  ○ Calculate arithmetic sequence (7.EE.B.4)
		  ○ Apply properties of exponents (8.EE.A)
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-

geometric recursive formula is multiplication of a repeated constant. ● Over time, a quadratic function will grow faster than a linear function, and an exponential function will grow faster than both a linear and a quadratic function.	technology and identify and label key features. ● Create and translate between recursive and explicit definitions of arithmetic and geometric sequences. ● Identify when a table, graph, equation, and/or verbal description exhibits a linear or exponential relationship.	monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Graphing calculator ○ Desmos ○ Graphic organizers ○ Sketch a graph ○ Create a table of values

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on constructing and comparing linear, quadratic, and exponential models and solving problems by critiquing student approaches/solutions to make connections through a short mini lesson because students will be given an opportunity to hear vocabulary and revisit concepts and skills needed to construct various models.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on constructing and comparing linear, quadratic, and exponential models and solving problems offering opportunities to understand and explore different strategies because when students are able to have various opportunities to understand and explore different strategies then they are able to think of the different models that they have learned about and use that connection to solve the problem by choosing the strategy that they understand the best.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when constructing and comparing linear, quadratic, and exponential models and solving problems because students can design their own learning path and select the resources, guides and information they will need to discover new information and think critically about it.

CCSS Domain	*CCSS Cluster*
Linear, Quadratic, and Exponential Models	Interpret expressions for functions in terms of the situation they model

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on interpreting expressions for functions in terms of the situation they model, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, think of a special event or activity that you have done. How would you represent it as a function and how would you model the event or activity? Using mathematics, you will create a stronger connection between your personal life and mathematics.	
Cross-Curricular Connections	Science: Colony Collapse Disorder refers to the drastic loss of honeybees and honeybee colonies, such as what has been observed around the world in recent decades. Consider providing a connection where students construct models based on the data and then use those models to describe factors affecting the bee colony populations.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in*	• Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of the classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time." For example, when interpreting expressions for functions in terms of the situation they model, eliciting and using student thinking is critical because students need to feel comfortable that their peers will validate their thinking so they can share what they did. Sharing is an opportunity to learn from our mistakes and from others.

	creating mathematical identities as capable mathematicians that can use mathematics within school and society?	

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to understanding slope is a rate of change expressed as the ratio of rise over run for any two distinct points on the same line. **(8.EE.6)** • Connect to relating the information gathered by the ratio of rise over run to the linear equation and understand that a change in slope will cause the steepness of the line to change. **(8.F.2)** • Connect to simplifying exponential expressions using the Rules of Exponents. **(8.EE.1)**	• Connect to using paper and pencil, graphing calculators, graphing programs, spreadsheets, or other graphing technologies to model and interpret parameters in linear, quadratic, or exponential functions. Parameters may include slope, *y*-intercept, base value, and vertical shifts. **(HSF.IF.7, HSF.BF.3)** • Connect to studying functions to develop contextual understanding on parameter changes in linear and exponential function situations. **(HSF.LE.1-3)**	• Connect to extending analysis to different types of functions and interpreting the key features in modeling situations. **(HSF.IF.4-6, 7)** •

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that analyzes common misconceptions when interpreting expressions for functions in terms of the situation. Modeling allows students to go over what they previously learned and think about the misconceptions they had about the process and skills needed to construct models of functions. This will benefit students when interpreting expressions.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	*8.F.B.4: This standard provides a foundation for working with interpreting expressions for functions in terms of the situation they model. Students learn to read values from a table or from a graph and interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.*
Universal Support Framework		
		Potential Scaffolds
• A function is a special relationship between two sets in which each domain value corresponds to one and only one range number. • The similarities and differences of linear, quadratic, and exponential functions. • That an arithmetic recursive formula is	• Use multiple representations (including graphs, tables, and symbols) to determine the domain and range and describe important behaviors of functions. • Graph linear, quadratic, and	• Build on students' experience with the following skills: ○ Graphing on the coordinate plane (6.NS.C.8) ○ Know and recognize linear functions (8.EE.C.A.7) ○ Calculate arithmetic sequence (7.EE.B.4) ○ Apply properties of exponents (8.EE.A) • Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies

addition of a repeated constant and a geometric recursive formula is multiplication of a repeated constant. ● Over time, a quadratic function will grow faster than a linear function, and an exponential function will grow faster than both a linear and a quadratic function.	exponential by hand and using technology and identify and label key features. ● Create and translate between recursive and explicit definitions of arithmetic and geometric sequences. ● Identify when a table, graph, equation, and/or verbal description exhibits a linear or exponential relationship.	○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Graphing calculator ○ Desmos ○ Graphic organizers ○ Sketch a graph ○ Create a table of values

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on interpreting expressions for functions in terms of the situation they model. By providing specific feedback to students on their work through a short mini-lesson, misconceptions can be addressed immediately.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit interpreting expressions for functions in terms of the situation they model by confronting student misconceptions because students will be aware of them and avoid them next time they are exposed to the same task.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the opportunity to interpret expressions for functions in terms of the situation they model because this allows students to make connections not only with mathematics but to other topics.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, **all** standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Perform arithmetic operations on polynomials.
  - HSA.APR.A.1

Grade	CCSS Domain	CCSS Cluster
A1	Arithmetic with Polynomials & Rational Expressions	Perform arithmetic operations on polynomials

Cluster Standard: HSA.APR.A.1

Standard	Standards for Mathematical Practice
Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	• **SMP5**: Use appropriate tools strategically. • **SMP7**: Look for and make use of structure.

Clarification Statement	Students Who Demonstrate Understanding Can...
• The development of polynomials and rational expressions in high school parallels the development of numbers in elementary and middle grades. In elementary school, students might initially see expressions for the same numbers 8 + 3 and 11, or 3/4 and 0.75, as referring to different entities: 8 + 3 might be seen as describing a calculation and 11 is its answer; 3/4 is a fraction and 0.75 is a decimal. They come to understand that these different expressions are different names for the same numbers, that properties of operations allow numbers to be written in different but equivalent forms, and that all these numbers can be represented as points on the number line. In middle grades, they come to see numbers as forming a unified system, the number system, still represented by points on the number line. The whole numbers expand to the integers—with extensions of addition, subtraction, multiplication, and division, and their properties. Fractions expand to the rational numbers—and the four operations and their properties are extended. A similar evolution takes place in algebra. At first algebraic expressions are simply numbers in which one or more letters are used to stand for a number which is either unspecified or unknown. Students learn to use the properties of operations to write expressions in different but	• Define 'closure' in terms of mathematical properties and operations. • Identify that the sum, difference, or product of two polynomials will always be a polynomial, which means polynomials are closed under addition, subtraction, and multiplication (but not division). • Apply arithmetic operations of add, subtract, and multiply multi-variable polynomials of any degree.

equivalent forms. At some point they see equivalent expressions, particularly polynomial and rational expressions, as naming some underlying thing.	
DOK	**Blooms**
1	Remember, Understand

Common Misconceptions

- Students might think polynomials are only monomial, binomial, or trinomial.
- Students may confuse the impact of adding and subtracting polynomials on the degree of the variable.
- Students may not fully distribute the multiplication of polynomials when applying the distributive property or squaring a binomial, and only multiply like terms.
- When adding and multiplying like terms, students may initially confuse $x + x$ as x^2 instead of $2x$, or $x \cdot x$ as $2x$ instead of x^2.
- Students may not think $x^2 \bullet x = x^3$ is not an example of closure for polynomial multiplication since the result has a different exponent than the factors.

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Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning, and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Arithmetic with Polynomials & Rational Expressions**	Strand: **Perform arithmetic operations on polynomials**

Suggested Student Discourse Questions

- What is the name of a polynomial with two terms?
- When you multiply polynomials what do you do with the exponents?
- Have students critique each other's work. (Various Approaches/Strategies)
- A new bakery offers decorated sheet cakes for children's birthday parties and other special occasions. The bakery wants the volume of a small cake to be 351 cubic inches. The cake is in the shape of a rectangular solid. They want the length of the cake to be four inches longer than the width of the cake and the height of the cake to be one-third of the width. What should the dimensions of the cake pan be?

ASSESSMENT GUIDE

- Perform arithmetic operations on polynomials.

Grade	CCSS Domain	CCSS Cluster
A1	Arithmetic with Polynomials & Rational Expressions	Perform arithmetic operations on polynomials

Sample Task #1 (Constructed Response)

$$(x^2y-3y^2+5xy^2)-(-x^2y+3xy^2-3y^2)$$

Write an equivalent expression to the one above.

SAT, #18093 (Modified)

Sample Task #2 (Multiple Choice)

$(ax+3)(5x^2-bx+4)=20x^3-9x^2-2x+12$

The equation above is true for all x, where a and b are constants. What is the value of ab ?

A. 18

B. 20

C. 24

D. 40

Rationale

Choice C is correct. If the equation is true for all x, then the expressions on both sides of the equation will be equivalent. Multiplying the polynomials on the left-hand side of the equation gives $5ax^3-abx^2+4ax+15x^2-3bx+12$. On the right-hand side of the equation, the only x^2-term is $-9x^2$. Since the expressions on both sides of the equation are equivalent, it follows that $-abx^2+15x^2=-9x^2$, which can be rewritten as $(-ab+15)x^2=-9x^2$. Therefore, $-ab+15=-9$, which gives $ab=24$.

Choice A is incorrect. If $ab=18$, then the coefficient of x^2 on the left-hand side of the equation would be $-18+15=-3$, which doesn't equal the coefficient of x^2, -9, on the right-hand side. Choice B is incorrect. If $ab=20$, then the coefficient of x^2 on the left-hand side of the equation would be $-20+15=-5$, which doesn't equal the coefficient of x^2, -9, on the right-hand side. Choice D is incorrect. If $ab=40$, then the coefficient of x^2 on the left-hand side of the equation would be $-40+15=-25$, which doesn't equal the coefficient of x^2, -9, on the right-hand side.

SAT, #5207088

NEW MEXICO Public Education Department

MLSS AND CLR GUIDE

- Perform arithmetic operations on polynomials

CCSS Domain	*CCSS Cluster*
Arithmetic with Polynomials and Rational Expressions	**Perform arithmetic operations on polynomials**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on understanding that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, discuss how their family culture celebrates an event. Use that event to show some elements of each families' celebration may be similar and different but each is valid and link it to learning. Some students may need to use area models, some may need colored pencils, some students may prefer to add horizontally, and some will prefer to add vertically. Although we learn in different ways, our learning is valid.	
Cross-Curricular Connections	Industrial Arts: Often construction makes use of multiplying polynomials in deciding how to design various aspects of a house or office to fit a predetermined area. Consider providing a connection for students to design something like a sliding door given a specific frame to height ratio and surrounding framework and then substituting in to find the total area for different input values. Art: Often students like to "play" with manipulatives. Consider having students make a work of art using algebra tiles and then create a polynomial expression to represent their artwork.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and*	• Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it hinders those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur

	reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	outside of school mathematics. For example, when understanding that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication, adding, subtracting, and multiplying polynomials. These types of mathematical tasks are critical because students need to build procedural fluency and their conceptual understanding. Students need to understand conceptually like terms, how and why the result of adding and multiplying polynomials is different, how multiplying polynomials is connected to an area model, and how adding polynomials connects to a linear model like perimeter. Algebra tiles, area models, and tasks involving perimeter and area will help students build conceptual understanding while improving their procedural fluency.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to combining like terms and simplifying expressions using the distributive property **(6.EE.3)**	• Connect to using the properties of operations to write expressions in different but equivalent forms. **(HSA.SSE.A.2)**	• Connect to dividing polynomials. **(HSA.APR.2-3)** • Connect to performing operations with rational expressions. **(HSA.APR.7)**

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching when performing arithmetic operations on polynomials, understanding the relationship between zeros and factors of polynomials, using polynomial identities to solve problems and rewriting rational expressions. Students may have unfinished learning when identifying and combining like terms, understanding the relationship between a zero and a factor, and division of numerical expressions. Students need to understand the connection between numerical and variable expressions. Also, students may have unfinished learning on identifying the parts, such as, coefficient, variable, constant of a variable expression and would benefit from targeted pre-teaching.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	6.EE.A.4 provides a foundation for work with performing arithmetic operations of polynomials because students learned to identify two expressions as equivalent written in the form y + y + y and 3y by substituting a fixed value for y. Students should use this same reasoning to add and subtract polynomials. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• Different forms of an expression can be equivalent and are useful in different contexts. • The addition, subtraction, or multiplication of polynomials results in	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form. • Perform the	• Build on students' experience with the following skills: ○ Connect to identifying and interpreting slope and *y*-intercept for linear representations. (8.F.3-4) ○ Connect to rewriting standard linear equation to slope intercept form for systems of equations. (8.EE.8)

another polynomial. ● When a situation and its potential constraints will be represented by a linear or quadratic, or exponential equation/inequality or a system of those equations/inequalities. ● The relationship between solutions of equations/ inequalities and their graphical representations.	operations of addition, subtraction, and multiplication with polynomials. ● Determine reasonable solutions based on the context of real-world problems from graphs of equations/inequalities and systems of equations/inequalities. ● Use the properties of mathematics to solve linear and quadratic equations/inequalities and systems of those equations/inequalities.	○ Connect to knowing and apply the properties of integer exponents to generate equivalent, simplified numerical expressions using the properties of exponents. (8.A.1) ○ Connect to combining like terms and simplifying expressions using the distributive property (6.EE.3) ○ Connect to creating and solving equations in one variable. (7.EE.4) ○ Connect to reasoning with inequalities. (7.EE.4) ○ Connect to solving real world problems involving two linear equations in two variables. (8.EE.8) ● Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos graphing calculator ○ Algebra tiles ○ Graphic Organizers ○ Sketch graph ○ Create table of values

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on performing arithmetic operations on polynomials, by examining tasks from a different perspective through a short mini-lesson because students need to understand the parts of the expression that are related to the outcome (i.e. Sum, difference, product, quotient). Given the outcome and one of its parts, students can find the other part. Example: (4x + 6) + ? = 8x -10
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit performing arithmetic operations on polynomials by offering opportunities to understand and explore different strategies because some students may need support strategies, such as using colored pencils to color code like terms, using algebra tiles to perform operations on polynomials, or the use of calculators to assist in the adding or subtracting of integers.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension performing arithmetic operations on polynomials because students need to expand their algebraic thinking to gain a deeper understanding of polynomials by generating their own equivalent expressions. Students' understanding of integer sums, differences, products and quotients will be reinforced when students are asked to use reasoning to generate their own equivalent expressions.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Build a function that models a relationship between two quantities.
  - HSF.BF.A.1
  - HSF.BF.A.2
- Build new functions from existing functions.
  - HSF.BF.B.3
  - HSF.BF.B.4

Grade	CCSS Domain	CCSS Cluster
A1	Building Functions	Build a function that models a relationship between two quantities

Cluster Standard: HSF.BF.A.1

Standard	Standards for Mathematical Practice
Write a function that describes a relationship between two quantities. • HSF.BF.A.1.A Determine an explicit expression, a recursive process, or steps for calculation from a context. • HSF.BF.A.1.B Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	• **SMP 4:** Model with mathematics. • **SMP7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should write functions for given relationships between quantities. Students can use functions to model real-life situations and make predictions. Students should be able to use functions to describe relationships between two quantities, usually x and $f(x)$, where $f(x)$ is some output value that depends on the input value x. Within a context, students should be able to express a given relationship as a function.	• Write an explicit expression to model linear, exponential and quadratic relationships. • Determine and explain which arithmetic operation(s) should be performed to build the desired combined function given a context or scenario. • Combine two functions by adding, subtracting, multiplying, or dividing, and evaluate the domain of the combined functions related to the context of the problem.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	Building Functions	Build a function that models a relationship between two quantities

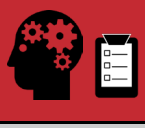
Cluster Standard: HSF.BF.A.2

Standard	Standards for Mathematical Practice
Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	• **SMP 4:** Model with mathematics. • **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students should write **formulas** for **arithmetic** and **geometric sequences** with both **explicit** and **recursive formulas**. They should be able to relate these to a context they represent and be able to **transition** from one form to the other. Students should know that they can write explicit functions recursively, too. For instance, with every year that passes, your age increases by 1. It can be interpreted as constantly adding 1 to the age you were before. In other words, write your age as $f(x) = f(x - 1) + 1$ starting with $f(1) = 1$. Students should know how to recognize that arithmetic functions that take the explicit form $A(n) = A(1) + (n - 1)d$ have the recursive form $A(n) = A(n - 1) + d$ and geometric functions with the form $G(n) = G(1) \times r^{n-1}$ have the recursive form $G(n) = G_{n-1} \times r$.	• Identify arithmetic and geometric patterns in given sequences. • Generate arithmetic or geometric sequences from recursive and explicit formulas. • Justify the translation of given and constructed arithmetic and geometric sequences between recursive and explicit formulas.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- Students may believe that arithmetic and geometric sequences are the same and may need more experience with both types to be able to recognize the difference and develop formulas to describe the patterns.
- Some students may interchange the input and the output values, which can lead to confusion about domain and range, or determining if a relation is a function. This can also affect student understanding and application of inverse functions.
- Students may want to try to use a linear function, specifically the slope-intercept form, for every situation.
- Students may tend to focus on the symbolic form of a function and may need additional support in working with other forms.

Grade	CCSS Domain	CCSS Cluster
A1	Building Functions	Build new functions from existing functions



Cluster Standard: HSF.BF.B.3

Standard	Standards for Mathematical Practice
Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	• **SMP 5:** Use appropriate tools strategically. • **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students should describe the effect of stretches, shrinkages, vertical and horizontal transformations of linear, quadratic and exponential functions. They should be able to find the value of the transformation when given a graph and be able to explain the effects of transformations using technology. Students should know that adding a constant *k* to a function will change the graph of the function depending not only on the value of the constant, but on where it is inserted as well. If $y = f(x)$ is changed to $y = f(x) + k$, the curve will shift vertically (up for $k > 0$, down if $k < 0$). Adding *k* to *x* such that $y = f(x + k)$ will shift the curve horizontally (left for $k > 0$, right for $k < 0$). Multiplying $f(x)$ by a constant *k* stretches ($k > 1$) or squishes ($0 < k < 1$) the graph vertically. If $k < 0$, the graph is also flipped over the *x*-axis. Multiplying *x* by *k* stretches ($k > 0$) or squishes ($k < 0$) the graph horizontally.	• Describe the effect of a single transformation on graphs of functions. • Find the value of k using the graphs of a parent function, f(x), and the transformed function: f(x)+k, kf(x), f(kx), or f(x+k)
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	Building Functions	Build new functions from existing functions

Cluster Standard: HSF.BF.A.4

Standard	Standards for Mathematical Practice
HSF.BF.B.4 Find inverse functions. HSF.BF.B.4.A • Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.*	• **SMP 6:** Attend to precision. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Students should be able to find the inverse of simple linear functions and recognize that other functions may not have an inverse unless there are restrictions placed on the domain. If $f(x) = y$ is a function, the inverse function can be found by switching the place of x and y ($f(y) = x$), and then solving for y so that $f^{-1}(x) = y$. For instance, if the function $f(x)$ is $y = 2x^3$, then the inverse function $f^{-1}(x)$ consists of switching the places of x and y ($x = 2y^3$) and then solving for y.	• Describe how to determine the input of a function when the output is known, using the idea of going backwards. • Determine restrictions on the domain of a function that are required for an inverse of that function to exist.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- Students often have difficulty determining the direction of the horizontal or vertical shifts, as well as understanding the difference between shrink and stretch
- Students often confuse the notation for the inverse and negative numbers.
- Students can easily get confused from traditional algorithms, such as “switch *x* and *y*”, as this can become problematic when *x* and *y* are representing real-world quantities.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Building Functions**	Strand: **Build a function that models a relationship between two quantities**

Suggested Student Discourse Questions

- What are the differences between a recursive and explicit formula? What are the similarities? Explain using mathematical vocabulary.
- Explain your thinking in writing the recursive formula for this _____ (arithmetic, geometric) sequence.
- Explain your algebraic process in converting the recursive formula to an explicit one.
- Imagine you are _____ (laying tile on a floor, creating a pattern for a beaded necklace). Would a recursive formula or an explicit formula better help you know how many (tiles of a certain color, beads of a certain color) to purchase for a small room / necklace? Which formula would better help you determine how many to purchase if you wanted to tile a very large room / make a very long necklace?

ASSESSMENT GUIDE

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

Grade	CCSS Domain	CCSS Cluster
A1	Building Functions	Build a function that models a relationship between two quantities

Sample Task #1 (Constructed Response)

The population of a town is currently 50,000, and the population is estimated to increase each year by 3% from the previous year. Write an equation that can be used to estimate the number of years, t, it will take for the population of the town to reach 60,000.

SAT, #4383286 (Modified)

Sample Task #2 (Multiple Choice)

A new savings account was opened with an initial deposit of $1,000. Each year, the account earns 2% interest on the amount of money in the account the previous year, and this interest is added to the account. If no additional deposits or withdrawals are made, which of the following functions gives the account value $A(t)$, in dollars, after t years?

A.

$A(t) = 1{,}000(1 + 0.02t)$

B.

$A(t) = 1{,}000(1 + 1.02t)$

C.

$A(t) = 1{,}000(0.02)^t$

D.

$A(t) = 1{,}000(1.02)^t$

Rationale

Choice D is correct. A model for a quantity that increases by a certain percentage per time period t is an exponential function in the form $A(t) = I\left(1 + \frac{r}{100}\right)^t$, where I is the initial value at time $t = 0$ for r% annual interest. It's given that $A(t)$ is the account value, in dollars, and t is the number of years after the account was opened. There was an initial deposit of $1,000 at time $t = 0$, so $I = 1{,}000$. This number increases 2% per year after year $t = 0$, so $r = 2$. Substituting these values into the function equation produces $A(t) = 1{,}000(1.02)^t$.

Choices A and B are incorrect and may result from setting up a linear function rather than an exponential function. Choice C is incorrect and may result from representing the exponential function as a decreasing function instead of an increasing function.

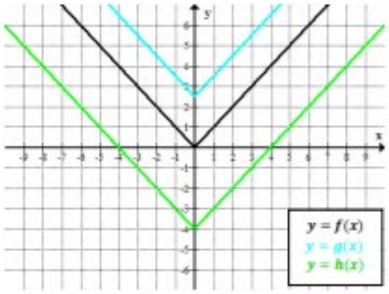
SAT, #4789285

Grade	CCSS Domain	CCSS Cluster
A1	Building Functions	Build new functions from existing functions

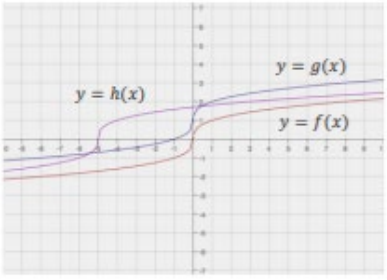
Sample Task #1 (Constructed Response)

For each of the following graphs, use the formula for the parent function f to write the formula of the translated function.

a.



b.



Sample Task #2 (Multiple Choice)

Write a function, g ,in terms of another function, f, such that the graph of g is a vertical shrink of the graph f by a factor of 0.75.

*Convert to Multiple-Choice

MLSS AND CLR GUIDE

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

CCSS Domain	*CCSS Cluster*
Building Functions	**Build a function that models a relationship between two quantities**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on building a function that models a relationship between two quantities, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, find something that you do in your family and create a function model to show someone can create a strong connection between your school tasks and your life tasks.	
Cross-Curricular Connections	Science: In high school the NGSS students should apply concepts of statistics and probability to explain the variation and distribution of expressed traits in a population. Consider providing a connection for students to examine scientific data and predict the effect of a change in one variable on another. https://www.nextgenscience.org/topic-arrangement/hsinheritance-and-variation-traits	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of*	• Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when building a function that models a relationship between two quantities the pattern of questions within the classroom is critical because by posing purposeful questions you

	your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	will be able to scaffold the activity to provide multiple entry points meeting students where they are at.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to interpreting the equation $y = mx + b$ as defining a linear function. **(8.F.3)** • Connect to comparing properties of two functions, each represented in a different way. **(8.F.2)**	• Connect to identifying patterns in the function's rate of change, specifying intervals of increase and decrease, and graphing to model functions. **(HSF.IF.4,6)** • Connect to discussing the relative strengths and weaknesses of each representation and which are most efficient to be able to assist them in making symbolic functions. **(HSF.IF.9)** • Connect to recognizing situations that grow by a constant rate or percent. **(HSF.LE.1)**	• Connect to continuing to write arithmetic and geometric sequences. **(HSF.LE.2)** • Connect to using geometric series to find the sum. **(HSA.SSE.4)** • Connect to performing operations with all parent functions and composing functions. **(HSF.BF.1)**

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that previews new contexts for tasks within the unit (e.g., cell phone plans) when building a function that models a relationship between two quantities and discussing possible strategies and viable solutions.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.F.A.3 This standard provides a foundation for work with building a function that models a relationship between two quantities because students identify the type of relationship the two quantities have (linear, non-linear, exponential). If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
		Potential Scaffolds

• A function is a special relationship between two sets in which each domain value corresponds to one and only one range number.
• The similarities and differences of linear, quadratic, and exponential functions.
• That an arithmetic recursive formula is addition of a repeated constant and a geometric recursive formula is
	• Use multiple representations (including graphs, tables, and symbols) to determine the domain and range and describe important behaviors of functions.
	• Graph linear, quadratic, and exponential by hand and using technology and identify and label
		• Build on students' experience with the following skills:
		  ○ Graphing on the coordinate plane (6.NS.C.8)
		  ○ Know and recognize linear functions (8.EE.C.A.7)
		  ○ Calculate arithmetic sequence (7.EE.B.4)
		  ○ Apply properties of exponents (8.EE.A)
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of

multiplication of a repeated constant. ● Over time, a quadratic function will grow faster than a linear function, and an exponential function will grow faster than both a linear and a quadratic function.	key features. ● Create and translate between recursive and explicit definitions of arithmetic and geometric sequences. ● Identify when a table, graph, equation, and/or verbal description exhibits a linear or exponential relationship.	representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Graphing calculator ○ Desmos ○ Graphic organizers ○ Sketch a graph ○ Create a table of values

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on building a function that models a relationship between two quantities by revisiting student thinking through a short mini lesson because some students have trouble writing their thinking and they just need more time to explain what they are thinking.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit on building a function that models a relationship between two quantities by helping students move from specific answers to generalizations for certain types of problems because students need to understand that the content used in this unit is not only useful for one relationship between two quantities. It is a concept that they will continue to use every time that they have a relationship between two quantities.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as an open-ended task linking multiple disciplines. With the problem being open-ended it allows the students to view multiple perspectives, explain and describe their thinking without feeling pressured to one specific answer.

CCSS Domain	*CCSS Cluster*
Building Functions	Build new functions from existing functions

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on building new functions from existing functions, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, compare functions that represent your community that you can find online. This can create a strong connection between your school tasks and your community.	
Cross-Curricular Connections	Science: The equation for velocity, $M(v) = 6v^2$, is one where the variable, v, has directions. Therefore, an inverse function of $M(v)$ cannot give back both a positive and negative velocity. Consider providing a connection for students to consider how they will handle this situation.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the*	• Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school

	cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when building new functions from existing functions, the use of mathematical representations within the classroom is critical because students will need different representations when creating new functions from existing functions. Students will need to make connections to their previous "mathematical and cultural "knowledge.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to recognizing and understanding that all linear functions can be written in the form $y = mx + b$. **(8.F.3)** • Connect to graphing linear relationships. **(8.F.5)**	• Connect to graphing linear, quadratic, and exponential relationships. **(HSF.IF.4)**	• Connect to extending transformation patterns to all functions. **(HSF.BF.3)** • Connect to graph transformations and compositions of transformations on a coordinate plane. **(HSF.BF.1)**

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that analyzes common misconceptions when building new functions from existing functions because students will need to make connections to the previous standard. If they still have misconceptions, it is better to address before the new standard is introduced to reduce the amount of future confusion.
Intensive	*What critical understandings will prepare students to*	8.F.A.3: This standard provides a foundation for work with building new functions from existing functions

	access the mathematics for this cluster?	because students identify the type of relationship the two quantities have (linear, non-linear, exponential) and they can create new functions. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
		Potential Scaffolds
• A function is a special relationship between two sets in which each domain value corresponds to one and only one range number. • The similarities and differences of linear, quadratic, and exponential functions. • That an arithmetic recursive formula is addition of a repeated constant and a geometric recursive formula is multiplication of a repeated constant. • Over time, a quadratic function will grow faster than a linear function, and an exponential function will grow faster than both a linear and a quadratic function.	• Use multiple representations (including graphs, tables, and symbols) to determine the domain and range and describe important behaviors of functions. • Graph linear, quadratic, and exponential by hand and using technology and identify and label key features. • Create and translate between recursive and explicit definitions of arithmetic and geometric sequences. • Identify when a table, graph, equation, and/or verbal description exhibits a linear or exponential relationship.	• Build on students' experience with the following skills: ○ Graphing on the coordinate plane (6.NS.C.8) ○ Know and recognize linear functions (8.EE.C.A.7) ○ Calculate arithmetic sequence (7.EE.B.4) ○ Apply properties of exponents (8.EE.A) • Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas • Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Graphing calculator ○ Desmos ○ Graphic organizers ○ Sketch a graph ○ Create a table of values

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on Build new functions from existing functions by critiquing student approaches/solutions to make connections through a short mini-lesson because by having students critiquing their work or others they are able to make connections which they can use to help them build new functions.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after the unit building new functions from existing functions by addressing conceptual understanding because this will inform the teacher what the student understands and why it is important to understand why building new functions from existing functions is useful.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the opportunity to build new functions from existing functions because some students can do the assignments but sometimes do not fully understand the concept. This will allow them to focus on the concept in greater depth and not just on finishing the problems.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

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Standards Breakdown

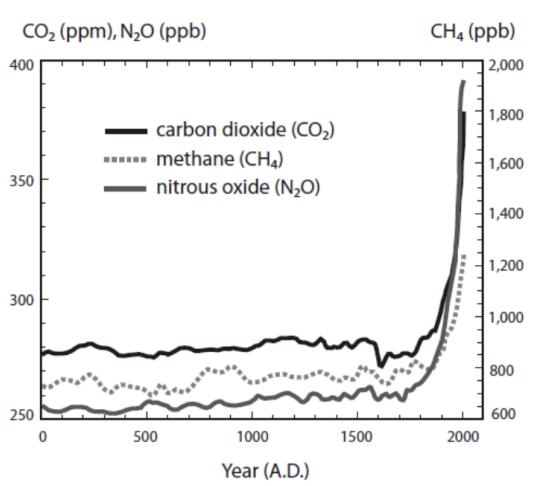
- Reason quantitatively and use units to solve problems.
  - HSN.Q.A.1
  - HSN.Q.A.2
  - HSN.Q.A.3

Grade	CCSS Domain	CCSS Cluster
A1	Quantities	Reason quantitatively and use units to solve problems.

Cluster Standard: HSN.Q.A.1

Standard	Standards for Mathematical Practice
Use units to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 2:** Reason abstractly and quantitatively.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Reasoning quantitatively includes knowing when and how to convert units in computations, such as when adding and subtracting quantities that measure the same attribute but are expressed in different units and other computations with measurements in different units or converting units for derived quantities such as density and speed. Reasoning quantitatively can also include analyzing the units in a calculation to reveal the units of the answer. This can help reveal a mistake if, for example, the answer comes out to be a distance when it should be a speed (SMP2). Students should specify units when defining variables and attend to units when writing expressions and equations (SMP6). In applications, formulas are often used, and errors can occur in the use of the formulas if units are not attended to carefully. The formula $d=vt$ notwithstanding, a car driving at 25 mph for 3 minutes does not cover 25 x 3 miles. Conversely, if the student does attend carefully to units, the result can be a deeper understanding of a formula or a situation. A good quantitative understanding of [a real-life situation] helps a student make sound choices for	• Choose the units in a formula. • Correctly scale a graph with unit increments and identify a quantity from a graph with a scale in unit increments of a specified measurement. • Use units to guide the solution of a familiar multi-step problem with scaffolding. • Make measurement conversions between compound units.

the scale and origin of a graph or a display. In a map of arable land area, for example, there is no sense in having a scale that extends to negative values, in a graph showing the concentration of atmospheric carbon dioxide over the past 2000 years, the choice of origin in the vertical scale is an important editorial decision. These considerations apply to graphs, data tables, scatter plots, and other visual displays of numerical data. It should go without saying that graphs and displays must be properly labeled, or else they are meaningless (SMP6)



Concentration of carbon dioxide and other gases in the atmosphere over the past 2,000 years. Source: Forster et al., 2007, Changes in Atmospheric Constituents and in Radiative Forcing. In Solomon et al. (Eds.), *Climate Change 2007: The Physical Science Basis*, Figure 1, p. 135, http://www.ipcc.ch/pdf/assessment-report/ar4/wg1/ar4-wg1-chapter2.pdf.

DOK	Blooms
1-2	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
A1	Quantities	Reason quantitatively and use units to solve problems.

Cluster Standard: HSN.Q.A.2

Standard	Standards for Mathematical Practice
Define appropriate quantities for the purpose of descriptive modeling.	• **SMP 1**: Make sense of problems and persevere in solving them. • **SMP 4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• In modeling situations (SMP.4), defining the key quantity of interest might be part of the task. For example, in a situation that involves crop productivity, a student might choose to examine the number of tons of fertilizer per acre as the variable of interest. In a situation that involves content development for a web site, a choice might arise as to whether the number of posts per day or the number of words per day is the key productivity variable.	• Identify important information, plan, and develop strategies to solve a problem in a context. • Define appropriate quantities to construct a model
DOK	**Blooms**
1-2	Understand, Apply

Grade	CCSS Domain	CCSS Cluster
A1	Quantities	Reason quantitatively and use units to solve problems.

Cluster Standard: HSN.Q.A.3

Standard	Standards for Mathematical Practice
Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	• **SMP 5:** Use appropriate tools strategically. • **SMP 6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Quantitative reasoning includes choosing an appropriate level of accuracy when reporting quantities. For example, if the doctor measures your height as 73 inches and your weight as 210 pounds, then your Body Mass Index (BMI) is (weight in pounds)/(height in inches^2) x 703 = (210)/(73^2) x 703 ≈ 27.7031 ≈ 28. There is no point in reporting a value more precise than 28 here, because any value between 25 and 30 is considered overweight.	• Determine whether a measurement is appropriate in each context. (e.g., measuring the length of a desk in inches versus yards). • Determine the appropriate level of precision of measurement in each context. • Write solutions using appropriate units and rounding techniques based on the context of the problem.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- Students may have difficulty with multi-step problems.
- Students frequently confuse precision with accuracy.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Quantities**	Strand: **Reason quantitatively and use units to solve problems**

Suggested Student Discourse Questions

- A sequence can be thought of as an ordered list of elements. The elements of the list are called_____ .
- Use an Exit ticket to have students fill in the missing blanks of a sequence or other quantitative expression.
- Compare the numerical approach to the algebra approach
- With given information How much longer would one printing press take to print a novel versus a cookbook?

ASSESSMENT GUIDE

- Reason quantitatively and use units to solve problems.

Grade	*CCSS Domain*	*CCSS Cluster*
A1	Quantities	Reason quantitatively and use units to solve problems.

Sample Task #1 (Constructed Response)

Jason is collecting data on the rate of water usage in the tallest skyscraper in the world during a typical day. The skyscraper contains both apartments and businesses. The electronic water meter for the building displays the total amount of water used in liters. At noon, Jason looks at the water meter and notes that the digit in the ones place on the water meter display changes too rapidly to read the digit and that the digit in the tens place changes every second or so.

a. Estimate the total number of liters used in the building during one 24-hour day. Take into account the time of day when he made his observation. (Hint: Will water be used at the same rate at 2:00 a.m. as at noon?) Explain how you arrived at your estimate.

b. To what level of accuracy can Jason reasonably report a measurement if he takes it at precisely 12:00 p.m.? Explain your answer.

Engage NY - Algebra 1
Module 1, Mid-Module Assessment, #2a/b

Sample Task #2 (Multiple Choice)

Experts say vitamin C is a nutrient that provides many health benefits. The amount of vitamin C, in milligrams (mg), found in 100 grams (g) of each of several fruits is shown in the table below.

Vitamin C Content in Fruits

Type of fruit	Amount of vitamin C in 100 g of fruit
Acerola cherries	1,678 mg
Black currants	181 mg
Guava	228 mg
Kiwifruit	105 mg
Pineapple	56 mg
Strawberries	59 mg

Which quantity of fruit contains an amount of vitamin C closest to the combined amount of vitamin C in 50 g of acerola cherries and 150 g of kiwifruit?

A. 2,000 g of black currants

B. 800 g of guava

C. 1,800 g of pineapple

D. 600 g of strawberries

MLSS AND CLR GUIDE

- Reason quantitatively and use units to solve problems.

CCSS Domain	*CCSS Cluster*
Quantities	**Reason quantitatively and use units to solve problems**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on using units as a way to understand problems and to guide the solution of multi-step problems, choose and interpret units consistently in formulas, choose and interpret the scale and the origin in graphs and data displays, defining appropriate quantities for the purpose of descriptive modeling, and choose a level of accuracy appropriate to limitations on measurement when reporting quantities. Consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, discussing how different cultures eat food will show that although certain cultures choose different tools, such as, forks, chopsticks, tortillas, etc., they are all possible approaches, but some may be more precise than others. Also, with practice other approaches can be useful. The connection can be made that although trying something new, as in a new approach to a mathematical task, may be uncomfortable, but with practice it becomes more useful.	
Cross-Curricular Connections	Science: In high school the NGSS states students should "carefully format data displays and graphs, attending to origin, scale, units, and other essential items." Consider providing a connection for students to choose and interpret the scale and the origin in graphs and data displays that they are working with in science. Social Studies: In high school the New Mexico Social Studies Standards state students should "explain how to use technological tools to research data, verify facts and information, and communicate findings." Consider providing a connection for students to look at the accuracy/precision of measurement data.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students*	• Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge concepts for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical

	and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when using units as a way to understand problems and to guide the solution of multi-step problems, choosing and interpreting units consistently in formulas, choosing and interpreting the scale and the origin in graphs and data displays, defining appropriate quantities for the purpose of descriptive modeling, and choosing a level of accuracy appropriate to limitations on measurement is critical because students approach as well as their solutions need to be validated. For example, multi-entry tasks allow students to choose the tools and approaches best suited for the situation. Allowing for discourse regarding the tools and approach selected provides students' knowledge that there are limitations to tools and approaches. When selecting an approach or tools to attempt a mathematical task, students use their reasoning skills to determine if their approach is valid for the situation and whether there are limits to the approach. Also, students are aware that many approaches or tools may be accurate for the situation, but some are more precise than others.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to rounding. **(4.NBT.3, 5.NBT.4)** • Connect to finding unit rates. • Connect to labeling *x*- and *y*-axes with appropriate scales and units. **(8.F.4-5)**	• Connect to application problems using linear, quadratic, and exponential models. **(HSF.IF.4- 6)**	• Continue to use and expand upon the use of units to make sense of problems and use the context of a problem to create and label graphs using appropriate scales. **(HSF.IF.4-7) -** *Focus on using key features to guide selection of appropriate type of function.*

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when using units to understand problems and to guide the solution of multi-step problems; Using descriptive modeling and choosing a level of accuracy appropriate to the limitations because students need time to determine relevant information and the unit's importance of the units given in the context to help guide their approach. Students also need to use reasoning skills to determine the level of accuracy appropriate to the limitations of their problem. Students need to make sense of the problem, use reasoning to create a plan and use precision to develop a solution that makes sense in the context of the problem. Students should be given multiple opportunities to apply these skills.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	6.RP.A.1 This standard provides a foundation for work with using units as a way to understand problems and to guide the solution of multi-step problems because understanding the concept of a ratio and use ratio language to describe a ratio relationship between two quantities is the building blocks for proportional reasoning and graphs. Students can gain confidence in their problem-solving ability by attempting a problem based on prior learning. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• An expression with a rational exponent can be rewritten as a radical expression.
• The denominator of the rational exponent is the index (root) of the radical expression and the numerator of the rational exponent is the exponent of the radical expression.
• How to select appropriate quantities and/or create appropriate labels for quantities for a real-world context.
• The appropriate levels of measurement precision when using digital and concrete tools, such as calculators, rulers, and protractors.
	• Translate fluently between expressions with rational exponents and radical expressions.
	• Simplify expressions with rational exponents and radical expressions using the properties of exponents.
	• Determine the correct units in multi-step and real-world problems.
	• Choose the appropriate level of precision to report based on the meaning of the quantities in a problem.
		• Build on students' experience with the following skills:
		  ○ Connect to using square root and cube root symbols . (8.EE.2)
		  ○ Connect to understanding and applying the properties of integer exponents. (8.EE.1)
		  ○ Writing and solving one-step and two-step equations
		
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of representation for mathematical ideas
		
		• Encourage students to use alternative tools to better access the grade level content. Examples include:
		  ○ **Desmos calculator**
		  ○ **Square calculator**

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on using units as a way to understand problems and to guide the solution of multi-step problems; Using descriptive modeling and choosing a level of accuracy appropriate to the limitations by critiquing student approaches/solutions to make connections through a short mini-lesson because providing students with feedback not only on their solution but on their approach will engage students in discussions that will lead to clarifying the best approach for a given context.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit using units as a way to understand problems and to guide the solution of multi-step problems; Using descriptive modeling and choosing a level of accuracy appropriate to the limitations by offering opportunities to understand and explore different strategies because students with unfinished learning need ample opportunities to explore different strategies to determine the validity of each strategy given a specific context. Students need opportunities to solve contextual problems that involve using units to understand and solve problems.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when using units as a way to understand problems and to guide the solution of multi-step problems, using descriptive modeling and choosing a level of accuracy appropriate to the limitations. Open-ended tasks linking multiple disciplines allow students to begin to understand the relationship between mathematics and other disciplines. Students engage in using problem solving approaches to address problems in a context other than mathematics. Students will extend their thinking to contextual situations to reinforce their understanding of using units to understand and persevere through all problems.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Understand solving equations as a process of reasoning and explain the reasoning.
  - HSA.REI.A.1
- Solve equations and inequalities in one variable.
  - HSA.REI.B.3
  - HSA.REI.B.4
- Solve systems of equations.
  - HSA.REI.C.5
  - HSA.REI.C.6
  - HSA.REI.C.7
- Represent and solve equations and inequalities graphically.
  - HSA.REI.D.10
  - HSA.REI.D.11
  - HSA.REI.D.12

Grade	CCSS Domain	CCSS Cluster
A1	Reasoning with Equations & Inequalities	Understand solving equations as a process of reasoning and explain the reasoning

Cluster Standard: HSA.REI.A.1

Standard	Standards for Mathematical Practice
Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 5:** Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• A written sequence of steps to solve an equation is code for a narrative line of reasoning using words like "if," "then," "for all," " and "there exists." In the process of learning to solve equations, students learn certain standard "if-then" moves, for example "if $x = y$ then $x + 2 = y + 2$." The danger in learning algebra is that students emerge with nothing but the moves, which may make it difficult to detect incorrect or made-up moves later. Thus, the first requirement in the standards in this domain is that students understand that solving equations is a process of reasoning. This does not necessarily mean that they always write out the full text; part of the advantage of algebraic notation is its compactness. Once students know what the code stands for, they can start writing in code.	• Explain why an equation is equivalent when performing operations to isolate a variable. • Construct arguments for equality using visual representations. • Justify reasoning for elimination of coefficients and/or constants and other steps using multiple types of operations, including multiplication of fractions.

Fragments of reasoning

$$x^2 = 4$$
$$x^2 - 4 = 0$$
$$(x-2)(x+2) = 0$$
$$x = 2, -2$$

This sequence of equations is short-hand for a line of reasoning:

> If x is a number whose square is 4, then $x^2 - 4 = 0$. Since $x^2 - 4 = (x-2)(x+2)$ for all numbers x, it follows that $(x-2)(x+2) = 0$. So either $x - 2 = 0$, in which case $x = 2$, or $x + 2 = 0$, in which case $x = -2$.

More might be said: a justification of the last step, for example, or a check that 2 and -2 actually do satisfy the equation, which has not been proved by this line of reasoning.

DOK	Blooms
1-3	Understand, Apply, Evaluate

Common Misconceptions

- Students do not recognize equality as a relationship between two quantities or, more generally, two mathematical expressions, asserting that the quantities have the same value, or that the expressions represent the same mathematical object.
- Students may perform inappropriate operations on polynomials. Students may subtract from coefficients and constants when subtracting on both sides of an equation or multiply only coefficients when multiplying both sides of an equation.

NEW MEXICO Public Education Department

Grade	CCSS Domain	CCSS Cluster
A1	Reasoning with Equations & Inequalities	Solve equations and inequalities in one variable

Cluster Standard: HSA.REI.B.3

Standard	Standards for Mathematical Practice
Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• With an understanding of solving equations as a reasoning process, students can organize the various methods for solving different types of equations into a coherent picture. For example, solving linear equations involves only steps that are reversible (adding a constant to both sides, multiplying both sides by a non-zero constant, transforming an expression on one side into an equivalent expression). Therefore, solving linear equations does not produce extraneous solutions.	• Solve linear equations, including ones that require using the distributive property, combining like terms, variables on both sides and rational coefficients. • Solve literal equations to isolate a specific variable (e.g., rewriting point slope form to solve for *m*). • Solve linear inequalities, including ones with negative coefficients.
DOK	**Blooms**
1-2	Understand, Apply

NEW MEXICO Public Education Department

Grade	CCSS Domain	CCSS Cluster
A1	Reasoning with Equations & Inequalities	Understand solving equations as a process of reasoning and explain the reasoning

Cluster Standard: HSA.REI.B.4

Standard	Standards for Mathematical Practice
Solve quadratic equations in one variable. • HSA.REI.B.4.A: Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. • HSA.REI.B.4.B: Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	• **SMP 5**: Use appropriate tools strategically. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• HSE.REI.B.4a: The key step in completing the square involves at its heart factoring. And the quadratic formula is nothing more than an encapsulation of the method of completing the square, expressing the actions repeated in solving a collection of quadratic equations with numerical coefficients with a single formula. (MP.8) • HSE.REI.B.4b: It is traditional for students to spend a lot of time on various techniques of solving quadratic equations, which are often presented as if they are completely unrelated (factoring, completing the square, the quadratic formula). Students with an understanding of the underlying reasoning behind all these methods are opportunistic in their application, choosing the method that best suits the situation at hand.	• Derive the quadratic formula from the general form of a quadratic equation. • Solve quadratic equations in one variable with real solutions by inspection, taking square roots, completing the square, using the quadratic formula and factoring. • Identify the number and types of solutions of a quadratic equation using the discriminant.

DOK	Blooms
1-3	Understand, Apply, Analyze, Evaluate

Common Misconceptions

- Since the steps for solving addition and subtraction equations and inequalities are similar, students often forget to change the direction of the inequality sign when multiplying or dividing by a negative coefficient.
- Students will often gravitate toward one solution method or another and try to use it in every possible situation given rather than paying attention to the structure of the equation and choosing the method that is most appropriate to use based on its structure.

Grade	CCSS Domain	CCSS Cluster
A1	Reasoning with Equations & Inequalities	Solve systems of equations

Cluster Standard: HSA.REI.C.5

Standard	Standards for Mathematical Practice
Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Student work with solving systems of equations starts the same way as work with solving equations in one variable; with an understanding behind the various techniques. An important step is realizing that a solution to a system of equations must be a solution to all the equations in the system simultaneously. Then the process of adding one equation to another is understood as "if the two sides of one equation are equal, and the two sides of another equation are equal, then the sum of the left sides of the two equations is equal to the sum of the right sides." Since this reasoning applies equally to subtraction, the process of adding one equation to another is reversible, and therefore leads to an equivalent system of equations.	• Transform a given system of two equations in two variables into an equivalent system that has the same solutions as the original system. • Prove that both systems have the same solution.
DOK	**Blooms**
2-3	Apply, Analyze, Evaluate

Grade	CCSS Domain	CCSS Cluster
A1	Reasoning with Equations & Inequalities	Solve systems of equations

Cluster Standard: HSA.REI.C.6

Standard	Standards for Mathematical Practice
Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	• **SMP 3**: Construct viable arguments and critique the reasoning of others. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Systems of two linear equations with two variables also have the advantage that a good graphical visualization is available; a pair (*x*,*y*) satisfies two equations in two variables if it is on both their graphs, and therefore an intersection point of the graphs.	• Solve a system of linear equations using substitution. • Solve a system of linear equations using elimination. • Solve a system of linear equations by graphing by hand. • Solve a system of linear equations using graphing technology (or Desmos) to estimate more complicated solutions (non-terminating rational solutions). • Differentiate among situations where one solution, no solutions or infinite solutions occur.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	Reasoning with Equations & Inequalities	Solve systems of equations

Cluster Standard: HSA.REI.C.7

Standard	Standards for Mathematical Practice
Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y = -3x and the circle $x^2 + y^2 = 3$.*	• **SMP 5:** Use appropriate tools strategically. • **SMP 7**: Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Systems of two linear equations with two variables also have the advantage that a good graphical visualization is available; a pair (*x*,*y*) satisfies two equations in two variables if it is on both their graphs, and therefore an intersection point of the graphs	• Solve a simple system of a linear equation and a quadratic equation algebraically. • Solve a simple system of a linear equation and a quadratic equation by graphing by hand. • Differentiate among situations where one solution, no solutions or two solutions occur.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	Reasoning with Equations & Inequalities	Represent and solve equations and inequalities graphically

Cluster Standard: HSA.REI.D.10

Standard	Standards for Mathematical Practice
Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Systems of two linear equations with two variables also have the advantage that a good graphical visualization is available; a pair (*x*,*y*) satisfies two equations in two variables if it is on both their graphs, and therefore an intersection point of the graphs	• Explain and verify that every point (x, y) on the graph of a linear or exponential equation represents all values for x and y that make the equation true. • Identify points that are solutions to an equation given a graph of a linear or exponential equation.
DOK	**Blooms**
1	Remember, Understand

Grade	CCSS Domain	CCSS Cluster
A1	Reasoning with Equations & Inequalities	Represent and solve equations and inequalities graphically

Cluster Standard: HSA.REI.D.11

Standard	Standards for Mathematical Practice
Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 5**: Use appropriate tools strategically. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Just as the algebraic work with equations can be reduced to a series of algebraic moves unsupported by reasoning, so can the graphical visualization of solutions. The simple idea that an equation $f(x) = g(x)$ can be solved (approximately) by graphing $y = f(x)$ and $y = g(x)$ and finding the intersection points involves a number of pieces of conceptual understanding. This method seeks to convert an equation in one variable, $f(x) = g(x)$, to a system of equations in two variables, $y = f(x)$ and $y = g(x)$, by introducing a second variable y and setting it equal to each side of the equation. If x is a solution to the original equation, then $f(x)$ and $g(x)$ are equal, and thus (x, y) is a solution to the new system.	• Recognize what the solution $y = f(x)$ and $y = g(x)$ means on a graph. • Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. • Find approximate solutions for the system $y = f(x)$ and $y = g(x)$ using graphs and tables. • Find successive approximations and use them to solve the system $y = f(x)$ and $y = g(x)$.
DOK	**Blooms**
1-3	Understand, Apply, Analyze, Evaluate

Grade	CCSS Domain	CCSS Cluster
A1	Reasoning with Equations & Inequalities	Represent and solve equations and inequalities graphically.

Cluster Standard: HSA.REI.D.12

Standard	Standards for Mathematical Practice
Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 5**: Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Just as the algebraic work with equations can be reduced to a series of algebraic moves unsupported by reasoning, so can the graphical visualization of solutions. The simple idea that an equation $f(x) = g(x)$ can be solved (approximately) by graphing $y = f(x)$ and $y = g(x)$ and finding the intersection points involves a number of pieces of conceptual understanding. This method seeks to convert an equation in one variable, $f(x) = g(x)$, to a system of equations in two variables, $y = f(x)$ and $y = g(x)$, by introducing a second variable y and setting it equal to each side of the equation. If x is a solution to the original equation, then $f(x)$ and $g(x)$ are equal, and thus (x, y) is a solution to the new system.	• Determine whether the boundary line of a linear inequality is inclusive (solid) or is exclusive (broken) of the solution. • Determine which half-plane is the solution to a linear inequality • Graph the solution set to a system of linear inequalities in two variables as the intersection. of the corresponding half-planes. • Identify points that are a solution or non-solution to a linear inequality or system of linear inequalities.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- Students often interpret the solutions to an equation or graphical representation of an equation as only integer values.
- Students may believe an estimate of a value between two integer points is sufficient, but the standard states that students should find successive approximations to approximate the solution.

NEW MEXICO
Public Education Department

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **Reasoning with Equations & Inequalities**	Strand: **Understand solving equations as a process of reasoning and explain the reasoning**

Suggested Student Discourse Questions

- Solve the equation. After each step, explain what you did and what mathematical property allows you to do it.
- Share your steps for solving the equation. Give feedback to other students on the steps they used.
- How many different ways can you solve this equation? Describe each method, step-by-step.
- Write an equation describing how you might spend money at a store for gift bags. You will have the same number of items in each gift bag, and you will spend the same amount of money for all the items you buy. Explain why you wrote the equation using specific numbers and variables.

Domain: **Reasoning with Equations & Inequalities**	Strand: **Solve equations and inequalities in one variable**

Suggested Student Discourse Questions

- Describe the structure of the literal equation. List the operations needed to rewrite the literal equation so it is defined by any of the variables within the literal equation.
- Each group should share the process used to solve the inequality. When they are finished, the other groups should give the presenting group feedback on their process.
- Compare how you solved the literal equation with others in your group. Whose seems to be the most efficient?
- Write an equation defining money you can spend at a store. Rewrite this equation so that it can represent buying various items at the store (i.e., one equation may include only shirts and pants, while another equation includes grocery items).

ASSESSMENT GUIDE

- Understand solving equations as a process of reasoning and explain the reasoning.
- Solve equations and inequalities in one variable.
- Solve systems of equations.
- Represent and solve equations and inequalities graphically.

Grade	*CCSS Domain*	*CCSS Cluster*
A1	Reasoning with Equations & Inequalities	Understand solving equations as a process of reasoning and explain the reasoning

Sample Task #1 (Constructed Response)

What value of x is the expression $\frac{-3}{x^2+3x-10}$ undefined?

SAT, #

Sample Task #2 (Multiple Choice)

$\frac{a-b}{a}=c$

In the equation above, if a is negative and b is positive, which of the following must be true?

A. $c>1$

B. $c=1$

C. $c=-1$

D. $c<-1$

SAT, #422064

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Grade	CCSS Domain	CCSS Cluster
A1	Reasoning with Equations & Inequalities	Solve equations and inequalities in one variable

Sample Task #1 (Constructed Response)

Solve for x in each of the equations or inequalities below, and name the property and/or properties used:

a. $\frac{3}{4}x = 9$

b. $10 + 3x = 5x$

c. $a + x = b$

d. $cx = d$

e. $\frac{1}{2}x - g < m$

f. $q + 5x = 7x - r$

Engage NY - Algebra 1
Module 1, End of Module Assessment, #4

	Sample Task #2 (Multiple Choice)
	$a = 1{,}052 + 1.08t$ The speed of a sound wave in air depends on the air temperature. The formula above shows the relationship between a, the speed of a sound wave, in feet per second, and t, the air temperature, in degrees Fahrenheit (°F). At which of the following air temperatures will the speed of a sound wave be closest to 1,000 feet per second? A. −46°F B. −48°F C. −49°F D. −50°F SAT, #18488

Grade	CCSS Domain	CCSS Cluster
A1	Reasoning with Equations & Inequalities	Solve systems of equations

Sample Task #1 (Constructed Response)

$x - y = 1$
$x + y = x^2 - 3$

Which ordered pair is a solution to the system of equations above?

A. $(1 + \sqrt{3}, \sqrt{3})$

B. $(\sqrt{3}, -\sqrt{3})$

C. $(1 + \sqrt{5}, \sqrt{5})$

D. $(\sqrt{5}, -1 + \sqrt{5})$

SAT, #5439728

Sample Task #2 (Multiple Choice)

$y = x^2 + 3x - 7$
$y - 5x + 8 = 0$

How many solutions are there to the system of equations above?

A. There are exactly 4 solutions.

B. There are exactly 2 solutions.

C. There is exactly 1 solution.

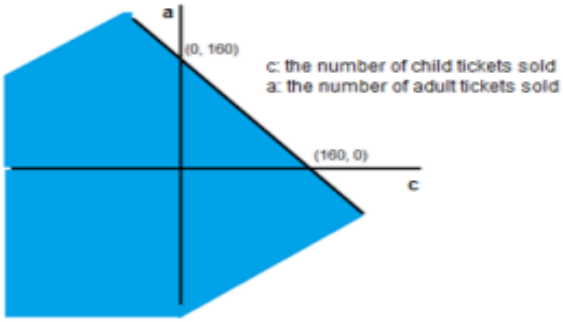
D. There are no solutions.

SAT, #1473185

Grade	CCSS Domain	CCSS Cluster
A1	Reasoning with Equations & Inequalities	Represent and solve equations and inequalities graphically

Sample Task #1 (Constructed Response)

The local theater in Jamie's home town has a maximum capacity of 160 people. Jamie shared with Venus the following graph and said that the shaded region represented all the possible combinations of adult and child tickets that could be sold for one show.



a. Venus objected and said there was more than one reason that Jamie's thinking was flawed. What reasons could Venus be thinking of?

Use equations, inequalities, graphs, and/or words to describe for Jamie the set of all possible combinations of adult and child tickets that could be sold for one show.

Engage NY - Algebra 1
Module 1, End of Module Assessment, #12

Sample Task #2 (Multiple Choice)

Which of the following ordered pairs (x, y) satisfies the inequality $5x - 3y < 4$?

1. $(1, 1)$
2. $(2, 5)$
3. $(3, 2)$

A. I only

B. II only

C. I and II only

D. I and III only

SAT, #422861

NEW MEXICO Public Education Department

MLSS AND CLR GUIDE

- Understand solving equations as a process of reasoning and explain the reasoning.
- Solve equations and inequalities in one variable.
- Solve systems of equations.
- Represent and solve equations and inequalities graphically.

CCSS Domain	*CCSS Cluster*
Reasoning with Equations and Inequalities	**Understand solving equations as a process of reasoning and explain the reasoning**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on solving equations as a process of reasoning, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, learn about the different ways of relating the steps of solving equations in real-life applications of using equations. Students working backward to solve for the unknown quantity is the same as students using inverse operations to solve the equation.	
Cross-Curricular Connections	Language Arts: Justifying reasoning is a form of persuasive writing, as students are trying to get others to agree that their solving process is appropriate and accurate. Consider providing a connection for students to write out the full text (as referenced above) in more of an essay format. Science: When students write up a lab report they often must detail how they tested their hypothesis and clarify why they performed their experiment in a specific way. Consider providing a connection where students must make some "prediction" or hypothesis about an equation prior to solving and then write up their solving method in a format like a lab report.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes*	• Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion, we position them as knowers and doers of mathematics. Using equitable talk moves students and the ways students talk about who is and isn't capable of mathematics, As a result, we can disrupt the negative images and stereotypes around

	regarding the mathematical abilities of students of marginalized cultures and languages? • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when understanding solving equations as a process of reasoning and explaining the mathematical discourse is critical because students practice expressing their mathematical thinking using the content language. Students compare and evaluate different entry points of solving equations. Students defend their strategies by constructing viable arguments and build confidence in math.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to applying the associative, commutative, distributive, and identity properties. **(3.OA.5)** • Connect to learning math properties and their names. **(7.NS.1-2)** • Connect to using variables to write expressions and equations. **(6.EE.2)** • Connecting to solving linear equations. **(7.EE.4, 8.EE.7)**	• Connect to creating and solving equations and inequalities in one variable. **(HSA.CED.1, HSA.REI.3)**	• Connect to justifying steps in solving rational and radical equations. **(HSA.REI.2)** • Connect to justifying steps in writing proofs for geometry. **(HSG.CO.9-11, HSG.SRT.4-5)**

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on solving equations and explaining each step because students may need to justify the inverse operation used in each step with viable arguments. Students may practice expressing their mathematical thinking verbally and symbolically.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	6.EE.B.5: This standard provides a foundation for work with reasoning and solving one-variable equations because students need to understand each step of solving one-variable equations and explain the reason for each step. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• Different forms of an expression can be equivalent and are useful in different contexts. • The addition, subtraction, or multiplication of polynomials results in another polynomial. • When a situation and its potential constraints will be represented by a linear or quadratic, or exponential equation/inequality or a system of those	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form. • Perform the operations of addition, subtraction, and multiplication with polynomials. • Determine reasonable	• Build on students' experience with the following skills: ○ Connect to identifying and interpreting slope and *y*-intercept for linear representations. (8.F.3-4) ○ Connect to rewriting standard linear equation to slope intercept form for systems of equations. (8.EE.8) ○ Connect to knowing and apply the properties of integer exponents to generate equivalent, simplified numerical expressions using the properties of exponents. (8.A.1) ○ Connect to combining like terms and simplifying expressions using the distributive property (6.EE.3)

equations/inequalities. ● The relationship between solutions of equations/ inequalities and their graphical representations.	solutions based on the context of real-world problems from graphs of equations/inequalities and systems of equations/inequalities. ● Use the properties of mathematics to solve linear and quadratic equations/inequalities and systems of those equations/inequalities.	○ Connect to creating and solving equations in one variable. (7.EE.4) ○ Connect to reasoning with inequalities. (7.EE.4) ○ Connect to solving real world problems involving two linear equations in two variables. (8.EE.8) ● Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos graphing calculator ○ Algebra tiles ○ Graphic Organizers ○ Sketch graph ○ Create table of values

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on explaining the reason of each step of solving equations by critiquing student approaches/solutions to make connections through a short mini-lesson because students need to understand why the specific inverse operation is used and develop the viable argument using properties of equality.
Intensive	What assessment data will help identify content needing to be revisited	For example, some students may benefit from intensive extra time during and after a unit explaining the steps of solving equations by offering opportunities to

	for intensive interventions?	understand and explore different strategies because students need to understand why some steps are interchangeable when solving the equations. Students need to explain the order of applying the inverse operations and how that relates to the order of operation of the equations.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as solving complex equations and explaining the steps because students may deepen their understanding of inverse operation, such as logarithm as the inverse operation of exponent. Students explore strategies of solving equations with complex operations and justify their reason in cooperative learning groups.

CCSS Domain	*CCSS Cluster*
Reasoning with Equations and Inequalities	Solve equations and inequalities in one variable

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on solving equations and inequalities in one variable, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, allowing students the autonomy to choose and create problems relevant to their home culture provides students a connection to the world of mathematics.
Cross-Curricular Connections	Science: Projectile motion is modeled by quadratic functions. Consider providing a connection for students to experiment with projectile motion by tossing objects themselves, possibly using technology, and then exchanging equations with another classmate or group to solve. Language Arts: Explaining a process is a form of expository writing, as students are trying to give facts and information. Consider providing a connection for students to write out the derivation of the quadratic formula from standard form to help them see and explain how the two forms are related.

Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	• Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when solving equations and inequalities in one variable, the pattern of questions within the classroom is critical because promoting student learning in It should connect students' lived experiences and interests (their only resources for learning something new) to disciplinary problems in the world. For example, how are verbal and algebraic models and formulas used to represent real life situations? This allows students to come up with their own ideas and make it personable.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to solving equations and inequalities in one variable. **(7.EE.4, 8.EE.7)** • Connect to solving equations involving squares and square roots. **(8.EE.2)**	• Connect to solving quadratic equations and relating solutions to the graph of the function. **(HSF.IF.7)** • Connect to use completing the square and factoring to rewrite quadratic functions in vertex and intercept form to identify key features of the graph. **(HSS.SSE.3)**	• Connect to solving additional types of nonlinear equations. **(HSA.REI.2)** • Connect to relating knowledge of solving quadratic equations to complex numbers, solving rational equations, trigonometric equations, and trigonometric form.

		(HSN.CN.7, HSA.REI.2, HSF.TF.5, 7) ● Connect to understanding the need for a variety of methods (factoring, completing the square, and using quadratic formula) when solving other types of equations, such as parabolas, hyperbolas, and ellipses. **(HSG.GPE.A)**

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on solving equations and inequalities in one variable because knowing this will help prevent errors when solving this type of problem. Students will know what to look for and be aware of when approaching the problems.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	6.EE.B.5Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.: This standard provides a foundation for work with solving the equations and inequalities because students will learn that solving is a process of reasoning to find the numbers which make an equation true, which can include checking if a given number, is a solution. Although the process of reasoning will eventually lead to standard methods for solving equations, students should study examples where looking for structure pays off. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• Different forms of an expression can be equivalent and are useful in different contexts. • The addition, subtraction, or multiplication of polynomials results in another polynomial. • When a situation and its potential constraints will be represented by a linear or quadratic, or exponential equation/inequality or a system of those equations/inequalities. • The relationship between solutions of equations/ inequalities and their graphical representations.	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form. • Perform the operations of addition, subtraction, and multiplication with polynomials. • Determine reasonable solutions based on the context of real-world problems from graphs of equations/inequalities and systems of equations/inequalities. • Use the properties of mathematics to solve linear and quadratic equations/inequalities and systems of those equations/inequalities.	• Build on students' experience with the following skills: ○ Connect to identifying and interpreting slope and *y*-intercept for linear representations. (8.F.3-4) ○ Connect to rewriting standard linear equation to slope intercept form for systems of equations. (8.EE.8) ○ Connect to knowing and apply the properties of integer exponents to generate equivalent, simplified numerical expressions using the properties of exponents. (8.A.1) ○ Connect to combining like terms and simplifying expressions using the distributive property (6.EE.3) ○ Connect to creating and solving equations in one variable. (7.EE.4) ○ Connect to reasoning with inequalities. (7.EE.4) ○ Connect to solving real world problems involving two linear equations in two variables. (8.EE.8) • Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas • Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos graphing calculator ○ Algebra tiles

		○ Graphic Organizers ○ Sketch graph ○ Create table of values
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on solving equations and inequalities in one variable by providing specific feedback to students on their work through a short mini-lesson because < completing a task that compares equations and inequalities side by side and using the previous learned steps in solving both problems allows them to practice the skills that they have learned previously and reinforce them.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit solving equations and inequalities in one variable by offering opportunities to understand and explore different strategies> because students need opportunities to explore different methods and find which one works best for them.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when studying solving equations and inequalities in one variable because making connections helps students appreciate learning the concept more and gives them opportunities to see where it may be going.

CCSS Domain	*CCSS Cluster*
Reasoning with Equations and Inequalities	**Solve systems of equations**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on solving systems of equations, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, connecting systems to their future will allow students the opportunity to understand there are many variables that influence their future goals and some variables are dependent on other variables. For example, whether they attend or where they attend college is dependent on money, grades, etc. By connecting systems of equations to their future goals, students learn how variables are connected and influence each other.	
Cross-Curricular Connections	Science: Projectile motion is modeled by quadratic functions and height is modeled by linear functions. Consider providing a connection for students to experiment with projectile motion by tossing objects themselves, possibly using technology, and then exchanging their system equations with another classmate or group to solve. Social Studies: In high school the New Mexico Social Studies Standards state students should "use quantitative data to analyze economic information". Consider providing a connection for students to work with systems of equations involving economic data.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school*	• Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when solving systems of equations, the pattern of questions within the classroom is critical because promoting student learning in It

	mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?	should connect students' lived experiences and interests (their only resources for learning something new) to disciplinary problems in the world. Systems can be used when trying to determine if you'll make more money at one job or another, taking multiple variables into account, such as salary, benefits and commissions. For example, how would you describe in writing the graphical and algebraic solutions to systems of linear equations using key, technical vocabulary in expanded and some complex sentences? This allows students to really see if they understand the concept. How can you create your own real-world problem?

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to solving systems of linear equations with a focus on graphing and substitution. **(8.EE.8)**	• Connect to creating a system of linear equations or inequalities in a real- world context. **(HSA.CED.3)**	• Connect to using matrices to solve systems of linear equations. **(HSA.REI.8-9)**

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on solving systems of equations because understanding common errors will help clarify understanding and avoid making the same mistakes.
Intensive	*What critical understandings will prepare students to access the mathematics for this*	8. EE.C.8 Analyze and solve pairs of simultaneous linear equations. This standard provides a foundation for work with solving equations simultaneously

	cluster?	graphically, algebraically, or with a matrix because understanding that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• Different forms of an expression can be equivalent and are useful in different contexts. • The addition, subtraction, or multiplication of polynomials results in another polynomial. • When a situation and its potential constraints will be represented by a linear or quadratic, or exponential equation/inequality or a system of those equations/inequalities. • The relationship between solutions of equations/ inequalities and their graphical representations.	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form. • Perform the operations of addition, subtraction, and multiplication with polynomials. • Determine reasonable solutions based on the context of real-world problems from graphs of equations/ inequalities and systems of equations/inequalities. • Use the properties of mathematics to solve linear and quadratic equations/inequalities and systems of those equations/inequalities.	• Build on students' experience with the following skills: ○ Connect to identifying and interpreting slope and *y*-intercept for linear representations. (8.F.3-4) ○ Connect to rewriting standard linear equation to slope intercept form for systems of equations. (8.EE.8) ○ Connect to knowing and apply the properties of integer exponents to generate equivalent, simplified numerical expressions using the properties of exponents. (8.A.1) ○ Connect to combining like terms and simplifying expressions using the distributive property (6.EE.3) ○ Connect to creating and solving equations in one variable. (7.EE.4) ○ Connect to reasoning with inequalities. (7.EE.4) ○ Connect to solving real world problems involving two linear equations in two variables. (8.EE.8) • Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students

		○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ **Desmos graphing calculator** ○ **Algebra tiles** ○ **Graphic Organizers** ○ **Sketch graph** ○ **Create table of values**
Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on solving system of equations by revisiting student thinking through a short mini lesson because sometimes students need a refresher in prior knowledge to help them continue in the task.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit system of equations by confronting student misconceptions because learning from other students' mistakes can help develop their own understanding and help them to not continue to make the same mistakes.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when solving systems of equations because they can relate the concept to a real-world problem

	and see how this will benefit in real life. Making connections with them and applying solving a system to a real-life situation will allow them to make connections to other concepts as well.

CCSS Domain	*CCSS Cluster*
Reasoning with Equations and Inequalities	**Represent and solve equations and inequalities graphically**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on representing and solving equations and inequalities graphically, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, bring in the different languages spoken in the home and connecting it to the tools available to translate different languages, (i.e. Google translate, closed captions on televisions, etc.) make connections that show that in the culture of mathematics, tools are used to translate mathematics and help us make sense of what we are seeing.	
Cross-Curricular Connections	Computer Science: Computer programs use functions to define the points used to graph the animation on a computer. Consider providing a connection where students can write from scratch or compile premade selections to create code that will result in their own animations. Social Studies: In high school the New Mexico Social Studies Standards state students should "use quantitative data to analyze economic information". Consider providing a connection for students to work with a context that compares two situations that each include a standard base fee and additional charges per unit of some quantity.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?*	• Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge concepts for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social,

	• *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when representing and solving equations and inequalities graphically the use of mathematical representations within the classroom is critical because students are given a situation in two variables and they must find the value of one variable given the value of the other, create an equation to represent the situation, use technology to create a graph, and interpret each representation. Understanding how lines and tables represent solution sets of linear relationships will help students make sense of graphs of and solutions to linear inequalities, and later, to make sense of solutions to systems of linear equations in their Algebra 1 class.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to using variables to write expressions, equations, and inequalities. **(6.EE.2)** • Connect to graphing one variable inequalities on a number line. **(7.EE.4)** • Connect to graphing linear equations. **(8.EE.5)** • Connect to graphing systems of linear equations. **(8.EE.8)**	• Connect to interpreting statements, key features, and solutions of linear, quadratic, and exponential functions in terms of context. **(HSA.CED.1,3)** • Connect to graphing linear, quadratic, and exponential functions. **(HSA.CED.2)** • Connect to creating linear, quadratic, and exponential functions. **(HSA.CED.1-2)** • Connect to using graphs of linear, quadratic, and exponential functions to solve real-world contexts.	• Connect to apply these principles to different types of functions. **(HSA.REI.11)**

	(HSA.CED.2)	

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	*Essential Question*	*Examples*
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on representing and solving equations and inequalities graphically because they will be taking the graphing of single points to graphing lines and equations as a set.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	6.EE.B.5: This standard provides a foundation for work to represent and solve equations and inequalities graphically because by substituting numerical values into an equation to determine if the equation is true, the student will comprehend that the answer is a solution. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework

A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• Different forms of an expression can be equivalent and are useful in different contexts. • The addition, subtraction, or multiplication of polynomials results in another polynomial. • When a situation and	• Use the structure of an expression and the properties of mathematics to rewrite it in a different form. • Perform the operations of addition, subtraction, and multiplication with polynomials.	• Build on students' experience with the following skills: ○ Connect to identifying and interpreting slope and *y*-intercept for linear representations. (8.F.3-4) ○ Connect to rewriting standard linear equation to slope intercept form for systems of equations. (8.EE.8) ○ Connect to knowing and apply the properties of integer exponents to generate equivalent, simplified

its potential constraints will be represented by a linear or quadratic, or exponential equation/inequality or a system of those equations/inequalities. ● The relationship between solutions of equations/ inequalities and their graphical representations.	● Determine reasonable solutions based on the context of real-world problems from graphs of equations/inequalities and systems of equations/inequalities. ● Use the properties of mathematics to solve linear and quadratic equations/inequalities and systems of those equations/inequalities.	numerical expressions using the properties of exponents. (8.A.1) ○ Connect to combining like terms and simplifying expressions using the distributive property (6.EE.3) ○ Connect to creating and solving equations in one variable. (7.EE.4) ○ Connect to reasoning with inequalities. (7.EE.4) ○ Connect to solving real world problems involving two linear equations in two variables. (8.EE.8) ● Cognitive Strategies ○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ Desmos graphing calculator ○ Algebra tiles ○ Graphic Organizers ○ Sketch graph ○ Create table of values

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on representing and solving equations and inequalities graphically by clarifying mathematical ideas and/or concepts through a short mini-lesson because helping students

		to understand what the different parts of the graph are telling them will help them to make better understanding of the graphs themselves.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit representing and solving equations and inequalities graphically by confronting student misconceptions because graphs can be misleading if read incorrectly and lead to quite a number of misconceptions, especially when it comes to how accurate the answers you are getting from them are.
Extension		
Essential Question		***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?		Some learners may benefit from an extension that addresses representing and solving equations and inequalities graphically. Some students will pick up on the nuances of graphing quite quickly, by comparison, and could investigate further along points of inquiry such as how changing windows, scaling, or other aspects of the graph affect the readability and usefulness of it as a tool.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Common Misconceptions
  - Identification of Priority Standards, as identified by NMPED.
  - Level of Rigor Identification
- Sample aligned assessment items
- Suggested Student Discourse Guide
- A multilayered system of supports (MLSS) and culturally and linguistically responsive instruction (CLR) guide

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Extend the properties of exponents to rational exponents.
  - HSN.RN.A.1
  - HSN.RN.A.2
- Use properties of rational and irrational numbers.
  - HSN.RN.B.3

Grade	CCSS Domain	CCSS Cluster
A1	The Real Number System	Extend the properties of exponents to rational exponents

Cluster Standard: HSN.RN.A.1

Standard	Standards for Mathematical Practice
Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)^3}$ to hold, so $(5^{1/3})^3$ must equal 5.*	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Exponent notation is a remarkable success story in the expansion of mathematical ideas. It is not obvious at first that a number such as sqrt(2) can be represented as a power of 2. But reflecting that $(\text{sqrt}(2))^2=2$ and thinking about the properties of exponents, it is natural to define $2^{(1/2)}=\text{sqrt}(2)$ since if we follow the rule $(a^b)^c=a^{(bc)}$ then $(2^{(1/2)})^2=2^{((1/2)*2)}=2^1=2$.	• Explain how integer exponent properties apply to rational exponent properties. • Show how a rational exponent (whose numerator is not one) can be expanded as a whole number multiplied by a fraction. • Justify that raising the base to a power and then taking the root is equivalent to taking the root and then raising the base to a power.
DOK	**Blooms**
2	Apply, Analyze

Grade	CCSS Domain	CCSS Cluster
A1	The Real Number System	Extend the properties of exponents to rational exponents

Cluster Standard: HSN.RN.A.2

Standard	Standards for Mathematical Practice
Rewrite expressions involving radicals and rational exponents using the properties of exponents.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 3**: Construct viable arguments and critique the reasoning of others. • **SMP 7**: Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• Because rational exponents have been introduced in such a way as to preserve the laws of exponents, students can now use those laws in a wider variety of situations. For example, they can rewrite the formula for the volume of a sphere of radius r, $V=(4/3)(\text{pi})(r^3)$ to express the radius in terms of the volume, $r=((3/4)(V/\text{pi}))^{(1/3)}$.	• Apply properties of exponents to simplify algebraic expressions with fractional exponents. • Apply power of zero, negative exponent, product, quotient, power to a power, product to a power, and quotient rules of exponents to simplify or write equivalent expressions. • Convert radical expression to expressions with rational exponents and vice versa. • Identify the exponent property used when rewriting expressions and recognize when laws of exponents cannot be used to rewrite an expression.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Common Misconceptions

- Struggle to connect rational exponents to its radical form. Students tend to multiply the number by the exponent.
- When using the Power of a Power Property some students may forget to multiply the entire quantity by the exponent and only multiply the variable.

rade	CCSS Domain	CCSS Cluster
A1	The Real Number System	Use properties of rational and irrational numbers

Cluster Standard: HSN.RN.B.3

Standard	Standards for Mathematical Practice
Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• An important difference between rational and irrational numbers is that rational numbers form a number system. If you add, subtract, multiply, or divide two rational numbers, you get another rational number (provided the divisor is not 0 in the last case). The same is not true of irrational numbers. Although in applications of mathematics the distinction between rational and irrational numbers is irrelevant, since we always deal with finite decimal approximations (and therefore with rational numbers), thinking about the properties of rational and irrational numbers is good practice for mathematical reasoning habits such as constructing viable arguments and attending to precision. (SMP3, SMP6).	• Identify the difference between a rational and an irrational number. • Perform operations on rational and irrational numbers. • Explain that the sum and product of two rational numbers is rational. • Explain that the sum and product of a rational number and a nonzero irrational number are irrational.
DOK	**Blooms**
2	Apply, Analyze

Common Misconceptions

- Students may think that the quotient of two rational numbers isn't always rational because some quotients do not appear to terminate or repeat.
- Students may wrongly believe that a single explanation is an explanation or proof of a property.

Student Discourse Guide

- **Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.**
- **Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)**

Domain: **The Real Number System**	Strand: **Extend the properties of exponents to rational exponents**

Suggested Student Discourse Questions

- Explain why the terms "base" and "power" represent things that have a close physical resemblance to their definitions. Can you think of other terms about exponents that have the same property?
- Share the method you used to simplify the rational exponent. Give feedback about their method to another student.
- How many steps are needed to simplify this rational exponent? Are there other ways to simplify it using the same number of steps?
- Look at measurements of various objects provided by NASA. How do the exponents used in the scientific notation representation of those measurements demonstrate the size of the objects?

ASSESSMENT GUIDE

- Extend the properties of exponents to rational exponents.
- Use properties of rational and irrational numbers.

Grade	*CCSS Domain*	*CCSS Cluster*
A1	The Real Number System	Extend the properties of exponents to rational exponents

Sample Task #1 (Constructed Response)

What is an equivalent expression to $\sqrt{(a+c)^3} \cdot \sqrt{a+c}$?
SAT, #4383735 (Modified)

Sample Task #2 (Multiple Choice)

Which of the following expressions is equivalent to $\sqrt[3]{b} \cdot b \cdot \sqrt[5]{b^2}$ for $b > 0$?

A. $b^{\frac{2}{15}}$

B. $b^{\frac{6}{15}}$

C. $b^{\frac{11}{15}}$

D. $b^{\frac{26}{15}}$

MLSS AND CLR GUIDE

- Extend the properties of exponents to rational exponents.
- Use properties of rational and irrational numbers.

CCSS Domain	*CCSS Cluster*
The Real Number System	**Extend the properties of exponents to rational exponents**

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on: (1) how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, and (2) allowing for a notation for radicals in terms of rational exponents and rewriting expressions involving radicals and rational exponents using the properties of exponents, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, learning about what systems are in place at home to organize and simplify the household can provide students with a powerful connection on why we simplify expressions.	
Cross-Curricular Connections	Science: Rational exponents can be applied to scientific notation. Consider providing a connection for students to explore rational exponents in this context, such as to determine the maximum distance of each planet from the sun. Music: The frequencies in the musical range of various instruments can be modeled using rational exponents. Consider providing a connection for students to find the highest and lowest frequencies.	
Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of*	• Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time." For example, rewriting expressions involving radicals and rational exponents using the properties of exponents elicits and uses student thinking, which is critical, because students need to experience

	marginalized cultures and languages? ● *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	mathematics that allow students to use different approaches to find the same end result. When rewriting expressions with rational exponents in order to simplify expressions, students will be provided the opportunity to see the order in which you perform the root and the power does not make a difference with the end result. Also, by using student thinking in regards to simplifying exponential expressions using the exponent properties, students are provided the opportunity to reason, communicate their reasoning and justify their solution. Students will have the opportunity to build on the knowledge that mathematics is a powerful tool and all approaches should be validated.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
● Connect to using square root and cube root symbols. **(8.EE.2)** ● Connect to understanding and applying the properties of integer exponents. **(8.EE.1)**	● Connect to applying the properties of exponents to rewrite exponential functions. **(HSA.SSE.3)**	● Connect to solving equations using rational exponents and radical operations. **(HSA.REI.2)** ● Connect to continuing to use exponent properties as they arise in various situations**.** **(HSF.IF.8, HSF.LE)**

Suggested Instructional Strategies		
Pre-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that focuses on the rewriting of expressions involving radicals and rational exponents by applying the properties of exponents. Students may have unfinished learning in regards to simplifying expressions with exponents or using the exponent properties and would benefit from the access of that prior learning. Re-visiting expanded form and connecting to exponent properties as well as anchor charts would be beneficial to provide students with access to this content.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.EE.A.1 This standard provides a foundation for work with extending exponent properties to rewriting expressions with radicals and rational exponents because this was the first-time students were introduced to applying the exponent properties in simplifying and generating equivalent expressions. Students will benefit from time to access and apply this prior knowledge. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***
• An expression with a rational exponent can be rewritten as a radical expression. • The denominator of the rational exponent is the index (root) of the radical expression and the numerator of the rational exponent is the exponent of the	• Translate fluently between expressions with rational exponents and radical expressions. • Simplify expressions with rational exponents and	• Build on students' experience with the following skills: ○ Connect to using square root and cube root symbols . (8.EE.2) ○ Connect to understanding and applying the properties of integer exponents. (8.EE.1) ○ Writing and solving one-step and two-step equations • Cognitive Strategies

radical expression. ● How to select appropriate quantities and/or create appropriate labels for quantities for a real-world context. ● The appropriate levels of measurement precision when using digital and concrete tools, such as calculators, rulers, and protractors.	radical expressions using the properties of exponents. ● Determine the correct units in multi-step and real-world problems. ● Choose the appropriate level of precision to report based on the meaning of the quantities in a problem.	○ Repeatedly model the strategies ○ Monitor the students' use of the strategies ○ Provide feedback to students ○ Teach self-questioning and self-monitoring strategies ○ Introduce multiple means of representation for mathematical ideas ● Encourage students to use alternative tools to better access the grade level content. Examples include: ○ **Desmos calculator** ○ **Square calculator**

Re-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on rewriting radical and rational exponent expressions by clarifying mathematical ideas and/or concepts through a short mini-lesson because students must build a conceptual understanding of the meaning of a rational exponent by decomposing the exponent into parts. Students must understand the parts of the rational exponent ⅔; ⅔ can be rewritten as 2 times ⅓; the numerator of 2 is the power and the denominator 3 of (⅓) is the cube root.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit of rewriting radical expressions and expressions with rational exponent by offering opportunities to understand and explore different strategies because students must make the connection that a rational exponent can be broken down to the root and the power. Students need time to explore performing different operations first to understand the mathematical relationships between inverse operations. <i.e. (5 ^3)^(⅓) is 5>

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension such as the application of and development of abstract thinking skills when rewriting radical expressions and expressions with rational exponents by using the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay.

CCSS Domain	*CCSS Cluster*
The Real Number System	Use properties of rational and irrational numbers

Culturally and Linguistically Responsive Instruction

Relevance to Families and Communities	During a unit focused on (1) why the sum or product of two rational numbers is rational; (2) that the sum of a rational number and an irrational number is irrational; and (3) that the product of a nonzero rational number and an irrational number is irrational, consider options for learning from your families and communities the cultural and linguistic ways mathematics exists outside of school to create stronger home to school connections for students. For example, learning about student interests, how they are able to explore their interests and make connections that provide meaning to their interests can help students understand that learning occurs from the ability to make connections and make sense of how it connects to their world.
Cross-Curricular Connections	Science: Two irrational numbers that are of great importance in physics are e and π. Consider providing a connection for students to explore irrational numbers in this context, and the fact that whenever we compute a number answer we must use rational numbers to do it, most generally a finite-precision decimal representation. Social Studies: In high school the New Mexico Social Studies Standards state students should explain and analyze "tension and cooperation between religion and new scientific discoveries". Consider providing a connection for students to learn about Hippias's who was rumored to have been murdered for divulging the existence of irrational numbers.

Validate/Affirm/Build/Bridge	• *How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?* • *How can you create connections between the cultural and linguistic behaviors of your students' home culture and language, the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?*	• Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice leads to more students viewing themselves as capable mathematicians. The types of mathematical tasks are critical because students must develop conceptual understanding of this concept making conjectures regarding the sum and products of rational and irrational numbers, conduct investigations by exploring many cases, providing counter examples, if possible, to refute the conjecture, and justifying their claims through verbal and written communication. Students who are given rules, do not remember them unless they make a personal connection to the rule. Discovery is the connection students need to truly understand and remember the outcomes of sums and products of rational and irrational numbers.

Planning for Multi-Layered System of Supports

Vertical Alignment

Previous Learning	*Current Learning*	*Future Learning*
• Connect to identifying and comparing rational and irrational numbers. **(8.NS.2)** • Connect to computing rational and irrational values when working with volume, surface area, and circles. **(7.G.4,6)**	• Connect to using the same strategies as classifying one number as rational or irrational to classify sums and products.	• Connect to rationalizing denominators using an understanding of products of irrational numbers. **(HSN.CN.5)** • Connect to working with irrational numbers when solving equations.

		(HSN.CN.7) ● Connect to simplifying radicals using an understanding of irrational numbers. **(HSA.REI.2)** ● Connect to calculating and interpreting measurements using irrational numbers. **(HSF.TF.1-3)**

Suggested Instructional Strategies

Pre-Teach

Level of Intensity	***Essential Question***	***Examples***
Targeted	*What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*	Some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying properties of rational and irrational numbers because some students have either unfinished learning or misconceptions such as confusing repeating with non-terminating regarding rational and irrational numbers. Assessing students' prior learning of rational and irrational numbers and addressing student misconceptions is imperative to avoid further misconceptions on classifying sums and products of rational and irrational numbers.
Intensive	*What critical understandings will prepare students to access the mathematics for this cluster?*	8.NS.A.1 This standard provides a foundation for work with the classification of sums and products of rational and irrational numbers because this standard introduces the concept that a number can't be both rational and irrational simultaneously. In prior grades, students were presented with only the rational number system. Students must understand the difference between a rational and irrational number before they can classify expressions. Allowing time for those discussions and addressing misconceptions regarding the real number system will diminish further misconceptions from developing. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Universal Support Framework		
A student should know/understand...	A student should be able to do...	***Potential Scaffolds***

• An expression with a rational exponent can be rewritten as a radical expression.
• The denominator of the rational exponent is the index (root) of the radical expression and the numerator of the rational exponent is the exponent of the radical expression.
• How to select appropriate quantities and/or create appropriate labels for quantities for a real-world context.
• The appropriate levels of measurement precision when using digital and concrete tools, such as calculators, rulers, and protractors.
	• Translate fluently between expressions with rational exponents and radical expressions.
	• Simplify expressions with rational exponents and radical expressions using the properties of exponents.
	• Determine the correct units in multi-step and real-world problems.
	• Choose the appropriate level of precision to report based on the meaning of the quantities in a problem.
		• Build on students' experience with the following skills:
		  ○ Connect to using square root and cube root symbols . (8.EE.2)
		  ○ Connect to understanding and applying the properties of integer exponents. (8.EE.1)
		  ○ Writing and solving one-step and two-step equations
		• Cognitive Strategies
		  ○ Repeatedly model the strategies
		  ○ Monitor the students' use of the strategies
		  ○ Provide feedback to students
		  ○ Teach self-questioning and self-monitoring strategies
		  ○ Introduce multiple means of representation for mathematical ideas
		• Encourage students to use alternative tools to better access the grade level content. Examples include:
		  ○ Desmos calculator
		  ○ Square calculator

Re-Teach		
Level of Intensity	***Essential Question***	***Examples***
Targeted	What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisited during a unit?	For example, students may benefit from re-engaging with content during a unit on Classifying Expressions as Rational or Irrational by revisiting student thinking through a short mini-lesson because students cannot classify expressions as rational or irrational before they have developed conceptual understanding of rational and irrational numbers. Revisiting student thinking before the presentation of this concept will show

		student's unfinished learning, student misconceptions, and students' level of reasoning. Understanding student thinking is essential to present this concept in a way that students can extend their thinking.
Intensive	What assessment data will help identify content needing to be revisited for intensive interventions?	For example, some students may benefit from intensive extra time during and after a unit Classifying Sums and Products as Rational or Irrational by helping students move from specific answers to generalizations for certain types of problems because students need time to reason and apply their thinking using generalizations to develop conceptual understanding. For example, students should be given time and tools (calculators) to investigate whether the sum of two irrational numbers are always, sometimes, or never rational. Students can explore sums of different irrational numbers to determine if they are always irrational or can a counterexample be found. Students then should be allowed time to communicate their thinking verbally and in writing to write a general statement regarding the posed question.

Extension	
Essential Question	***Examples***
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?	Some learners may benefit from an extension addressing the classification of sums and products of rational and irrational numbers because students use reasoning skills to make conjectures and provide counterexamples to disprove conjectures and develop deep understanding of the concept. For example, "Does the product of a rational and irrational number always produce an irrational product? If not, can you provide a case where it does NOT hold true."