New Mexico Instructional Scope for Supporting Equitable and Culturally Responsive Mathematics Instruction

Overview

This Instructional Scope for Mathematics 3.0 was created by a cohort of New Mexico educators and the New Mexico Public Education Department.

The intention of this document is to act as a companion during the planning process alongside High Quality Instructional Materials (HQIM). A sample template is provided to show a snapshot of planning supports provided within each cluster of standards in the mathematics instructional scope.

During the creation of this document, we leveraged the work of other states, organizations, and educators from across the country and world. This work would not have been possible without all that came before it, and we wish to express our sincerest gratitude for everyone that contributed to the resources listed within our references.

To better understand the planning support provided in the mathematics instruction scope, this section provides a brief description of each planning support. This includes *what* support is provided, *why* the planning support is critical for equitable and culturally responsive mathematics instruction, and *how* to use the planning support with HQIM.

Cluster Statement

What: The New Mexico Mathematics Standards are grouped by domains, with somewhere between 4 to 10 domains per grade level. Within each domain, the standards are arranged into clusters. Cluster statements summarize groups of related standards.

Why: The New Mexico Mathematics Standards require a stronger focus on the way time and energy are spent in mathematics classrooms. Students should spend the majority of their time (65-85%) working on content within the major clusters of the grade/course. Supporting clusters and–where appropriate–additional clusters should only comprise 15-35% of the time spent in classes and be covered when they are connected to and engage students in the major work of the grade/course.

How: When planning with your HQIM, consider the time being devoted to major clusters versus additional or supporting clusters. The major work of each grade/course should be designed to provide students with strong foundations for future mathematical work, which will require more time than additional or supporting clusters. Also consider the ways your HQIM makes these connections between major clusters and additional and supporting clusters explicit for students.

Standard Text

What: Each cluster-level support document contains the text of each standard within that cluster.

Why: The cluster statement and standards are meant to be read together to understand the structure of the standards. By grouping the standards within the cluster, the connectedness of the standards is reinforced.

How: The text of the standards should ground all planning with your HQIM. Reading the standards within a cluster intentionally enables us to focus on the connections within and between the standards.

Standards for Mathematical Practice

What: The Standards for Mathematical Practice describe the expertise and habits of mind that mathematics educators at all levels should develop in their students.

Why: Equitable and excellent mathematics instruction supports students in becoming confident and competent mathematicians. By engaging with the Standards for Mathematical Practice, students engage in the practice of doing mathematics and developing mathematical habits of mind. These include the ability to think mathematically, analyze situations, understand relationships, and adapt what they know to solve a wide range of problems, including problems that do not look like any they have encountered before.

How: When planning with HQIM, it is critical to consider the connections between the content standards and the Standards for Mathematical Practice. The planning supports highlight a few practices students can engage in when learning the content of the standard. It is not necessary, or even appropriate, to engage in all of the practices every day. Instead, teachers should choose which Standards for Mathematical Practice to focus on within a given lesson, depending on the content standards being covered and the activities students will be engaging with. When teachers spend time intentionally supporting students in learning both the what (content standards) and the how (Standards for Mathematical Practice), students will have a stronger foundation of mathematical learning.

Clarification Statement

What: The clarification statement provides greater clarity for teachers in understanding the purpose of the standard.

Why: The New Mexico Mathematics Standards illustrate how progressions support student learning within each major domain of mathematics. The clarification statement provides additional context about the ways each cluster of standards supports student learning of the larger learning progression.

How: When planning with HQIM, use the clarification statement to support your understanding of how the materials use specific types of representations or change the learning sequence.

Demonstration Statements, Webb's Depth of Knowledge, Bloom's Taxonomy, and Aspect of Rigor

What: The New Mexico Mathematics Standards include one, two, or all three of the aspects of mathematical rigor: conceptual understanding, procedural skill and fluency, and application to the real world. These planning supports considers which aspect(s) of rigor are within each standard and then identifies academic skills students need to demonstrate comprehension of the standard and associated mathematical practices. The planning supports also provide information about two common classifications on cognitive complexity, Webb's Depth of Knowledge and Bloom's Taxonomy.

Why: Analyzing standards alongside the standards for mathematical practice provide a fuller picture of the mathematical competencies demanded in the standard.

How: When planning for a cluster of standards with your HQIM, a critical first step is to analyze the content and language demands of the standards and the associated Standards for Mathematical Practice. This analysis can be used to plan/design appropriate formative assessment, as well as interpret student data from formative assessments. The planning supports provide a breakdown of the standard that can serve as the basis for this sort of analysis.

Definitions of the Components of Rigor

Rigorous teaching in mathematics does not simply mean increasing the difficulty or complexity of practice problems. Incorporating rigor into classroom instruction and student learning means exploring at a greater depth, the standards and ideas with which students are grappling. There are **three** components of rigor that will be expanded upon in this document, and each is equally important to student mastery: **Conceptual Understanding, Procedural Skill and Fluency,** and **Application**.

- **Conceptual Understanding** refers to understanding mathematical concepts, operations, and relations. It is more than knowing isolated facts and methods. Students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. It also allows students to connect prior knowledge to new ideas and concepts.
- **Procedural Skill and Fluency** is the ability to apply procedures accurately, efficiently, and flexibly. It requires speed and accuracy in calculation while giving students opportunities to practice basic skills. Students' ability to solve more complex application tasks is dependent on procedural skill and fluency.
- **Application** provides valuable context for learning and the opportunity to solve problems in a relevant and a meaningful way. It is through real-world application that students learn to select an efficient method to find a solution, determine whether the solution makes sense by reasoning, and develop critical thinking skills.

A Special Note on Procedural Skill and Fluency

While speed is a component of fluency, it is not necessarily speed in producing an answer; rather, fluency can be observed by watching the speed with which a student engages with a particular problem. Furthermore, fluency does not require the most efficient strategy. The standards specify grade-level appropriate strategies or types of strategies with which students should demonstrate fluency (e.g., 1.OA.C.6 allows for students to use counting on, making ten, creating equivalent but easier or known sums, etc.). It should also be noted that teachers should expect some procedures to take longer than others (e.g., fluency with the standard algorithm for division, 6.NS.B.2, as compared to fluently adding and subtracting within 10, 1.OA.C.6).

Standards identified as targeting procedural skill and fluency do not all have an expectation of automaticity and/or rote recall. Only two standards, 2.OA.B.2 and 3.OA.C.7, have explicit expectations of students knowing facts from memory. Other standards targeting procedural skill and fluency do not require students to reach automaticity. For example, in 4.G.A.2, students do not need to reach automaticity in classifying two-dimensional figures.

*Adapted from Louisiana Department of Education

Assessment Items

What: Formative assessment is the planned, ongoing process used by teachers during learning and teaching to assess student learning in order to improve student understanding and support students in their learning. This planning support provides one or more sample items to assist teachers in their planning of formative assessments for each standard. These are intended to be used as a guide for what students should be able to complete and can be used alongside assessments provided within your HQIM. A link to additional assessment items from Illustrative Mathematics is also provided with each standard.

Why: When student thinking is made visible, the teacher can examine the progression of learning towards the goals of the standards and adjust instruction as necessary. By including students in the assessment and analysis process, teachers allow them to become strategic and goal-directed with their learning.

How: The sample items address the aspect(s) of rigor that aligns with each standard. This example can be used to discuss possible responses by students and next steps for instruction. A similar process can then be used to identify additional items from the formative assessment resources provided by your HQIM.

For additional information, see Standards Aligned Instructionally Embedded Formative Assessment Resources.

Common Misconceptions

What: This planning support identifies some of the common misconceptions students might have when engaging in learning about each mathematical topic.

Why: Students might have misconceptions based on an overgeneralization of patterns they notice or overly relying on rules rather than understanding the underlying concepts. Tips and tricks in mathematics expire over time as students move up through the grade levels. It is critical to understand some of the common misconceptions students can develop so we can address them directly with students and continue to build a strong foundation for their mathematical learning.

How: When planning with your HQIM, look for ways to directly address common misconceptions students have. This planning support provides some of the possible misconceptions, and your HQIM might include additional ones. The goal is not to avoid misconceptions–they are a natural part of the learning process. But we do want to support students in exploring their misconceptions and modifying incorrect or partial understanding.

Planning for Multi-Layered System of Support & Universal Design for Learning

What: The section on planning for Multi-Layered Systems of Supports (MLSS) and Universal Design for Learning (UDL) is designed to support teachers in planning for the needs of all students. Each section includes the three pillars of UDL, as well as options for pre-teaching and re-teaching. Some students might benefit from targeted pre-teaching and re-teaching supports, which improve students' acquisition of the knowledge and skills identified in the New Mexico Mathematics Standards. Intensive pre-teaching and re-teaching supports may also be helpful for smaller groups of students with more intensive needs. Progress monitoring should occur to assess students' responses to these additional supports.

Why: MLSS is a holistic framework that guides educators to intervene quickly when students need additional supports. The framework moves away from the "wait to fail" model and empowers teachers to use their professional judgment to make data-informed decisions regarding the students in their classrooms to ensure academic success with the grade-level expectations of the New Mexico Mathematics Standards.

How: When planning with your HQIM, use the suggestions for pre-teaching as a starting point to determine if some or all of the students in your classroom need targeted or intensive pre-teaching at the start of a unit to ensure they can access the grade-level material within the unit. The core instruction (which is grounded in the UDL Framework) and re-teach sections work together to support planning within a unit, looking for ways the materials support greater access for all students and provide options to revisit concepts based on formative assessment data. Additional information about MLSS can be found on the PED's website, and guidance and tools can be found here.

Vertical Alignment

What: The New Mexico Mathematics Standards are designed around coherent progressions of learning. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but rather an extension of previous learning. The connections to previous, current, and future learning make this coherence visible. A link to Achieve the Core's grade-level coherence maps is provided with each standard.

Why: Students build stronger foundations for learning when they see mathematics as an interconnected discipline of relationships rather than discrete skills and knowledge that are only applicable in current situations. The intentional inclusion of connections to previous, current, and future learning can support a more interconnected understanding of mathematics.

How: When planning with HQIM, use the vertical alignment supports to find ways to help students make explicit connections within their study of mathematics.

Culturally and Linguistically Responsive Instruction

What: Culturally and Linguistically Responsive Instruction (CLRI) requires educators to contribute to a positive school climate by validating and affirming students' home languages and cultures, building and bridging mathematics to previous learning (both formal and informal), and using linguistic vocabulary supports. These actions legitimize students' home culture and language, making it clear that students' culture and language are positive assets in the classroom. It is also an intentional effort to reverse negative stereotypes of non-dominant cultures and languages and must be purposeful, consistent, and authentic, as well as both proactive and reactive. By building and bridging, students learn to toggle between the behaviors and expectations of home and those of school. By focusing on creating connections between the culture and language expectations of home and the expectations at school, we help students be set up for success in school. Linguistic supports are important within this structure as well, and help students learn new, academic vocabulary in ways that allow students to bring in their prior knowledge and connect it to new experiences. Linguistic supports for helping students acquire new academic vocabulary are provided from Pathways2Careers.

Why: The mathematical identities of students are shaped by the messages they receive about their ability to do mathematics and the power of mathematics in their lives outside of school. Educators must intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages. In addition, creating connections between the cultural and linguistic behaviors of students' home culture and language supports students in creating identities as capable mathematicians within school and society.

How: When planning instruction, it is critical to consider ways to validate/affirm and build/bridge students' cultural and linguistic assets. The planning supports for each cluster provide information

and examples of how to engage with equity-based teaching practices. There may be additional support available in your HQIM to ensure all students develop strong mathematical identities.

Student Discourse Guide

What: This guide provides educators with suggested questions to engage students in discourse about mathematical concepts. Purposeful, rich classroom discourse offers students the opportunity to express their ideas and thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Ideas for supporting student discourse are also provided from Pathways2Careers.

Why: Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding.

How: When students have frequent opportunities for discussion, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences, and evolving math identities.

Cross-Curricular Connections

What: Cross-curricular connections are connections between knowledge and/or skills that can be made between two or more areas of study. These connections can be made by teachers or students.

Why: The purpose of planning cross-curricular connections in an instructional sequence is to ensure that students build connections and recognize the relevance of mathematics beyond the mathematics classroom.

How: When planning with HQIM, look for opportunities to make explicit connections to other content areas, including the examples provided for each standard.

Career and Skill Connections

What: The knowledge and/or skills students are mastering within each standard can be directly connected to various careers. These connections can be made by teachers or students and can provide motivation for students as they work through mathematical topics.

Why: Understanding and recognizing the relevance of mathematics beyond the mathematics classroom is important. This is particularly valid for students who may not plan on working in a

"mathematics" field.

How: When planning with HQIM, look for opportunities to make explicit connections to careers, including the examples provided for each standard.

Template of the New Mexico Cluster Level Planning Support for the New Mexico Mathematics Standards

Grade	*CCSS Domain*	*CCSS Cluster*
Grade level	CCSS domain	CCSS cluster statement summarizing the group of related standards
Standard and icons that indicate which aspect of rigor it aligns with		
Standard		**Standards for Mathematical Practice**
Full text of the standard.		Correlation of the standard to the Standards for Mathematical Practice to which it aligns, including a link to a descriptor of what teachers and students should be doing.
Clarification Statement		**Students Who Demonstrate Understanding Can...**
Clarifies the language of the standard.		The skills students perform to demonstrate comprehension of the standard.
DOK		**Blooms**
Correlation of the standard to Webb's Depth of Knowledge.		Correlation of the standard to Bloom's Taxonomy.
Conceptual Understanding, Procedural Skill and Fluency, and/or Application		
Highlights the aspect(s) of rigor the standard is aligned to, and descriptors for which portions of the standard fall under each aspect of rigor.		
Assessment Items		
When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.		
Provides at least one high-quality formative assessment item aligned to the standard, as well as a link to more items.		
Common Misconceptions		
Provides guidance on where student misconceptions might occur.		

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)		
Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Layer 1 ensures that all students receive strong instruction in a high-quality differentiated core curriculum that is based on the principles of UDL. This includes school-wide implementation of positive behavioral interventions and supports, data-driven instruction, targeted interventions in small group instruction, universal screening, and English Language Development (ELD) for English Learners (ELs).	Layer 2 interventions should be focused on delivering individualized and targeted support (pre-teaching and re-teaching) for students on a grade-level trajectory. The interventions must be aligned with Layer 1 skills. Students should be provided with additional time and intensity in a small-group setting.	Layer 3 interventions should be provided individually or in small groups. Students are grouped according to their skill needs. The goal is for each student to acquire academic skills that will persist and transfer when the student returns to core instruction. If needed, specialized teachers may provide specific intervention instruction based on the needs identified by the data.

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/7/30/308/308		
Previous Learning	*Current Learning*	*Future Learning*
Lists skills relevant to current learning that students should have already mastered.	Lists skills within the current learning that students will master.	Lists skills from upcoming learning that students will need to be able to master based on what they are learning now.
Culturally and Linguistically Responsive Instruction		
Consider these resources for vocabulary from Pathways2Careers: • https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf • https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf		
Consider these questions as you plan for instruction that is culturally and linguistically responsive: • How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
Provides ways to build connections between the families in the community and the mathematical content, as well as examples that connect the math to students' home lives.	Provides information on supporting students as they learn mathematics by starting with conceptual knowledge that students can make connections to based on their prior knowledge. Also provides information on helping students build positive mathematical identities.	Provides ideas and supports for helping students learn new academic vocabulary and making connections to their prior knowledge.

Suggested Student Discourse Questions
Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf
Provides questions teachers can employ to increase student discourse.
Cross-Curricular Connections
Provides various connections between the standard and the knowledge and skills that students might use in other content areas.
Career and Skill Connections
Provides various connections between the standard and future careers/skills students will need for future careers.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards of Mathematical Practice
    - Procedural and Conceptual Understanding and Application
    - Sample Assessment Items
    - Common Misconceptions
    - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
    - Vertical Alignment
    - Culturally and Linguistically Responsive Instruction (CLRI)
    - Suggested Student Discourse Questions
    - Cross-Curricular and Career and Skill Connections
- A Student Discourse Guide
- Planning for a Multi-Layer System of Support (MLSS) and Universal Design for Learning (UDL) for behavioral and social and emotional supports

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

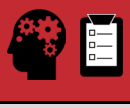
Standards Breakdown

- Use equivalent fractions as a strategy to add and subtract fractions
  - 5.NF.A.1
  - 5.NF.A.2
- Apply and extend previous understandings of multiplication and division
  - 5.NF.B.3
  - 5.NF.B.4
  - 5.NF.B.5
  - 5.NF.B.6
  - 5.NF.B.7

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)

Grade	CCSS Domain	CCSS Cluster
5	Fractions	Use equivalent fractions as a strategy to add and subtract fractions.



Cluster Standard: 5.NF.A.1

Standard	Standards for Mathematical Practice
Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)	• **SMP 4**: Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Builds on the work in fourth grade where students add fractions with denominators. In fifth grade, the example provided in the standard 2⁄3 + 3⁄4 has students finding a common denominator by finding the product of both denominators. This process should come after students have used visual fraction models (area models, number lines, etc.) to build understanding of equivalent fractions before moving into the standard algorithm described in the standard. The use of these visual fraction models allows students to use reasonableness to find a common denominator prior to using the algorithm. Fifth grade students will need to express both fractions in terms of a new denominator with adding unlike denominators.	• Explain why fractions with unlike denominators need to be replaced with equivalent fractions with like denominators when adding or subtracting. • Generate equivalent fractions to find the denominator. • Solve addition and subtraction problems involving fractions (including mixed numbers) with like and unlike denominators using an equivalent fraction strategy.
DOK	**Blooms**
1	Apply

Procedural and Conceptual Understanding and Application

Procedural:

- Students demonstrate fluency in performing addition and subtraction operations with fractions, including adding fractions by whole numbers, adding fractions by fractions, subtracting fractions by whole numbers, and subtracting fractions by fractions. Students also apply appropriate procedures and algorithms to solve problems involving fraction addition and subtraction accurately and efficiently.

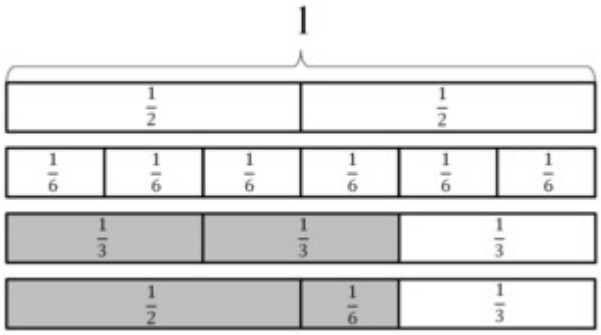
Conceptual Understanding:

- Students understand the concept of fractions as numbers representing parts of a whole or parts of a set and the meaning of addition and subtraction of fractions, including the relationship between the operations and their inverse relationship.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Ancient Egyptians used unit fractions, such as ½ and ⅓, to represent all fractions. For example, they might write the number ⅔ as ½ + ⅙.



We often think of ⅔ as ⅓ + ⅓, but the ancient Egyptians would not write it this way because they didn't use the same unit fraction twice.

a. Write each of the following Egyptian fractions as a single fraction:
   i. ½ + ⅓
   ii. ½ + ⅓ + ⅕
   iii. ¼ + ⅕ + 1/12

b. How might the ancient Egyptians have written the fraction we write as ¾?

Common Misconceptions

- **Bigger Denominator Means Bigger Fraction:** Students may mistakenly believe that a fraction with a larger denominator is always larger than a fraction with a smaller denominator. This misconception overlooks the relationship between the numerator and denominator and can lead to errors when comparing or ordering fractions.
- **Adding Numerators and Denominators Separately:** Students may believe that to add fractions, they should add the numerators together and the denominators together separately.
- **Adding Whole Numbers and Fractions the Same Way:** Students may incorrectly generalize their understanding of addition from whole numbers to fractions, assuming that the process is the same.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1	*Layer 2*	*Layer 3*

Core Instruction + UDL	*Core + UDL + Targeted*	*Core + UDL + Targeted + Intensive*
• Provide visual models such as fraction bars, fraction circles, and pictorial representations to help all students understand the concept of adding and subtracting fractions visually. • Use concrete examples from real-life situations, such as sharing food items or dividing a pizza, to demonstrate the addition and subtraction of fractions. **Universal Design for Learning** • Continue to allow students to explore fractions by using visual models to represent fractions with different denominators • Incorporate videos animations and interaction simulations to explain the process of finding common denominators and performing the required operation. • Provide step-by-step written instructions alongside verbal explanations to accommodate different learning preferences. • Offer multiple ways for students to practice, such as through hands-on activities, digital games, or solving real-world relevant problems.	• Provide targeted small group instruction for students who need additional support in understanding fraction operations. Offer differentiated activities based on individual student needs, focusing on hands-on practice and peer collaboration. **Universal Design for Learning** • Provide step-by-step written instructions alongside verbal explanations to accommodate different learning preferences. • Allow students to demonstrate their understanding in various ways—creating a video tutorial, writing a math journal entry, or presenting their work to the class. • Provide templates, sentence starters, or graphic organizers to help students organize their thinking, especially when explaining their reasoning.	• For students who require intensive support, provide one-on-one instruction tailored to their specific learning needs. Offer additional practice opportunities, reteaching sessions, and personalized learning experiences to address misconceptions and build foundational skills in fraction operations. • Implement multisensory teaching techniques, such as using manipulatives, movement, or tactile materials, to engage students with diverse learning preferences and provide multiple pathways for understanding fraction operations. **Universal Design for Learning** • Incorporate videos animations and interaction simulations to explain the process of finding common denominators and performing the required operation. • Help and align personal goals for students with goals or benchmarks in existence.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/5/23

Previous Learning	*Current Learning*	*Future Learning*
• Connect to comparing fractions	• Connect to making a line plot to	• Connect to solving algebraic

with different denominators by creating common denominators. ● Connect to adding and subtracting fractions with like denominators. ● Connect to making a line plot to display a data set of measurements in fractions of a unit.	display a data set and will add and subtract fractions of a unit to solve problems involving the information presented in the line plot.	equations and real-world problems using rational numbers.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
● Incorporate examples of adding and subtracting fractions from various cultural contexts, such as recipes from different cultures, traditional music rhythms, or cultural artwork that involves fractions.	● Scaffold instruction by building on students' prior knowledge and experiences related to fractions. ● Start with concrete examples, such as using visual models like fraction bars or circles, to demonstrate the addition and subtraction of fractions. Gradually transition to more abstract representations, such as numerical expressions, while providing opportunities for students to make connections between the concrete and abstract concepts.	● Provide linguistic supports to ensure that all students understand key mathematical vocabulary related to adding and subtracting fractions. ● Offer clear explanations of mathematical terms such as "numerator," "denominator," "common denominator," and "least common denominator." Use visual aids, gestures, and bilingual resources to support students' comprehension of mathematical vocabulary.

Suggested Student Discourse Questions

- How do fractions relate to parts of a whole or parts of a set?
- Can you provide examples of real-world situations where multiplying fractions is necessary?
- Why is it important to understand the relationship between multiplication and division of fractions?
- What strategies can we use to solve complex problems involving fraction operations?

- Can you explain the connection between fraction multiplication and division and other mathematical concepts we've learned?

Cross-Curricular Connections

STEM: Students divide fractions from data given or collected data to find the total.

History: When learning about timelines, students can connect knowledge of fractions on number lines to have a deeper understanding of reading historical timelines.

Music: Using fractions to divide musical whole notes into parts (halves, quarters, eighths, sixteenths)

Career and Skill Connections

- Baker
- Barista
- Accountant
- Budget Analyst
- Educator

Grade	*CCSS Domain*	*CCSS Cluster*
5	Fractions	Use equivalent fractions as a strategy to add and subtract fractions.

Cluster Standard: 5.NF.A.2

Standard	Standards for Mathematical Practice
Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2. Use equivalent fractions as a strategy to add and subtract fractions.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 2:** Reason abstractly and quantitatively.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This standard refers to number sense, which means students' understanding of fractions as numbers that lie between whole numbers on a number line. Number sense in fractions also includes moving between decimals and fractions to find equivalents, also being able to use reasoning such as 7⁄8 is greater than 3⁄4 because 7⁄8 is missing only 1⁄8 and 3⁄4 is missing 1⁄4 so 7⁄8 is closer to a whole. Also, students should use benchmark fractions to estimate and examine the reasonableness of their answers. Examples such as 5⁄8 are greater than 6⁄10 because 5⁄8 is 1⁄8 larger than 1⁄2 (4⁄8) and 6⁄10 is only 1⁄10 larger than 1⁄2 (5⁄10).	• Assess the reasonableness of answers, using mental estimation. • Add and subtract fractions, including those with unlike denominators. • Solve word problems using addition and subtraction of fractions, including those with unlike denominators.
DOK	**Blooms**
1	Apply

Procedural and Conceptual Understanding and Application

Application:

- Students apply their understanding of adding and subtracting fractions within real-world contexts, enhancing their problem-solving skills. They use visual fraction models or equations to explain difficult problems, which helps them understand and solve them more effectively.

Conceptual Understanding:

- Students utilize their understanding of benchmark fractions, i.e. (1/2) to determine the reasonableness of their answers as they solve word problems.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

For each of the following word problems, determine whether or not (⅖ + 3/10) represents the problem. Explain your decision.

a. A farmer planted ⅖ of his forty acres in corn and another 3/10 of his land in wheat. Taken together, what fraction of the 40 acres had been planted in corn or wheat?
b. Jim drank ⅖ of his water bottle and John drank 3/10 of his water bottle. How much water did both boys drink?
c. Allison has a batch of eggs in the incubator. On Monday ⅖ of the eggs hatched, By Wednesday, 3/10 more of the original batch hatched. How many eggs hatched in all?
d. Two fifths of the cross-country team arrived at the weight room at 7 a.m. Ten minutes later, 3/10 of the team showed up. The rest of the team stayed home. What fraction of the team made it to the weight room that day?
e. Andy made 2 free throws out of 5 free throw attempts. Jose made 3 free throws out of 10 free throw attempts. What is the fraction of free throw attempts that the two boys made together?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Confusion with Terminology:** Students may misunderstand the roles of numerators and denominators, leading to confusion about which part represents the whole and which part represents the parts being added or subtracted.
- **Difficulty with Common Denominators:** Many students struggle to find common denominators when adding or subtracting fractions, especially with fractions that have large or unfamiliar denominators.
- **Reliance on Algorithms:** Some students rely solely on memorized algorithms or procedures for adding and subtracting fractions, without truly understanding the underlying concepts. This can lead to errors, particularly in complex or non-standard problems.
- **Misapplication of Strategies:** Students may use incorrect strategies, such as adding numerators and denominators separately or adding/subtracting whole numbers separately from fractions, resulting in inaccurate solutions.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
• Provide diverse instructional approaches including the use of fraction tiles, fraction	• Identify student specific learning needs related to addition and subtraction of	• Offer individualized instruction tailored to address specific learning needs and

circles, or fraction bars to provide hands-on experiences for students to solve word problems involving addition and subtraction of fractions • Incorporate physical objects like food items or building blocks to represent fractions concretely, allowing students to engage with solving word problem fractions in a tangible way. • Break down fraction addition and subtraction into manageable steps, using concrete examples and real-world contexts to enhance understanding and relevance for students. **Universal Design for Learning (UDL)** • Connect word problems to real-life situations that are meaningful to students, such as shopping, cooking, or sports. This increases engagement and helps students see the relevance of the math they're learning. • Use diagrams, fraction bars, and number lines to visually represent the fractions in word problems. Students can visually map out the problem to understand how fractions are being added or subtracted. • Allow students to solve word problems using different methods, such as drawing, using manipulatives, or working through the problem algebraically. Providing options helps students find the approach that works best for them. • Encourage students to	fraction through formative assessment data. • Provide targeted instruction in small groups to address individual learning gaps, offering additional practice and support tailored to students' needs. • Develop learning activities that target areas of difficulty in solving word problems involving fractions with unlike denominators. • Tailor tasks, worksheets, or projects to address individual learning gaps and provide opportunities for focused practice and reinforcement. **Universal Design for Learning (UDL)** • Give students a choice of different word problems to solve, allowing them to select the ones that interest them most or that they feel confident tackling.	challenges related to fraction operations. Provide personalized guidance, feedback, and support to help students master fraction addition and subtraction concepts. • Develop personalized learning plans in collaboration with students, families, and support staff. Outline specific goals, strategies, and interventions to address areas of weakness and accelerate progress toward mastery. • Continuously monitor student progress through frequent assessment and data analysis. Adjust intervention strategies as needed based on student performance and feedback, ensuring that students receive the support necessary to achieve mastery of solving word problems involving addition and subtraction of fractions with unlike denominators. **Universal Design for Learning (UDL** • Incorporate videos animations and interaction simulations to explain the process to solve word problems with visual models. Provide several videos which showcase different models. • Help and align personal goals for students with goals or benchmarks in existence

explain their problem-solving process in writing or verbally. Providing sentence starters or graphic organizers can help students articulate their thinking.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/5/23

Previous Learning	*Current Learning*	*Future Learning*
• Connect to comparing fractions with different denominators by creating common denominators. • Connect to adding and subtracting fractions with like denominators. • Connect to making a line plot to display a data set of measurements in fractions of a unit.	• Connect to making a line plot to display a data set and will add and subtract fractions of a unit to solve problems involving the information presented in the line plot.	• Connect to solving algebraic equations and real-world problems using rational numbers.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Use word problems involving fractions that reflect students' cultural backgrounds and experiences. For example, incorporate scenarios related to traditional recipes, cultural celebrations, or community events that resonate with students' cultural identities.	• Relate fraction concepts to culturally familiar contexts, such as food, art, or traditional practices. For instance, use examples of sharing food among family members or creating artwork using fractional measurements to illustrate fraction addition and subtraction.	• Use visual aids, such as charts, diagrams, and graphic organizers, to support students in understanding mathematical language related to fraction addition and subtraction. Provide labeled diagrams and illustrations to reinforce vocabulary and concepts. • Facilitate collaborative

	• Offer explicit instruction on key mathematical vocabulary and concepts, taking into account students' cultural and linguistic backgrounds. Use visual representations, gestures, and real-life examples to bridge the gap between students' backgrounds and mathematical content, ensuring comprehension for all.	discussions and peer interactions where students can engage in conversations using mathematical language. Encourage students to explain their problem-solving strategies and justify their reasoning, promoting language development in a supportive environment.

Suggested Student Discourse Questions

- What does it mean to add or subtract fractions? How are these operations related?
- How do we determine if fractions have the same whole or refer to different wholes in a problem?
- How do we find a common denominator when adding or subtracting fractions? Why is this important?
- How do we know if our solution is reasonable? What checks can we perform to verify our answer?
- How can understanding addition and subtraction of fractions help us solve practical problems or make decisions in everyday life?

Cross-Curricular Connections

STEM: Students divide fractions from data given or collected data to find the total.

History: When learning about timelines, students can connect knowledge of fractions on number lines to have a deeper understanding of reading historical timelines.

Music: Using fractions to divide musical whole notes into parts (halves, quarters, eighths, sixteenths)

Career and Skill Connections

- Statistician
- Scientist
- Lawyer
- Judge
- Educator
- Computer Programmer

Grade	CCSS Domain	CCSS Cluster
5	Fractions	Apply and extend previous understandings of multiplication and division.

Cluster Standard: 5.NF.B.3

Standard	Standards for Mathematical Practice
Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size ¾.	• **SMP 2:** Reason abstractly and quantitatively.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Fifth grade students should connect fractions with division, understanding that 5 ÷ 3 = 5⁄3 * Students should explain this by working with their understanding of division as equal sharing. Students should also create story contexts to represent problems involving division of whole numbers. This standard calls for students to extend their work of partitioning a number line from third and fourth grade. Students need ample experiences to explore the concept that a fraction is a way to represent the division of two quantities.	• Interpret a fraction as division of the numerator by the denominator. • Interpret the remainder as a fractional part of the problem. • Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers.
DOK	**Blooms**
1-2	Apply

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Students develop an understanding of fractions as a division of the numerator by the denominator, realizing that the fraction a/b represents the result of dividing a by b. They understand the concept of solving word problems involving division of whole numbers and recognize that the quotient can be expressed as a fraction or mixed number and understand the relationship between division and fractions.

Application:

- Students engage in solving practical word problems requiring division of whole numbers, such as dividing a set of objects equally among a certain number of groups or sharing resources among individuals. They use visual fraction models or equations to represent the division problems, helping students to understand better and solve problems more accurately, leading to a deeper understanding and correct solutions.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

After a class potluck, Emily has three equally sized apple pies left and she wants to divide them into eight equal portions to give to eight students who want to take some pie home.

a. Draw a picture showing how Emily might divide the pies into eight equal portions. Explain how your picture shows eight equal portions.
b. What fraction of a pie will each of the eight students get?
c. Explain how the answer to (b) is related to the division problem 3 ÷ 8.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Fraction Misinterpretation:** Students may misunderstand fractions as only representing parts of a whole, rather than recognizing them as a division operation where the numerator is divided by the denominator.
- **Understanding of Division:** Some students may struggle to understand the concept of division as the inverse of multiplication, leading to difficulties in interpreting fractions as division and vice versa.
- **Confusion with Mixed Numbers:** Students might find it challenging to convert between improper fractions and mixed numbers accurately, leading to errors in representing solutions in the appropriate form.
- **Application of Visual Models:** Some students may rely solely on memorized procedures for solving division problems, neglecting the use of visual fraction models or equations to represent and solve problems accurately.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
• Provide a range of instructional methods, including visual aids like fraction bars, manipulatives such as fraction tiles, and interactive technology tools. Incorporate cooperative learning activities where	• Identify students' specific learning needs related to interpreting fractions as division and solving word problems involving division of whole numbers. • Offer targeted instruction in small groups to address	• Provide focused, one-on-one instruction to students who require intensive support in understanding fractions as division and solving division word problems. • Develop personalized learning plans in collaboration with

students work collaboratively to solve division word problems, ensuring engagement and participation from all learners. • Offer alternative representations of fraction division problems, such as using diagrams, models, and real-world examples. **Universal Design for Learning** • Continue to allow students to explore fractions by using visual models to represent fractions • Use manipulatives like counters, fraction tiles, or digital tools that allow students to physically divide sets of objects or virtual representations, reinforcing their understanding through action. • Incorporate videos animations and interaction simulations to explain the process of finding dividing a whole into parts. • Provide step-by-step written instructions alongside verbal explanations to accommodate different learning preferences. • Offer multiple ways for students to practice, such as through hands-on activities, digital games, or solving real-world relevant problems. • Create challenges or puzzles that require students to apply their understanding of division and fractions in new ways, fostering a deeper level of engagement.	individual learning gaps, providing additional practice and support tailored to students' needs. • Develop learning activities that target areas of difficulty in understanding fractions as division and solving division word problems. • Tailor tasks or projects to address individual learning gaps and provide opportunities for focused practice and reinforcement. **Universal Design for Learning** • Provide step-by-step written instructions alongside verbal explanations to accommodate different learning preferences. • Allow students to demonstrate their understanding in various ways—creating a video tutorial, writing a math journal entry, or presenting their work to the class. • Provide templates, sentence starters, or graphic organizers to help students organize their thinking, especially when explaining their reasoning.	students, families, and support staff, outlining targeted goals and strategies for improvement in understanding fractions as division. • Offer targeted interventions and resources designed to address specific areas of difficulty and accelerate progress toward mastery of fraction division skills. **Universal Design for Learning** • Incorporate videos animations and interaction simulations to explain the process. • Help and align personal goals for students with goals or benchmarks in existence.

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/5/23		
Previous Learning	*Current Learning*	*Future Learning*
• Connect to understanding concepts of area and relate area to multiplication and to addition. • Connect to using the four operations with whole numbers to solve problems. • Connect to understanding the concept of equivalent fractions by using visual fraction models. • Connect to multiplying a fraction by a whole number.	• Connect to understanding of tenths and hundredths to perform operations with multi-digit whole numbers and with decimals to hundredths. • Connect to knowledge of writing simple expressions to solve real problems with fractions. They will also interpret expressions without actually evaluating them. • Connect to using operations on fractions of a unit (1/2, 1/4, 1/8) to solve problems involving information presented in line plots.	• Connect to using ratios written as fractions and divide into decimal form 3 ÷ 4 = 3⁄4 = 0.75. • Connect to solving multiplication equations that include non negative rational numbers. • Connect to multiplying and divide fractions by fractions.
Culturally and Linguistically Responsive Instruction		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Enhance students' appreciation of their cultural backgrounds and experiences by integrating fraction division examples that align with their heritage. For instance, utilize scenarios from diverse cultural practices like cooking recipes or communal resource distribution to illustrate fraction concepts. • Acknowledge students' diverse perspectives by acknowledging the varied approaches to fraction	• Strengthen students' understanding of fraction concepts by linking fraction division to culturally familiar contexts. Utilize examples from students' cultural backgrounds, such as traditional recipes or cultural festivities, to illustrate fraction division concepts in a relatable and meaningful manner. • Bridge the gap between students' cultural and linguistic backgrounds and mathematical content by	• Provide language support strategies to help students comprehend and use mathematical language related to fraction division effectively. Offer visual aids, such as labeled diagrams and illustrations, to reinforce vocabulary and concepts. • Incorporate opportunities for language development through collaborative discussions and peer interactions that promote the use of mathematical language in context. Encourage students to explain their

division originating from different cultural and linguistic backgrounds. Encourage students to share their cultural insights and problem-solving strategies, fostering a sense of inclusivity and appreciation within the classroom community.	providing explicit instruction and support in understanding key mathematical vocabulary related to fraction division. Offer explanations and examples tailored to students' cultural experiences, enabling better comprehension and engagement with the material.	problem-solving strategies using both mathematical and cultural vocabulary, fostering language acquisition and mathematical understanding simultaneously.

Suggested Student Discourse Questions

- What does it mean to interpret a fraction as division? How are fractions related to division?
- Can you explain why the numerator represents the dividend and the denominator represents the divisor in a fraction?
- What do the quotient and remainder represent in the context of a fraction?
- How do equivalent fractions relate to the concept of interpreting fractions as division?
- Can you explain why multiplying or dividing the numerator and denominator of a fraction by the same number does not change the value of the fraction?

Cross-Curricular Connections

STEM: Students divide fractions from data given or collected data to find the total.

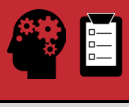
History: When learning about timelines, students can connect knowledge of fractions on number lines to have a deeper understanding of reading historical timelines.

Music: Using fractions to divide musical whole notes into parts (halves, quarters, eighths, sixteenths)

Career and Skill Connections

- Computer Programmer
- Statistician
- Mechanical Engineer
- Educator
- Health and Fitness Coach

Grade	CCSS Domain	CCSS Cluster
5	Fractions	Apply and extend previous understandings of multiplication and division.



Cluster Standard: 5.NF.B.4

Standard	Standards for Mathematical Practice
Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. A. Interpret the product (a/b) × q as a part of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.) B. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas.	● **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students need to develop a fundamental understanding that the multiplication of a fraction by a whole number could be represented as repeated addition of a unit fraction (e.g., 2 x (1⁄4) = 1⁄4 + 1⁄4). This standard extends student's work of multiplication from earlier grades. In fourth grade, students worked by recognizing that a fraction such as 3⁄5 could be represented as 3 pieces that are each one-fifth (3 x (1⁄5)). These standards reference both the multiplication of a fraction by a whole number and the multiplication of two fractions. Visual fraction models (area models, tape diagrams, number lines) should be used and created by students during their work with this standard. This standard extends students' work with area. In third grade students determine the area of rectangles and composite rectangles. In fourth grade	● Extend previous understandings of multiplication to multiply a fraction or a whole number by a fraction. ● Explain that the product (a/b) x q is the same as ○ a x q ÷ b. ● Multiply a fraction or a whole number by a fraction. ● Create a story context to multiply a fraction or a whole number by a fraction. ● Explain that finding the area of a rectangle with fractional side lengths by filling with tiles is the same as would be found by multiplying the side lengths. ● Find the area of a rectangle by tiling it with unit squares.

students continue this work. The fifth-grade standard calls students to continue the process of covering (with tiles).	• Multiply fractional side lengths to find the area of a rectangle.
DOK	**Blooms**
1-2	Apply

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Students grasp the concept that multiplying a fraction by a whole number greater than 1 results in a product that represents a larger quantity or amount compared to the original fraction, showing an understanding of scaling and multiplication as a form of repeated addition.

Procedural Skills and Fluency:

- Students develop fluency in multiplying fractions by whole numbers by applying the concept of repeated addition and interpreting the multiplication as scaling the fraction by the whole number factor.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

The 5th graders want to raise money for their overnight camping trip by selling cornbread during the school Chili Cook-Off contest. All the pans of cornbread are square. A pan of cornbread costs $12. The customers can buy any fractional part of a pan of cornbread and pay that fraction of $12. For example, ½ of a full pan costs ½ of $12.

a. Mrs. Farmer buys cornbread from a pan that is ¼ full. She buys ⅓ of the remaining cornbread in the pan.
   i. What fraction of the whole pan of cornbread does she buy? Use objects and/or a diagram to show how much of the pan of cornbread she buys.
   ii. What does she pay for the cornbread she bought? Use objects and/or a diagram to show how much pays.
b. The next customer is the school principal. He buys cornbread from a different pan that is ½ full. He buys ⅝ of the remaining cornbread in the pan.
   i. What fraction of the whole pan of cornbread does he buy? Use a diagram to show how much of the pan of cornbread he buys.
   ii. What does he pay for the cornbread he bought? Use a diagram to show how much he will pay for his part of the pan.
c. What would be the cost of the cornbread the principal bought if the price of the entire pan changed to:
   i. $24
   ii. $6
   iii. $18

Use a diagram to explain each answer.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions
• **Multiplication is the Same as Adding:** Students may confuse multiplication of fractions with addition of fractions. • **Multiplying Numerator and Denominator Separately:** Students may mistakenly believe that to multiply a fraction by a whole number, they should multiply the numerator and denominator separately. • **Whole Number Denominator After Multiplication:** Students may believe that multiplying a fraction by a whole number will always result in a whole number denominator. • **Multiplying Fractions Always Results in a Smaller Value:** Students may mistakenly believe that multiplying a fraction by a whole number always results in a fraction with a smaller value

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning		
Layer 1 ***Core Instruction + Universal***	***Layer 2*** ***Core + Targeted***	***Layer 3*** ***Core + Targeted + Intensive***
• Provide visual models such as manipulatives, provide multimedia tools, number lines , tape diagrams. **Universal Design for Learning** • Continue to allow students to explore fractions by using visual models to represent fractions • Use manipulatives like counters, fraction tiles, or digital tools that allow students to physically divide sets of objects or virtual representations, reinforcing their understanding through action. • Use area models, fraction bars, and number lines to visually represent the multiplication of fractions. For example, show how multiplying 1/2 by 1/3 can be represented as shading 1/3 of a 1/2 piece.	• Provide targeted small group instruction for students who need additional support, level grouping through diversity of participants to learn from their perspectives, lived experiences, and feedback, foster collaboration within community of learners **Universal Design for Learning** • Provide step-by-step written instructions alongside verbal explanations to accommodate different learning preferences. • Allow students to demonstrate their understanding in various ways—creating a video tutorial, writing a math journal entry, or presenting their work to the class. • Provide templates, sentence	• For students who require intensive support, provide one on one instruction and build confidence through levels of support and guided appropriateness goal setting development **Universal Design for Learning** • Incorporate videos animations and interaction simulations to explain the process. • Help and align personal goals for students with goals or benchmarks in existence.

• Incorporate videos animations and interaction simulations to explain the process of finding dividing a whole into parts. • Provide step-by-step written instructions alongside verbal explanations to accommodate different learning preferences. • Offer multiple ways for students to practice, such as through hands-on activities, digital games, or solving real-world relevant problems. • Create challenges or puzzles that require students to apply their understanding of division and fractions in new ways, fostering a deeper level of engagement.	starters, or graphic organizers to help students organize their thinking, especially when explaining their reasoning.	

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/5/23

Previous Learning	*Current Learning*	*Future Learning*
• Connect to understanding concepts of area and relate area to multiplication and to addition. • Connect to using the four operations with whole numbers to solve problems. • Connect to understanding the concept of equivalent fractions by using visual fraction models. • Connect to multiplying a fraction by a whole number.	• Connect to understanding of tenths and hundredths to perform operations with multi-digit whole numbers and with decimals to hundredths. • Connect to knowledge of writing simple expressions to solve real problems with fractions. They will also interpret expressions without actually evaluating them. • Connect to using operations on fractions of a unit (1/2, 1/4, 1/8) to solve problems involving information presented in line plots.	• Connect to using ratios written as fractions and divide into decimal form $3 \div 4 = 3/4 = 0.75$. • Connect to solving multiplication equations that include non negative rational numbers. • Connect to multiplying and divide fractions by fractions.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Incorporate examples of the concepts that relate to multiplication and addition of fractions through perceived knowledge.	• Scaffold instruction, by building upon students prior knowledge and use of existing knowledge, allows learners to participate in the design of classroom activities and academic tasks.	• Provide linguistic support to ensure students understand key mathematical words related to subject at hand, support decoding of text, mathematical notation, and symbols and promote understanding across languages.

Suggested Student Discourse Questions

- What does it mean to multiply a fraction by a whole number? How is this operation different from multiplying two fractions?
- Can you explain why multiplying a fraction by a whole number results in repeated addition of the fraction?
- How is fraction multiplication related to division? Can you explain the connection between the two operations?
- What strategies can we use to multiply fractions by whole numbers efficiently? Can you demonstrate with examples?
- How can we decompose a whole number into factors to simplify the multiplication of fractions?

Cross-Curricular Connections

Social Studies: Connect fractions to studies of geography including scaling graphs and cross-sections, changes in measure (population, GDP)

Health: Connect fractions to food sharing, cooking, serving portions, nutrition, medical doses, heart beats per minute, steps per day. Present students with real-world problems using these topics.

Career and Skill Connections

- Accountant
- Bookkeeping
- Finance
- Scientist
- Economist

Grade	CCSS Domain	CCSS Cluster
5	Fractions	Apply and extend previous understandings of multiplication and division.

Cluster Standard: 5.NF.B.5

Standard	Standards for Mathematical Practice
Interpret multiplication as scaling (resizing), by: A. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. B. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n×a)/(n×b) to the effect of multiplying a/b by 1.	● **SMP 2:** Reason abstractly and quantitatively.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
These standards ask students to examine how numbers change when we multiply by fractions. Students should have ample opportunities to examine both cases in the standard: a) when multiplying by a fraction greater than 1, the number increases and b) when multiplying by a fraction less than one, the number decreases. This standard should be explored and discussed while students are working with 5.NF.4, and should not be taught in isolation.	● Interpret multiplication by scaling, comparing the size of a product to the size of one factor based on the size of the other factor. ● Explain why multiplying a given number by a fraction greater than one results in a product greater than the given number and why multiplying a given number by a fraction less than one results in a product smaller than the given number.
DOK	**Blooms**
1-3	Apply, Analyze
Procedural and Conceptual Understanding and Application	
Conceptual Understanding: ● Students understand how multiplication works by comparing how the size of the product relates to the size of	

one factor compared to the other factor, without actually doing the multiplication, which helps them see how multiplication scales or resizes quantities.

- Students understand how multiplication works by explaining that multiplying a whole number by a fraction greater than 1 makes the product larger, while multiplying by a fraction less than 1 makes the product smaller. They also connect the idea that $\frac{a}{b} = \frac{n \times a}{n \times b}$ shows how multiplying a fraction by 1 keeps it equivalent, demonstrating a strong understanding of how multiplication affects quantities and fractions.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Luke had a calculator that would only display numbers less than or equal to 999,999,999. Which of the following products will his calculator display? Explain.

a. 792 × 999,999,999
b. ½ × 999,999,999
c. 15/4 × 999,999,999
d. 0.67 × 999,999,999

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Multiplying by a Fraction Decreases the Value:** Students may mistakenly believe that multiplying a whole number by a fraction will always result in a smaller product.
- **All Multiplication by Fractions Reduces Quantity:** Students may generalize that any multiplication involving a fraction will result in a smaller quantity.
- **Multiplying by 1 Always Yields the Same Number:** Students might believe that multiplying any number by 1 will always yield the same number.
- **Multiplying by a Fraction Always Increases the Value:** Students might mistakenly think that multiplying a whole number by a fraction always leads to a larger product.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
• Provide clear explanations and demonstrations of how multiplication affects the size of numbers, particularly when multiplying by	• Use scaffolded activities that gradually increase in complexity. Start with simple problems involving whole numbers and	• Provide intensive one-on-one instruction for students who require individualized support. Tailor lessons to the student's specific learning needs, using

fractions. Use visual aids such as number lines, fraction bars, and area models to illustrate these concepts. **Universal Design for Learning** ● Use visual aids like fraction bars, area models, and number lines to help students visualize multiplication as scaling. ● Provide manipulatives such as fraction tiles, cubes, and grids to give students hands-on experience with the concepts. ● Use visual representations such as scaling diagrams, grids, or number lines to illustrate how multiplying by a fraction changes the size of a number. For example, show how multiplying by 1/2 reduces a quantity, while multiplying by 2 increases it. ● Use examples from everyday life where scaling occurs, such as resizing images, recipes, or maps, to make the abstract concept more concrete and relatable. ● Frame scaling problems in contexts that are meaningful to students, such as adjusting a recipe for a different number of people or resizing a drawing for a project. This makes learning more relevant and engaging.	fractions and progress to more complex problems involving fractions and mixed numbers. ● Provide graphic organizers and step-by-step guides to help students organize their thoughts and approach problems systematically. **Universal Design for Learning** ● Provide step-by-step written instructions alongside verbal explanations to accommodate different learning preferences. ● Allow students to demonstrate their understanding in various ways—creating a video tutorial, writing a math journal entry, or presenting their work to the class. ● Provide students with choices in the problems they solve or the projects they undertake, allowing them to explore scaling in ways that interest them.	additional explanations, examples, and practice opportunities. ● Develop personalized learning plans that include targeted goals, strategies, and supports for students with significant learning gaps. Collaborate with special education teachers, ELL specialists, and other support staff to address individual needs. ● Incorporate specific accommodations and modifications as necessary, such as extended time, additional practice opportunities, or alternative assessments. **Universal Design for Learning** ● Incorporate videos animations and interaction simulations to explain the process of scaling ● Incorporate digital tools that allow students to manipulate numbers and see the effects of multiplying by fractions in real-time, reinforcing the concept of scaling. ● Help and align personal goals for students with goals or benchmarks in existence.

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/5/23		
Previous Learning	*Current Learning*	*Future Learning*
● Connect to understanding	● Connect to understanding	● Connect to using ratios

concepts of area and relate area to multiplication and to addition. • Connect to using the four operations with whole numbers to solve problems. • Connect to understanding the concept of equivalent fractions by using visual fraction models. • Connect to multiplying a fraction by a whole number.	of tenths and hundredths to perform operations with multi-digit whole numbers and with decimals to hundredths. • Connect to knowledge of writing simple expressions to solve real problems with fractions. They will also interpret expressions without actually evaluating them. • Connect to using operations on fractions of a unit (1/2, 1/4, 1/8) to solve problems involving information presented in line plots.	written as fractions and divide into decimal form 3 ÷ 4 = 3⁄4 = 0.75. • Connect to solving multiplication equations that include non negative rational numbers. • Connect to multiplying and divide fractions by fractions.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Incorporate examples of problems that makes connection to their understanding of whole numbers, fractions, illustrate through multiple media outlets	• By scaffolding instruction you are building upon students prior knowledge, activate or give background knowledge	• Provide linguistic support to ensure all students understand key mathematical vocabulary related to, clarify vocabulary and of mathematical symbols, promote understanding of mathematical languages

Suggested Student Discourse Questions

- What does it mean to scale a quantity? How does scaling relate to multiplication?
- Can you explain why multiplying a fraction by another fraction results in scaling the original quantity by a fraction of its size?
- How can we use models or visual representations to illustrate the concept of scaling with fractions?
- How is fraction multiplication related to scaling? Can you describe the relationship between the factors and the scaled result?

- What strategies can we use to scale quantities using fractions? Can you demonstrate with examples?

Cross-Curricular Connections

Social Studies: Connect fractions to studies of geography including scaling graphs and cross-sections, changes in measure (population, GDP)

Health: Connect fractions to food sharing, cooking, serving portions, nutrition, medical doses, heart beats per minute, steps per day. Present students with real-world problems using these topics.

Career and Skill Connections

- Computer Programmer
- Accountant
- Budget Analyst
- Educator
- Lawyer
- Judge

Grade	*CCSS Domain*	*CCSS Cluster*
5	Fractions	Apply and extend previous understand of multiplication and division.

Cluster Standard: 5.NF.B.6

Standard	Standards for Mathematical Practice
Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	• **SMP 2:** Reason abstractly and quantitatively.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This standard builds on all of the work done in this cluster. Students should be given ample opportunities to use various strategies to solve word problems involving the multiplication of a fraction by a mixed number. This standard could include fraction by a fraction, fraction by a mixed number or mixed number by a mixed number.	• Represent word problems involving multiplication of fractions and mixed numbers. • Solve real world problems involving multiplication of fractions and mixed numbers
DOK	**Blooms**

1-2	Apply

Procedural and Conceptual Understanding and Application

Application:

- Students demonstrate strong problem-solving skills by effectively using visual models, equations, and other strategies to solve complex real-world problems that involve multiplying fractions and mixed numbers, showing they can apply fraction multiplication concepts in practical situations with accuracy and proficiency.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

The Burj Khalifa (Dubai) is about 2 ½ times as tall as the Eiffel Tower (Paris) . The Eiffel Tower is about 5/7 as tall as the Willis Tower (Chicago).

a. Which of these buildings is the tallest? Which is the shortest? Explain.
b. Draw pictures to illustrate.

Alisa had ½ a liter of juice in a bottle. She drank ¾ of the juice that was in the bottle. How many liters of juice did she drink?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Whole Numbers Cannot be Multiplied by Fractions:** Students might think that whole numbers cannot be multiplied by fractions or mixed numbers.
- **Multiplying Fractions Always Results in a Fraction with a Smaller Numerator:** Students may believe that when multiplying fractions, the resulting product will always have a smaller numerator than the original fractions.
- **Using Visual Models is Only Necessary for Simple Problems:** Students might think that visual fraction models are only needed for basic problems and not for more complex real-world scenarios.
- **Mixed Numbers Cannot Be Used in Real-World Applications:** Students might think that mixed numbers are not applicable in real-world problem-solving involving multiplication.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Provide visual representations such as fraction bars, diagrams, or	• Offer targeted small group instruction for students who require additional support.	• Provide intensive one-on-one instruction for students who need individualized support.

models to illustrate the concept of multiplying fractions. • Incorporate real-life examples and scenarios into lessons to make multiplication of fractions and mixed numbers relevant and engaging for all students. **Universal Design for Learning** • Continue to allow students to explore fractions by using visual models to represent fractions and mixed numbers • Use area models, fraction bars, and number lines to visually represent the multiplication of fractions and mixed numbers in real-world contexts. For example, show how to multiply fractions when calculating the area of a rectangle or scaling a recipe. • Incorporate videos animations and interaction simulations to explain the process of finding common denominators and performing the required operation. • Provide step-by-step written instructions alongside verbal explanations to accommodate different learning preferences. • Offer multiple ways for students to practice, such as through hands-on activities, digital games, or solving real-world relevant problems	Provide differentiated activities and materials based on individual needs. **Universal Design for Learning** • Provide step-by-step written instructions alongside verbal explanations to accommodate different learning preferences. • Allow students to demonstrate their understanding in various ways—creating a video tutorial, writing a math journal entry, or presenting their work to the class. • Provide templates, sentence starters, or graphic organizers to help students organize their thinking, especially when explaining their reasoning.	Offer additional explanations, examples, and practice opportunities tailored to the student's learning needs **Universal Design for Learning** • Incorporate videos animations and interaction simulations to explain the process. • Help and align personal goals for students with goals or benchmarks in existence.
Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/5/23		

Previous Learning	*Current Learning*	*Future Learning*
• Connect to understanding concepts of area and relate area to multiplication and to addition. • Connect to using the four operations with whole numbers to solve problems. • Connect to understanding the concept of equivalent fractions by using visual fraction models. • Connect to multiplying a fraction by a whole number.	• Connect to understanding of tenths and hundredths to perform operations with multi-digit whole numbers and with decimals to hundredths. • Connect to knowledge of writing simple expressions to solve real problems with fractions. They will also interpret expressions without actually evaluating them. • Connect to using operations on fractions of a unit (1/2, 1/4, 1/8) to solve problems involving information presented in line plots.	• Connect to using ratios written as fractions and divide into decimal form $3 \div 4 = 3/4 = 0.75$. • Connect to solving multiplication equations that include non negative rational numbers. • Connect to multiplying and divide fractions by fractions.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Incorporate examples of real-world problems involving fractions and mixed numbers from various cultural contexts, such as recipes, cultural celebrations, or traditional practices, to validate students' identities and experiences.	• Use concrete examples and real-life contexts to introduce the concept of solving real-world problems involving multiplication of fractions and mixed numbers, then gradually transition to more abstract representations. • Provide opportunities for students to make connections between their cultural backgrounds and the mathematical content, such as exploring how	• Provide linguistic supports to ensure that all students understand key mathematical vocabulary related to solving real-world problems involving multiplication of fractions and mixed numbers.

	fractions and mixed numbers are used in cultural practices or traditions.	

Suggested Student Discourse Questions

- Can you summarize the problem in your own words? What are we trying to find or solve?
- How does this problem relate to situations we encounter in everyday life?
- How does understanding the relationship between multiplication and addition help us solve the problem?
- Can you explain how we can break down a complex problem into smaller, manageable steps using multiplication and addition?
- How can we use models, diagrams, or visual representations to illustrate and justify our solutions?

Cross-Curricular Connections

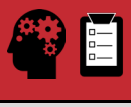
Social Studies: Connect fractions to studies of geography including scaling graphs and cross-sections, changes in measure (population, GDP)

Health: Connect fractions to food sharing, cooking, serving portions, nutrition, medical doses, heart beats per minute, steps per day. Present students with real-world problems using these topics.

Career and Skill Connections

- Bookkeepers
- Finance
- Accountants
- Economist
- Urban Planner

Grade	CCSS Domain	CCSS Cluster
5	Fractions	Apply and extend previous understandings of multiplication and division.



Cluster Standard: 5.NF.B.7

Standard	Standards for Mathematical Practice
Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. A. Interpret division of a unit fraction by a non-zero whole number and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3. B. Interpret division of a whole number by a unit fraction and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that a. 4 ÷ (1/5) = 20 because 20 × (1/5) = 4. C. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?	● **SMP 1:** Make sense of problems and persevere in solving them. ● **SMP 3:** Construct viable arguments and critique the reasoning of others.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This is the first time that students are dividing fractions. In fourth grade students divided whole numbers, and multiplied a whole number by a fraction. The concept unit fraction is a fraction that has a one in the denominator. For example, the fraction 3⁄5 is 3 copies of the unit fraction 1⁄5. 1⁄5 + 1⁄5 + 1⁄5 = 3⁄5 = 1⁄5 x 3 or 3 x 1⁄5. This standard asks students to work with story contexts where	● Know the relationship between multiplication and division. ● Interpret division of a unit fraction by a whole number and justify your answer using the relationship between multiplication and division, by creating story problems, using visual models, and relationship to multiplication.

a unit fraction is divided by a non-zero whole number. Students should use various fraction models and reasoning about fractions.	• Interpret division of a whole number by a unit fraction and justify your answer using the relationship between multiplication and division, and by representing the quotient with a visual fraction model. • Solve real world problems involving division of unit fractions by whole numbers other than 0 and division of whole numbers by unit fractions using strategies such as visual fraction models and equations.
DOK	**Blooms**
1-2	Apply

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Students develop a thorough understanding of dividing unit fractions and whole numbers by recognizing division as the process of breaking a whole into equal parts and understanding the connection between division and multiplication of fractions.

Procedural Skills and Fluency:

- Students enhance their ability to divide unit fractions and whole numbers efficiently by employing specific strategies, such as using visual models to represent division, converting mixed numbers to improper fractions for calculation, and applying these skills to solve practical problems accurately and with flexibility.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Carolina's Banana Pudding Recipe

2 cups sour cream
5 cups whipped cream
3 cups vanilla pudding mix
4 cups milk
8 bananas

Carolina is making her special banana pudding recipe. She is looking for her cup measure, but can only find her quarter cup measure.

a. How many quarter cups does she need for the sour cream? Draw a picture to illustrate your solution, and write an equation that represents the situation.
b. How many quarter cups does she need for the milk? Draw a picture to illustrate your solution, and write an equation that represents the situation.

c. Carolina does not remember in what order she added the ingredients, but the last ingredient added required 12 quarter cups. What was the last ingredient Carolina added to the pudding? Draw a picture to illustrate your solution and write an equation that represents the situation.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Dividing a Unit Fraction by a Whole Number Always Results in a Larger Fraction:** Students may mistakenly believe that when dividing a unit fraction by a whole number, the result will always be a larger fraction.
- **Dividing Whole Numbers by Unit Fractions Always Yields a Larger Whole Number:** Students might erroneously think that when dividing a whole number by a unit fraction, the result will always be a larger whole number.
- **Division of Fractions Always Requires Finding a Common Denominator:** Students may believe that dividing fractions always involves finding a common denominator.
- **Visual Fraction Models Are Unnecessary for Solving Division Problems:** Students might think that division involving fractions and whole numbers can be solved purely with equations and mental math, without using visual fraction models.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
• Provide visual representations such as fraction bars, diagrams, or models to illustrate the concept of dividing fractions. • Use real-life examples and contexts to demonstrate division of fractions, such as sharing food items or dividing ingredients in a recipe. **Universal Design for Learning** • Continue to allow students to explore fractions by using visual models to represent fractions and mixed numbers • Use fraction bars, area models, and number lines to	• Offer targeted small group instruction for students who require additional support. Provide differentiated activities and materials based on individual needs. **Universal Design for Learning** • Provide step-by-step written instructions alongside verbal explanations to accommodate different learning preferences. • Allow students to demonstrate their understanding in various ways—creating a video	• Provide intensive one-on-one instruction for students who need individualized support. Offer additional explanations, examples, and practice opportunities tailored to the student's learning needs. **Universal Design for Learning** • Incorporate videos animations and interaction simulations to explain the process. • Help and align personal goals for students with goals or benchmarks in existence.

visually represent the division of fractions by whole numbers and vice versa. For instance, illustrate how dividing 1/2 by 3 can be seen as finding how many times 3 fits into 1/2. • Incorporate videos animations and interaction simulations to explain the process of finding common denominators and performing the required operation. • Provide step-by-step written instructions alongside verbal explanations to accommodate different learning preferences. • Offer multiple ways for students to practice, such as through hands-on activities, digital games, or solving real-world relevant problems	tutorial, writing a math journal entry, or presenting their work to the class. • Use manipulatives like fraction tiles, measuring cups, or counters to physically divide fractions by whole numbers and whole numbers by fractions. This hands-on approach helps solidify understanding through action. • Provide templates, sentence starters, or graphic organizers to help students organize their thinking, especially when explaining their reasoning.	

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/5/23

Previous Learning	*Current Learning*	*Future Learning*
• Connect to understanding concepts of area and relate area to multiplication and to addition. • Connect to using the four operations with whole numbers to solve problems. • Connect to understanding the concept of equivalent fractions by using visual fraction models. • Connect to multiplying a fraction by a whole number.	• Connect to understanding of tenths and hundredths to perform operations with multi-digit whole numbers and with decimals to hundredths. • Connect to knowledge of writing simple expressions to solve real problems with fractions. They will also interpret expressions without actually evaluating them. • Connect to using operations on fractions of a unit (1/2, 1/4, 1/8) to solve problems	• Connect to using ratios written as fractions and divide into decimal form $3 \div 4 = 3/4 = 0.75$. • Connect to solving multiplication equations that include non-negative rational numbers. • Connect to multiplying and divide fractions by fractions.

	involving information presented in line plots.	

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Incorporate examples of real-world problems involving division of fractions from various cultural contexts, such as recipes from different cultures, cultural celebrations, or traditional practices, to validate students' identities and experiences.	• Use concrete examples and real-life contexts to introduce the concept, then gradually transition to more abstract representations. • Provide opportunities for students to make connections between their cultural backgrounds and the mathematical content, such as exploring how division of fractions is used in cultural practices or traditions.	• Provide linguistic supports to ensure that all students understand key mathematical vocabulary related to division of fractions and mathematical symbols. Promote understanding of mathematical languages.

Suggested Student Discourse Questions

- What does it mean to divide a whole number by a unit fraction? How is this operation different from dividing whole numbers?
- Can you explain why dividing a whole number by a unit fraction results in the whole number being partitioned into equal-sized groups?
- How does understanding the relationship between division and multiplication help us solve problems involving unit fractions and whole numbers?
- What strategies can we use to divide a whole number by a unit fraction or a unit fraction by a whole number? Can you demonstrate with examples?
- How do the factors in a division problem affect the number of groups or the size of each group?

Cross-Curricular Connections

Social Studies: Connect fractions to studies of geography including scaling graphs and cross-sections, changes in measure (population, GDP)

Health: Connect fractions to food sharing, cooking, serving portions, nutrition, medical doses, heart beats per minute, steps per day. Present students with real-world problems using these topics.

Career and Skill Connections

- Statistician
- Mechanical Engineer
- Computer Programmer
- Quantitative Analyst
- Economist

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Procedural and Conceptual Understanding and Application
  - Sample Assessment Items
  - Common Misconceptions
  - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
  - Vertical Alignment
  - Culturally and Linguistically Responsive Instruction (CLRI)
  - Suggested Student Discourse Questions
  - Cross-Curricular and Career and Skill Connections
- A Student Discourse Guide
- Planning for a Multi-Layer System of Support (MLSS) and Universal Design for Learning (UDL) for behavioral and social and emotional supports

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Graph points on the coordinate plane to solve real-world and mathematical problems
  - 5.G.A.1
  - 5.G.A.2
- Classify two-dimensional figures into categories based on their properties
  - 5.G.B.3
  - 5.G.B.4

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008).

Grade	CCSS Domain	CCSS Cluster
5	Geometry	Graph points on the coordinate plane to solve real-world and mathematical problems.

Cluster Standard: 5.G.A.1

Standard	Standards for Mathematical Practice
Graph points on the coordinate plane to solve real world mathematical problems. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y axis and y-coordinate).	• **SMP 6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
These standards deal with only the first quadrant (positive numbers) in the coordinate plane. Although students can often "locate a point," these understandings are beyond simple skills. For example, initially, students often fail to distinguish between two different ways of viewing the point (2, 3), say, as instructions: "right 2, up 3"; and as the point defined by being a distance 2 from the y-axis and a distance 3 from the x-axis. In these two descriptions the 2 is first associated with the x-axis, then with the y-axis.	• Graph points in the first quadrant. • Interpret coordinate values of points in real world context and mathematical problems. • Represent real world and mathematical problems by graphing points in the first quadrant.
DOK	**Blooms**
1-2	Understand, Apply
Procedural and Conceptual Understanding and Application	

Application: Students apply and practice plotting points in the coordinate plane. Students apply coordinates in real world situations using maps, and directions.

Conceptual Understanding: Students develop a thorough conceptual understanding of the coordinate plane by recognizing it as a system composed of perpendicular axes (x-axis and y-axis) intersecting at the origin, where coordinates of a point (ordered pair) indicate distances traveled from the origin along each axis, thus establishing a framework for locating points and solving real-world mathematical problems using graphical representations.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Materials
The students will need grid paper and colored pencils; some color for the ships and (for example) red for explosions on their ships and their enemy's ships. This is how they will keep track of what ordered pairs have been called.

Setup
Students begin by folding the grid paper in half. They need to draw coordinate axes on both the top half and the bottom half and label the x and y axes with the numbers 1–10 on each axis. The students will need to draw in 5 ships on ordered pairs and label the ordered pairs. They should draw:

Two ships that are sitting on 2 ordered pairs,
One ship that is sitting on 3 ordered pairs,
One ship that is sitting on 4 ordered pairs, and
One ship sitting on 5 ordered pairs.
Remind them the bottom half has their boats or (Navy) and the top half has their opponent's boats.
NOTE: Remind students that the bottom half of the grid has one student's boat markings, while the top half holds their opponent's boat markings.

Actions
Students play in pairs sitting opposite each other and take turns calling out ordered pairs (without the ability to see their partner's boats). Players should keep a list of the ordered pairs they call out written in (x,y) form on a piece of paper that both players can see so there is no disagreement later on about what has been called (it is common for students to transpose the coordinates). Then they are to mark the ordered pair they call out on the top coordinate plane. They should mark in black if they missed and red if they hit their opponent's boat. On the bottom half of the grid paper, they are to color black for the ordered pairs their opponent calls out and color red for the ordered pairs that hit their ship.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Confusion Between Axes and Coordinates:** Students may incorrectly associate the terms "x-axis" and "y-axis" with specific coordinates (e.g., thinking the x-coordinate is on the x-axis) rather than understanding that these are separate concepts within the coordinate system.

- **Misinterpreting Positive and Negative Coordinates:** Students might struggle with understanding that

coordinates can be positive or negative, leading to errors in plotting points and interpreting locations on the coordinate plane.

- **Incorrectly Identifying the Origin:** Students may mistakenly believe that the origin (0,0) is located at the intersection of the positive x-axis and positive y-axis, rather than understanding that it is where both axes intersect, with positive and negative directions extending from this point.

- **Difficulty with Distance Interpretation:** Students might have difficulty interpreting coordinates as distances from the origin along each axis, leading to challenges in accurately graphing points and understanding their positions within the coordinate plane.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
• Provide visual models through multimedia tools, hands on manipulatives to enhance learning of graphing points on a coordinate plane and define a coordinate system. Know x and y coordinates and understand the placement of indicates where they go on the axes. **Universal Design for Learning (UDL)** • Design multiple means of representation by connecting prior knowledge to learning by highlighting and exploring patterns, critical features, big ideas and relationships.	• Provide targeted small group instruction for students that need additional support, level grouping through diversity of participants to learn from their perspectives, lived experiences and feedback. Foster collaboration within the community of learners. **Universal Design for Learning (UDL)** • Design multiple means of representation by connecting prior knowledge to learning by highlighting and exploring patterns, critical features, big ideas and relationships.	• Provide students who require intensive support, one on one instruction to build confidence with graduated levels of support and guide appropriate goal setting development, clarify vocabulary and symbols. **Universal Design for Learning (UDL)** • Design multiple means of representation by connecting prior knowledge to new learning by pre-teaching critical concepts through demonstration or models.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/5/20

Previous Learning	*Current Learning*	*Future Learning*
• Connect to plotting points on a number line and construct perpendicular lines.	• Connect to forming ordered pairs from given rules and graph points on a coordinate plane.	• Connect to extending understanding of a coordinate plane to the negative number coordinates.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Incorporate examples of problems that make connections to their understanding of graphing points on a coordinate plane relates to real world mathematical problems.	• By scaffolding instruction you are building upon students prior knowledge, activate or give background knowledge, such as prior work experience related to what is being learned.	• Provide linguistic support to ensure all students understand key mathematical vocabulary and mathematical symbols, promote understanding of mathematical languages.

Suggested Student Discourse Questions

- What do the terms "x-coordinate" and "y-coordinate" represent in a coordinate pair?
- How does the position of a point on a coordinate plane relate to its coordinates?
- What is the significance of the origin (0, 0) on the coordinate plane?
- In what ways can coordinates be used to solve problems involving distance, direction, or position?

Cross-Curricular Connections

Social Studies:

Align geographic representations on a coordinate system.

STEM:

Plot on a coordinate system. For example: Plot stars, planets, moons, asteroids, and other celestial bodies on a diorama of the solar system. Plot stars of a constellation on a coordinate system. Identify the location of stars on a system map using ordered pairs.

Career and Skill Connections

- Urban Planners
- Engineers
- Surveyors
- Architects
- Utility Workers

Grade	CCSS Domain	CCSS Cluster
5	Geometry	Graph points on the coordinate plane to solve real-world and mathematical problems.

Cluster Standard: 5.G.A.2

Standard	Standards for Mathematical Practice
Graph points on the coordinate plane to solve real world mathematical problems. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 2:** Reason abstractly and quantitatively.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This standard references real-world and mathematical problems, including the traveling from one point to another and identifying the coordinates of missing points in geometric figures, such as squares, rectangles, and parallelograms.	• Graph points in the first quadrant. • Interpret coordinate values of points in real world context and mathematical problems. • Represent real world and mathematical problems by graphing points in the first quadrant.
DOK	**Blooms**
1	Apply

Procedural and Conceptual Understanding and Application

Application:

- Students gain a strong understanding of how to plot points on a graph with two intersecting lines (the coordinate plane). They learn that each point's position on the graph represents its distance to the left or right (x-axis) and up or down (y-axis) from a starting point (the origin). This helps them solve problems in the real world and in math class.

Procedural Skills and Fluency:

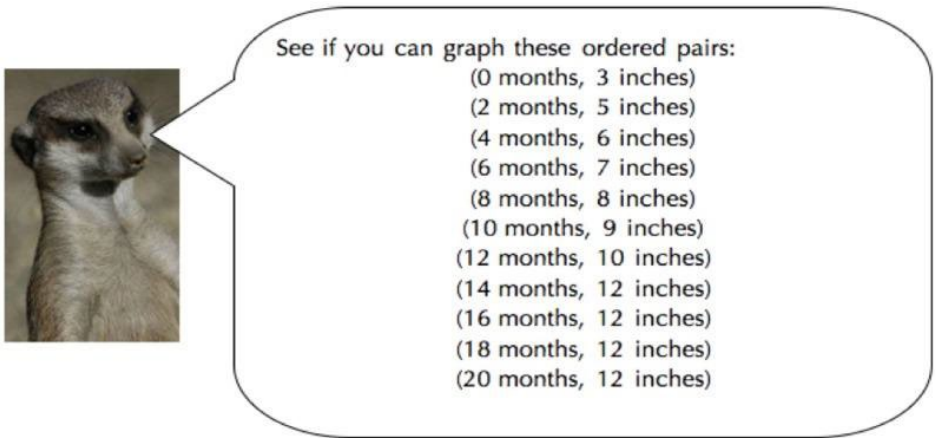
- Students practice drawing accurate points on graphs that show positive numbers on both the x-axis and y-axis (the first quadrant). They use these graphs to show situations from the real world or math problems. They also learn to read the x and y coordinates of these points quickly and accurately.

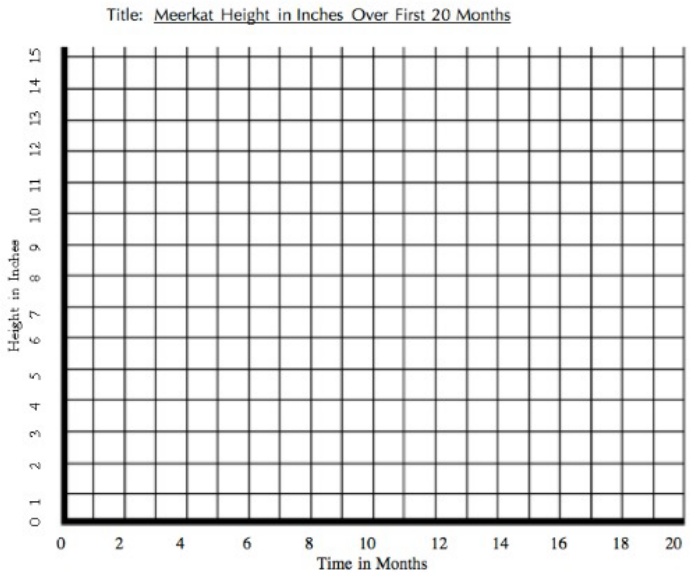
Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Greetings from the Kalahari Desert in South Africa! In this activity, you will learn a lot about the Kalahari's most playful residents: meerkats.

a. The following ordered pairs show the height of a typical meerkat at different times during the first 20 months of life. Graph the corresponding points and see what you can discover about meerkats. Once you have graphed them all, connect the points in the order they are given to form a line graph.





b. What does the point (0 months, 3 inches) mean for a typical meerkat's height?

c. How tall do you think a typical meerkat gets? Why?

d. At what age do meerkats reach their full height? How do you know from this graph?

e. If this graph were about a human instead of a meerkat, at what age do you think the height would stop getting larger.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions
• **Coordinate Values as Absolute Distance:** Students may mistakenly believe that the coordinate values of a point represent its distance from the origin rather than its position relative to the axes. • **Confusion Between Axes and Quadrants:** Students might confuse the x-axis and y-axis with the different quadrants of the coordinate plane, leading to errors in plotting points and interpreting their positions. • **Misinterpretation of Point Placement:** Students could misunderstand how to accurately place points on the coordinate plane, resulting in incorrect graphing and misinterpretation of coordinate values. • **Limited Understanding of Contextual Interpretation:** Students may struggle to interpret coordinate values in the context of real-world or mathematical problems, making it difficult to apply graphing skills effectively to solve problems.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning		
Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Use graph paper, coordinate grids, and interactive whiteboards to visually represent the coordinate plane. Provide manipulatives such as movable points or magnetic grids for hands-on exploration. • Incorporate real-life examples and problems that require graphing points on a coordinate plane, such as mapping a treasure hunt, planning a garden, or designing a game board. This helps students see the relevance and application of the skill. • Teach and model the process of plotting points on the coordinate plane step-by-step. Use clear, explicit language and provide visual examples to explain concepts such as the x-axis, y-axis, origin, and coordinates. **Universal Design for Learning (UDL)** • Provide multiple means of	• Provide targeted small group instruction for students who need additional support in understanding coordinate graphing. Use differentiated activities to address specific areas of difficulty and offer opportunities for peer collaboration. • Offer guided practice sessions where students receive immediate feedback and support from the teacher or a peer mentor while working on coordinate graphing tasks. Use formative assessments to monitor progress and adjust instruction as needed. **Universal Design for Learning (UDL)** • Provide multiple means of engagement by incorporating student interests, cultural	• Provide intensive one-on-one instruction for students who require individualized support. Tailor lessons to the student's specific learning needs, using additional explanations, examples, and practice opportunities. Focus on building foundational skills and addressing specific areas of difficulty. • Develop personalized learning plans that include targeted goals, strategies, and supports for students with significant learning gaps. Collaborate with special education teachers, ELL specialists, and other support staff to address individual needs. Incorporate specific accommodations and modifications as necessary. **Universal Design for Learning (UDL)** • Provide multiple means of representation by using visual

engagement by incorporating student interests, cultural relevance, and real-world connections into the lesson.	relevance, and real-world connections into the lesson.	aids, manipulatives, and verbal explanations to accommodate diverse learning styles and preferences.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/5/20

Previous Learning	*Current Learning*	*Future Learning*
• Connect to plotting points on a number line and construct perpendicular lines.	• Connect to forming ordered pairs from given rules and graph points on a coordinate plane.	• Connect to extending understanding of a coordinate plane to the negative number coordinates.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Incorporate culturally relevant contexts in graphing problems. For example, use examples from students' communities or cultural events, such as plotting points related to a local festival layout or a map of significant places in their culture. • Encourage students to create their own coordinate graphing problems based on their interests and cultural backgrounds. This allows students to see the relevance of math in their own lives and cultures and affirms their	• Begin lessons by connecting to students' prior knowledge and experiences with maps, grids, and navigation. Use familiar contexts, such as navigating a school map or plotting locations in their neighborhood, to introduce coordinate graphing concepts. • Bridge new content with real-world examples that are culturally relevant. For example, use a coordinate plane to map out a garden layout, including culturally significant plants or items.	• Provide visual aids, such as diagrams of coordinate planes with labeled axes and origin points. Use color-coding to help students distinguish between the x-axis and y-axis. • Create a word wall with key vocabulary terms and their definitions, including visual representations. Update it regularly as new terms are introduced. • Use sentence frames to support students in articulating their understanding of coordinate graphing (e.g., "The point (x, y) is located..."). This helps

identities.	• Engage students in interactive activities that connect coordinate graphing to real-life applications. For instance, use a large floor grid for students to physically plot points, representing real-world locations or cultural sites.	students practice using academic language in context.

Suggested Student Discourse Questions

- How do we interpret the coordinates of a point (x, y) in the context of a real-world problem?
- How can we use coordinates to find solutions or make predictions about a situation?
- In what ways does graphing points on a coordinate plane enhance our ability to solve mathematical problems?
- Can you provide examples of how coordinate geometry is used in fields such as navigation, engineering, or economics?

Cross-Curricular Connections

Social Studies:

Align geographic representations on a coordinate system.

STEM:

Plot on a coordinate system. For example: Plot stars, planets, moons, asteroids, and other celestial bodies on a diorama of the solar system. Plot stars of a constellation on a coordinate system. Identify the location of stars on a system map using ordered pairs.

Career and Skill Connections

- Engineers, Construction Engineer
- Architects
- Physics Professors
- Surveyors

Grade	CCSS Domain	CCSS Cluster
5	Geometry	Classify two-dimensional figures into categories based on their properties.

Cluster Standard: 5.G.B.3

Standard	Standards for Mathematical Practice
Classify two-dimensional figures into categories based on their properties. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.	• **SMP 6:** Attend to precision. • **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This standard calls for students to reason about the attributes (properties) of shapes. Students should have experiences discussing the property of shapes and reasoning. The notion of congruence ("same size and same shape") may be part of classroom conversation, but the concepts of congruence and similarity do not appear until middle school.	• Recognize that some two-dimensional shapes can be classified into more than one category based on their attributes. • Recognize if a two-dimensional shape is classified into a category, that it belongs to all subcategories of that category.
DOK	**Blooms**
1-3	Understand

Procedural and Conceptual Understanding and Application

Procedural: Students demonstrate fluency by knowing the properties of two-dimensional figures and can categorize them based on their properties.

Conceptual Understanding: Students build a solid understanding of how to group and categorize two-dimensional shapes by recognizing that certain characteristics shared by a group of shapes also apply to all the subgroups within that category, demonstrated with examples such as the connection between rectangles and squares.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Decide whether each of these statements is always, sometimes, or never true. If it is sometimes true, draw and describe a figure for which the statement is true and another figure for which the statement is not true.

a. A rhombus is a square
b. A triangle is a parallelogram
c. A square is a parallelogram
d. A square is a rhombus
e. A parallelogram is a rectangle
f. A trapezoid is a quadrilateral

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Assuming All Traits Apply Universally:** Students might wrongly think that every characteristic of a larger group always applies to every smaller subgroup, without considering exceptions or differences.
- **Mixing Up Main Groups and Subgroups:** Students may struggle to tell the difference between broader categories (like rectangles) and their subcategories (like squares), which can lead to mistakes when classifying figures based on their properties.
- **Difficulty with Specific Characteristics:** Students might have trouble identifying and understanding the specific traits that define a category or subcategory of two-dimensional figures, resulting in errors when categorizing shapes.
- **Misunderstanding Relationships:** Some students may misunderstand how different categories and subcategories of two-dimensional figures are related, which can lead to confusion about which attributes are shared between them.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Use diagrams, shape charts, and graphic organizers to visually represent different categories and subcategories of two-dimensional figures. Provide manipulatives such as cut-out shapes or pattern blocks for hands-on exploration. • Provide clear and explicit	• Conduct small group sessions for students who need additional support in understanding shape attributes and categorization. Use differentiated activities to address specific areas of difficulty and provide opportunities for guided	• Provide intensive one-on-one instruction for students who require individualized support. Tailor lessons to the student's specific learning needs, using additional explanations, examples, and practice opportunities. • Develop personalized learning plans that include targeted

instruction on the attributes of various two-dimensional figures and their subcategories. Model the process of identifying and categorizing shapes, using think-aloud strategies to demonstrate your thought process. • Use real-world examples and non-examples to highlight the attributes that define each category and subcategory. **Universal Design of Learning (UDL)** • Provide multiple means of representation by building knowledge. Connect prior knowledge to learning, highlight and explore patterns.	practice. • Use structured group activities, such as sorting shapes into categories and subcategories or creating posters that illustrate shape attributes. • Use scaffolded activities that gradually increase in complexity. Start with basic shape identification and attribute recognition, then progress to more complex categorization tasks. **Universal Design of Learning (UDL)** • Provide multiple means of representation by building knowledge. Connect prior knowledge to learning, highlight and explore patterns.	goals, strategies, and supports for students with significant learning gaps. Collaborate with special education teachers, ELL specialists, and other support staff to address individual needs. • Incorporate specific accommodations and modifications as necessary, such as extended time, additional practice opportunities, or alternative assessments. **Universal Design of Learning (UDL)** • Provide multiple means of representation by building knowledge. Connect prior knowledge to learning, highlight and explore patterns.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/5/20

Previous Learning	*Current Learning*	*Future Learning*
• Connect to learning that shapes in different categories share attributes. • Connect to learning that shared attributes can define a larger category. For example, rectangles, squares, and rhombuses are all examples of quadrilaterals. • Connect to classifying two dimensional figures based on lines and angles.	• Connect to understanding and classifying two-dimensional shapes by their attributes.	• Connect to drawing shapes with given conditions.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:
- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities

of students of marginalized cultures and languages?

- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Use examples of two-dimensional figures from various cultures, such as traditional art, architecture, and designs. For instance, show geometric patterns in quilts, tile work, or cultural symbols from different parts of the world. • Encourage students to create projects that involve identifying and categorizing shapes found in their cultural artifacts or daily lives. This allows students to see the connection between geometry and their own cultural experiences.	• Begin lessons by connecting to students' prior knowledge and experiences with shapes. Use familiar contexts, such as drawing shapes they see in their environment or cultural settings, to introduce new geometric concepts. • Discuss shapes found in everyday items and cultural artifacts to make the learning process more relatable and meaningful. • Use culturally relevant contexts for problems, such as designing a pattern for a traditional festival or creating a floor plan for a community event, to help students see the practical applications of geometry.	• Provide visual aids such as diagrams, charts, and graphic organizers that visually represent different categories and subcategories of two-dimensional figures. • Offer bilingual resources, including vocabulary lists and explanations of key terms like "attributes," "categories," "subcategories," "quadrilaterals," and "triangles," in both English and students' home languages. • Create a word wall with key geometric vocabulary terms and their definitions, including visual representations. Update it regularly as new terms are introduced. • Use sentence frames to support students in articulating their understanding of shape attributes and categorization (e.g., "A rectangle is a type of quadrilateral because..."). This helps students practice using academic language in context.

Suggested Student Discourse Questions

- What does it mean to classify two-dimensional figures into categories based on their properties?
- How can we use properties to classify unfamiliar figures into known categories?
- Why is it important to recognize that subcategories inherit the attributes of their parent categories?
- What are some shared attributes of all rectangles? How do these attributes apply to subcategories like squares and parallelograms?
- How do attributes of a category of figures relate to all subcategories within that category?

Cross-Curricular Connections
Art: Provide students with multiple colors and textures of paper. Have them work in groups to create collages based on the attributes of different shapes. Give students strips of paper that give examples of different shapes. Allow them to create their collages based on the attributes given. History and Architecture: Have students study the shapes of historical dwellings/buildings. Have students make connections to the building in their communities. Discuss why certain shapes may have been more fitting than others for various buildings. Have students describe the dwellings/buildings based on their attributes.
Career and Skill Connections
• Construction Worker • Interior Designer • Mathematics Teacher • CAD Engineer • Plumber • Animator

Grade	CCSS Domain	CCSS Cluster
5	Geometry	Classify two-dimensional figures into categories based on their properties.

Cluster Standard: 5.G.B.4

Standard	Standards for Mathematical Practice
Classify two-dimensional figures into categories based on their properties. Classify two dimensional figures in a hierarchy based on properties.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This standard builds on what was done in 4th grade. Figures from previous grades: polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle, kite. A kite is a quadrilateral whose four sides can be grouped into two pairs of equal-length sides that are beside (adjacent to) each other. Students should be able to reason about the attributes of shapes by examining: What are ways to classify triangles? Why can't trapezoids and kites be classified as parallelograms? Which quadrilaterals have opposite angles congruent and why is this true of certain quadrilaterals, and How many lines of symmetry does a regular polygon have? Note, in the U.S., the term "trapezoid" may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with at least one pair of parallel sides. The exclusive definition states: A trapezoid is a quadrilateral with exactly one pair of parallel sides. With this definition, a parallelogram is not a trapezoid. North Carolina has adopted the exclusive definition. (Progressions for the CCSSM: Geometry, The Common Core Standards Writing Team, June 2012.)	• Recognize the hierarchy of two-dimensional shapes based on their attributes. • Analyze properties of two-dimensional figures in order to place them into a hierarchy. • Classify two-dimensional figures into categories and/or subcategories based on their attributes.

DOK	Blooms
1-2	Understand

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

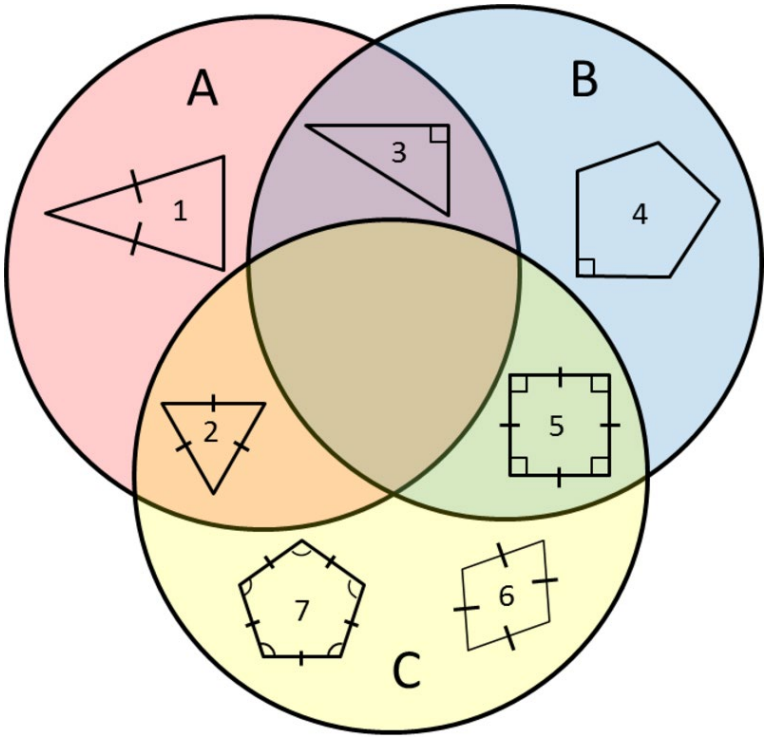
- Students develop a comprehensive understanding of classifying two-dimensional figures by organizing them into categories and a hierarchical structure based on shared properties, enabling them to recognize relationships and distinctions among different types of figures.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

The picture below is called a Venn Diagram. Each circle (A, B, and C) contain shapes that all share at least one characteristic. Some shapes are contained in more than one circle because they share more than one characteristic. For example, shape 3 fits the rule for circles A and B, but not circle C. It lies within circles A and B, but not circle C.

a. What are the characteristics shared by shapes within circle A? Within circle B? Within circle C? Double check to make sure that any shapes that have that characteristic are contained within the circle and any shapes that don't lie outside of the circle.



Characteristics of all shapes contained in Circle A:

Characteristics of all shapes contained in Circle B:

Characteristics of all shapes contained in Circle C:

b. Where would you place a rectangle that does not have four sides of the same length? Why?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Limited Scope of Classification:** Students may incorrectly believe that classification of two-dimensional figures is based solely on a single property rather than considering multiple properties that define different categories.
- **Misunderstanding Hierarchy:** Students might misunderstand the concept of hierarchy in classification, thinking that all figures must fit into a strict linear hierarchy rather than recognizing the flexibility and overlap between different categories.
- **Overemphasis on Specific Properties:** Students could focus excessively on one particular property (e.g., number of sides) when classifying figures, leading to oversimplification and incorrect categorization of more complex shapes.
- **Ignoring Exceptional Cases:** Students may overlook exceptions or unique cases within classification, assuming that all figures neatly fit into predefined categories without acknowledging variations or special characteristics that defy typical classification rules.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning		
Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Use diagrams, shape charts, and graphic organizers to visually represent different properties and classifications of two-dimensional figures. • Provide manipulatives such as cut-out shapes, pattern blocks, or geometric solids for hands-on exploration. • Provide clear and explicit instruction on the properties of various two-dimensional figures and their classifications. Model the process of identifying and classifying shapes, using think-aloud strategies to demonstrate your thought process. **Universal Design of Learning (UDL)** • Provide multiple means of representation by building knowledge. Connect prior knowledge to learning, highlight and explore patterns.	• Conduct small group sessions for students who need additional support in understanding shape properties and classification. Use differentiated activities to address specific areas of difficulty and provide opportunities for guided practice. • Use scaffolded activities that gradually increase in complexity. Start with basic shape identification and property recognition, then progress to more complex classification tasks. **Universal Design of Learning (UDL)** • Provide multiple means of representation by building knowledge. Connect prior knowledge to learning, highlight and explore patterns.	• Provide intensive one-on-one instruction for students who require individualized support. Tailor lessons to the student's specific learning needs, using additional explanations, examples, and practice opportunities. • Develop personalized learning plans that include targeted goals, strategies, and supports for students with significant learning gaps. Collaborate with special education teachers, ELL specialists, and other support staff to address individual needs. • Incorporate specific accommodations and modifications as necessary, such as extended time, additional practice opportunities, or alternative assessments. **Universal Design of Learning (UDL)** • Provide multiple means of representation by building knowledge. Connect prior knowledge to learning, highlight and explore patterns.
Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/5/20		
Previous Learning	*Current Learning*	*Future Learning*
• Connect to learning that shapes in different categories	• Connect to understanding and classifying two-	• Connect to drawing shapes with given conditions.

share attributes. ● Connect to learning that shared attributes can define a larger category. For example, rectangles, squares, and rhombuses are all examples of quadrilaterals. ● Connect to classifying two dimensional figures based on lines and angles.	dimensional shapes by their attributes.	

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
● Use examples of two-dimensional figures found in various cultural artifacts, such as traditional art, architecture, and designs. Show how different cultures use shapes and patterns in their artistic expressions and daily life. ● Discuss the significance of these shapes in different cultural contexts to validate students' cultural backgrounds and make learning relevant.	● Start lessons by connecting to students' prior knowledge and experiences with shapes and patterns. Use familiar contexts, such as household items, cultural decorations, or community landmarks, to introduce geometric classification. ● Incorporate real-world problems that involve classifying shapes based on their properties. For example, have students sort and classify shapes in cultural patterns, decorations, or symbols found in their communities. ● Use culturally relevant contexts for problems, such as designing a traditional costume pattern or classifying tiles in cultural art, to show practical applications of geometric	● Provide visual aids, such as diagrams, charts, and graphic organizers, that visually represent different properties and classifications of two-dimensional figures. ● Offer bilingual resources, including vocabulary lists and explanations of key terms like "properties," "classification," "polygons," "angles," "sides," and "vertices," in both English and students' home languages. ● Create a word wall with key geometric vocabulary terms and their definitions, including visual representations. Update it regularly as new terms are introduced. ● Use sentence frames to support students in articulating their understanding of shape properties and classification

	classification.	(e.g., "A square is a type of quadrilateral because..."). This helps students practice using academic language in context.

Suggested Student Discourse Questions

- What does it mean to classify two-dimensional figures into categories based on their properties?
- How can properties of figures help us organize them into a hierarchy?
- What are some key properties we can use to classify two-dimensional figures?
- How do properties like angles, side lengths, and symmetry contribute to classification?
- Can you propose a method to organize figures into a hierarchy based on specific properties?

Cross-Curricular Connections

Art:

Provide students with multiple colors and textures of paper. Have them work in groups to create collages based on the attributes of different shapes. Give students strips of paper that give examples of different shapes. Allow them to create their collages based on the attributes given.

History and Architecture:

Have students study the shapes of historical dwellings/buildings. Have students make connections to the building in their communities. Discuss why certain shapes may have been more fitting than others for various buildings. Have students describe the dwellings/buildings based on their attributes.

Career and Skill Connections

- Cartographer
- Fashion Designer
- Urban Planner
- Game Developer
- Surveyor

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards of Mathematical Practice
    - Procedural and Conceptual Understanding and Application
    - Sample Assessment Items
    - Common Misconceptions
    - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
    - Vertical Alignment
    - Culturally and Linguistically Responsive Instruction (CLRI)
    - Suggested Student Discourse Questions
    - Cross-Curricular and Career and Skill Connections
- A Student Discourse Guide
- Planning for a Multi-Layer System of Support (MLSS) and Universal Design for Learning (UDL) for behavioral and social and emotional supports

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Convert like measurement units within a given measurement system
  - 5.MD.A.1
- Represent and interpret data
  - 5.MD.B.2
- Geometric measurement: understand concepts of volume
  - 5.MD.C.3
  - 5.MD.C.4
  - 5.MD.C.5

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)

Grade	CCSS Domain	CCSS Cluster
5	Measurement and Data	Convert like measurement units within a given measurement system.

Cluster Standard: 5.MD.A.1

Standard	Standards for Mathematical Practice
Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 2:** Reason abstractly and quantitatively. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
In Grade 5, students extend their abilities from Grade 4 (4.MD.A.1) to express measurements in larger or smaller units within a measurement system. This is an excellent opportunity to reinforce notions of place value for whole numbers and decimals, and make connections between fractions and decimals (e.g., 2 1/2 meters can be expressed as 2.5 meters or 250 centimeters).	• Recognize units of measurement within the same system. • Convert units of measurement within the same system by multiplying or dividing. • Solve multi-step, real world problems that involve converting units
DOK	**Blooms**
1-2	Remember, Understand, Apply

Procedural and Conceptual Understanding and Application

Procedural: Students demonstrate fluency in converting measurements between units within the same measurement system and apply appropriate procedures for converting measurements, including multiplying and dividing by conversion factors.

Application: Students apply measurement conversion skills to solve real-world problems and scenarios and recognize situations where measurement conversions are necessary and apply appropriate strategies to solve them.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available

a. Five brothers are going to take turns watching their family's new puppy. How much time will each brother spend watching the puppy in a single day if they all watch him for an equal length of time? Write your answer:
   i. Using only hours,
   ii. Using a whole number of hours and a whole number of minutes, and
   iii. Using only minutes.

b. Mrs. Hinojosa had 75 feet of ribbon. If each of the 18 students in her class gets an equal length of ribbon, how long will each piece be? Write your answer:
   i. Using only feet,
   ii. Using a whole number of feet and a whole number of inches, and
   iii. Using only inches.

c. Wesley walked 11 miles in 4 hours. If he walked the same distance every hour, how far did he walk in one hour? Write your answer:
   i. Using only miles,
   ii. Using a whole number of miles and a whole number of feet, and
   iii. Using only feet.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Misunderstanding the Purpose of Conversion Factors:** Students may mistakenly believe that conversion factors are random numbers to be memorized, rather than understanding that they represent the relationship between different units of measurement. This can lead to difficulties in applying conversion factors correctly.
- **Confusion with Multiplication and Division:** Some students may struggle with knowing whether to multiply or divide when using conversion factors. They may incorrectly assume that multiplying is always the correct operation or vice versa, leading to errors in their conversions.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Provide clear explanations and demonstrations of how to convert between different measurement units (e.g., inches to feet, centimeters to meters). Use visual aids, such as conversion charts and number lines, to support understanding. • Model multi-step problems involving measurement conversions, using think-aloud strategies to demonstrate problem-solving	• Conduct small group sessions for students who need additional support in understanding measurement conversions. Use differentiated activities to address specific areas of difficulty and provide guided practice. • Focus on collaborative problem-solving tasks and hands-on activities to reinforce concepts. • Use scaffolded activities	• Provide intensive one-on-one instruction for students who require individualized support. Tailor lessons to the student's specific learning needs, using additional explanations, examples, and practice opportunities. • Develop personalized learning plans that include targeted goals, strategies, and supports for students with significant learning gaps. Collaborate with special education

processes. • Use visual aids, such as conversion charts, rulers, measuring tapes, and graduated cylinders, to help students understand different units of measurement. • Provide manipulatives, such as measuring cups and scales, to give students hands-on experience with measurement tools and conversions. **Universal Design for Learning (UDL)** • Design multiple means of representation by providing various perception opportunities in presenting or displaying different ways for information to be taught, provide captions, charts, diagrams, and manipulatives, and physical models.	that gradually increase in complexity. Start with simple one-step conversion problems and progress to more complex multi-step problems. • Provide graphic organizers and step-by-step guides to help students organize their thoughts and approach multi-step problems systematically. **Universal Design for Learning (UDL)** • Design multiple means of representation by providing various perception opportunities in presenting or displaying different ways for information to be taught, provide captions, charts, diagrams, and manipulatives, and physical models.	teachers, ELL specialists, and other support staff to address individual needs. • Incorporate specific accommodations and modifications as necessary, such as extended time, additional practice opportunities, or alternative assessments. **Universal Design for Learning (UDL)** • Design multiple means of representation by providing various perception opportunities in presenting or displaying different ways for information to be taught, provide captions, charts, diagrams, and manipulatives, and physical models.
Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/5/21		
Previous Learning	*Current Learning*	*Future Learning*
• Connect to understanding the relative sizes of measurement units within a system. • Connect to using the four operations to solve word problems including problems involving fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit.	• Connect to the powers of 10, which relates to converting metric measurements. • Connect to working to perform operations with multi-digit whole numbers and with decimals to hundredths.	• Connect to using ratios to convert measurement units. Connect to manipulating and transforming units appropriately when multiplying or dividing quantities.
Culturally and Linguistically Responsive Instruction		
Consider these questions as you plan for instruction that is culturally and linguistically responsive:		

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Use measurement examples from different cultures, such as traditional recipes, construction methods, or clothing patterns, to illustrate the importance and application of measurement conversions. • Highlight how various cultures measure and convert units in their everyday lives, validating students' backgrounds and showing the relevance of the concept. • Encourage students to create projects that involve measuring and converting units based on their cultural experiences, such as family recipes, traditional crafts, or community events.	• Begin lessons by connecting to students' prior knowledge and experiences with measurement in their homes and communities. Use familiar contexts, such as cooking, shopping, or building, to introduce new measurement conversion concepts. • Discuss how measurement is used in various cultural practices and everyday activities to make learning more relatable and meaningful. • Incorporate real-world problems that involve measurement conversions relevant to students' lives, such as cooking a meal, planning a trip, or organizing a cultural event. • Engage students in interactive and collaborative activities that require them to measure and convert units. For instance, have students work in groups to solve real-world problems involving measurement conversions, using materials and contexts from their cultural backgrounds.	• Provide visual aids, such as conversion charts, diagrams, and graphic organizers, that visually represent different measurement units and conversions. • Offer bilingual resources, including vocabulary lists and explanations of key terms like "conversion," "unit," "measurement," "length," "volume," and "weight," in both English and students' home languages. • Create a word wall with key measurement vocabulary terms and their definitions, including visual representations. Update it regularly as new terms are introduced. • Use sentence frames to support students in articulating their understanding of measurement conversions (e.g., "To convert from inches to feet, I need to..."). This helps students practice using academic language in context.
Suggested Student Discourse Questions		

- What does it mean to convert between different-sized standard measurement units within the same system?
- Why is it important to be able to convert measurements in real-world scenarios?
- How do you convert measurements from one unit to another within the metric system (e.g., centimeters to meters)?
- Discuss the importance of understanding the relationships between different units in the conversion process.

Cross-Curricular Connections

Science:

In fifth grade the NGSS recommends students work with measurement related to conservation of mass. Consider providing a connection for students to determine the mass of an object in different states in two different units and then convert one unit unto the other to discover that they are equivalent.

Art:

Making a model of an object involves having to convert from larger to small units. Consider providing a connection for students to make a scaled model of something involving simple polygons or polyhedrons.

Career and Skill Connections

- Controls Engineer
- Metrology Technician
- Quality Engineer
- Measurement Technician
- Instrumentation Technician
- Civil Engineer
- Architect
- Construction Manager
- Interior Designer
- Surveyor
- Logistics Coordinator

Grade	CCSS Domain	CCSS Cluster
5	Measurement and Data	Represent and interpret data.

Cluster Standard: 5.MD.B.2

Standard	Standards for Mathematical Practice
Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 5:** Use appropriate tools strategically. • **SMP 1**: Make sense of problems and persevere in solving them.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Grade 5 students grow in their skill and understanding of fraction arithmetic, including multiplying a fraction by a fraction, dividing a unit fraction by a whole number or a whole number by a unit fraction, and adding and subtracting fractions with unlike denominators. Students can use these skills to solve problems, including problems that arise from analyzing line plots. For example, given five graduated cylinders with different measures of liquid in each, students might find the amount of liquid each cylinder would contain if the total amount in all the cylinders were redistributed equally. (Students in Grade 6 will view the answer to this question as the mean value for the data set in questions.)	• Identify benchmark fractions. • Make a line plot to display a data set of measurements in fractions of a unit. • Solve problems involving information presented in line plots which use fractions of a unit by adding, subtracting, multiplying, and dividing fractions.
DOK	**Blooms**
1-2	Remember, Apply

Procedural and Conceptual Understanding and Application

Procedural: Students will construct a line plot accurately displaying a data set of measurements in fractions of a unit (e.g., 1/2, 1/4, 1/8) by organizing data points along a number line and labeling each fractional measurement

appropriately.

Application: Using their understanding of operations on fractions, students will analyze the data presented in the line plot to solve problems involving comparisons, additions, or subtractions of fractional measurements represented on the plot.

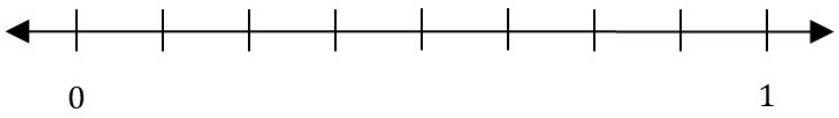
Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

You and your partner will need fraction cards made from this set:

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

a. Label the line-plot below with 1/8's. Cut out and divide the cards evenly between the two players, laying them face-down. Each partner will choose one of their face-down cards and turn it over. The team will then add their fractions together. For each turn, each team will record their sum on the line plot.



Each team should have 12 data points marked on their line plot.

b. Look at the line plot. Which values came up the most? Which values did not come up?

c. The tick marks on the number line correspond to eighths. Which of the eighths will never come up as a sum of two of these cards? Why?

d. You want to improve the game so that it is possible for two fractions to sum to ⅞. Name one fraction card that you could add to the deck and explain why your new card would now make it possible to have 7/8 as a sum of two cards.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Misinterpreting Line Plot Positioning:** Students might mistakenly think that the positions of data points on a line plot directly represent specific fractional values like 1/2, 1/4, or 1/8, leading to confusion about the plot's purpose in showing frequency distribution rather than exact measurements.
- **Uniform Fractional Spacing:** There could be a misconception that all fractional measurements (e.g., 1/2, 1/4, 1/8) should be evenly spaced along the line plot axis, ignoring the need for varied intervals to accurately show the distribution of the data set.
- **Length Misinterpretation:** Students may misunderstand that the length of marks on a line plot indicates the size of the fractional measurement, rather than representing the frequency or occurrence of each measurement within the data.
- **Limitation to Whole Numbers:** There might be a belief that line plots are only meant for displaying whole number measurements and cannot effectively display or analyze fractional measurements like 1/2, 1/4, or 1/8, which limits students' ability to use line plots accurately for fractions in data analysis.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Provide clear, step-by-step instructions and demonstrations for creating and interpreting line plots. Use visual aids, such as charts and diagrams, to support understanding. • Model the process of plotting data points, labeling axes, and interpreting the information presented in the line plot. • Use visual aids such as conversion charts, diagrams, and graphic organizers to	• Conduct small group sessions to provide additional support and practice for students who need help with creating and interpreting line plots. Use differentiated activities tailored to the specific needs of the group. • Focus on collaborative problem-solving tasks and guided practice to reinforce concepts. • Provide scaffolded activities that gradually increase in complexity. Start with simple	• Provide intensive one-on-one instruction for students who require individualized support. Tailor lessons to the student's specific learning needs, using additional explanations, examples, and practice opportunities. • Develop personalized learning plans that include targeted goals, strategies, and supports for students with significant learning gaps. Collaborate with special education teachers,

help students understand the relationship between fractions and data points on a line plot. • Provide manipulatives like fraction strips, number lines, and measuring tools to give students hands-on experience with measurement and fractions. **Universal Design for Learning (UDL)** • Design multiple means of representation by providing various perception opportunities in presenting or displaying different ways for information to be taught, provide captions, charts, diagrams, and manipulatives, and physical models.	data sets and basic line plots, then progress to more complex data and multi-step problems involving fractions. **Universal Design for Learning (UDL)** • Design multiple means of representation by providing various perception opportunities in presenting or displaying different ways for information to be taught, provide captions, charts, diagrams, and manipulatives, and physical models.	ELL specialists, and other support staff to address individual needs. • Incorporate specific accommodations and modifications as necessary, such as extended time, additional practice opportunities, or alternative assessments. **Universal Design for Learning (UDL)** • Design multiple means of representation by providing various perception opportunities in presenting or displaying different ways for information to be taught, provide captions, charts, diagrams, and manipulatives, and physical models.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/5/21

Previous Learning	*Current Learning*	*Future Learning*
• Connect to generating data by measuring lengths and making line plots using that data**.** • Connect to solving addition and subtraction problems using the data on line plots.	• Connect to growing in their skill and understanding of fraction arithmetic.	• Connect to displaying numerical data in plots on number lines, dot plots, histograms, and boxplots and choosing the most appropriate graph/plot for the data.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:
- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*

• Use data sets that reflect the cultural backgrounds and interests of the students. For example, if students are interested in sports, use data related to culturally relevant sports statistics. If there are cultural events or practices involving measurements, use those as data sets. • Highlight traditional or culturally significant ways of recording and interpreting data from different cultures, such as how certain indigenous communities track environmental data or agricultural yields. • Encourage students to collect data from their own cultural experiences or communities and create line plots to represent this data. For example, they might measure ingredients for a family recipe or track the time spent on different cultural activities.	• Begin by discussing students' prior experiences with measurement and data collection in their everyday lives. Relate these experiences to the concept of creating and interpreting line plots. • Use familiar and culturally relevant contexts, such as cooking, gardening, or sports, to introduce the concept of line plots and data analysis. • Incorporate real-world problems that involve interpreting and creating line plots with culturally relevant data sets. For example, students could analyze data on community events, school activities, or local weather patterns. • Show how line plots are used in various professions and everyday activities, such as tracking exercise routines, recording academic progress, or monitoring environmental changes.	• Provide visual aids, such as charts, diagrams, and graphic organizers, that visually represent the steps in creating and interpreting line plots. • Offer bilingual resources, including vocabulary lists and explanations of key terms like "line plot," "data set," "fraction," "measurement," "axis," and "scale," in both English and students' home languages. • Create a word wall with key vocabulary terms related to line plots and data analysis, including visual representations and definitions. Update it regularly as new terms are introduced. • Use sentence frames to support students in articulating their understanding of line plots and data interpretation (e.g., "The line plot shows that the most common measurement is..."). This helps students practice using academic language in context.

Suggested Student Discourse Questions

- What is a line plot, and how is it used to display a data set of measurements?
- Why is it beneficial to use line plots, especially when dealing with measurements in fractions of a unit?
- How do you read and interpret a line plot that displays measurements in fractions (e.g., 1/2, 1/4, 1/8)?
- How can operations on fractions (like addition, subtraction, and finding equivalent fractions) be applied to solve problems based on information from a line plot?
- What patterns or trends can be observed by examining the data on a line plot?
- Why is it important to accurately represent measurements in fractions on a line plot?

Cross-Curricular Connections

Science:

In fifth grade the NGSS recommends students work with measurement related to conservation of mass. Consider providing a connection for students to determine the mass of various objects in different states in that measure in fractional units. Then have students graph and analyze that data.

Social Studies:

In fifth grade the New Mexico Social Studies Standards state students should "gather, organize and interpret information using a variety of media and technology". Consider having students gather, graph and analyze data that contains measurements in fractions of a unit.

Career and Skill Connections

- Computer Programmer
- Statistician
- Economist
- Urban Planner
- Scientist
- Market Research Analyst
- Data Analyst
- Quality Control Inspector
- Food Scientist
- Chemical Engineer
- Baker

Grade	CCSS Domain	CCSS Cluster
5	Measurement and Data	Geometric measurement: understand concepts of volume.

Cluster Standard: 5.MD.C.3

Standard	Standards for Mathematical Practice
Recognize volume as an attribute of solid figures and understand concepts of volume measurement. A. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. B. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.	• **SMP 6:** Attend to precision. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
"Packing" volume is more difficult than iterating a unit to measure length and measuring area by tiling. Students learn about a unit of volume, such as a cube with a side length of 1 unit, called a unit cube.	• Explain that volume is the measurement of the space inside a solid three-dimensional figure. • Explain that a unit cube has 1 cubic unit of volume and is used to measure volume of three-dimensional shapes. • Explain that any solid figure packed without gaps or overlaps and filled with n unit cubes indicates the total cubic units or volume.
DOK	**Blooms**
1	Remember

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Students will develop a deep understanding of volume as an inherent property of solid objects, exploring how volume is measured by visualizing it as the space occupied within three-dimensional figures and grasping the relationship between dimensions and volume.
- Students will develop a deep understanding of volume measurement by comprehending that a unit cube, with each side measuring 1 unit, represents precisely one cubic unit of volume, thus serving as a foundational tool for comparing and measuring volumes of other geometric shapes based on the concept of filling space with these standardized units.

- Students will enhance their comprehension of volume measurement by understanding that the volume of a solid figure, measured in cubic units, corresponds directly to the number of unit cubes needed to fill the figure completely without gaps or overlaps, illustrating the relationship between the figure's size and the quantity of unit cubes used.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

A box 2 centimeters high, 3 centimeters wide, and 5 centimeters long can hold 40 grams of clay. A second box has twice the height, three times the width, and the same length as the first box. How many grams of clay can it hold?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Confusing Volume with Surface Area:** Students might incorrectly believe that volume refers to the surface area or perimeter of a solid figure, rather than understanding it as the measure of space occupied inside the figure.
- **Understanding Volume Inside a Unit Cube:** Students may think that the volume of a unit cube (1 cubic unit) is determined by its surface area or perimeter, rather than understanding that volume is a measure of the space inside the cube.
- U**nrealistic Ideas about Packing Volume:** Students could mistakenly assume that any solid figure can be perfectly packed with unit cubes without any gaps or overlaps, leading to confusion about the concept of volume and the characteristics of different shapes.
- **The Role of Depth in Volume:** Students might believe that the volume of a solid figure is solely determined by the number of faces it has, overlooking the role of depth or thickness in defining volume and leading to misunderstandings about how to measure and compare volumes accurately.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Provide clear explanations and demonstrations of volume concepts. Use visual aids like diagrams and physical models of cubic units and solid figures. • Model the process of measuring and calculating volume, using think-aloud strategies to demonstrate how to count cubic units and	• Conduct small group sessions for students who need additional support in understanding volume concepts. Use differentiated activities to address specific areas of difficulty and provide guided practice. • Focus on collaborative problem-solving tasks and hands-on activities to	• Provide intensive one-on-one instruction for students who require individualized support. Tailor lessons to the student's specific learning needs, using additional explanations, examples, and practice opportunities. • Use diagnostic assessments to identify specific areas of misunderstanding and target

understand volume. ● Use visual aids, such as charts, diagrams, and graphic organizers, to help students understand and remember the concept of volume. ● Provide manipulatives, such as unit cubes and 3D shapes, for hands-on exploration. Allow students to build and measure the volume of various shapes using these tools. **Universal Design for Learning** ● Provide multiple means of representations by anchoring comprehension with prior knowledge using visuals, concept models, diagrams, and providing real world application of volume.	reinforce volume concepts. ● Use scaffolded activities that gradually increase in complexity. Start with simple volume measurement tasks and progress to more complex problems. **Universal Design for Learning** ● Provide multiple means of representations by anchoring comprehension with prior knowledge using visuals, concept models, diagrams, and providing real world application of volume.	those areas with focused instruction. ● Develop personalized learning plans that include targeted goals, strategies, and supports for students with significant learning gaps. Collaborate with special education teachers, ELL specialists, and other support staff to address individual needs. ● Incorporate specific accommodations and modifications as necessary, such as extended time, additional practice opportunities, or alternative assessments. **Universal Design for Learning** ● Provide multiple means of representations by anchoring comprehension with prior knowledge using visuals, concept models, diagrams, and providing real world application of volume.
Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/5/21		
Previous Learning	*Current Learning*	*Future Learning*
● Connect to creating 3-D shapes. ● Connect to learning to measure area using unit squares ● Connect to applying the formulas to determine area and perimeter of rectangles.	● Connect to fluently multiplying multi-digit whole numbers.	● Connect to finding the volume of right rectangular prisms with fractional dimensions in the context of solving real-world and mathematical problems.
Culturally and Linguistically Responsive Instruction		
Consider these questions as you plan for instruction that is culturally and linguistically responsive: ● How can you design your mathematics classroom to intentionally and purposefully legitimize the home		

culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Use examples of containers, buildings, or artifacts from different cultures to illustrate volume measurement. For instance, discuss the volume of traditional pottery, baskets, or storage units from various cultures. • Share stories or videos about how different cultures historically measured and utilized volume in everyday life, such as how ancient civilizations stored grains or transported water.	• Begin lessons by discussing students' prior experiences with volume, such as filling containers, cooking, or packing. Relate these experiences to the concept of volume measurement. • Use familiar and culturally relevant contexts, like measuring the volume of a traditional recipe or determining the amount of soil needed for a cultural garden, to introduce new concepts. • Incorporate real-world problems that involve measuring and calculating volume using culturally relevant examples, such as designing a cultural monument or creating a model of a traditional house.	• Provide visual aids, such as diagrams, charts, and graphic organizers, that visually represent volume concepts and calculations. • Offer bilingual resources, including vocabulary lists and explanations of key terms like "volume," "cubic units," "solid figure," "length," "width," and "height," in both English and students' home languages. • Create a word wall with key vocabulary terms related to volume measurement, including visual representations and definitions. Update it regularly as new terms are introduced. • Use sentence frames to support students in articulating their understanding of volume concepts (e.g., "The volume of the object is... cubic units because..."). This helps students practice using academic language in context.

Suggested Student Discourse Questions

- What does "volume" refer to when describing solid figures?
- How is volume different from other measurements like area or perimeter?
- What is a unit cube, and why is it useful for understanding volume measurement?
- If a solid figure can be packed without gaps or overlaps using n unit cubes, what does this tell us about its volume?

- In what real-life situations is understanding volume measurement important (e.g., packing boxes, filling containers)?

Cross-Curricular Connections

Science:

In fifth grade the NGSS states students should "describe and graph quantities such as area and volume to address scientific questions." Consider providing a connection for students to determine the volume of cubes or rectangular prisms as part of their investigation.

Art:

Drawing boxes is connected to developing the ability to indicate perspective in a drawing. Consider providing an opportunity for students to sketch various boxes with the same volume but different dimensions. Also, consider allowing students to make boxes to pack inside of larger boxes (measuring 1in. X 1in. X 1in. or 1 cm. X 1cm. X 1cm.). Have students predict how many boxes can fit inside of the premade larger boxes. Connect the number of boxes used to the volume of the box. Allowing students to cut, and construct boxes will help with their fine motor skills.

Career and Skill Connections

- Engineers
- Graphic Designers
- Animators
- CAD Engineer
- Cartographer
- Architect
- Civil Engineer
- Interior Designer
- Construction Manager
- Product Designer
- Urban Planner

Grade	CCSS Domain	CCSS Cluster
5	Measurement and Data	Geometric measurement: understand concepts of volume.

Cluster Standard: 5.MD.C.4

Standard	Standards for Mathematical Practice
Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	• **SMP 4:** Model with mathematics. • **SMP 5:** Use appropriate tools strategically. • **SMP 6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
They pack cubes (without gaps) into right rectangular prisms and count the cubes to determine the volume or build right rectangular prisms from cubes and see the layers as they build.	• Measure volume by counting unit cubes, cubic cm, cubic in, cubic ft, and improvised units.
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Procedural Skills and Fluency: Students will measure volumes by counting unit cubes in cubic centimeters, cubic inches, cubic feet, and improvised units, gaining proficiency in applying this technique across various measurement systems.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

A box 2 centimeters high, 3 centimeters wide, and 5 centimeters long can hold 40 grams of clay. A second box has twice the height, three times the width, and the same length as the first box. How many grams of clay can it hold?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Equating Volume with Area:** Students might confuse volume (measured in cubic units) with area (measured in square units) and incorrectly apply area formulas to calculate volume, leading to inaccurate measurements.
- **Ignoring Unit Conversion:** Students might overlook the need to convert measurements to the appropriate units (e.g., converting cubic inches to cubic feet) when working with different volume units, resulting in incorrect volume calculations.
- **Misinterpreting Cubic Units:** Students might misunderstand the concept of cubic units and believe that the numerical value represents a linear measurement (e.g., confusing cubic inches with inches), leading to errors in volume calculations.
- **Not Recognizing the Need for Standard Units:** Students might rely solely on improvised units (e.g., counting arbitrary cubes) without understanding the importance of using standard units like cubic centimeters or cubic inches for accurate volume measurement, which can affect the precision of their calculations.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + Universal***	***Layer 2*** ***Core + Targeted***	***Layer 3*** ***Core + Targeted + Intensive***
• Provide clear and explicit instruction on measuring volume by counting unit cubes. Demonstrate how to fill a three-dimensional shape with unit cubes and count them to determine the volume. • Use visual aids, such as diagrams, videos, and interactive simulations, to illustrate the concept of volume and how it can be measured. • Provide students with physical manipulatives, such as unit cubes, to explore and measure the volume of various containers. Allow students to build and count the cubes to determine volume. • Use real-world objects, such as boxes or containers, and have students fill them with unit cubes to measure their volume. **Universal Design for Learning** • Provide multiple means of representations by anchoring	• Conduct small group sessions for students who need additional support in understanding and applying the concept of volume measurement. Use differentiated activities to address specific areas of difficulty and provide guided practice. • Use scaffolded activities that gradually increase in complexity. Start with simple tasks involving measuring volume with unit cubes and progress to more complex problems involving different units and irregular shapes. **Universal Design for Learning** • Provide multiple means of representations by anchoring comprehension with prior knowledge using visuals, concept models, diagrams, and providing real world application of volume.	• Provide students who require intensive support, one on one instruction, build confidence with graduated levels of support and guide appropriate goal setting development, clarify vocabulary and symbols. • Develop personalized learning plans that include targeted goals, strategies, and supports for students with significant learning gaps. Collaborate with special education teachers, ELL specialists, and other support staff to address individual needs. • Incorporate specific accommodations and modifications as necessary, such as extended time, additional practice opportunities, or alternative assessments. **Universal Design for Learning** • Provide multiple means of representations by anchoring comprehension with prior knowledge using visuals,

comprehension with prior knowledge using visuals, concept models, diagrams, and providing real world application of volume.		concept models, diagrams, and providing real world application of volume.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/5/21

Previous Learning	*Current Learning*	*Future Learning*
• Connect to creating 3-D shapes. • Connect to learning to measure area using unit squares • Connect to applying the formulas to determine area and perimeter of rectangles.	• Connect to fluently multiplying multi-digit whole numbers.	• Connect to finding the volume of right rectangular prisms with fractional dimensions in the context of solving real-world and mathematical problems.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Use examples and problems that reflect the students' cultural backgrounds and experiences. For instance, use objects and structures from different cultures when discussing and measuring volume. • Highlight how different cultures measure and use volume in everyday life. For example, discuss traditional storage methods and containers used in various	• Start lessons by connecting to students' prior experiences with volume measurement in their everyday lives. Use familiar contexts, such as cooking, shopping, or building, to introduce new volume concepts. • Incorporate real-world problems that involve measuring volume in culturally relevant contexts. For example, have students	• Provide visual aids, such as charts, diagrams, and graphic organizers, that visually represent different volume units and measurement processes. • Offer bilingual resources, including vocabulary lists and explanations of key terms like "volume," "cubic centimeter," "cubic inch," "cubic foot," and "unit cube," in both English and students' home languages.

cultures. ● Validate and affirm ALL students background experience, culture, and language before, during, and after the instructional lesson.	calculate the volume of a culturally significant building or object. ● Build and bridge previous knowledge, skills, and lesson standards.	● Provide students with print out cards of targeted terminologies accompanied with an example.

Suggested Student Discourse Questions

- What does it mean to measure volume, and why is it important in mathematics and everyday life?
- How can counting unit cubes help us determine the volume of a three-dimensional object?
- What are some common units used to measure volume, such as cubic centimeters (cm^3), cubic inches (in^3), and cubic feet (ft^3)?
- Imagine scenarios where you need to measure volumes of irregular objects or containers. How would you approach these measurements?
- How can understanding volume measurement help us solve problems related to capacity, space, or quantity?
- In what professions or fields is volume measurement particularly important (e.g., construction, cooking, manufacturing)?

Cross-Curricular Connections

Science:

In fifth grade the NGSS states students should "describe and graph quantities such as area and volume to address scientific questions." Consider providing a connection for students to determine the volume of cubes or rectangular prisms as part of their investigation.

Art:

Drawing boxes is connected to developing the ability to indicate perspective in a drawing. Consider providing an opportunity for students to sketch various boxes with the same volume but different dimensions. Also, consider allowing students to make boxes to pack inside of larger boxes (measuring 1in. X 1in. X 1in. or 1 cm. X 1cm. X 1cm.). Have students predict how many boxes can fit inside of the premade larger boxes. Connect the number of boxes used to the volume of the box. Allowing students to cut, and construct boxes will help with their fine motor skills.

Career and Skill Connections

- Construction Worker
- Interior Designer
- Mathematics Teacher
- Civil Engineer
- Architect
- Interior Designer
- Construction Manager
- Land Surveyor
- Materials Scientist

Grade	CCSS Domain	CCSS Cluster
5	Measurement and Data	Geometric measurement: understand concepts of volume.

Cluster Standard: 5.MD.C.5

Standard	Standards for Mathematical Practice
Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. A. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole number products as volumes, e.g., to represent the associative property of multiplication. B. Apply the formulas V = l × w × h and V = b × h for rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. C. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems	● **SMP 1:** Make sense of problems and persevere in solving them. ● **SMP 4:** Model with mathematics. ● **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students understand that multiplying the length times the width of a right rectangular prism can be viewed as determining how many cubes would be in each layer if the prism were packed with or built up from unit cubes. They also learn that the height of the prism tells how many layers would fit in the prism.	● Identify a right rectangular prism. ● Multiply the three dimensions in any order to calculate volume (Commutative and Associative properties). ● Recognize that “B” refers to the area of the base. ● Recognize volume as additive. ● Develop a volume formula for a rectangle prism by comparing volume when filled with cubes to volume by multiplying the height by the area of the base, or when multiplying the edge lengths (l x w x h).

	• Apply the following formulas to right rectangular prisms having whole number edge lengths in the context of real-world mathematical problems: Volume = length x width x height or Volume = area of base x height. • Solve real world problems by decomposing a solid figure into two non-overlapping right rectangular prisms and adding their volumes. • Find the volume of a right rectangular prism with whole number side lengths by packing it with unit cubes.
DOK	**Blooms**
2-3	Understand, Apply, Analyze

Procedural and Conceptual Understanding and Application

Conceptual Understanding: Students develop a deep understanding that volume is related to multiplication and addition, recognizing that the volume of a right rectangular prism can be conceptualized by packing it with unit cubes and understanding that volume is the product of length, width, and height.

Procedural Skills and Fluency: Students demonstrate application of the volume formulas $V=l\times w\times h$ and $V=b\times h$ for rectangular prisms with whole-number dimensions, showcasing their ability to calculate volumes accurately and efficiently in both mathematical and real-world contexts.

Application: Students apply their understanding and skills by recognizing volume as additive, finding volumes of composite solid figures composed of non-overlapping right rectangular prisms, and solving real-world problems by adding the volumes of individual components.

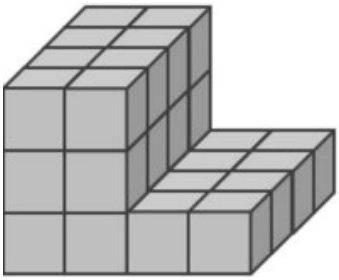
Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.
You can find the task above, as well as others aligned to this task,

Students will need two different color markers or crayons to complete this task.

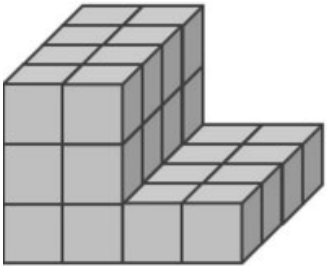
John was finding the volume of this figure. He decided to break it apart into two separate rectangular prisms. John found the volume of the solid below using this expression: (4 x 4 x 1) + (2 x 4 x 2).

Decompose the figure into two rectangular prisms and shade them in different colors to show one way John might have thought about it.



Phillis also broke this solid into two rectangular prisms, but she did it differently than John. She found the volume of the solid below using this expression: (2 x 4 x 3) + (2 x 4 x 1).

Decompose the figure into two rectangular prisms and shade them in different colors to show one way Phillis might have thought about it.



You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Confusing Volume with Area:** Students might mistakenly believe that volume and area are the same concept, leading to errors when attempting to calculate or understand volume in three-dimensional space.
- **Misunderstanding the Role of Unit Cubes:** Students may think that the purpose of using unit cubes to find volume is merely a visual aid or a way to count, rather than understanding that volume is the measure of space occupied in three dimensions.
- **Incorrect Application of Volume Formulas:** Students might apply the volume formulas $V=l\times w\times h$ and $V=b\times h$ inappropriately, such as using the wrong dimensions or misinterpreting which measurements correspond to length, width, and height.
- **Not Recognizing Volume as Additive:** Students may struggle to grasp the concept that volume is additive, leading to difficulties when finding volumes of composite shapes by adding volumes of non-overlapping parts.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning		
Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Provide clear and explicit instruction on how to calculate the volume of rectangular prisms using multiplication (length × width × height) and addition (adding the volumes of decomposed parts). • Model solving volume problems step-by-step using real-world examples, such as packing boxes or filling containers with water. **Universal Design of Learning (UDL)** Expose all students to the core standard using various UDL approaches. For example, kinesthetic presentation, visual, and auditory, including discovery and philosophical teaching styles.	• Conduct small group sessions for students who need additional support in understanding volume concepts. Use differentiated activities to address specific areas of difficulty and provide guided practice. • Use scaffolded activities that gradually increase in complexity. Start with simple one-step volume problems and progress to more complex multi-step problems. • Provide graphic organizers and step-by-step guides to help students organize their thoughts and approach multi-step problems systematically. **Universal Design of Learning (UDL)** • Differentiate ALL students accordingly utilizing pretest scores, informal teacher assessments, and anecdotal records.	• Provide intensive one-on-one instruction for students who require individualized support. Tailor lessons to the student's specific learning needs, using additional explanations, examples, and practice opportunities. • Develop personalized learning plans that include targeted goals, strategies, and supports for students with significant learning gaps. Collaborate with special education teachers, ELL specialists, and other support staff to address individual needs. • Incorporate specific accommodations and modifications as necessary, such as extended time, additional practice opportunities, or alternative assessments. **Universal Design of Learning (UDL)** • For students having misunderstanding of the lesson, provide targeted support such as one-to-one instruction, reteaching the concept, and/or utilize anchor charts, manipulatives, and teacher-student discourse.

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/5/21		
Previous Learning	*Current Learning*	*Future Learning*
• Connect to creating 3-D shapes. • Connect to learning to measure area using unit squares. • Connect to applying the formulas to determine area and perimeter of rectangles.	• Connect to fluently multiplying multi-digit whole numbers.	• Connect to finding the volume of right rectangular prisms with fractional dimensions in the context of solving real-world and mathematical problems.
Culturally and Linguistically Responsive Instruction		
Consider these questions as you plan for instruction that is culturally and linguistically responsive: • How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Use examples of volume in cultural contexts, such as traditional cooking recipes that involve measuring ingredients or cultural artifacts like containers and tools that have specific volumes. • Highlight how different cultures use volume in various practices, such as water storage, food preparation, or construction, to validate students' backgrounds and show the relevance of volume in their lives. • Encourage students to create projects that involve measuring and calculating volume based on their	• Begin lessons by connecting to students' prior knowledge and experiences with volume in their daily lives. Use familiar contexts, such as cooking, building, or playing with toys, to introduce the concept of volume. • Discuss how volume is used in various cultural practices and everyday activities to make learning more relatable and meaningful. • Incorporate real-world problems that involve calculating volume in contexts relevant to students' lives, such as filling a swimming pool, packing a box, or planning a garden.	• Provide visual aids, such as diagrams, charts, and graphic organizers, that visually represent volume measurements and calculations. • Offer bilingual resources, including vocabulary lists and explanations of key terms like "volume," "cubic units," "length," "width," "height," and "capacity," in both English and students' home languages. • Create a word wall with key volume vocabulary terms and their definitions, including visual representations. Update it regularly as new terms are

cultural experiences. For example, students could measure the volume of traditional cooking pots or containers used in their homes.		introduced. ● Use sentence frames to support students in articulating their understanding of volume calculations (e.g., "The volume of a box is found by multiplying..."). This helps students practice using academic language in context.

Suggested Student Discourse Questions

- How does the volume of a right rectangular prism relate to the operations of multiplication?
- Why is the volume calculated in this way equivalent to multiplying the edge lengths of the prism?
- Explain the formulas V = l × w × h and V = b × h for finding the volume of rectangular prisms. How are these formulas derived?
- Explain why volume is considered additive. How do we find the volume of composite figures composed of two non-overlapping rectangular prisms?
- Can you demonstrate this technique to solve a real-world problem involving additive volume?
- What skills and concepts are necessary to accurately apply volume formulas and solve volume-related problems in various contexts?

Cross-Curricular Connections

Science:

In fifth grade the NGSS states students should “describe and graph quantities such as area and volume to address scientific questions.” Consider providing a connection for students to determine the volume of cubes or rectangular prisms as part of their investigation.

Art:

Drawing boxes is connected to developing the ability to indicate perspective in a drawing. Consider providing an opportunity for students to sketch various boxes with the same volume but different dimensions. Also, consider allowing students to make boxes to pack inside of larger boxes (measuring 1in. X 1in. X 1in. or 1 cm. X 1cm. X 1cm.). Have students predict how many boxes can fit inside of the premade larger boxes. Connect the number of boxes used to the volume of the box. Allowing students to cut, and construct boxes will help with their fine motor skills.

Career and Skill Connections

- Baker
- Barista
- Educator
- Civil Engineer
- Architect

- Construction Manager
- Interior Designer
- Urban Planner
- Materials Scientist

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Procedural and Conceptual Understanding and Application
  - Sample Assessment Items
  - Common Misconceptions
  - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
  - Vertical Alignment
  - Culturally and Linguistically Responsive Instruction (CLRI)
  - Suggested Student Discourse Questions
  - Cross-Curricular and Career and Skill Connections
- A Student Discourse Guide
- Planning for a Multi-Layer System of Support (MLSS) and Universal Design for Learning (UDL) for behavioral and social and emotional supports

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Understand the place value system
  - 5.NBT.A.1
  - 5.NBT.A.2
  - 5.NBT.A.3
  - 5.NBT.A.4
- Perform operations with multi-digit whole numbers and with decimals to hundredths
  - 5.NBT.B.5
  - 5.NBT.B.6
  - 5.NBT.B.7

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)

Grade	CCSS Domain	CCSS Cluster
5	Numbers and Operations in Base Ten	Understand the place value system.

Cluster Standard: 5.NBT.A.1

Standard	Standards for Mathematical Practice
Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students extend their understanding of the base-ten system to the relationship between adjacent places, how numbers compare, and how numbers round for decimals to thousandths. This standard calls for students to reason about the magnitude of numbers. Students should work with the idea that the tens place is ten times as much as the ones place, and the ones place is 1⁄10th the size of the tens place. Based on the base-10 system, digits to the left are 10 times as great as digits to the right; likewise, digits to the right are 1⁄10th of digits to the left.	• Explain that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1⁄10 of what it represents in the place to its left.
DOK	**Blooms**
1	Understand

Procedural and Conceptual Understanding and Application

Conceptual Understanding: Students gain a strong understanding of place value in multi-digit numbers, understanding that each digit's position represents a value that is ten times larger than the value of the digit to its right and one-tenth of the value of the digit to its left.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

a. Kipton has a digital scale. He puts a marshmallow on the scale and it reads 7.2 grams. How much would you expect 10 marshmallows to weigh? Why?
b. Kipton takes the marshmallows off the scale. He then puts on 10 jellybeans and then the scale reads 12.0 grams. How much would you expect 1 jellybean to weigh? Why?
c. Kipton then takes off the jellybeans and puts on 10 brand-new pink erasers. The scale reads 312.4 grams. How much would you expect 1,000 pink erasers to weigh? Why?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Equal Value across Places:** Students may incorrectly believe that digits in different places of a number have the same value, disregarding the increasing or decreasing value based on their position.
- **Confusion with Absolute Values:** Students might think that a digit's value is fixed and doesn't change based on its position within a multi-digit number.
- **Reversing the Relationship:** Students could mistakenly reverse the understanding of place value, thinking that a digit represents 1/10 of its value in the place to its right and 10 times its value in the place to its left.
- **Neglecting Decimal Place Value:** Students may overlook the concept of decimal place value and apply the same understanding of multiplying by 10 and dividing by 10 solely to whole numbers, leading to difficulties in comprehending decimal numbers.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
• Provide visual models through multimedia tools, hands on manipulatives to enhance learning of multi-digit numbers, and that a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. **Universal Design for Learning (UDL)** • Allow for representation by using student perception to provide different ways to	• Provide targeted small group instruction for students that need additional support, level grouping through diversity of participants to learn from their perspectives, through lived experiences and feedback. Foster collaboration within the community of learners. **Universal Design for Learning (UDL)** • Allow for representation by using student perception to provide different ways to	• Provide students who require intensive support, one on one instruction, build confidence with graduated levels of support and guide appropriate goal setting development, clarify vocabulary and symbols. **Universal Design for Learning (UDL)** • Allow for representation by using student perception to provide different ways to present or display information being taught, for visual information such as physical

present or display information being taught, for visual information such as physical models. Use clarity in language and symbols to provide vocabulary.	present or display information being taught, for visual information such as physical models. Use clarity in language and symbols to provide vocabulary.	models. Use clarity in language and symbols to provide vocabulary.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/5/22

Previous Learning	*Current Learning*	*Future Learning*
• Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division. • Express a fraction with denominator 10 as an equivalent fraction with denominator 100 and use this technique to add two fractions with respective denominators 10 and 100. * For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100. • Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram. • Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of	• Fluently multiply multi digit whole numbers using the standard algorithm. • Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. • Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	• Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. • Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. • Write and evaluate numerical expressions involving whole-number exponents.

comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.		

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Use examples of numeracy from various cultures to illustrate place value. For instance, explore how different number systems, like Roman numerals, Mayan numerals, or Chinese counting rods, represent values. • Discuss the historical and cultural significance of different number systems to validate students' cultural backgrounds and show the diversity in mathematical concepts. • Encourage students to create projects that involve representing numbers using place value concepts based on cultural numeracy practices. For instance, students can research and present how different cultures historically represented large numbers.	• Begin lessons by connecting to students' prior knowledge and experiences with numbers and counting. Use familiar contexts, such as everyday transactions, cooking measurements, or sports scores, to introduce place value concepts. • Relate the concept of place value to students' real-life experiences and cultural backgrounds to make the learning process more relatable and meaningful. • Incorporate real-world problems that involve place value, such as comparing prices, estimating large quantities, or understanding population statistics in different countries.	• Provide visual aids, such as place value charts, diagrams, and graphic organizers, that visually represent different place value concepts and positions (e.g., ones, tens, hundreds, thousands). • Offer bilingual resources, including vocabulary lists and explanations of key terms like "place value," "digit," "position," "expanded form," and "base-ten," in both English and students' home languages.

Suggested Student Discourse Questions

- What does place value mean in a multi-digit number and how does the position of a digit affect its value within a number?
- Explain the concept that a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left and can you provide examples to illustrate this relationship?
- How do we determine the value of a digit based on its position in a number (e.g., units, tens, hundreds)?
- What are the roles of place value positions like tenths and hundredths in decimal numbers?
- How does the concept of place value extend to decimal numbers and explain how to interpret and compare decimal numbers based on their place value positions.
- How does understanding place value impact our ability to manipulate and interpret numbers effectively?

Cross-Curricular Connections

STEM:

Using given or collected data, round numbers to a given whole number or decimal place to solve real-world problems.

Science:

Provide students opportunities to take precise measurements. Have students round these measurements to the nearest tenth, hundredth, or thousandths.

Career and Skill Connections

- Accountant
- Data Analyst
- Software Developer
- Engineer
- Financial Analyst

Grade	CCSS Domain	CCSS Cluster
5	Numbers and Operations in Base Ten	Understand the place value system.

Cluster Standard: 5.NBT.A.2

Standard	Standards for Mathematical Practice
Explain patterns in the number of zeros of the product when multiplying a number by powers of 10 and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	• **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Multiplying by a power of 10 shifts the digits of a whole number or decimal that many places to the left. Patterns in the number of 0s in products of a whole number and a power of 10 and the location of the decimal point in products of decimals with powers of 10 can be explained in terms of place value. Because students have developed their understanding of and computations with decimals in terms of multiples rather than powers, connecting the terminology of multiples with that of powers affords connections between understanding of multiplication and exponentiation. (Progressions for the CCSSM, Number and Operation in Base Ten, CCSS Writing Team, April 2011, page 16) This standard includes multiplying by multiples of 10 and powers of 10, including 102 which is 10 x 10=100, and 103 which is 10 x 10 x 10=1,000. Students should have experiences working with connecting the pattern of the number of zeros in the product when you multiply by powers of 10. Students should reason that the exponent above the 10 indicates how many places the decimal point is moving (not just that the decimal point is moving but that you are multiplying or making the number 10 times greater three times) when you multiply by a power of 10. Since we are multiplying by a power of 10 the decimal point moves to the right.	• Represent powers of 10 using whole number exponents. • Translate between powers of 10 written as 10 raised to a whole number exponent, the expanded form, and standard notation. • Explain the patterns in the number of zeros of the product when multiplying a number by powers of 10. • Explain the relationship of the placement of the decimal point when a decimal is multiplied or divided by a power of 10.

DOK	Blooms
1	Understand, Apply

Procedural and Conceptual Understanding and Application

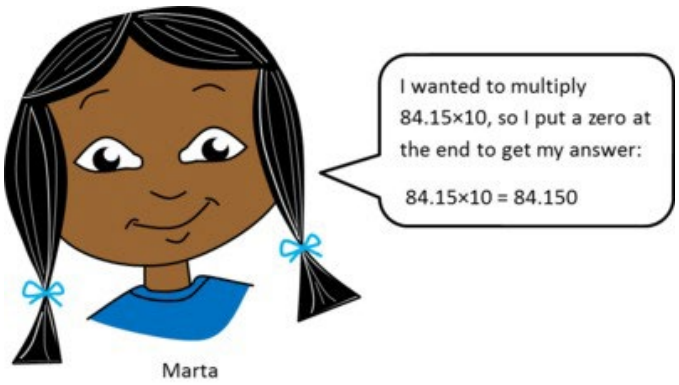
Conceptual Understanding: Students gain a thorough understanding of how multiplication or division by powers of 10 affects the number of zeros in the result and the placement of the decimal point, recognizing the predictable changes based on the exponent used to represent powers of 10.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Marta made an error while finding the product 84.15 × 10

.



In your own words, explain Marta's misunderstanding. Please explain what she should do to get the correct answer and include the correct answer in your response.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Incorrect Multiplication with Powers of 10:** Students may mistakenly believe that multiplying any number by a power of 10 always results in adding zeros to the end of the number, regardless of the number's value or the exponent used.
- **Decimal Point Movement Misunderstanding:** Students might misunderstand how the placement of the decimal point changes when multiplying or dividing decimals by powers of 10, leading to errors in predicting the correct position of the decimal in the product or quotient.
- **Inconsistent Use of Exponents:** Students could misuse whole-number exponents when denoting powers of

10, leading to confusion about the magnitude of the multiplication or division operation and its effect on the number of zeros and decimal point placement.

- **Lack of Generalization:** Students may struggle to generalize the patterns observed in multiplying or dividing by powers of 10 to different scenarios, such as applying the concepts to numbers with varying digits or decimal placements.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Provide clear explanations and demonstrations of the patterns in the number of zeros and the placement of the decimal point when multiplying or dividing by powers of 10. Use visual aids such as place value charts and number lines to illustrate these concepts. • Model the process using multiple examples and think-aloud strategies to make your thought process visible. • Use visual aids such as place value charts, base-ten blocks, and number lines to help students understand how the digits shift when multiplying or dividing by powers of 10. • Provide manipulatives such as digit cards and decimal grids for hands-on practice in shifting digits and understanding the impact on place value. **Universal Design for Learning (UDL)** • Allow for representation by using student perception to provide different ways to present or display information being taught, for visual information such as physical models. Use clarity in language and symbols to	• Conduct small group sessions for students who need additional support in understanding multiplication as scaling. Use differentiated activities to address specific areas of difficulty and provide guided practice. • Use scaffolded activities that gradually increase in complexity. • Provide graphic organizers and step-by-step guides to help students organize their thoughts and approach problems systematically. **Universal Design for Learning (UDL)** • Allow for representation by using student perception to provide different ways to present or display information being taught, for visual information such as physical models. Use clarity in language and symbols to provide vocabulary.	• Provide students who require intensive support, one on one instruction, build confidence with graduated levels of support and guide appropriate goal setting development, clarify vocabulary and symbols. • Develop personalized learning plans that include targeted goals, strategies, and supports for students with significant learning gaps. Collaborate with special education teachers, ELL specialists, and other support staff to address individual needs. • Incorporate specific accommodations and modifications as necessary, such as extended time, additional practice opportunities, or alternative assessments. **Universal Design for Learning (UDL)** • Allow for representation by using student perception to provide different ways to present or display information being taught, for visual information such as physical models. Use clarity in language and symbols to provide vocabulary.

provide vocabulary.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/5/22

Previous Learning	*Current Learning*	*Future Learning*
• Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division. • Express a fraction with denominator 10 as an equivalent fraction with denominator 100 and use this technique to add two fractions with respective denominators 10 and 100. * For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100. • Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram. • Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	• Fluently multiply multi digit whole numbers using the standard algorithm. • Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. • Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	• Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. • Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. • Write and evaluate numerical expressions involving whole-number exponents.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Use examples from different cultures to explain place value concepts, such as traditional counting systems, trade systems, or historical numeration methods. • Highlight contributions from various cultures to the development of the number system, such as the use of zero from ancient India or the decimal system from the Middle East, to validate students' cultural backgrounds and show the relevance of mathematical concepts.	• Begin lessons by connecting to students' prior knowledge and experiences with numbers and place value in their homes and communities. Use familiar contexts, such as money, time, or measurements, to introduce new place value concepts. • Discuss how numbers and place value are used in various cultural practices and everyday activities to make learning more relatable and meaningful.	• Provide visual aids, such as place value charts, diagrams, and graphic organizers, that visually represent different place value concepts and operations. • Offer bilingual resources, including vocabulary lists and explanations of key terms like "place value," "digit," "decimal," "multiplication," and "division," in both English and students' home languages.

Suggested Student Discourse Questions

- What does it mean to multiply or divide a number by powers of 10 and how does whole-number exponents denote different powers of 10?
- Explain the patterns observed in the number of zeros when multiplying a number by powers of 10 and can you predict how many zeros will be in the product when multiplying by 10, 100, 1000, and so on?
- Describe the pattern in the placement of the decimal point when a decimal is multiplied or divided by a power of 10 and how does the position of the decimal point change?
- How can we use whole-number exponents to represent different powers of 10 (e.g., 10^2, 10^3, 10^4)?
- How can recognizing patterns in decimal movements and zero placement simplify calculations and problem-solving?
- How does this understanding enhance our ability to perform calculations efficiently and accurately?

Cross-Curricular Connections

STEM:

Using given or collected data, round numbers to a given whole number or decimal place to solve real-world problems.

Science:

Provide students opportunities to take precise measurements. Have students round these measurements to the nearest tenth, hundredth, or thousandths.

Career and Skill Connections

- Financial Analyst
- Engineers
- Data Scientist
- Physicist
- Accountant
- Technical Writer

Grade	CCSS Domain	CCSS Cluster
5	Numbers and Operations in Base Ten	Understand the place value system.

Cluster Standard: 5.NBT.A.3

Standard	Standards for Mathematical Practice
Read, write, and compare decimals to thousandths A. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). B. Compare two decimals to the thousandths based on the meanings of the digits in each place, using >, =, < symbols to record results of comparisons.	● **SMP 5:** Use appropriate tools strategically.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This standard reference expanded the form of decimals with fractions included and compared decimals builds on work from fourth grade. This standard refers to rounding. Students should go beyond simply applying an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line to support their work with rounding. Students should use benchmark numbers to support this work. Benchmarks are convenient numbers for comparing and rounding numbers. 0., 0.5, 1, 1.5 are examples of benchmark numbers.	● Read and write decimal to thousandths using base-ten numerals, number names, and expanded form. ● Use >, =, and < symbols to record the results of comparisons between decimals. ● Compare two decimals to the thousandths, based on the place value of each digit. ● Use knowledge of base ten and place value to round decimals to any place.
DOK	**Blooms**
1	Understand, Analyze

Procedural and Conceptual Understanding and Application

Conceptual Understanding: Students develop a solid understanding of decimals to the thousandths, recognizing how each digit's place value (units, tenths, hundredths, thousandths) contributes to the overall value of the decimal

number and its representation in various forms (base-ten numerals, number names, and expanded form).

Procedural Skills and Fluency: Students exhibit proficiency in reading, writing, and comparing decimals up to the thousandths using different formats (base-ten numerals, number names, expanded form) and accurately use symbols (> = <) to compare the values of two decimals based on their respective place values.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Isaiah is thinking of the number 9.52 in his head. Decide whether each of these has the same value as 9.52 and discuss your reasoning.

a. Nine and fifty-two tenths
b. 9 + 0.5 + 0.02
c. 9 ones + 5 tenths + 2 hundredths
d. (9×1) + (5× 1/10) + (2×1/100)
e. 952 tenths
f. 952 hundredths

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Ignoring Place Value Significance:** Students may mistakenly believe that the digits in decimal numbers contribute equally to the overall value, without understanding the critical role of place value in determining the magnitude of each digit.
- **Misunderstanding Expanded Form:** Students might think that expanded form representations of decimals involve simple addition of the product of digits and place values, without grasping the hierarchical structure and significance of each term (units, tenths, hundredths, thousandths).
- **Confusion with Number Names:** Students could struggle with correctly writing decimal numbers in number names (words), leading to errors in pronunciation and comprehension of the value represented by each digit.

- **Incorrect Comparison of Decimals:** Students may compare decimals based solely on the total number of digits or without considering the significance of each place value position, resulting in inaccurate use of comparison symbols (> = <) and incorrect conclusions about the relative values of decimals.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
• Use number lines and place value charts to visually represent decimal values and their place values. This helps students see the relative size of decimals and understand the concept of tenths, hundredths, and thousandths. • Use real-world scenarios, such as money, measurements, and data, to make decimal concepts relevant and meaningful. For instance, comparing prices, distances, or times can help students understand decimals in context. **Universal Design for Learning (UDL)** • Allow for representation by using student perception to provide different ways to present or display information being taught, for visual information such as physical models. Use clarity in language and symbols to provide vocabulary.	• Conduct small group sessions to provide additional support for students who need help with specific aspects of decimals. Tailor instruction to address their particular misconceptions or difficulties. • Start with simpler tasks and gradually increase the complexity. For example, begin with comparing decimals to the tenths place before moving to hundredths and thousandths. **Universal Design for Learning (UDL)** • Allow for representation by using student perception to provide different ways to present or display information being taught, for visual information such as physical models. Use clarity in language and symbols to provide vocabulary.	• Provide intensive one-on-one instruction for students who need significant support. Tailor lessons to address their specific learning gaps and needs. • Develop personalized learning plans with specific, measurable goals for students with significant learning gaps. Collaborate with special education teachers, ELL specialists, and other support staff to ensure comprehensive support. **Universal Design for Learning (UDL)** • Allow for representation by using student perception to provide different ways to present or display information being taught, for visual information such as physical models. Use clarity in language and symbols to provide vocabulary.

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/5/22		
Previous Learning	*Current Learning*	*Future Learning*
• Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division. • Express a fraction with denominator 10 as an equivalent fraction with denominator 100 and use this technique to add two fractions with respective denominators 10 and 100. * For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100. • Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram. • Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	• Fluently multiply multi digit whole numbers using the standard algorithm. • Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. • Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	• Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. • Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. • Write and evaluate numerical expressions involving whole-number exponents.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Use examples of decimals in various cultural contexts, such as currency from different countries or measurements in recipes from diverse cuisines. This helps validate students' cultural backgrounds and shows the relevance of decimals in their daily lives. • Highlight the use of decimals in traditional cultural activities, such as market transactions or measurement in traditional crafts, to show the practical applications of decimals.	• Begin lessons by connecting to students' prior knowledge and experiences with numbers and measurements. Use familiar contexts, such as comparing prices in different currencies or measuring ingredients in cooking, to introduce decimals. • Discuss how decimals are used in everyday life, such as in sports statistics or fuel prices, to make the learning process more relatable and meaningful. • Incorporate real-world problems that involve reading, writing, and comparing decimals. For example, ask students to compare the prices of items from different countries or to calculate the total cost of a shopping list in decimals.	• Provide visual aids such as number lines, place value charts, and decimal grids to help students understand the concept of decimals. Use color-coding to distinguish between whole numbers and decimal fractions. • Offer bilingual resources, including vocabulary lists and explanations of key terms like "decimal," "tenths," "hundredths," and "thousandths," in both English and students' home languages.

Suggested Student Discourse Questions

- What does it mean to write decimals to the thousandths place and how do we represent decimals using base-ten numerals, number names, and expanded form?
- Demonstrate how to write a decimal like 347.392 using base-ten numerals, number names, and expanded form. Why is it important to be able to express decimals in various representations?
- Explain the significance of each digit's place value in a decimal to the thousandths place and how does the value of a digit change as it moves to different places in a decimal?

- How do we compare two decimals to the thousandths place?
- What role do the digits in each place (tenths, hundredths, thousandths) play when comparing decimals using >, =, < symbols?
- Use the expanded form of decimals to facilitate comparisons between two numbers (e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000)) and how does breaking down a decimal into its place value components help in comparing decimals?
- How does comparing decimals contribute to our understanding of numerical relationships and mathematical reasoning?

Cross-Curricular Connections

STEM:

Using given or collected data, round numbers to a given whole number or decimal place to solve real-world problems.

Science:

Provide students opportunities to take precise measurements. Have students round these measurements to the nearest tenth, hundredth, or thousandths.

Career and Skill Connections

- Financial Analyst
- Accountant
- Data Scientist
- Market Research Analyst
- Data Interpreter

Grade	CCSS Domain	CCSS Cluster
5	Numbers and Operations in Base Ten	Understand the place value system.

Cluster Standard: 5.NBT.A.4

Standard	Standards for Mathematical Practice
Use place value understanding to round decimals to any place.	• **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students have a deep understanding of place value and number sense by explaining and giving reasons about the answers they get when they round. Students should have numerous experiences using a number line to support their work with rounding. When rounding a decimal to a given place, students may identify the two possible answers, and use their understanding of place value to compare the given number to the possible answers.	• Explain why the value of digits depends on its place. • Round decimals to any place.
DOK	**Blooms**

Procedural and Conceptual Understanding and Application

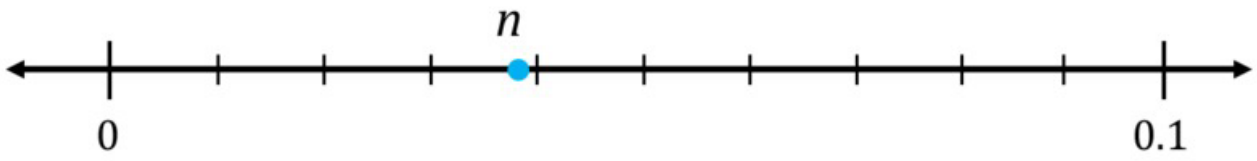
Conceptual Understanding: Students develop a solid understanding of rounding decimals by applying place value concepts, recognizing how the value of digits in different place positions determines whether to round up or down.

Procedural Skills and Fluency: Students exhibit competence and fluency by effectively applying their understanding of place value to round decimals to specified place values, such as tenths, hundredths, or thousandths, thereby demonstrating accuracy and efficiency in their calculations.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

A number *n* is shown on the number line.



1. The tick marks are evenly spaced. Label them.
2. What is *n* rounded to the nearest hundredth?
3. What is *n* rounded to the nearest tenth?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Rounding to the Nearest Whole Number:** Students may mistakenly believe that rounding decimals always involves rounding to the nearest whole number, disregarding the need to round to specific decimal places.
- **Rounding Based on the First Digit:** Students might think that rounding is determined solely by the first digit after the decimal point, overlooking the significance of digits in subsequent decimal places.
- **Rounding to the Closest Neighbor:** Students could misunderstand rounding as always rounding to the closest neighboring number, neglecting the rules for rounding up or down based on the value of the digit in the specified place.
- **Inconsistent Rounding Rules:** Students may apply inconsistent rounding rules, such as always rounding down or always rounding up, without considering the value of the digits following the specified rounding place.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Use visual aids such as place value charts, number lines, and rounding posters to help students visualize the rounding process. • Provide manipulatives like base-ten blocks and decimal grids to give students a hands-on understanding of place value. • Model the process of rounding decimals step-by-step using clear and explicit	• Provide targeted small group instruction for students who need additional support in understanding rounding decimals. Focus on specific areas of difficulty and use differentiated activities to address individual learning needs. • Use scaffolded activities that gradually increase in complexity. Start with	• Provide intensive one-on-one instruction for students who need significant support. Tailor lessons to address their specific learning gaps and needs. • Develop personalized learning plans with specific, measurable goals for students with significant learning gaps. Collaborate with special education teachers, ELL specialists, and other support

language. Demonstrate how to identify the place value to which the number is being rounded and how to determine whether to round up or down. **Universal Design for Learning (UDL)** • Allow for representation by using student perception to provide different ways to present or display information being taught, for visual information such as physical models. Use clarity in language and symbols to provide vocabulary.	rounding whole numbers and then progress to rounding decimals to different place values. Provide graphic organizers and checklists to help students organize their work and approach. **Universal Design for Learning (UDL)** • Allow for representation by using student perception to provide different ways to present or display information being taught, for visual information such as physical models. Use clarity in language and symbols to provide vocabulary.	staff to ensure comprehensive support. **Universal Design for Learning (UDL)** • Allow for representation by using student perception to provide different ways to present or display information being taught, for visual information such as physical models. Use clarity in language and symbols to provide vocabulary.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/5/22

Previous Learning	*Current Learning*	*Future Learning*
• Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division. • Express a fraction with denominator 10 as an equivalent fraction with denominator 100 and use this technique to add two fractions with respective denominators 10 and 100. * For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100. • Use decimal notation for	• Fluently multiply multi digit whole numbers using the standard algorithm. • Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. • Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and	• Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. • Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. • Write and evaluate numerical expressions involving whole-number exponents.

fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram. • Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Use real-life examples that are culturally relevant to illustrate rounding decimals. For instance, discuss prices in different currencies, distances in travel, or measurements in cooking from various cultures. This can make the concept more relatable and meaningful to students. • Highlight cultural practices that involve numerical precision, such as traditional crafts, trades, or recipes that require precise measurements.	• Begin lessons by connecting to students' prior knowledge and experiences with whole numbers and rounding. Discuss how they use rounding in their daily lives, such as estimating time, money, or quantities. • Use familiar contexts, such as rounding prices at a store or distances on a map, to introduce the concept of rounding decimals.	• Provide visual aids such as number lines, place value charts, and rounding guides to visually represent the concept of rounding decimals. • Offer bilingual resources, including vocabulary lists and explanations of key terms like "place value," "rounding," "decimal," "nearest whole number," "tenth," and "hundredth," in both English and students' home languages.

Suggested Student Discourse Questions
• What does place value mean when working with decimals? How do the positions of digits (tenths, hundredths, thousandths, etc.) determine their value in a decimal number? • Explain the concept of rounding decimals. Why do we round numbers, and in what situations might we need to round decimals? • How can we use place value understanding to round decimals to different place values (e.g., tenths, hundredths, thousandths)? • When rounding decimals, which digit do we focus on to determine the rounding rule? • Compare and contrast the concepts of precise decimal values and rounded estimates. • When is it appropriate to use rounded numbers instead of exact decimal values?
Cross-Curricular Connections
STEM: Using given or collected data, round numbers to a given whole number or decimal place to solve real-world problems. **Science:** Provide students opportunities to take precise measurements. Have students round these measurements to the nearest tenth, hundredth, or thousandths.
Career and Skill Connections
• Statistician • Market Analyst • Accountant • Data Scientist • Financial Analyst

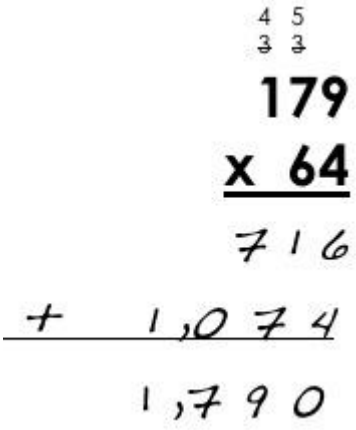
Grade	*CCSS Domain*	*CCSS Cluster*
5	Numbers and Operations in Base Ten	Perform operations with multi-digit whole numbers and with decimals to hundredths.

Cluster Standard: 5.NBT.B.5

Standard	Standards for Mathematical Practice
Fluently multiply multi-digit whole numbers using the standard algorithm.	• **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
In fifth grade, students fluently compute products of whole numbers using the standard algorithm. Underlying this algorithm are the properties of operations and the base-ten system. Division strategies in fifth grade involve breaking the dividend apart into like base-ten units and applying the distributive property to find the quotient place by place, starting from the highest place. (Division can also be viewed as finding an unknown factor: the dividend is the product, the divisor is the known factor, and the quotient is the unknown factor.) Students continue their fourth-grade work on division, extending it to computation of whole number quotients with dividends of up to four digits and two-digit divisors. Estimation becomes relevant when extending to two-digit divisors. Even if students round appropriately, the resulting estimate may need to be adjusted.	• Multiply multi-digit whole numbers using the standard algorithm.
DOK	**Blooms**
1	Apply
Procedural and Conceptual Understanding and Application	
Procedural Skills and Fluency: Students exhibit proficiency and accuracy in applying the standard algorithm to multiply multi-digit whole numbers.	
Assessment Items	

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

This is Elmer's work on a multiplication problem:



a. Use estimation to explain why Elmer's answer is not reasonable.

b. What error do you think Elmer made? Why do you think he made that error?

c. Find 179 × 64 using a correct version of Elmer's method. Then show another way of doing it to help Elmer see why your answer is correct.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Focusing Only on Speed:** Students may mistakenly believe that fluency in multiplication means solely focusing on speed without emphasizing accuracy or understanding of the algorithm.
- **Rote Memorization of Steps:** Students might think that fluency requires memorizing steps of the standard algorithm without understanding the underlying mathematical concepts of multiplication.
- I**nflexible Algorithm Use:** Students could misunderstand fluency as using only one specific method (standard algorithm) for all multiplication tasks, without recognizing alternative strategies or approaches.
- **Equating Fluency with Automaticity:** Students may equate fluency with automaticity, assuming that quick recall of multiplication facts alone constitutes fluency, rather than a combination of efficiency, accuracy, and

understanding.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning		
Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Use visual aids such as place value charts, multiplication grids, and step-by-step algorithm posters to help students understand the multiplication process. • Provide manipulatives like base-ten blocks or place value disks to allow hands-on exploration of multi-digit multiplication. • Provide explicit instruction and modeling of the standard algorithm for multi-digit multiplication. Use clear, step-by-step explanations and visual examples to demonstrate the process. • **Universal Design for Learning (UDL)** • Allow for representation by using student perception to provide different ways to present or display information being taught, for visual information such as physical models. Use clarity in language and symbols to provide vocabulary.	• Provide targeted small group instruction for students who need additional support. Focus on specific areas of difficulty and use differentiated activities to address individual learning needs. • Use scaffolded practice activities that gradually increase in complexity. Start with simpler problems and build up to more complex multi-digit multiplication tasks. Provide checklists or graphic organizers to help students organize their work. **Universal Design for Learning (UDL)** • Allow for representation by using student perception to provide different ways to present or display information being taught, for visual information such as physical models. Use clarity in language and symbols to provide vocabulary.	• Provide intensive one-on-one instruction for students who require individualized support. Tailor lessons to the student's specific learning needs, using additional explanations, examples, and practice opportunities. • Develop personalized learning plans that include targeted goals, strategies, and supports for students with significant learning gaps. Collaborate with special education teachers, ELL specialists, and other support staff to address individual needs. Incorporate specific accommodations and modifications as necessary. **Universal Design for Learning (UDL)** • Allow for representation by using student perception to provide different ways to present or display information being taught, for visual information such as physical models. Use clarity in language and symbols to provide vocabulary.
Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/5/22		
Previous Learning	*Current Learning*	*Future Learning*
• Connect to using place value understanding and properties of operations to perform	• Connect to understanding the place value concept that the number to the left is 10	• Connect to fluently adding, subtracting, multiplying, and dividing decimals using the

multi-digit arithmetic.	times larger and the number to the right is 10 times smaller, will use exponents to express powers of 10 and can understand the patterns of zeros and decimal placement related to powers of 10. ● Connect to applying and extend their previous understandings of multiplication and division to multiply and divide fractions. ● Connect to converting customary and metric measurement units within a given measurement system.	standard algorithm. ● Connect to recognizing that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. ● Connect to read, write, and compare decimals to thousandths.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
● Incorporate examples of multiplication from various cultural contexts, such as traditional methods of multiplication used in different cultures or historical contexts. Validate and affirm students' cultural backgrounds by acknowledging different approaches to solving multiplication problems. ● Highlight the relevance of multiplication in students' everyday lives, including cultural contexts. For	● Build on students' prior knowledge of multiplication strategies. Start with concrete manipulatives or visual representations to reinforce the concept, then gradually transition to the standard algorithm. Relate new learning to students' existing understanding and cultural experiences. ● Provide opportunities for students to explore how multiplication is used in cultural contexts relevant to their backgrounds. For	● Use visual aids such as posters, charts, or diagrams to illustrate key multiplication terms like "product," "multiplicand," "multiplier," and "algorithm." Include visuals that represent these terms in students' home languages, if applicable. ● Provide sentence frames or language supports to help students express their understanding of multiplication concepts. Encourage students to explain their thinking in their home

example, relate multiplication to traditional practices like weaving patterns, cooking recipes, or financial transactions in different cultures.	example, discuss how traditional crafts, agricultural practices, or cultural celebrations involve multiplication concepts.	language and provide translations or explanations as needed.

Suggested Student Discourse Questions

- What is the standard algorithm for multiplying multi-digit whole numbers?
- How does the standard algorithm differ from other methods of multiplication, such as using arrays or partial products?
- Can you explain the step-by-step process of the standard algorithm for multiplying two multi-digit numbers?
- What are the key components of the algorithm, and why is each step important?
- How does place value play a role in the standard multiplication algorithm and why do we align numbers by place value when using this method?
- Why is fluency in using the standard algorithm important for multiplying large numbers?
- How does practicing this method improve both speed and accuracy in multiplication?

Cross-Curricular Connections

STEM:

Using given or collected data, round numbers to a given whole number or decimal place to solve real-world problems.

Career and Skill Connections

- Financial Analyst
- Engineer
- Data Analyst
- Economist
- Inventory Manager

Grade	CCSS Domain	CCSS Cluster
5	Numbers and Operations in Base Ten	Perform operations with multi-digit whole numbers and with decimals to hundredths.

Cluster Standard: 5.NBT.B.6

Standard	Standards for Mathematical Practice
Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	• **SMP 1:** Make sense of problems and persevere in solving them.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This standard references various strategies for division. Division problems can include remainders. Even though this standard leads more towards computation, the connection to story contexts is critical. Make sure students are exposed to problems where the divisor is the number of groups and where the divisor is the size of the groups. In fourth grade, students' experiences with division were limited to dividing by one-digit divisors. This standard extends students' prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a "familiar" number, a student might decompose the dividend using place value.	• Explain calculations using equations or models that represent understanding of division. • Find whole number quotients of whole numbers with four-digit dividends and two-digit divisors. • Use multiple strategies to solve division problems.
DOK	**Blooms**
1	Understand, Apply

Procedural and Conceptual Understanding and Application

Conceptual Understanding: Students develop a deep understanding of division by finding whole-number quotients of up to four-digit dividends and two-digit divisors, utilizing place value concepts, properties of operations, and connections between multiplication and division to solve problems.

Procedural Skills and Fluency: Students proficiently calculate whole-number quotients using strategies based on place value, properties of operations, and the relationship between multiplication and division, illustrating and explaining their calculations through equations, rectangular arrays, and area models.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

What time was it 2011 minutes after the beginning of January 1, 2011?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Relying Solely on Memorization:** Students may mistakenly believe that division involves memorizing procedures without understanding the underlying concepts of place value, properties of operations, or the relationship between multiplication and division.
- **Ignoring Strategic Approaches:** Students might think that division is only about getting the correct answer quickly, overlooking the importance of using strategic approaches based on place value and mathematical properties to solve problems.
- **Overlooking Visual Representations:** Students could misunderstand division as a purely abstract process, neglecting the value of using visual representations like equations, rectangular arrays, or area models to illustrate and explain calculations.
- **Focusing Only on Correct Answers:** Students may focus solely on finding the right quotient without considering the process or reasoning behind the division, missing the opportunity to deepen their understanding of division concepts and strategies.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
• Provide visual models through multimedia tools, hands on manipulatives to enhance learning of whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors. **Universal Design of Learning (UDL)** • Use representation by	• Provide targeted small group instruction for students that need additional support, level grouping through diversity of participants to learn from their perspectives, through lived experiences and feedback. Foster collaboration within the community of learners.	• Provide students who require intensive support, one on one instruction, build confidence with graduated levels of support and guide appropriate goal setting development, clarify vocabulary and symbols. **Universal Design of Learning (UDL)** • Use representation by

providing real world, community relevant application of math projects, and science investigations.	**Universal Design of Learning (UDL)** • Use representation by providing real world, community relevant application of math projects, and science investigations.	providing real world, community relevant application of math projects, and science investigations.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/5/22

Previous Learning	*Current Learning*	*Future Learning*
• Connect to using place value understanding and properties of operations to perform multi-digit arithmetic.	• Connect to understanding the place value concept that the number to the left is 10 times larger and the number to the right is 10 times smaller, will use exponents to express powers of 10 and can understand the patterns of zeros and decimal placement related to powers of 10. • Connect to applying and extend their previous understandings of multiplication and division to multiply and divide fractions. • Connect to converting customary and metric measurement units within a given measurement system.	• Connect to fluently adding, subtracting, multiplying, and dividing decimals using the standard algorithm. • Connect to recognizing that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. • Connect to read, write, and compare decimals to thousandths.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Incorporate examples of problems that make connections to their understanding of quotients of whole numbers and their relation to dividends and divisors.	• By scaffolding instruction you are building upon students' prior knowledge, activate or give background knowledge, such as prior work experience related to what is being learned.	• Provide linguistic support to ensure all students understand key mathematical vocabulary and mathematical symbols, promote understanding of mathematical languages.

Suggested Student Discourse Questions

- What strategies can be used to find the quotient of a division problem involving up to four-digit dividends and two-digit divisors?
- How do place value, properties of operations, and the relationship between multiplication and division help in solving such problems?
- How does understanding place value assist in dividing larger numbers?
- How can we use rectangular arrays or area models to illustrate the division of large numbers?
- Demonstrate how to represent and explain a division problem using a visual model.
- How does using multiple representations (equations, models, etc.) enhance understanding and support problem-solving?

Cross-Curricular Connections

STEM:

Using given or collected data, round numbers to a given whole number or decimal place to solve real-world problems.

Career and Skill Connections

- Teacher (Elementary or Middle School)
- Math Curriculum Developer
- Education Consultant
- Tutor
- Educational Content Writer
- Mathematics Instructional Coach

Grade	CCSS Domain	CCSS Cluster
5	Numbers and Operations in Base Ten	Perform operations with multi-digit whole numbers and with decimals to hundredths.

Cluster Standard: 5.NBT.B.7

Standard	Standards for Mathematical Practice
Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	• **SMP 3:** Construct viable arguments and critique the reasoning of others.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
In fourth grade, students' experiences with division were limited to dividing by one-digit divisors. This standard extends students' prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a "familiar" number, a student might decompose the dividend using place value.	• Justify reasoning with written explanation. • Explain how place value affects how to use the four operations. • Use the four operations with decimals to the hundredths. • Use models or drawings.
DOK	**Blooms**
1	Apply, Analyze

Procedural and Conceptual Understanding and Application

Conceptual Understanding: Students develop a deep understanding of operations with decimals to hundredths by using concrete models or drawings and applying strategies based on place value, properties of operations, and the relationships between addition and subtraction, multiplication, and division, and they explain the reasoning behind their chosen strategy.

Procedural Skills and Fluency: Students demonstrate proficiency and fluency in adding, subtracting, multiplying, and dividing decimals to hundredths by utilizing concrete models or drawings and employing strategies based on place value, properties of operations, and the relationship between different operations, and they relate their chosen strategy to a written method, articulating the reasoning behind their approach.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

The table shows four people who earn the typical amount for their education level.

Name	Level of Education	Weekly Income
Miley	High School Dropout	$440.50
Niko	High School Graduate	$650.35
Taylor	2-Year College Graduate	$771.25
Pinky	4-Year College Graduate	$1,099.20

a. How much more does Niko earn than Miley in one week?
b. If Taylor and Miley both work for 2 weeks, how much more will Taylor earn?
c. How much money will Pinky earn in a month? About how long will Miley have to work to earn the same amount?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Over Reliance on Concrete Models:** Students may mistakenly believe that using concrete models or drawings is the only effective way to solve decimal operations, overlooking the importance of mental strategies and abstract reasoning.
- **Ignoring Place Value Concepts:** Students might think that decimals can be added, subtracted, multiplied, or divided without considering the significance of place value, leading to errors in understanding and manipulating decimal numbers.
- **Limited Understanding of Properties:** Students could misunderstand the properties of operations (e.g., commutative, associative, distributive) in the context of decimals, resulting in ineffective or incorrect strategies for solving decimal operations.
- **Inflexible Reliance on Concrete Drawings:** Students may believe that concrete models or drawings must be used for all decimal operations, failing to transition to more efficient mental strategies or written methods as they develop fluency with decimals.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
• Provide visual models through multimedia tools, hands on manipulatives to enhance learning of adding, subtracting, multiplying and dividing decimals, using concrete models or drawing and strategies to explain the strategy used. **Universal Design of Learning (UDL)** • Use representation by providing real world, community relevant application of math projects, and science investigations.	• Provide targeted small group instruction for students that need additional support, level grouping through diversity of participants to learn from their perspectives, through lived experiences and feedback. Foster collaboration within the community of learners. **Universal Design of Learning (UDL)** • Use representation by providing real world, community relevant application of math projects, and science investigations.	• Provide students who require intensive support, one on one instruction, build confidence with graduated levels of support and guide appropriate goal setting development, clarify vocabulary and symbols. **Universal Design of Learning (UDL)** • Use representation by providing real world, community relevant application of math projects, and science investigations.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/5/22

Previous Learning	*Current Learning*	*Future Learning*
• Connect to using place value understanding and properties of operations to perform multi-digit arithmetic.	• Connect to understanding the place value concept that the number to the left is 10 times larger and the number to the right is 10 times smaller, will use exponents to express powers of 10 and can understand the patterns of zeros and decimal placement related to powers of 10. • Connect to applying and extend their previous understandings of multiplication and division to multiply and divide fractions.	• Connect to fluently adding, subtracting, multiplying, and dividing decimals using the standard algorithm. (6.NS.2,3) • Connect to recognizing that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. • Connect to read, write, and compare decimals to thousandths.

	• Connect to converting customary and metric measurement units within a given measurement system.	

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Incorporate examples of problems that make connections to their understanding of decimals in how to add, divide, subtract, and multiply.	• By scaffolding instruction you are building upon students' prior knowledge, activate or give background knowledge, such as prior work experience related to what is being learned.	• Provide linguistic support to ensure all students understand key mathematical vocabulary and mathematical symbols, promote understanding of mathematical languages.

Suggested Student Discourse Questions

- How does place value play a role when adding, subtracting, multiplying, and dividing decimals?
- Why is it important to consider the position of digits (tenths and hundredths) in decimal operations?
- How can concrete models or drawings (such as base-ten blocks, grids, or number lines) help in visualizing and solving decimal operations?
- Explain how strategies based on place value and properties of operations can be related to written methods (such as the standard algorithm for addition, subtraction, multiplication, and division).
- When working with decimals, how can mental math and estimation help in verifying the reasonableness of answers?
- How do decimal operations support problem-solving in everyday situations?

Cross-Curricular Connections

STEM:

Using given or collected data, round numbers to a given whole number or decimal place to solve real-world problems.

Career and Skill Connections
• Financial Analyst • Accountant • Data Analyst • Market Research Analyst • Operations Manager • Supply Chain Analyst

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Procedural and Conceptual Understanding and Application
  - Sample Assessment Items
  - Common Misconceptions
  - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
  - Vertical Alignment
  - Culturally and Linguistically Responsive Instruction (CLRI)
  - Suggested Student Discourse Questions
  - Cross-Curricular and Career and Skill Connections
- A Student Discourse Guide
- Planning for a Multi-Layer System of Support (MLSS) and Universal Design for Learning (UDL) for behavioral and social and emotional supports

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Write and interpret numerical expressions
  - 5.OA.A.1
  - 5.OA.A.2
- Analyze patterns and relationships
  - 5.OA.B.3

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)

Grade	CCSS Domain	CCSS Cluster
5	Operations and Algebraic Thinking	Write and interpret numerical expressions.

Cluster Standard: 5.OA.A.1

Standard	Standards for Mathematical Practice
Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	• **SMP 6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• In fourth grade, students used comparison of multiplication and division problems; thinking about solutions in terms of reasonableness by using estimation in order to determine if the solutions were reasonable. Listening to others and gathering a variety of strategies to solve problems. Used appropriate mathematical vocabulary and accurate units of measure begin solving more sophisticated problems. • The order of operations is introduced in third grade and is continued in fourth. This standard calls for students to evaluate expressions with parentheses ( ), brackets [ ] and braces { }. In upper levels of mathematics, evaluate means to substitute for a variable and simplify the expression. However, at this level students are only to simplify the expressions because there are no variables. • In fifth grade, students work with exponents only dealing with powers of ten (5.NBT.2). Students are expected to evaluate an expression that has a power of ten in it.	• Understand the use of parentheses, expressions inside parentheses/brackets must be completed first when solving the equation. • Apply rules and solve problems for orders of operations (not to include exponents). • Solve problems and equations that employ parentheses, brackets, or braces. • Students will be able to work with components, with powers of ten • Use of exponents using the power of ten to evaluate an expression
DOK	**Blooms**
1-2	Remember, Understand, Apply

Procedural and Conceptual Understanding and Application

Procedural: Students learn to identify parentheses, brackets, and braces in expressions, recognize their role in applying the changing the order of operations, apply the order of operations within grouping symbols, and evaluate expressions with these symbols by following the order of operations.

Conceptual Understanding: Students understand that grouping symbols create mini-expressions within a larger one and that these symbols control the order in which calculations are performed.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

The school cafeteria is making pizzas. They need 4 cups of flour for each large pizza dough and 2 cups of flour for each small pizza dough. Today, they made 3 large pizzas and 5 small pizzas.

- **Write** an expression to show how much flour they used in total.
- **Explain** how the use of parentheses affects the answer.
- **Evaluate** the expression to find the total flour used.

Evaluate the following numerical expressions.

a. $2\times5+3\times2+4$
b. $2\times(5+3\times2+4)$
c. $2\times5+3\times(2+4)$
d. $2\times(5+3)\times2+4$
e. $(2\times5)+(3\times2)+4$
f. $2\times(5+3)\times(2+4)$

Can the parentheses in any of these expressions be removed without changing the value of the expression?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Order of Operations Within vs. Outside Grouping Symbols**: Students might misunderstand that the order of operations (PEMDAS/BODMAS) applies uniformly throughout the entire expression, regardless of parentheses.
  For instance, they might evaluate 8 / (2 + 3) as 8 / 5 (treating it like 8 / 2 + 3) instead of correctly solving it as 8 / (5) = 1.6.
- **Ignoring Precedence Between Multiplication and Division:** Some students might treat multiplication and division as having equal weight, evaluating them from left to right. This can lead to mistakes like solving 3 * (4 / 2) as 12 (treating it like 3 * 4 / 2) instead of the correct answer, 6 (3 * 2 = 6).

- **Grouping Symbols Don't Change Anything:** Beginners might not grasp the power of grouping symbols. They might believe the expression doesn't change with or without parentheses, leading to incorrect evaluations.
- **Focus on Just Parentheses:** Students might be comfortable with parentheses but struggle with brackets or braces (if introduced). It's important to clarify that all grouping symbols follow the same principle.
- **Misinterpreting the Expression:** In word problems, students might get confused about what operations to perform within parentheses based on the wording. Emphasize translating the problem accurately into a mathematical expression.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
• Provide visual representations such as diagrams, number lines, and manipulatives to help all students understand the concept of numerical expressions with parentheses, brackets, and braces. **Universal Design for Learning** • Explain the concept using relatable analogies, such as how parentheses can be seen as "groups" or "containers" that need to be addressed first, making the abstract concept more accessible. • Use color-coded visuals to highlight parentheses, brackets, and braces within numerical expressions. This helps students see the hierarchy of operations clearly. • Encourage students to work in pairs or small groups to solve expressions, promoting discussion and the exchange of strategies. Students can take turns creating expressions for each other to solve. • Use digital tools or online math platforms that allow	• Provide targeted small group instruction for students who need additional support in understanding numerical expressions. Offer differentiated activities based on individual student needs and learning styles. • Offer guided practice sessions where students receive immediate feedback and support from the teacher or a peer mentor while working on problems related to numerical expressions. **Universal Design for Learning** • Encourage students to work in pairs or small groups to solve expressions, promoting discussion and the exchange of strategies. Students can take turns creating expressions for each other to solve. • Provide step-by-step written instructions alongside verbal explanations to accommodate different learning preferences. • Allow students to demonstrate their	• For students who require intensive support, provide one-on-one instruction tailored to their specific learning needs. Offer additional practice opportunities, reteaching sessions, and personalized learning experiences. **Universal Design for Learning** • Incorporate videos animations and interaction simulations to explain the process. • Help and align personal goals for students with goals or benchmarks in existence.

students to manipulate expressions by adding or removing parentheses, brackets, or braces, seeing in real-time how it changes the outcome.	understanding in various ways—creating a video tutorial, writing a math journal entry, or presenting their work to the class. • Provide manipulatives, such as cards with numbers and operation symbols, where students physically arrange and group items to create and solve expressions.	

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/5/24

Previous Learning	*Current Learning*	*Future Learning*
• Connect to fluently adding and subtracting within 1,000. • Connect to recalling from memory products of two 1-digit numbers.	• Connect to using knowledge of parentheses as a building block for order of operations.	• Connect to performing arithmetic operations following the order of operations with and without parentheses, including those involving whole number exponents. • Connect to applying the properties of operations to generate equivalent expressions with an emphasis on the distributive property. • Connect to writing, reading, and evaluating expressions in which letters stand for numbers. • Connect to applying the properties of operations to generate equivalent expressions. • Connect to identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they

		name the same number regardless of which number y stands for. ● Connect to finding the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
● Begin by acknowledging the diverse backgrounds and experiences of your students. Incorporate culturally relevant examples and contexts into the lessons to make mathematical concepts more relatable and engaging. For example, when introducing numerical expressions with parentheses, you could use scenarios or problems that resonate with the cultural backgrounds of your students. This validates their experiences and helps them	● Scaffold instruction by starting with concrete, real-life examples before moving to abstract mathematical representations. ● Provide opportunities for students to explore numerical expressions using familiar contexts from their own cultures or communities. As you progress, gradually introduce more abstract representations and guide students in making connections between the concrete examples and the	● Provide language support to ensure that students understand key mathematical vocabulary related to numerical expressions. This may include using visual aids such as graphic organizers, charts, or diagrams to illustrate the meaning of mathematical terms like "parentheses," "brackets," and "braces." ● Additionally, offer opportunities for students to practice using these terms in context through structured discussions, cooperative

see the relevance of the mathematical concept in their lives.	mathematical symbols. Building on prior knowledge and bridging to new concepts ensures that all students can access the content regardless of their background knowledge or language proficiency.	learning activities, or writing exercises.

Suggested Student Discourse Questions

- What is the role of parentheses (or brackets/braces) in this expression?
- How does using parentheses change the order in which we solve the expression?
- Imagine we don't have any grouping symbols in this problem. How would the answer change, if at all?
- Have you ever encountered grouping symbols outside of math class? Where might they be used?
- How can using parentheses help us communicate our ideas clearly in math problems?
- Can someone share a strategy they use to remember the order of operations within parentheses?

Cross-Curricular Connections

Science: Students can create numerical expressions from data displayed in a table or graph.

Career and Skill Connections

- Computer Programmer
- Financial Analyst
- Mathematician
- Software Developer
- Statistician
- Data Scientist

Grade	CCSS Domain	CCSS Cluster
5	Operations and Algebraic Thinking	Write and interpret numerical expressions.

Cluster Standard: 5.OA.A.2

Standard	Standards for Mathematical Practice
Write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.	• **SMP 7:** Look for and make use of structure.

Clarification Statement	Students Who Demonstrate Understanding Can...
• In fourth grade, students used quantitative reasoning to solve single and multi-step problems that included all four operations using models, pictures, words, and numbers. Students continue to develop problem solving strategies by using various representations and models and selecting appropriate tools. They started writing equations to represent the mathematics of the situation. • This standard refers to expressions. Expressions are a series of numbers and symbols (+, -, x, ÷) without an equal's sign. Equations result when two expressions are set equal to each other (2 + 3 = 4 + 1). • This standard calls for students to verbally describe the relationship between expressions without calculating them. This standard calls for students to apply their reasoning of the four operations as well as place value while describing the relationship between numbers. The standard does not include the use of variables, only numbers and signs for operations.	• Write simple expressions that record calculations. • Interpret numerical expressions.

DOK	Blooms
1-2	Apply

Procedural and Conceptual Understanding and Application
Conceptual Understanding: Students interpret and understand the meaning and structure of each numerical expression, such as how different operations interact and the value of each term contributes, without solving the expression. They need to recognize patterns and relationships within the expressions.

Assessment Items
When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Eric is playing a video game. At a certain point in the game, he has 31500 points. Then the following events happen, in order:

- He earns 2450 additional points.
- He loses 3310 points.
- The game ends, and his score doubles.

a. Write an expression for the number of points Eric has at the end of the game. Do not evaluate the expression. The expression should keep track of what happens in each step listed above.

b. Eric's sister Leila plays the same game. When she is finished playing, her score is given by the expression:

3(24500 +3610) –6780

Describe a sequence of events that might have led to Leila earning this score.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Misunderstanding the Order of Operations (PEMDAS/BODMAS):** Students may believe that operations should always be performed from left to right in a numerical expression, regardless of the type of operation.
- **Confusion with Parentheses and Grouping Symbols**: Students may struggle to understand how parentheses and other grouping symbols affect the evaluation of an expression.
- **Misunderstanding the Concept of "Without Evaluating":** Students may struggle to grasp the idea of interpreting numerical expressions without calculating the final numerical value.
- **Confusion with Distributive Property:** Students might overlook or misunderstand how to apply the distributive property when simplifying expressions.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning		
Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***

• Use visual aids, such as flowcharts, to show the steps involved in writing and interpreting numerical expressions. • Provide manipulatives like number cards and operation symbols for hands-on practice in constructing numerical expressions. **Universal Design for Learning** • Use diagrams, flowcharts, and graphic organizers to illustrate how expressions are constructed and how they relate to verbal descriptions. For example, show a diagram that connects a word problem to its corresponding numerical expression. • Use math apps or online platforms that let students input expressions and receive instant feedback on their construction and interpretation, promoting active learning and self-correction.	• Conduct small group sessions for students who need additional support in understanding numerical expressions. Use differentiated activities to address specific areas of difficulty and provide guided practice. • Focus on collaborative problem-solving tasks and hands-on activities to reinforce concepts. • Use scaffolded activities that gradually increase in complexity. Start with simple expressions and progress to more complex ones, providing step-by-step guidance and support. • Provide graphic organizers and checklists to help students organize their thoughts and approach problems systematically. **Universal Design for Learning** • Encourage students to work in pairs or small groups to solve expressions, promoting discussion and the exchange of strategies. Students can take turns creating expressions for each other to solve. • Break down the process of writing and interpreting expressions into clear, manageable steps, providing visual aids or written guides that highlight each step.	• For students who require intensive support, provide one-on-one instruction tailored to their specific learning needs. Offer additional practice opportunities, reteaching sessions, and personalized learning experiences to address misconceptions and build foundational skills in writing and interpreting numerical expressions. • Develop personalized learning plans that include targeted goals, strategies, and supports for students with significant learning gaps. Collaborate with special education teachers, ELL specialists, and other support staff to address individual needs. **Universal Design for Learning** • Incorporate videos animations and interaction simulations to explain the process. • Help and align personal goals for students with goals or benchmarks in existence.
Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/5/24		

Previous Learning	*Current Learning*	*Future Learning*
• Connect to fluently adding and subtracting within 1,000. • Connect to recalling from memory products of two 1-digit numbers.	• Connect to using knowledge of parentheses as a building block for order of operations.	• Connect to performing arithmetic operations following the order of operations with and without parentheses, including those involving whole number exponents. • Connect to applying the properties of operations to generate equivalent expressions with an emphasis on the distributive property. • Connect to writing, reading, and evaluating expressions in which letters stand for numbers. • Connect to applying the properties of operations to generate equivalent expressions. • Connect to identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for. • Connect to finding the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express

		36 + 8 as 4 (9 + 2).

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Incorporate examples of numerical expressions from various cultural contexts, such as traditional practices, cultural celebrations, or historical events, to validate students' identities and experiences.	• Start with concrete examples and real-life contexts to introduce the concept of writing and interpreting numerical expressions, then gradually transition to more abstract representations. • Provide opportunities for students to make connections between their cultural backgrounds and the mathematical content, such as exploring how numerical expressions are used in cultural practices or traditions	• Provide linguistic supports to ensure that all students understand key mathematical vocabulary related to writing and interpreting numerical expressions. • Offer clear explanations of mathematical terms such as "expression," "variable," "operation," and "evaluate." Use visual aids, gestures, and bilingual resources to support students' comprehension of mathematical vocabulary.

Suggested Student Discourse Questions

- What operation(s) do you think are necessary to solve this problem? Why?
- How do we decide whether to add, subtract, multiply, or divide in this situation?
- Can you explain how the context of the problem helps us determine which operation to use?
- Can you think of real-life situations where we might encounter similar problems?
- How can we use the skills learned from solving this problem in everyday life?

Cross-Curricular Connections

Science: Students can create numerical expressions from data displayed in a table or graph.

Career and Skill Connections

- Accountant
- Financial Analyst
- Data Analyst
- Market Research Analyst
- Economist
- Operations Manager

Grade	CCSS Domain	CCSS Cluster
5	Operations and Algebraic Thinking	Analyze patterns and relationships.

Cluster Standard: 5.OA.B.3

Standard	Standards for Mathematical Practice
Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
• This standard is closely related to graphing points in the first quadrant of a coordinate plane (5.G.1-2) This standard extends the work from Fourth Grade, where students generate numerical patterns when they are given one rule. • In Fifth Grade, students are given two rules and generate two numerical patterns. The graphs that are created should be line graphs to represent the pattern. This is a linear function which is why we get the straight lines.	• Identify the relationship between two patterns. • Given a starting point, apply two math rules to that number. • Graph data on a coordinate plane (positive numbers only).
DOK	**Blooms**
1	Analyze, Understand

Procedural and Conceptual Understanding and Application

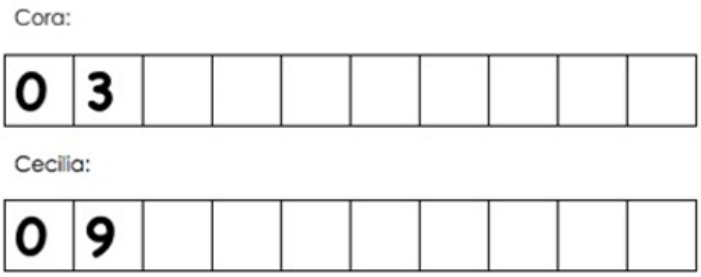
Procedural: Students develop procedural fluency by effectively applying given rules to generate sequences, establishing ordered pairs of corresponding terms, and accurately graphing them on a coordinate plane, demonstrating precision and competence in procedural execution.

Conceptual Understanding: Students develop a deep conceptual understanding of numerical patterns and relationships by exploring sequences generated from specific rules and recognizing proportional connections between corresponding terms in different sequences.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Cora and Cecilia each use chalk to make their own number patterns on the sidewalk. They make each of their patterns 10 boxes long and line their patterns up so they are next to each other. Cora puts 0 in her first box and decides that she will add 3 every time to get the next number. Cecilia puts 0 in her first box and decides that she will add 9 every time to get the next number.



a. Complete each girl's sidewalk pattern.
b. How many times greater is Cecilia's number in the 5th box than Cora's number in the 5th box? What about the numbers in the 8th box? The 10th box?
c. What pattern do you notice in your answers for part b? Why do you think that pattern exists?
d. If Cora and Cecilia kept their sidewalk patterns going, what number would be in Cora's box when Cecilia's corresponding box shows 153?

Common Misconceptions

- **Assuming All Patterns Follow Simple Arithmetic Rules:** Students may believe that all numerical patterns can be explained by straightforward arithmetic operations (e.g., addition, multiplication).
- **Overlooking Starting Points:** Students might overlook the impact of different starting numbers on the relationships between corresponding terms.
- **Expecting Uniform Relationships Across Different Rules:** Students may expect consistent relationships between corresponding terms regardless of the specific rules used.
- **Ignoring Complex Patterns and Variations**: Students might generalize observations based on simple examples (like adding 3 and adding 6) to all numerical patterns, assuming uniformity across different scenarios.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1	*Layer 2*	*Layer 3*

Core Instruction + UDL	*Core + UDL + Targeted*	*Core + UDL + Targeted + Intensive*
• Provide visual models such as tables, graphs, or diagrams to represent numerical patterns and rules visually. • Use concrete examples and real-world contexts to demonstrate the application of generating numerical patterns using rules. **Universal Design for Learning** • Incorporate technology that allows students to generate and manipulate patterns, providing instant visual feedback on how changing the rule affects the pattern. • Use visual aids like charts, graphs, and tables to represent numerical patterns. This helps students see the relationships and regularities more clearly. • Present problems in contexts that students find interesting or relevant, such as patterns in sports scores, video games, or coding, to make learning more engaging.	• Provide targeted small group instruction for students who need additional support in understanding numerical patterns and rules. • Offer differentiated activities based on individual student needs and learning styles, focusing on hands-on practice and peer collaboration. **Universal Design for Learning** • Offer students a choice of rules to generate patterns from or let them create their own rules and patterns. This fosters a sense of ownership and creativity. • Encourage students to work in pairs or groups to generate patterns and explore relationships. This promotes discussion, collaboration, and the sharing of different strategies.	• For students who require intensive support, provide one-on-one instruction tailored to their specific learning needs. • Offer additional practice opportunities, reteaching sessions, and personalized learning experiences to address misconceptions and build foundational skills in generating numerical patterns using rules. **Universal Design for Learning** • Incorporate videos animations and interaction simulations to explain the process. • Help and align personal goals for students with goals or benchmarks in existence.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/5/24

Previous Learning	*Current Learning*	*Future Learning*
• Connect to following one rule and then determine what happened in that pattern.	• Connect to graphing points on a coordinate plane.	• Connect to applying the use of variables to represent two quantities in real world problems. Students will write equations to represent the dependent and independent variables. • Connect to describing the

		relationship in ratio rates to solve real world problems.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Incorporate examples of numerical patterns from various cultural contexts, such as traditional counting methods, cultural celebrations, or historical events, to validate students' identities and experiences.	• Start with concrete examples and real-life contexts to introduce the concept of generating numerical patterns using rules, then gradually transition to more abstract representations. • Provide opportunities for students to make connections between their cultural backgrounds and the mathematical content, such as exploring how numerical patterns are used in cultural practices or traditions.	• Provide linguistic supports to ensure that all students understand key mathematical vocabulary related to generating numerical patterns. • Offer clear explanations of mathematical terms such as "numerical pattern," "rule," "sequence," and "generate." Use visual aids, gestures, and bilingual resources to support students' comprehension of mathematical vocabulary.

Suggested Student Discourse Questions

- What is a pattern, and how do we recognize patterns in numbers, shapes, or objects?
- Can you identify examples of patterns in everyday life or in mathematical contexts?
- What strategies can we use to predict the next term or terms in a pattern?
- What are some ways we can represent a pattern visually, such as through a table, a graph, or a diagram?
- How do we interpret the information presented in a table or a graph that represents a pattern?

Cross-Curricular Connections

Science: Give students data represented in a table. Have students discuss the relationship between the numbers in the table.

Career and Skill Connections

- Data Analyst
- Statistician
- Economist
- Financial Analyst
- Market Research Analyst
- Math Curriculum Developer

Section 3: Resources, References, and Glossary

Resources

Evidence-Based Resources	English Learner Resources	MLSS Resources	Mathematics Standard Resources
What Works Clearinghouse Best Evidence Encyclopedia Evidence for Every Students Succeeds Act Evidence in Education Lab	World-Class Instructional Design and Assessment (WIDA) Standards USCALE Language Routines for Mathematics English Language Development Standards Spanish Language Development Standards	NM Multi-Layered System of Supports (MLSS) Universal Design for Learning Guidelines Achieve the Core: Instructional Routines for Mathematics Project Zero Thinking Routines	Focus by Grade Level and Widely Applicable Prerequisites High school Coherence Map College-and Career Ready Math Shifts Fostering Math Practices: Routines for the Mathematical Practices

Planning Guidance for Multi-Layered Systems of Support: Core Instruction[9]

Core Instructional Planning must reflect and leverage scientific insights into how humans learn in order to ensure all students are ready for success, thus the following guidance for optimizing teaching and learning is grounded in the **Universal Design Learning (UDL) Framework**

Key design questions, planning actions, and potential strategies are provided below, with respect to guidance for minimizing barriers to learning and optimizing (1) universal ACCESS to learning experiences, (2) opportunities for students to BUILD their understanding of the **Learning Goal**, and (3) INTERNALIZATION of the Learning Goal.

Optimizing Universal ACCESS to Learning Experiences	
ENGAGEMENT ? How will you provide multiple options for recruiting interest?	**Recruiting Student Interest:** ? What do you anticipate in the range of student interest for this lesson? ➢ Plan for options for recruiting student interest: ❒ provide choice (e.g. sequence or timing of task completion) ❒ set personal academic goals ❒ provide contextualized examples connected to their lives ❒ support culturally relevant connections (i.e home culture) ❒ create socially relevant tasks ❒ provide novel & relevant problems to make sense of complex ideas in creative ways

[9] Adapted from: CAST (2018). *Universal Design for Learning Guidelines version 2.2*. Retrieved from http://udlguidelines.cast.org

	❒ provide time for self-reflection about content & activities ❒ create accepting and supportive classroom climate ❒ utilize **instructional routines** to involve all students
REPRESENTATION [?] How will you reduce barriers to perceiving the information presented in this lesson?	**Perception:** [?] What do you anticipate about the range in how students will perceive information presented in this lesson? ➢ Plan for different modalities and formats to reduce barriers to learning: ❒ display information in a flexible format to vary perceptual features ❒ offer alternatives for auditory information ❒ offer alternatives for visual information
ACTION & EXPRESSION [?] How will the learning for students provide a variety of methods for navigation to support access?	**Physical Action:** [?] What do you anticipate about the range in how students will physically navigate and respond to the learning experience? ➢ Plan a variety of methods for response and navigation of learning experiences by offering alternatives to: ❒ requirements for rate, timing, speed, and range of motor action with instructional materials, manipulatives, and technologies ❒ physically indicating selections ❒ interacting with materials by hand, voice, keyboard, etc.

Opportunities for Students to BUILD their Understanding

ENGAGEMENT [?] How will the learning for students provide options for sustaining effort and persistence?	**Sustaining Effort & Persistence:** [?] What do you anticipate about the range in student effort? ➢ Plan multiple methods for attending to student attention and affect by: ❒ prompting learners to explicitly formulate or restate learning goals ❒ displaying the learning goals in multiple ways ❒ using prompts or scaffolds for visualizing desired outcomes ❒ engaging assessment discussions of what constitutes excellence ❒ generating relevant examples with students that connect to their cultural background and interests ❒ providing alternatives in the math representations and scaffolds ❒ creating cooperative groups with clear goals, roles, responsibilities ❒ providing prompts to guide when and how to ask for help ❒ supporting opportunities for peer interactions and supports (e.g. peer tutors) ❒ constructing communities of learners engaged in common interests ❒ creating expectations for group work (e.g., rubrics, norms, etc.) ❒ providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge ❒ providing feedback that: ❒ emphasizes effort, improvement, and achieving a standard rather than on relative performance ❒ is frequent, timely, and specific ❒ is informative rather than comparative or competitive

	❒ models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success
REPRESENTATION ⍰ How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners?	**Language & Symbols:** ⍰ What do you anticipate about the range of student background experience and vocabulary? ➢ Plan multiple methods for attending to linguistic and nonlinguistic representations of mathematics to ensure universal clarity by: ❒ pre-teaching vocabulary and symbols in ways that promote connection to the learners' experience and prior knowledge ❒ graphic symbols with alternative text descriptions ❒ highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to structure ❒ embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) ❒ embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) ❒ highlighting structural relations or make them more explicit ❒ making connections to previously learned structures ❒ making relationships between elements explicit (e.g., highlighting the transition words in an argument, links between ideas, etc.) ❒ allowing the use of text-to-speech and automatic voicing with digital mathematical notation (math ml) ❒ allowing flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) ❒ clarification of notation through lists of key terms ❒ making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf ❒ linking key vocabulary words to definitions and pronunciations in both dominant and heritage languages ❒ defining domain-specific vocabulary (e.g., "map key" in social studies) using both domain-specific and common terms ❒ electronic translation tools or links to multilingual web glossaries ❒ embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc) ❒ presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) ❒ making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams
ACTION & EXPRESSION ⍰ How will the learning provide multiple	**Expression & Communication:** ⍰ What do you anticipate about the range in how students will express their thinking in the learning environment? ➢ Plan multiple methods for attending to the various ways in which students can express knowledge, ideas, and concepts by providing:

modalities for students to easily express knowledge, ideas, and concepts in the learning environment?	❒ options to compose in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video ❒ use of social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) ❒ flexibility in using a variety of problem solving strategies ❒ spell or grammar checkers, word prediction software ❒ text-to-speech software, human dictation, recording ❒ calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper ❒ sentence starters or sentence strips ❒ concept mapping tools ❒ Computer-Aided-Design (CAD) or mathematical notation software ❒ virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) ❒ multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches) ❒ multiple examples of novel solutions to authentic problems ❒ different approaches to motivate, guide, feedback or inform students of progress towards fluency ❒ scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) ❒ differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners)

Optimizing INTERNALIZATION of the Learning Goal

ENGAGEMENT ⍰ How will the design of the learning strategically support students to effectively cope and engage with the environment?	**Self-Regulation:** ⍰ What do you anticipate about barriers to student engagement? ➢ Plan to address barriers to engagement by promoting healthy responses and interactions, and ownership of learning goals: ❒ metacognitive approaches to frustration when doing mathematics ❒ increase length of on-task orientation through distractions ❒ frequent self-reflection and self-reinforcements ❒ address subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") ❒ offer devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the math practices for the purpose of monitoring and improving ❒ use activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely
REPRESENTATION ⍰ How will the learning support transforming accessible information into usable knowledge	**Comprehension:** ⍰ What do you anticipate about barriers to student comprehension? ➢ Plan to address barriers to comprehension by intentionally building connections to prior understandings and experiences, relating meaningful information to learning goals,

that is accessible for future learning and decision-making?	providing a process for meaning making of new learning, and applying learning to new contexts: ❒ incorporate explicit opportunities for review and practice ❒ note-taking templates, graphic organizers, concept maps ❒ scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) ❒ explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) ❒ opportunities over time to revisit key ideas and connections ❒ make explicit cross-curricular connections ❒ highlight key elements in tasks, graphics, diagrams, formulas ❒ outlines, graphic organizers, unit organizer routines, concept organizer routines, and concept mastery routines to emphasize key ideas and relationships ❒ multiple examples & non-examples ❒ cues and prompts to draw attention to critical features ❒ highlight previously learned skills that can be used to solve unfamiliar problems ❒ options for organizing and possible approaches (tables and representations for processing mathematical operations) ❒ interactive representations that guide exploration and new understandings ❒ introduce graduated scaffolds that support information processing strategies ❒ tasks with multiple entry points and optional pathways ❒ "Chunk" information into smaller elements ❒ remove unnecessary distractions unless essential to learning goal ❒ anchor instruction by linking to and activating relevant prior knowledge (e.g., using visual imagery, concept anchoring, or concept mastery routines) ❒ pre-teach critical prerequisite concepts via demonstration or representations ❒ embed new ideas in familiar ideas and contexts (e.g., use of analogy, metaphor, drama, music, film, etc.) ❒ advanced organizers (e.g., KWL methods, concept maps) ❒ bridge concepts with relevant analogies and metaphors
ACCESS ACTION & EXPRESSION ? How will the learning for students support the development of executive functions to allow them to take advantage of their environment?	**Executive Functions:** ? What do you anticipate about barriers to students demonstrating what they know? ➢ Plan to address barriers to demonstrating understanding by providing opportunities for students to set goals, formulate plans, use tools and processes to support organization and memory, and analyze their growth in learning and how to build from it: ❒ prompts and scaffolds to estimate effort, resources, difficulty ❒ models and examples of process and product of goal-setting ❒ guides and checklists for scaffolding goal-setting ❒ post goals, objectives, and schedules in an obvious place ❒ embed prompts to "show and explain your work" ❒ checklists and project plan templates for understanding the problem, prioritization, sequences, and schedules of steps ❒ embed coaches/mentors to demonstrate think-alouds of process ❒ guides to break long-term goals into short-term objectives ❒ graphic organizers/templates for organizing information & data ❒ embed prompts for categorizing and systematizing ❒ checklists and guides for note-taking ❒ asking questions to guide self-monitoring and reflection ❒ showing representations of progress (e.g., before and after photos, graphs/charts showing progress, process portfolios)

	❒ prompt learners to identify type of feedback or advice they seek ❒ templates to guide self-reflection on quality & completeness ❒ differentiated models of self-assessment strategies (e.g., role-playing, video reviews, peer feedback) ❒ assessment checklists, scoring rubrics, and multiple examples of annotated student work/performance examples

Planning Guidance for Culturally and Linguistically Responsive Instruction[10]

In order to ensure our students from marginalized cultures and languages view themselves as confident and competent learners and doers of mathematics within and outside of the classroom, educators must intentionally plan ways to counteract the negative or missing images and representations that exist in our curricular resources. The guiding questions below support the design of lessons that validate, affirm, build, and bridge home and school culture for learners of mathematics:

> **Validate/Affirm:** How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
>
> **Build/Bridge:** How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

In addition, Aguirre and her colleagues[11] define **mathematical identities** as the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives. Many students see themselves as "not good at math" and approach math with fear and lack of confidence. Their identity, developed through earlier years of schooling, has the potential to affect their school and career choices.

Five Equity-Based Mathematics Teaching Practices[12]

> **Go deep with mathematics.** Develop students' conceptual understanding, procedural fluency, and problem solving and reasoning.
>
> **Leverage multiple mathematical competencies.** Use students' different mathematical strengths as a resource for learning.
>
> **Affirm mathematics learners' identities.** Promote student participation and value different ways of contributing.

[10] This resource relied heavily on the work of: Hollie, S. (2011). Culturally and linguistically responsive teaching and learning. Teacher Created Materials. (see also, https://www.culturallyresponsive.org/vabb)

[11] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices. Reston, VA: National Council of Teachers of Mathematics (p. 14).

[12] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc. (p.6). (adapted from Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013) (p. 43).

Challenge spaces of marginality. Embrace student competencies, value multiple mathematical contributions, and position students as sources of expertise.

Draw on multiple resources of knowledge (mathematics, language, culture, family). Tap students' knowledge and experiences as resources for mathematics learning.

The following lesson design strategies support Culturally and Linguistically Responsive Instruction, specific examples for each cluster of standards can be found in part 2 of the document. These were adapted from the Promoting Equity section of the Taking Action series published by NCTM.[13]

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.

Mathematical Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher.

Modifying Mathematical Tasks: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)."

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics.

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages.

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we

[13] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians.

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others."

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time."

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students.

References

Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics.

Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

Cardone, T. (n.d.). Nix the Tricks. Retrieved from https://nixthetricks.com/

CAST (2018). *Universal Design for Learning Guidelines version 2.2*. Retrieved from http://udlguidelines.cast.org

Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/

English Learners Success Forum. (2020). ELSF | Resource: Analyzing Content and Language Demands. Retrieved from https://www.elsuccessforum.org/resources/math-analyzing-content-and-language-demands

Gifted and talented | NZ Maths. (n.d.). Retrieved from https://nzmaths.co.nz/gifted-and-talented

Gojak, L. M. (Ed.) (2015-2018). The Common Core Mathematics Companion: The Standards Decoded: What They Say, What They Mean, How to Teach Them (Corwin Mathematics Series) (1st ed.). Thousands Oak, CA: Corwin.

Gonzales, Lorenzo (2004). *Building personal knowledge of rational numbers through mental images, concepts, language, facts, and procedures.* Unpublished Doctoral Dissertation, New Mexico State University, College of Education, Las Cruces, New Mexico.

Hollie, S. (2011). Culturally and linguistically responsive teaching and learning. Teacher Created Materials.

Howard County Public School System: Mathematics. (n.d.). Mathematics – HCPSS. Retrieved from https://www.hcpss.org/academics/mathematics/

Illustrative Mathematics. (2014, February 14). Standards for Mathematical Practice: Commentary and Elaborations for K–5. Retrieved from http://commoncoretools.me/wp-content/uploads/2014/02/Elaborations.pdf

Illustrative Mathematics. (2014b, May 6). Standards for Mathematical Practice: Commentary and Elaborations for 6–8. Retrieved from http://commoncoretools.me/wp-content/uploads/2014/05/2014-05-06-Elaborations-6-8.pdf

Kansas State Department of Education. (2018). 2018 Kansas Mathematics Flip Books. Retrieved from https://community.ksde.org/Default.aspx?tabid=5646

Kaplinsky, R. (n.d.). SIOP Content and Language Objectives for Math. Retrieved from http://robertkaplinsky.com/wp-content/uploads/2017/04/SIOP-Content-and-Language-Objectives-for-Math.pdf

Louisiana Department of Education . (n.d.). K-12 Math Planning Resources. Retrieved from https://www.louisianabelieves.com/resources/library/k-12-math-year-long-planning

Los Angeles, CAUSA. (n.d.). VABB™. Retrieved from https://www.culturallyresponsive.org/vabb New Visions for Public Schools. (2020). Re-engagement Lessons. Retrieved from https://curriculum.newvisions.org/math/nv-math-team/curriculum-components/re-engagement-lessons/

Seeley, C. L. (2016). Math is Supposed to Make Sense. In Making sense of math: How to help every student become a mathematical thinker and problem solver. Alexandria, VA, USA: ASCD. (P. 13)

Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

Glossary[14]

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (- 3/4) = (- 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[15]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. See: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances
from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten.
For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

---

[14] Glossary and tables taken from: Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/
[15] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[16] See also: median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.
Integer. A number expressible in the form a or -a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line.
Also known as a dot plot.[17]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[18] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 Ã· 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 Ã— 4/3 = 4/3 Ã— 3/4 = 1.

---

[16] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

[17] Adapted from Wisconsin Department of Public Instruction, op. cit.

[18] To be more precise, this defines the arithmetic mean.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin,
selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also*: uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form *a/b* or – *a/b* for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also*: terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[19]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar
model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

[19] Adapted from Wisconsin Department of Public Instruction, op. cit.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also*: median, first quartile, interquartile range.

Table 1: Common addition and subtraction.[1]

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 =5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now?5-2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?? -2 = 3
	TOTAL UNKNOWN	**ADDEND UNKNOWN**	**BOTH ADDENDS UNKNOWN[2]**
PUT TOGETHER / TAKE APART[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5-3 = ?	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? 5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1, 5 = 2 + 3, 5 = 3 + 2
COMPARE	**DIFFERENCE UKNOWN**	**BIGGER UNKNOWN**	**SMALLER UNKNOWN**
	("How many more?" version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2: Common multiplication and division situations.[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	3 x ? = 18, and 18 ÷ 3 = ?	? x 6 = 18, and 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	a x b = ?	a x ? = p and p ÷ a = ?	? x b = p, and p ÷ b = ?

[1]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3: The properties of operations.

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	(a +b) + c = a + (b+c)
Commutative property of addition	a + b = b + a

Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists *-a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) x c = a x (b x c)
Commutative property of multiplication	a x b = b x a
Multiplicative identity property 1	a x 1 = 1 x a = a
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that a x $1/a$ = $1/a$ x a = 1
Distributive property of multiplication over additions	a x (b + c) = a x b + a x c

Table 4: The properties of equality.

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a.
Symmetric property of equality	If a = b, then b = a.
Transitive property of equality	If a = b and b = c , then a = c.
Addition property of equality	If a = b, then a +c = b + c.
Subtraction property of equality	If a = b then a – c = b – c.
Multiplication property of equality	If a = b, then a x c = b x c.
Division property of equality	If a = b and $c \neq 0$, then a ÷ c = b ÷ c.
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality.

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b.
If a > b and b > c then a > c.
If a > b, b < a.
If a > b, then -a <-b.
If a > b, then a ± c > b ± c.
If a > b and c > 0, then a x c > b x c.
If a > b and c < 0, then a x c < b x c.
If a > b and c > 0, then a ÷ c > b ÷ c.
If a > b and c < 0, then a ÷ c < b ÷ c.