New Mexico Instructional Scope for Supporting Equitable and Culturally Responsive Mathematics Instruction

Overview

This Instructional Scope for Mathematics 3.0 was created by a cohort of New Mexico educators and the New Mexico Public Education Department.

The intention of this document is to act as a companion during the planning process alongside High Quality Instructional Materials (HQIM). A sample template is provided to show a snapshot of planning supports provided within each cluster of standards in the mathematics instructional scope.

During the creation of this document, we leveraged the work of other states, organizations, and educators from across the country and world. This work would not have been possible without all that came before it, and we wish to express our sincerest gratitude for everyone that contributed to the resources listed within our references.

To better understand the planning support provided in the mathematics instruction scope, this section provides a brief description of each planning support. This includes *what* support is provided, *why* the planning support is critical for equitable and culturally responsive mathematics instruction, and *how* to use the planning support with HQIM.

Cluster Statement

What: The New Mexico Mathematics Standards are grouped by domains, with somewhere between 4 to 10 domains per grade level. Within each domain, the standards are arranged into clusters. Cluster statements summarize groups of related standards.

Why: The New Mexico Mathematics Standards require a stronger focus on the way time and energy are spent in mathematics classrooms. Students should spend the majority of their time (65-85%) working on content within the major clusters of the grade/course. Supporting clusters and–where appropriate–additional clusters should only comprise 15-35% of the time spent in classes and be covered when they are connected to and engage students in the major work of the grade/course.

How: When planning with your HQIM, consider the time being devoted to major clusters versus additional or supporting clusters. The major work of each grade/course should be designed to provide students with strong foundations for future mathematical work, which will require more time than additional or supporting clusters. Also consider the ways your HQIM makes these connections between major clusters and additional and supporting clusters explicit for students.

Standard Text

What: Each cluster-level support document contains the text of each standard within that cluster.

Why: The cluster statement and standards are meant to be read together to understand the structure of the standards. By grouping the standards within the cluster, the connectedness of the standards is reinforced.

How: The text of the standards should ground all planning with your HQIM. Reading the standards within a cluster intentionally enables us to focus on the connections within and between the standards.

Standards for Mathematical Practice

What: The Standards for Mathematical Practice describe the expertise and habits of mind that mathematics educators at all levels should develop in their students.

Why: Equitable and excellent mathematics instruction supports students in becoming confident and competent mathematicians. By engaging with the Standards for Mathematical Practice, students engage in the practice of doing mathematics and developing mathematical habits of mind. These include the ability to think mathematically, analyze situations, understand relationships, and adapt what they know to solve a wide range of problems, including problems that do not look like any they have encountered before.

How: When planning with HQIM, it is critical to consider the connections between the content standards and the Standards for Mathematical Practice. The planning supports highlight a few practices students can engage in when learning the content of the standard. It is not necessary, or even appropriate, to engage in all of the practices every day. Instead, teachers should choose which Standards for Mathematical Practice to focus on within a given lesson, depending on the content standards being covered and the activities students will be engaging with. When teachers spend time intentionally supporting students in learning both the what (content standards) and the how (Standards for Mathematical Practice), students will have a stronger foundation of mathematical learning.

Clarification Statement

What: The clarification statement provides greater clarity for teachers in understanding the purpose of the standard.

Why: The New Mexico Mathematics Standards illustrate how progressions support student learning within each major domain of mathematics. The clarification statement provides additional context about the ways each cluster of standards supports student learning of the larger learning progression.

How: When planning with HQIM, use the clarification statement to support your understanding of how the materials use specific types of representations or change the learning sequence.

Demonstration Statements, Webb's Depth of Knowledge, Bloom's Taxonomy, and Aspect of Rigor

What: The New Mexico Mathematics Standards include one, two, or all three of the aspects of mathematical rigor: conceptual understanding, procedural skill and fluency, and application to the real world. These planning supports considers which aspect(s) of rigor are within each standard and then identifies academic skills students need to demonstrate comprehension of the standard and associated mathematical practices. The planning supports also provide information about two common classifications on cognitive complexity, Webb's Depth of Knowledge and Bloom's Taxonomy.

Why: Analyzing standards alongside the standards for mathematical practice provide a fuller picture of the mathematical competencies demanded in the standard.

How: When planning for a cluster of standards with your HQIM, a critical first step is to analyze the content and language demands of the standards and the associated Standards for Mathematical Practice. This analysis can be used to plan/design appropriate formative assessment, as well as interpret student data from formative assessments. The planning supports provide a breakdown of the standard that can serve as the basis for this sort of analysis.

Definitions of the Components of Rigor

Rigorous teaching in mathematics does not simply mean increasing the difficulty or complexity of practice problems. Incorporating rigor into classroom instruction and student learning means exploring at a greater depth, the standards and ideas with which students are grappling. There are **three** components of rigor that will be expanded upon in this document, and each is equally important to student mastery: **Conceptual Understanding, Procedural Skill and Fluency,** and **Application**.

- **Conceptual Understanding** refers to understanding mathematical concepts, operations, and relations. It is more than knowing isolated facts and methods. Students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. It also allows students to connect prior knowledge to new ideas and concepts.
- **Procedural Skill and Fluency** is the ability to apply procedures accurately, efficiently, and flexibly. It requires speed and accuracy in calculation while giving students opportunities to practice basic skills. Students' ability to solve more complex application tasks is dependent on procedural skill and fluency.
- **Application** provides valuable context for learning and the opportunity to solve problems in a relevant and a meaningful way. It is through real-world application that students learn to select an efficient method to find a solution, determine whether the solution makes sense by reasoning, and develop critical thinking skills.

A Special Note on Procedural Skill and Fluency

While speed is a component of fluency, it is not necessarily speed in producing an answer; rather, fluency can be observed by watching the speed with which a student engages with a particular problem. Furthermore, fluency does not require the most efficient strategy. The standards specify grade-level appropriate strategies or types of strategies with which students should demonstrate fluency (e.g., 1.OA.C.6 allows for students to use counting on, making ten, creating equivalent but easier or known sums, etc.). It should also be noted that teachers should expect some procedures to take longer than others (e.g., fluency with the standard algorithm for division, 6.NS.B.2, as compared to fluently adding and subtracting within 10, 1.OA.C.6).

Standards identified as targeting procedural skill and fluency do not all have an expectation of automaticity and/or rote recall. Only two standards, 2.OA.B.2 and 3.OA.C.7, have explicit expectations of students knowing facts from memory. Other standards targeting procedural skill and fluency do not require students to reach automaticity. For example, in 4.G.A.2, students do not need to reach automaticity in classifying two-dimensional figures.

*Adapted from Louisiana Department of Education

Assessment Items

What: Formative assessment is the planned, ongoing process used by teachers during learning and teaching to assess student learning in order to improve student understanding and support students in their learning. This planning support provides one or more sample items to assist teachers in their planning of formative assessments for each standard. These are intended to be used as a guide for what students should be able to complete and can be used alongside assessments provided within your HQIM. A link to additional assessment items from Illustrative Mathematics is also provided with each standard.

Why: When student thinking is made visible, the teacher can examine the progression of learning towards the goals of the standards and adjust instruction as necessary. By including students in the assessment and analysis process, teachers allow them to become strategic and goal-directed with their learning.

How: The sample items address the aspect(s) of rigor that aligns with each standard. This example can be used to discuss possible responses by students and next steps for instruction. A similar process can then be used to identify additional items from the formative assessment resources provided by your HQIM.

For additional information, see Standards Aligned Instructionally Embedded Formative Assessment Resources.

Common Misconceptions

What: This planning support identifies some of the common misconceptions students might have when engaging in learning about each mathematical topic.

Why: Students might have misconceptions based on an overgeneralization of patterns they notice or overly relying on rules rather than understanding the underlying concepts. Tips and tricks in mathematics expire over time as students move up through the grade levels. It is critical to understand some of the common misconceptions students can develop so we can address them directly with students and continue to build a strong foundation for their mathematical learning.

How: When planning with your HQIM, look for ways to directly address common misconceptions students have. This planning support provides some of the possible misconceptions, and your HQIM might include additional ones. The goal is not to avoid misconceptions–they are a natural part of the learning process. But we do want to support students in exploring their misconceptions and modifying incorrect or partial understanding.

Planning for Multi-Layered System of Support & Universal Design for Learning

What: The section on planning for Multi-Layered Systems of Supports (MLSS) and Universal Design for Learning (UDL) is designed to support teachers in planning for the needs of all students. Each section includes the three pillars of UDL, as well as options for pre-teaching and re-teaching. Some students might benefit from targeted pre-teaching and re-teaching supports, which improve students' acquisition of the knowledge and skills identified in the New Mexico Mathematics Standards. Intensive pre-teaching and re-teaching supports may also be helpful for smaller groups of students with more intensive needs. Progress monitoring should occur to assess students' responses to these additional supports.

Why: MLSS is a holistic framework that guides educators to intervene quickly when students need additional supports. The framework moves away from the "wait to fail" model and empowers teachers to use their professional judgment to make data-informed decisions regarding the students in their classrooms to ensure academic success with the grade-level expectations of the New Mexico Mathematics Standards.

How: When planning with your HQIM, use the suggestions for pre-teaching as a starting point to determine if some or all of the students in your classroom need targeted or intensive pre-teaching at the start of a unit to ensure they can access the grade-level material within the unit. The core instruction (which is grounded in the UDL Framework) and re-teach sections work together to support planning within a unit, looking for ways the materials support greater access for all students and provide options to revisit concepts based on formative assessment data. Additional information about MLSS can be found on the PED's website, and guidance and tools can be found here.

Vertical Alignment

What: The New Mexico Mathematics Standards are designed around coherent progressions of learning. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but rather an extension of previous learning. The connections to previous, current, and future learning make this coherence visible. A link to Achieve the Core's grade-level coherence maps is provided with each standard.

Why: Students build stronger foundations for learning when they see mathematics as an interconnected discipline of relationships rather than discrete skills and knowledge that are only applicable in current situations. The intentional inclusion of connections to previous, current, and future learning can support a more interconnected understanding of mathematics.

How: When planning with HQIM, use the vertical alignment supports to find ways to help students make explicit connections within their study of mathematics.

Culturally and Linguistically Responsive Instruction

What: Culturally and Linguistically Responsive Instruction (CLRI) requires educators to contribute to a positive school climate by validating and affirming students' home languages and cultures, building and bridging mathematics to previous learning (both formal and informal), and using linguistic vocabulary supports. These actions legitimize students' home culture and language, making it clear that students' culture and language are positive assets in the classroom. It is also an intentional effort to reverse negative stereotypes of non-dominant cultures and languages and must be purposeful, consistent, and authentic, as well as both proactive and reactive. By building and bridging, students learn to toggle between the behaviors and expectations of home and those of school. By focusing on creating connections between the culture and language expectations of home and the expectations at school, we help students be set up for success in school. Linguistic supports are important within this structure as well, and help students learn new, academic vocabulary in ways that allow students to bring in their prior knowledge and connect it to new experiences. Linguistic supports for helping students acquire new academic vocabulary are provided from Pathways2Careers.

Why: The mathematical identities of students are shaped by the messages they receive about their ability to do mathematics and the power of mathematics in their lives outside of school. Educators must intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages. In addition, creating connections between the cultural and linguistic behaviors of students' home culture and language supports students in creating identities as capable mathematicians within school and society.

How: When planning instruction, it is critical to consider ways to validate/affirm and build/bridge students' cultural and linguistic assets. The planning supports for each cluster provide information

and examples of how to engage with equity-based teaching practices. There may be additional support available in your HQIM to ensure all students develop strong mathematical identities.

Student Discourse Guide

What: This guide provides educators with suggested questions to engage students in discourse about mathematical concepts. Purposeful, rich classroom discourse offers students the opportunity to express their ideas and thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Ideas for supporting student discourse are also provided from Pathways2Careers.

Why: Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding.

How: When students have frequent opportunities for discussion, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences, and evolving math identities.

Cross-Curricular Connections

What: Cross-curricular connections are connections between knowledge and/or skills that can be made between two or more areas of study. These connections can be made by teachers or students.

Why: The purpose of planning cross-curricular connections in an instructional sequence is to ensure that students build connections and recognize the relevance of mathematics beyond the mathematics classroom.

How: When planning with HQIM, look for opportunities to make explicit connections to other content areas, including the examples provided for each standard.

Career and Skill Connections

What: The knowledge and/or skills students are mastering within each standard can be directly connected to various careers. These connections can be made by teachers or students and can provide motivation for students as they work through mathematical topics.

Why: Understanding and recognizing the relevance of mathematics beyond the mathematics classroom is important. This is particularly valid for students who may not plan on working in a

"mathematics" field.

How: When planning with HQIM, look for opportunities to make explicit connections to careers, including the examples provided for each standard.

Template of the New Mexico Cluster Level Planning Support for the New Mexico Mathematics Standards

Grade	*CCSS Domain*	*CCSS Cluster*
Grade level	CCSS domain	CCSS cluster statement summarizing the group of related standards
Standard and icons that indicate which aspect of rigor it aligns with		
Standard		**Standards for Mathematical Practice**
Full text of the standard.		Correlation of the standard to the Standards for Mathematical Practice to which it aligns, including a link to a descriptor of what teachers and students should be doing.
Clarification Statement		**Students Who Demonstrate Understanding Can...**
Clarifies the language of the standard.		The skills students perform to demonstrate comprehension of the standard.
DOK		**Blooms**
Correlation of the standard to Webb's Depth of Knowledge.		Correlation of the standard to Bloom's Taxonomy.
Conceptual Understanding, Procedural Skill and Fluency, and/or Application		
Highlights the aspect(s) of rigor the standard is aligned to, and descriptors for which portions of the standard fall under each aspect of rigor.		
Assessment Items		
When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.		
Provides at least one high-quality formative assessment item aligned to the standard, as well as a link to more items.		
Common Misconceptions		
Provides guidance on where student misconceptions might occur.		

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)		
Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Layer 1 ensures that all students receive strong instruction in a high-quality differentiated core curriculum that is based on the principles of UDL. This includes school-wide implementation of positive behavioral interventions and supports, data-driven instruction, targeted interventions in small group instruction, universal screening, and English Language Development (ELD) for English Learners (ELs).	Layer 2 interventions should be focused on delivering individualized and targeted support (pre-teaching and re-teaching) for students on a grade-level trajectory. The interventions must be aligned with Layer 1 skills. Students should be provided with additional time and intensity in a small-group setting.	Layer 3 interventions should be provided individually or in small groups. Students are grouped according to their skill needs. The goal is for each student to acquire academic skills that will persist and transfer when the student returns to core instruction. If needed, specialized teachers may provide specific intervention instruction based on the needs identified by the data.

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/7/30/308/308		
Previous Learning	*Current Learning*	*Future Learning*
Lists skills relevant to current learning that students should have already mastered.	Lists skills within the current learning that students will master.	Lists skills from upcoming learning that students will need to be able to master based on what they are learning now.
Culturally and Linguistically Responsive Instruction		
Consider these resources for vocabulary from Pathways2Careers: • https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf • https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf		
Consider these questions as you plan for instruction that is culturally and linguistically responsive: • How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
Provides ways to build connections between the families in the community and the mathematical content, as well as examples that connect the math to students' home lives.	Provides information on supporting students as they learn mathematics by starting with conceptual knowledge that students can make connections to based on their prior knowledge. Also provides information on helping students build positive mathematical identities.	Provides ideas and supports for helping students learn new academic vocabulary and making connections to their prior knowledge.

Suggested Student Discourse Questions
Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf
Provides questions teachers can employ to increase student discourse.
Cross-Curricular Connections
Provides various connections between the standard and the knowledge and skills that students might use in other content areas.
Career and Skill Connections
Provides various connections between the standard and future careers/skills students will need for future careers.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards of Mathematical Practice
    - Procedural and Conceptual Understanding and Application
    - Sample Assessment Items
    - Common Misconceptions
    - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
    - Vertical Alignment
    - Culturally and Linguistically Responsive Instruction (CLRI)
    - Suggested Student Discourse Questions
    - Cross-Curricular and Career and Skill Connections
- A Student Discourse Guide
- Planning for a Multi-Layer System of Support (MLSS) and Universal Design for Learning (UDL) for behavioral and social and emotional supports

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Apply and extend previous understandings of arithmetic to algebraic expressions.
  - 6.EE.A.1
  - 6.EE.A.2
  - 6.EE.A.3
  - 6.EE.A.4
- Reason about and solve one-variable equations and inequalities.
  - 6.EE.B.5
  - 6.EE.B.6
  - 6.EE.B.7
  - 6.EE.B.8
- Represent and analyze quantitative relationships between dependent and independent variables.
  - 6.EE.C.9

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)

Grade	*CCSS Domain*	*CCSS Cluster*
6	Expressions and Equations	Apply and extend previous understandings of arithmetic to algebraic expressions.

Cluster Standard: 6.EE.A.1

Standard	Standards for Mathematical Practice
Write and evaluate numerical expressions involving whole-number exponents	• **SMP 6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
The focus for this cluster is writing and evaluating numerical expressions involving whole number exponents, finding the value of an expression using exponential notation such as 42= 4 x 4 or d 3 = d x d x d, and using the appropriate terminology to explain how to evaluate an expression. Students are applying the properties of operations to generate equivalent expressions including the distributive property to produce equivalent representation.	• Write and evaluate numerical expressions involving whole number exponents using the correct terminology • Evaluate numerical expressions using their knowledge of order of operations from previous years.
DOK	**Blooms**
2	Understand, Apply

Procedural and Conceptual Understanding and Application

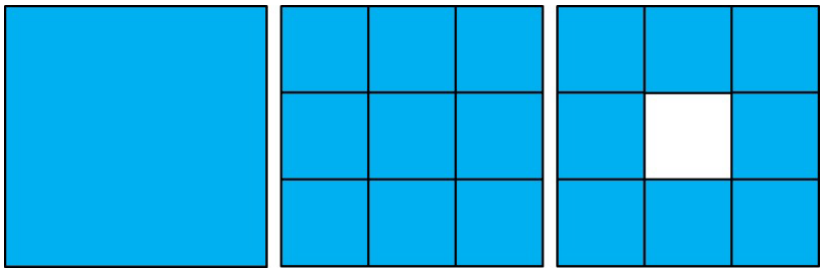
Procedural Skill and Fluency: Students demonstrate fluency in interpreting and evaluating expressions, including substituting values for variables and simplifying expressions and apply appropriate procedures to solve problems involving numeric expressions, including order of operations and properties of operations.

Conceptual Understanding: Students understand the concept of expressions as mathematical phrases that combine numbers, variables, and operations and the difference between expressions and equations, recognizing that equations contain an equal sign and expressions do not. Students must also understand the meaning of whole-number exponents, recognizing that, for example, 3^4 means multiplying 3 by itself four times ($3 \times 3 \times 3 \times 3$). This conceptual understanding is essential for students to correctly interpret and evaluate expressions involving exponents.

Assessment Items

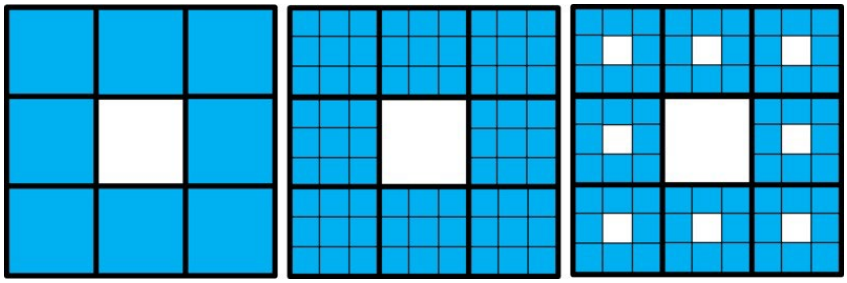
When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

a. Take a square with area 1. Divide it into 9 equal-sized squares. Remove the middle one.



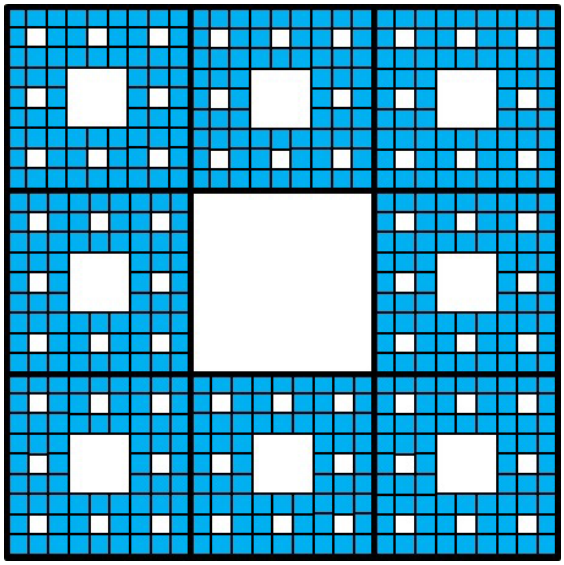
What is the area of the figure now?

b. Take the remaining 8 squares. Divide each one into 9 equal squares. Remove the middle one from each group of 9.



What is the area of the figure now?

c. Take the remaining squares. (How many are there?) Divide each one into 9 equal squares. Remove the middle one from each group of 9.



What is the area of the figure now?

d. Imagine you follow this same process until you have removed "the middle square from each group of 9" 10 times. How many squares will there be? What will the area of each little square be? What will the area of the entire figure be?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Equating Expressions with Equations:** Students may mistakenly believe that expressions and equations are the same thing. They might confuse the concept of evaluating an expression with solving an equation, leading to confusion when working with different types of mathematical statements.
- **Misinterpreting the Equal Sign:** Students may misunderstand the meaning of the equal sign in an equation. Instead of recognizing it as a symbol indicating equivalence between two expressions, they may interpret it as a signal to perform an operation, such as addition or multiplication, between the expressions.
- **Treating Variables as Fixed Numbers:** Students may have difficulty understanding that variables represent unknown quantities or placeholders for numbers. They might mistakenly treat variables as fixed numbers and apply arithmetic operations to them without considering their variable nature.
- **Confusing Terms and Factors:** Students may confuse terms and factors within an expression, leading to errors in simplification and evaluation. They might fail to recognize the distinction between the parts of an

expression and incorrectly apply operations to entire expressions rather than individual terms or factors.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning		
Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Provide visual representations such as diagrams, charts, or graphic organizers to help all students understand the concept of numerical expressions involving exponents visually. • Use concrete examples and real-world contexts to demonstrate the application of numerical expressions with exponents. Relate mathematical concepts to students' everyday experiences to make the content more accessible and relatable for all learners. **Universal Design for Learning (UDL)** • Provide multiple means of representation by using visual aids, manipulatives, and verbal explanations to accommodate diverse learning styles and preferences.	• Provide targeted small group instruction for students who need additional support in understanding numerical expressions with exponents. Offer differentiated activities based on individual student needs and learning styles, focusing on hands-on practice and peer collaboration. • Implement peer tutoring where students work in pairs or small groups to solve numerical expression problems involving exponents together. Pair students with varying levels of proficiency to provide peer support and encouragement.	• For students who require intensive support, provide one-on-one instruction tailored to their specific learning needs. Offer additional practice opportunities, reteaching sessions, and personalized learning experiences to address misconceptions and build foundational skills in numerical expressions with exponents. **Universal Design for Learning (UDL)** • Provide multiple means of representation by using visual aids, manipulatives, and verbal explanations to accommodate diverse learning styles and preferences.

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/6/25/249/249		
Previous Learning	*Current Learning*	*Future Learning*
• Students will connect their prior knowledge on using whole-number exponents to denote the powers of 10 in order to properly set-up	• Students will connect what they were previously taught in 6th grade finding the greatest common factor of two whole numbers and	• In 7th grade, learners will learn to apply properties of operations as strategies to add, subtract, factor, and expand linear expressions

exponents and identify the base. Additionally, in 5th grade learners have already been taught the commutative and associative property of both addition and multiplication.	using the distributive property to express sums of whole numbers to this cluster. These skills will be needed when students create and identify equivalent expressions.	with rational coefficients. Learners will develop an understanding of operations with rational numbers when working with expressions and linear equations. In 8th grade, students will know and apply the properties of integer exponents to generate equivalent numerical expressions. In high school, students will need to interpret parts of an expression, such as terms, factors, and coefficients.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Incorporate examples of numerical expressions with exponents from various cultural contexts, such as traditional practices, cultural celebrations, or historical events, to validate students' identities and experiences.	• Scaffold instruction by building on students' prior knowledge and experiences related to numerical expressions and exponents. Start with concrete examples and real-life contexts to introduce the concept of exponents, then gradually transition to more abstract representations. Provide opportunities for students to make connections between their cultural backgrounds and the mathematical content, such as exploring how numerical expressions with exponents are used in cultural practices or	• Provide linguistic supports to ensure that all students understand key mathematical vocabulary related to numerical expressions and exponents. Offer clear explanations of mathematical terms such as "exponent," "base," "power," and "evaluate." Use visual aids, gestures, and bilingual resources to support students' comprehension of mathematical vocabulary.

	traditions.	

Suggested Student Discourse Questions

- How do expressions differ from equations?
- How do you evaluate an expression when given specific values for the variables?
- Why is it important to understand how to evaluate expressions?
- How do you decide which operations to perform first when evaluating a complex expression?
- Can you think of examples where understanding expressions would be useful outside of the classroom?

Cross-Curricular Connections

Science:

Students can work to create, read, and evaluate expressions that result from the forces at work. Students will have to be able to create and support their argument.

English:

Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

Demonstrating the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.

Distinguish among facts, reasoned judgment based on research findings, and speculations in a text.

Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

Career and Skill Connections

- Bookkeeper
- Accountant
- Auditor
- Payroll Clerk
- Tax Preparer

Grade	CCSS Domain	CCSS Cluster
6	Expressions and Equations	Apply and extend previous understandings of arithmetic to algebraic expressions.

Cluster Standard: 6.EE.A.2

Standard	Standards for Mathematical Practice
Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 - y *A.* Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 - y* *B.* Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms* C. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = ½*	● **SMP 1:** Make sense of problems and persevere in solving them. ● **SMP 6**: Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
The focus for this cluster is writing and evaluating numerical expressions involving whole number exponents, finding the value of an expression using exponential notation such as 42= 4 x 4 or d 3 = d x d x d, and using the appropriate terminology to explain how to evaluate an expression. Students are applying the properties of operations to generate equivalent expressions including the distributive property to produce equivalent representation.	● Express orally and in writing that variables represent unknown quantities. ● Write expressions using variables that represent unknown numbers. ● Identify context to write algebraic expressions. ● Translate verbal expressions into numerical expressions. ● Use information from real world examples to evaluate expressions with variables

DOK	Blooms
2	Understand, Apply

Procedural and Conceptual Understanding and Application

Procedural Skill and Fluency: Students demonstrate fluency in writing and evaluating numerical expressions that involve both numbers and letters accurately and quickly. Further demonstrating fluency in writing and evaluating numerical expressions finding the value of the expression using exponential notation, while using appropriate terminology explaining evaluating the expression efficiently.

Conceptual Understanding: Students will be able to identify parts of an expression accurately and efficiently. Students will be able to evaluate the value of a variable within an expression. Students will need to use a higher order of thinking to reason when the expressions are equivalent, based on the value being substituted in for the variable. Students need to understand how to translate verbal descriptions of operations into algebraic expressions.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Some of the students at Kahlo Middle School like to ride their bikes to and from school. They always ride unless it rains. Let *d* be the distance in miles from a student's home to the school. Write two different expressions that represent how far a student travels by bike in a four-week period if there is one rainy day each week.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Incorrectly multiplying base:** When students work with exponents, they often multiply the exponent by the base rather than extend the base.
- **Variable substitution and operation performed incorrectly:** Students may not correctly substitute the variable value in correctly to the expression and perform the correct operation once the value is placed into the expression.
- **Order of operation process:** Students may not follow the order of operations correctly, lacking that conceptual understanding of the order of operations and why the order works mathematically.
- **Fractions viewed as division:** Students may misunderstand that a fraction problem can also be viewed as a division problem.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Standards aligned instruction. • Rigorous assessments. • Teacher modeled instruction. • Collaborative learning. • Differentiated Instruction. • Scaffolding. • Data Driven Instruction. **Universal Design for Learning (UDL)** • Provide multiple means of representation by using visual aids, manipulatives, and verbal explanations to accommodate diverse learning styles and preferences.	• Increased Instruction time with Modeling or tutoring. • Positive Reinforcement. • Academic Support, providing multiple examples. **Universal Design for Learning (UDL)** • Expose students to multiple means of presentation by anchor charts, manipulatives, auditory, and verbal explanations to accommodate diverse learning styles and preferences.	• Individualized one on one tutoring. • Progress Monitoring based on individualized student needs. • Provide notes and examples to students daily on lessons. • Explicit and Systematic Instruction. **Universal Design for Learning (UDL)** • Provide strategic systems of support by utilizing small group instruction, one-to-one, and direct teacher-to-student discourse to accommodate diverse learning styles and preferences.

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/6/25/249/249		
Previous Learning	*Current Learning*	*Future Learning*
• Students will connect their prior knowledge on using whole-number exponents to denote the powers of 10 in order to properly set-up exponents and identify the base. Additionally, in 5th grade learners have already been taught the commutative and associative property of both addition and multiplication.	• Students will connect what they were previously taught in 6th grade finding the greatest common factor of two whole numbers and using the distributive property to express sums of whole numbers to this cluster. These skills will be needed when students create and identify equivalent expressions.	• In 7th grade, learners will learn to apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. Learners will develop an understanding of operations with rational numbers when working with expressions and linear equations. In 8th grade, students will know and apply the properties of integer

		exponents to generate equivalent numerical expressions. In high school, students will need to interpret parts of an expression, such as terms, factors, and coefficients.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Creating a welcoming and affirming environment supporting all students in the classroom through a rich curriculum. Create a classroom that allows all students to feel comfortable. Materials that represent and affirm student identities. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". High expectations and rigorous instruction for all students will be provided at all times in the classroom.	• Legitimize the language and home culture of students through a collective responsibility of learning about students' cultures and communities through a close relationship with the students and their families. Build on prior knowledge to show a connection to current learning. Inclusion of current events into instruction related to varied communities to build and bridge relation to the identities of the various cultures within the classroom. Students as co-teachers and designers of the lessons. Inclusion of Instructional materials that adapt to diverse learning styles.	• Provide for a classroom that allows for all students equally to be comfortable regardless of their language. Provide appropriate strategies and check for understanding often. Label items around the room in both English and other languages to support the native languages in the classroom. Provide manipulatives to help students understand concepts with hands-on materials. Create a word wall. Cooperative learning such as small groups can be used to maximize learning. Building on background knowledge has also been found to be useful. Technology can also be a valuable tool.

Suggested Student Discourse Questions

- Can you provide examples of real-world situations where expressions or equations would be used to solve problems or make decisions?
- How do we interpret the solution to an equation in the context of the problem? What does it represent?
- What does it mean for two expressions to be equivalent? How can we determine if two expressions represent the same quantity?
- Can you explain why we can manipulate expressions using properties of operations and still maintain equivalence?
- How can we use equivalent expressions to simplify or solve problems more efficiently?

Cross-Curricular Connections

Science:

Students can work to create, read, and evaluate expressions that result from the forces at work. Students will have to be able to create and support their argument. (MS-PS2-1, Motion and Stability: Forces and Interactions)

English:

Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

Demonstrating the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.

Distinguish among facts, reasoned judgment based on research findings, and speculations in a text.

Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

Career and Skill Connections

- Bill and Account Collector
- Database Architect
- Computer Programmer
- Software Developer
- High School Algebra Teacher

Grade	CCSS Domain	CCSS Cluster
6	Expressions and Equations	Apply and extend previous understandings of arithmetic to algebraic expressions.

Cluster Standard: 6.EE.A.3

Standard	Standards for Mathematical Practice
Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y*	• **SMP 2:** Reason abstractly and quantitatively.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
The focus for this cluster is writing and evaluating numerical expressions involving whole number exponents, finding the value of an expression using exponential notation such as 42= 4 x 4 or d 3 = d x d x d, and using the appropriate terminology to explain how to evaluate an expression. Students are applying the properties of operations to generate equivalent expressions including the distributive property to produce equivalent representation.	• Create an equivalent expression through the use of properties of operations • Apply the distributive, commutative, identity, and distributives properties to expressions that include variables
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Procedural Skill and Fluency: Involves students' ability to perform mathematical procedures accurately and efficiently. In this standard, procedural rigor could entail students' proficiency in applying the rules of arithmetic operations (addition, subtraction, multiplication, division) to simplify expressions and solve equations.

Conceptual Understanding: Involves students' understanding of the underlying concepts and principles behind mathematical procedures. In this standard, conceptual rigor might involve students' comprehension of the properties of operations and their application in manipulating expressions and solving equations. Students need to understand the properties of operations, such as the distributive property and the idea of combining like terms. This understanding allows them to recognize how expressions can be rewritten in equivalent forms. For example,

recognizing that *y+y+y* is equivalent to *3y* requires understanding that adding a variable multiple times is the same as multiplying that variable by a number.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Anna enjoys dinner at a restaurant in Washington, D.C., where the sales tax on meals is 10%. She leaves a 15% tip on the price of her meal before the sales tax is added, and the tax is calculated on the pre-tip amount. She spends a total of $27.50 for dinner. What is the cost of her dinner without tax or tip?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Treating Variables as Constant Values -** Some students may mistakenly view variables in expressions or equations as fixed, constant values rather than placeholders for unknown quantities. For instance, in the expression "3X + 5," students may erroneously interpret "X" as a fixed number rather than recognizing it as a variable representing an unknown quantity.
- **Misapplying the Order of Operations -** Students may misunderstand or misapply the order of operations when simplifying expressions, leading to incorrect solutions. For example, they might perform addition or subtraction before multiplication or division, resulting in errors in the final answer. This misconception can arise when students fail to apply the correct sequence of operations according to PEMDAS (Parenthesis, Exponents, Multiplication and Division, Addition, and Subtraction).
- **Failing to Recognize Equivalent Expressions** - Students may struggle to identify equivalent expressions that represent the same mathematical relationship but are written differently. For example, they may fail to recognize that "2(X +4)" and "2X + 8" are equivalent expressions, leading to errors in simplification or substitution. This misconception can impede students' ability to manipulate expressions effectively and solve equations efficiently.
- **Equating Expressions to Equations** - Students may conflate (merge) expressions with equations and incorrectly treat them as equivalent. In particular, students may mistake the expression "2X + 3" for an equation and attempt to solve it for a specific value of "X," disregarding the absence of an equal sign. This misconception can lead to confusion about the distinction between expressions and equations and hinder students' ability to solve equations accurately.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Teach all students regardless	• Use formative assessment	• Scaffold executive function

of who the students are with this standard, 6.EE.A.3 by explicitly exposing instructional lessons in multiple ways. For example, provide multiple means of representation- offer multiple representations of mathematical expressions and equations, such as visual models, manipulatives, and diagrams to accommodate diverse learning styles and preferences. Use visual aids, graphic organizers, and multimedia resources to present information in varied formats, making mathematical concepts accessible to ALL STUDENTS. **Universal Design for Learning (UDL)** • Identity learning Goals and Objectives - Clarify the learning goals and objectives related to this particular standard to ALL STUDENTS, specifying the essential knowledge and skills that ALL STUDENTS need to demonstrate mastery. Align instructional activities and assessments with these goals to ensure coherence and relevance.	data, standardized test scores, and teacher observations to identify students who are struggling to meet the learning goals outlined in this standard. Look for patterns of misunderstanding, gaps in foundational knowledge, and areas of difficulty related to expressions and equations. • Tailor instruction to meet the diverse needs of students' identified as requiring additional support. Provide targeted interventions, small-group instruction, or one-on-one tutoring sessions that focus on specific areas of difficulty within cluster standard 6.EE.A.3, such as simplifying expressions, solving equations, or understanding mathematical properties. Other ideas include: Use research-based interventions, provide supplemental materials and resources, monitor progress and adjust instruction, collaborate with support staff, and provide structured support time. **Universal Design for Learning (UDL)** • Differentiate Instruction Using UDL Practices- Design targeted interventions and supports that incorporate UDL principles to accommodate diverse learner needs. For example, provide options for representation, expression, and engagement to ensure accessibility and flexibility	skills such as organization, planning, and self-regulation to support students in managing mathematical tasks effectively. Provide clear instructions, checklist, and graphic organizers to help students plan and organize their problem-solving process and monitor their progress towards goals. • Provide flexible options for students to demonstrate their understanding of mathematical concepts and problem-solving strategies. Allow for multiple modes of expression, such as written responses, verbal explanations, visual representations, or digital media to accommodate diverse communication preferences and strengths. • Provide Personalized Supports: Offer personalized support and accommodations to meet the individual student needs within the context of UDL. Adjust instructional materials, resources, and assessments to provide appropriate levels of challenge, support, and differentiation for each student. Consider students' learning preferences, interests, and strengths when designing interventions. **Universal Design for Learning (UDL)** • Monitor Progress and Adjust Interventions: Regularly monitor students' progress and response to interventions using formative assessment data and progress monitoring tools. Use this data to make

	for ALL STUDENTS. Offer multiple means of accessing content, demonstrating understanding, and engaging with learning activities.	informed decisions about the effectiveness of interventions and adjust supports as needed to ensure student success.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/6/25/249/249

Previous Learning	*Current Learning*	*Future Learning*
• Students will connect their prior knowledge on using whole-number exponents to denote the powers of 10 in order to properly set-up exponents and identify the base. Additionally, in 5th grade learners have already been taught the commutative and associative property of both addition and multiplication.	• Students will connect what they were previously taught in 6th grade finding the greatest common factor of two whole numbers and using the distributive property to express sums of whole numbers to this cluster. These skills will be needed when students create and identify equivalent expressions.	• In 7th grade, learners will learn to apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. Learners will develop an understanding of operations with rational numbers when working with expressions and linear equations. In 8th grade, students will know and apply the properties of integer exponents to generate equivalent numerical expressions. In high school, students will need to interpret parts of an expression, such as terms, factors, and coefficients.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Affirming Classroom Environment: Foster a	• Build on Prior Knowledge - Begin instruction by tapping	• Pre-Teach Key Vocabulary - Introduce and explicitly teach

classroom environment that is inclusive, affirming, and respectful of students' cultural identities and linguistic backgrounds. For example, create opportunities for students to share their cultural traditions, stories, and perspectives, and actively listen and validate their contributions. ● Additionally, incorporate diverse representations of individuals, families, and communities in instructional materials, illustrations, and examples. Include images, stories, and cultural references that reflect the diversity of students' identities and backgrounds, fostering a sense of representation and belonging. ● Affirm Culturally Responsive Problem-Solving by designing problem-solving tasks that draw students' cultural knowledge, experiences, and perspectives. Encourage collaborative problem-solving activities that value diverse ways of thinking, problem-solving strategies, and mathematical approaches; therefore, recognizing the richness of cultural diversity in mathematical reasoning. ● Acknowledge and respect students' diverse linguistic backgrounds and validate the use of multiple languages in the classroom.	into students' prior knowledge and experiences related to expressions and equations. Encourage students to share their own strategies, cultural perspectives, and problem-solving approaches, fostering a sense of relevance and ownership of the learning process. ● Make Connections: Facilitate connections between mathematical concepts and students' cultural backgrounds, interests, and lived experiences. Draw parallels between mathematical ideas and familiar contexts, stories, or cultural practices; thus, helping students see the relevance and applicability of the concepts to their own lives. ● Provide Multiple Modalities: Offer instruction and learning activities that cater to diverse learning styles and preferences, including visual, auditory, kinesthetic, and tactile modalities. Use a variety of instructional materials, manipulatives, multimedia resources, and interactive technology tools to engage students and reinforce key concepts through multiple sensory channels.	key mathematical vocabulary words BEFORE engaging in activities related to this particular standard. Provide definitions, examples, and non-examples of mathematical terms to ensure clarity and comprehension. ● Utilize Visual Supports- Pair mathematical vocabulary words with visual supports such as Visual representations can help reinforce vocabulary learning and provide context for understanding. ● Provide Contextual Examples: Embed mathematical vocabulary words within contextual examples and real-life situations that are meaningful and relevant to students' experiences. Relating vocabulary to familiar contexts helps students make connections and deepen their understanding of the terms. ● Model Language Use: Model the use of mathematical vocabulary in oral and written communication during instruction. Use precise mathematical language when explaining concepts, solving problems, and facilitating discussions, and encourage students to use the same vocabulary in their responses.
Suggested Student Discourse Questions		
● What does it mean for two expressions to be equivalent? How can we determine if two expressions represent		

the same quantity?

- Can you provide examples of equivalent expressions? How do we know they are equivalent?
- How can we use properties of operations to generate equivalent expressions?
- What strategies can we use to simplify expressions by combining like terms, distributing, or factoring?
- How can we use simplification to make expressions easier to work with or to solve problems?

Cross-Curricular Connections

Science:

Students can work to create, read, and evaluate expressions that result from the forces at work. Students will have to be able to create and support their argument. (MS-PS2-1, Motion and Stability: Forces and Interactions)

English:

Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

Demonstrating the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.

Distinguish among facts, reasoned judgment based on research findings, and speculations in a text.

Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

Career and Skill Connections

- Architect
- Budget Analyst
- Air Traffic Controller
- Insurance Underwriter
- STEM Educator

Grade	CCSS Domain	CCSS Cluster
6	Expressions and Equations	Apply and extend previous understandings of arithmetic to algebraic expressions.

Cluster Standard: 6.EE.A.4

Standard	Standards for Mathematical Practice
Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for*	• **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
The focus for this cluster is writing and evaluating numerical expressions involving whole number exponents, finding the value of an expression using exponential notation such as 42= 4 x 4 or d 3 = d x d x d, and using the appropriate terminology to explain how to evaluate an expression. Students are applying the properties of operations to generate equivalent expressions including the distributive property to produce equivalent representation.	• Identify equivalent expressions. • Combine like terms • Reason that two expressions are equivalent through the use of substitution
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Conceptual Understanding: Students must grasp the concept of equivalence in algebraic expressions, understanding that two expressions are equivalent if they yield the same value for any substitution of the variables involved. For example, recognizing that *y+y+y* and *3y* are equivalent requires an understanding that these expressions represent the same mathematical relationship, regardless of the value of *y*. This deepens their understanding of algebraic structure and the meaning of expressions.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Which of the following expressions are equivalent? Why? If an expression has no match, write 2 equivalent expressions to match it.

a. 2(x+4)
b. 8+2x
c. 2x+4
d. 3(x+4)−(4+x)
e. X+4

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Misinterpret base and exponent:** When given an expression with an exponent, students may misinterpret the base and the exponent as factors and multiply the two numbers. For example, show that 5 x 3=15, which is much smaller than 5x5x5 which equals 125.
- **Distributive property understanding:** Students may use distributive property incorrectly in that students will often multiply the first term, but forget to do the same to the second term.
- **Commutative property understanding:** Students may misuse the commutative property by applying it to subtraction and/or division problems.
- **Variable representation:** Students confuse variables with letters for units of measure.
- **Order of operation procedure:** Students may confuse the order of operations and the order in which they are applied within the problem.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Begin by presenting students with two expressions, such as 2x + 3 and x + x + 3, and ask them to determine if they are equivalent. • Facilitate a brief discussion about what it means for expressions to be equivalent and why it is important in *mathematics.*	• Review key concepts from the core instruction lesson, including the definition of equivalent expressions and strategies for identifying them. • Provide differentiated practice problems based on	• Administer a diagnostic assessment to identify specific areas of difficulty for individual students related to identifying equivalent expressions. • Group students based on their assessment results and provide targeted instruction to address their individual

• Define the concept of equivalent expressions as expressions that represent the same value for any given value of the variable(s). • Model how to identify equivalent expressions by simplifying and comparing them, focusing on common terms and properties of operations. • Provide examples of equivalent expressions and guide students through the process of determining their equivalence. • Provide guided practice problems where students identify whether pairs of expressions are equivalent. • Walk students through each problem step-by-step, demonstrating how to simplify and compare the expressions to determine their equivalence. • Assign independent practice problems for students to complete, identifying equivalent expressions on their own. • Encourage students to use the strategies and techniques learned during the lesson to justify their answers.	students' individual needs, offering additional support for struggling students and extension activities for advanced learners. • Offer visual aids or manipulatives to support students who need additional help with abstract concepts. • Present multi-step word problems that require students to apply their understanding of equivalent expressions in real-world contexts. • Encourage students to explain their problem-solving strategies and justify their reasoning. • Provide multiple means of representation by using visual aids, manipulatives, and verbal explanations to accommodate diverse learning styles and preferences. • Offer multiple means of engagement by incorporating real-world examples, interactive activities, and student choice to increase motivation and engagement. • Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.	needs. • Use a variety of instructional strategies, including hands-on activities and peer tutoring, to support students' understanding. • Provide one-on-one support for students who require intensive intervention, focusing on addressing their specific learning needs. • Offer additional practice opportunities and scaffolded support to help students build confidence and mastery. • Provide multiple means of representation by using visual aids, manipulatives, and verbal explanations to accommodate diverse learning styles and preferences. • Offer multiple means of engagement by incorporating real-world examples, interactive activities, and student choice to increase motivation and engagement. • Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/6/25/249/249		
Previous Learning	*Current Learning*	*Future Learning*
• Students will connect their prior knowledge on using whole-number exponents to denote the powers of 10 in order to properly set-up exponents and identify the base. Additionally, in 5th grade learners have already been taught the commutative and associative property of both addition and multiplication.	• Students will connect what they were previously taught in 6th grade finding the greatest common factor of two whole numbers and using the distributive property to express sums of whole numbers to this cluster. These skills will be needed when students create and identify equivalent expressions.	• In 7th grade, learners will learn to apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. Learners will develop an understanding of operations with rational numbers when working with expressions and linear equations. In 8th grade, students will know and apply the properties of integer exponents to generate equivalent numerical expressions. In high school, students will need to interpret parts of an expression, such as terms, factors, and coefficients.
Culturally and Linguistically Responsive Instruction		
Consider these questions as you plan for instruction that is culturally and linguistically responsive: • How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Acknowledge the diverse backgrounds and experiences of the students by asking them to discuss in small groups or pairs how their cultural backgrounds might influence their understanding of mathematical concepts like equivalence. Encourage	• Connect the concept of equivalent expressions to real-world scenarios or contexts that students can relate to. For example, discuss how different languages may have multiple ways of expressing the same idea, similar to	• Introduce and reinforce key vocabulary words related to the lesson, such as "equivalent," "expressions," "substitute," and "variable." Provide definitions and examples and encourage students to use these words in their discussions and

students to share examples if they feel comfortable. ● Display the Cluster Standard 6.EE.A.4 on the board and explain it in simple terms to the class. Emphasize the importance of recognizing when two expressions are equivalent and how this understanding is fundamental in algebraic problem-solving. ● Provide examples of equivalent expressions, such as y + y + y and 3y, and demonstrate how they represent the same number regardless of the value of y. ● Distribute index cards with different expressions written on them to each student or group. Have students work together to identify pairs of expressions that are equivalent. ● Encourage students to discuss their reasoning and strategies for determining equivalence, emphasizing that the expressions must represent the same value regardless of the variable used. ● Walk around the classroom to provide support and guidance	how algebraic expressions can be equivalent. ● Encourage students to make connections between mathematical concepts and their own cultural backgrounds, highlighting the diversity of perspectives and approaches to problem-solving. ● Provide additional practice problems or challenges for students to identify equivalent expressions independently or in groups. ● Encourage students to create their own examples of equivalent expressions and share them with the class. ● Explore cultural expressions or idioms from different languages that have equivalent meanings, drawing parallels to the concept of equivalent expressions in algebra. NOTE: Relate instructional activities to local cultures, geography of the district, and demographics, including background of all students in the classroom.	explanations. ● Offer linguistic support for students who may be English language learners by providing translations or additional explanations in their native languages if needed. ● Present students with real-world scenarios or images that require them to identify equivalent expressions. Encourage them to apply the concept they have learned to solve these problems and discuss their solutions with their peers. NOTE: Relate instructional activities to local cultures, geography of the district, and demographics, including background of all students in the classroom.

as needed and facilitate a whole-class discussion to review the pairs of equivalent expressions identified by the students. NOTE: Relate instructional activities to local cultures, geography of the district, and demographics, including background of all students in the classroom.		

Suggested Student Discourse Questions

- What are variables, and how are they used in expressions and equations?
- Can you explain how changes in one variable affect another variable in an expression or equation?
- How do we interpret the relationship between variables in the context of a problem or situation?
- How can we use the values of variables to predict or analyze the behavior of the expression or equation?
- What strategies can we use to graph or visualize relationships between variables

Cross-Curricular Connections

Science:

Students can work to create, read, and evaluate expressions that result from the forces at work. Students will have to be able to create and support their argument. (MS-PS2-1, Motion and Stability: Forces and Interactions)

English:

Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

Demonstrating the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.

Distinguish among facts, reasoned judgment based on research findings, and speculations in a text.

Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

Career and Skill Connections

- Auditing Clerk
- Civil Engineer
- Chemist
- Auditor
- Accountant

Grade	CCSS Domain	CCSS Cluster
6	Expressions and Equations	Reason about and solve one-variable equations and inequalities.

Cluster Standard: 6.EE.B.5

Standard	Standards for Mathematical Practice
Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 2:** Reason abstractly and quantitatively. • **SMP 4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students focus on the meaning of an equation and use reasoning and prior knowledge to solve it. They use variables to represent numbers and write expressions when solving problems. Students learn to write inequalities in the form of $x > c$, $x \geq c$, $x < c$ or $x \leq c$ and use number line representation to show the solutions of inequalities.	• Reason to find the single value that makes an equation true • Explain what a variable is representing in a particular situation or context • Use substitution to simplify numerical expressions and determine if the solution is true.
DOK	**Blooms**
2	Understand, Apply

Procedural and Conceptual Understanding and Application

Procedural Skill and Fluency: Students will solve equations and inequalities using prior knowledge and reasoning. Students will use the substitution method to find solution sets for an inequalities or equations. This requires fluency in substituting values into expressions and accurately performing the necessary calculations to determine if the equation or inequality holds true.

Conceptual Understanding: Students must understand that solving an equation or inequality is about finding which values, if any, satisfy the condition expressed by the equation or inequality. This involves grasping the idea that an equation or inequality represents a relationship between quantities, and solving it means determining which values make that relationship true.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

A theme park has a log ride that can hold 12 people. They also have a weight limit of 1500 lbs. per log for safety reasons. If the average adult weighs 150 lbs., the average child weighs 100 lbs. and the log itself weighs 200, the ride can operate safely if the inequality

$$150A+100C+200\leq 1500$$

is satisfied (*A* is the number of adults and *C* is the number of children in the log ride together). There are several groups of children of differing numbers waiting to ride. Group one has 4 children, group two has 3 children, group three has 9 children, group four 6 children while group five has 5 children.

If 4 adults are already seated in the log, which groups of children can safely ride with them?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Difference between symbols:** Some students may confuse the inequalities symbol with the equal sign.
- **Inequality and equal sign use:** Students treating the inequalities the same as equations is a common error.
- **Understanding finding an element of solution sets:** Students may not be able to check whether or not an element is in the solution set of the inequality.
- **Satisfying both statements and overlapping of solutions:** Confusion can come from thinking that the solution set to an inequality must be an inequality. In some inequalities there are no solutions when no numbers satisfy both statements or when the inequalities overlap.
- **Understanding infinity meaning:** A general misunderstanding of the meaning of infinity may prevent students from understanding the concept of infinitely many solutions.
- **Conceptualizing number of solutions:** Students may have difficulty conceptualizing that an inequality can have more than one solution.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Begin by presenting students with a real-world scenario involving an equation or inequality, such as "Samantha has $20. She wants to buy a toy that costs $8. How much money will she have left?" • Facilitate a brief discussion about the question being	• Review key concepts from the core instruction lesson, including the process of solving equations and inequalities using substitution. • Provide differentiated practice problems based on students' individual needs,	• Administer a diagnostic assessment to identify specific areas of difficulty for individual students related to solving equations and inequalities. • Group students based on their assessment results and provide targeted instruction

asked and what values would make the equation or inequality true. • Define the process of solving an equation or inequality as determining which values from a specified set, if any, make it true. • Model how to use substitution to test whether a given value makes an equation or inequality true, emphasizing the importance of plugging in the value and checking the result. • Provide examples of equations and inequalities and guide students through the process of determining their solutions using substitution. • Provide guided practice problems where students use substitution to determine whether given values make equations or inequalities true. • Walk students through each problem step-by-step, demonstrating how to substitute the value and check the result. • Assign independent practice problems for students to complete, using substitution to solve equations and inequalities. • Encourage students to check their work and ask questions if they encounter difficulties.	offering additional support for struggling students and extension activities for advanced learners. • Offer manipulatives or visual aids to support students who need additional help with abstract concepts. • Present multi-step word problems that require students to apply their understanding of solving equations and inequalities in real-world contexts. **Universal Design for Learning (UDL)** • Provide multiple means of representation by using visual aids, manipulatives, and verbal explanations to accommodate diverse learning styles and preferences. • Offer multiple means of engagement by incorporating real-world examples, interactive activities, and student choice to increase motivation and engagement. • Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations. • Ask students to explain their problem-solving strategies and justify their reasoning.	to address their individual needs. • Use a variety of instructional strategies, including hands-on activities and peer tutoring, to support students' understanding. • Provide one-on-one support for students who require intensive intervention, focusing on addressing their specific learning needs. • Offer additional practice opportunities and scaffolded support to help students build confidence and mastery. **Universal Design for Learning (UDL)** • Provide multiple means of representation by using visual aids, manipulatives, and verbal explanations to accommodate diverse learning styles and preferences. • Offer multiple means of engagement by incorporating real-world examples, interactive activities, and student choice to increase motivation and engagement. • Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.
Vertical Alignment		

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/6/25/249/249

Previous Learning	*Current Learning*	*Future Learning*
• Students connect their previous understandings of what the equal sign is and that it shows equivalence to this cluster. The idea of equivalence is most aligned to their work in grades 4 and 5 with visual fraction models and understanding basic properties of operations to solve	• This cluster really expands on the previous cluster where students learned how to read, write and evaluate expressions in which letters stand for numbers.	• In Grade 7, students begin to formally apply the properties of operations. They will solve two step equations in the form of px + q = r and p(x + q) = r. In Grade 8, students solve linear equations in one variable that include one solution, no solution, or infinitely many solutions. They include equations that require the distributive property or combining like terms. In Grade 8, the variable can be on both sides of the equation. In high school, students further their knowledge of solving equations with multi step equations that require the distributive property or combining like terms

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Validate and affirm students' diverse backgrounds by inviting them to share their cultural perspectives on problem-solving. Ask questions such as: "How does your cultural background influence the way you approach solving problems?" Encourage students to share	• Connect the concept of equation solving to real-world scenarios or contexts that students can relate to. For example, discuss how determining solutions to equations or inequalities can help solve practical problems such as budgeting, scheduling, or	• Introduce and reinforce key vocabulary words related to the lesson, such as "equation," "inequality," "substitution," and "solution set." Provide definitions and examples and encourage students to use these words in their discussions and explanations.

examples if they feel comfortable. • Display the Cluster Standard 6.EE.B.5 on the board and explain it in simple terms to the class. Emphasize that solving equations or inequalities involves finding which values from a specified set make them true. • Provide examples of equations and inequalities and demonstrate how substitution can be used to determine whether a given value makes them true. • Distribute index cards with equations and inequalities written on them to each student or group. Have students work together to identify which values from a specified set make the equations or inequalities true by substituting different values and checking the results. • Encourage students to discuss their reasoning and strategies for determining the solutions, emphasizing the importance of systematic problem-solving techniques. • Walk around the classroom to provide support and guidance as needed and facilitate a whole-class discussion to review the solutions identified by the students.	planning events. • Encourage students to make connections between mathematical concepts and their own cultural backgrounds, highlighting the diversity of perspectives and approaches to problem-solving. • Provide additional practice problems or challenges for students to solve equations or inequalities independently or in groups. • Encourage students to create their own real-world scenarios that can be solved using equations or inequalities and share them with the class. • Explore cultural perspectives on problem-solving techniques and mathematical approaches from different regions or communities around the world. In particular, utilize students' background, skills, and local instructional resources.	• Offer linguistic support for students who may be English language learners by providing translations or additional explanations in their native languages if needed. • Provide additional practice problems or challenges for students to solve equations or inequalities independently or in groups. • Encourage students to create their own real-world scenarios that can be solved using equations or inequalities and share them with the class. • Explore cultural perspectives on problem-solving techniques and mathematical approaches from different regions or communities around the world. In particular, utilize students' background, skills, and local instructional resources.

Suggested Student Discourse Questions

- What does it mean for an equation to have variables on both sides? How is it different from equations with variables on one side?

- Can you explain why it's important to isolate the variable when solving equations with variables on both sides?
- How do we determine the solution(s) to an equation with variables on both sides?
- How do we interpret the meaning of our solutions in the context of the problem?
- What strategies can we use to check our solutions for accuracy and reasonableness?

Cross-Curricular Connections

Science:

Students can work to create, read, and evaluate expressions that result from the forces at work. Students will have to be able to create and support their argument. (MS-PS2-1, Motion and Stability: Forces and Interactions)

English:

Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

Demonstrating the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.

Distinguish among facts, reasoned judgment based on research findings, and speculations in a text.

Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

Career and Skill Connections

- Architectural designer
- Application/ Software Developer
- Database Architect
- Biologist
- Chemist

Grade	*CCSS Domain*	*CCSS Cluster*
6	Expressions and Equations	Reason about and solve one-variable equations and inequalities.

Cluster Standard: 6.EE.B.6

Standard	Standards for Mathematical Practice
Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 2**: Reason abstractly and quantitatively. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students focus on the meaning of an equation and use reasoning and prior knowledge to solve it. They use variables to represent numbers and write expressions when solving problems. Students learn to write inequalities in the form of $x > c$, $x \geq c$, $x < c$ or $x \leq c$ and use number line representation to show the solutions of inequalities.	• Explain that a variable represents a number or a specified set of numbers. • Identify what the variable represents quantitatively and in context. • Represent real world scenarios with variable expressions.
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Procedural Skill and Fluency: Practice with speed and accuracy solving equations using reasoning and prior knowledge. Decide whether the variables represented are solutions to the expressions when solving problems. Analyze the meaning of solutions to an equation and inequality and defend whether the solution makes sense in real-world context.

Conceptual Understanding: Students must grasp the concept of a variable as a symbol that can represent an unknown number or any number within a specified set. This involves understanding the role of variables in mathematical expressions and equations, and how they can be used to model real-world and mathematical situations.

Application: Students are expected to apply their understanding of variables to solve real-world or mathematical problems. This includes using variables to represent numbers and writing expressions that model these situations. The ability to translate a real-world scenario into a mathematical expression using variables is a key skill in applying mathematical concepts to practical problems.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

A town's total allocation for firefighter's wages and benefits in a new budget is $600,000. If wages are calculated at $40,000 per firefighter and benefits at $20,000 per firefighter, write an equation whose solution is the number of firefighters the town can employ if they spend their whole budget. Solve the equation.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Variable values:** The idea that the value for a certain variable will always be the same for every instance of that variable in future problems in the worksheet they are working on; for instance, if x= 2 on question 1 it will be the same for question 4 if it has 'x' in it.
- **Variable meaning within the expression:** Students can sometimes confuse the variables used in expressions with variables used in formulas.
- **Meaning of infinity:** Students may not fully understand the concept of infinity which will inhibit their understanding of the concept of the inequality solution set.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Begin by presenting students with a real-world scenario involving an unknown quantity, such as "Samantha has some money. She wants to buy a toy that costs $8. How much money does she have left?" • Facilitate a brief discussion about the unknown quantity and how it can be represented using a variable. • Define the concept of variables as symbols used to represent unknown quantities or values that can change. • Model how to use variables to write expressions and equations that represent real-world or mathematical problems, emphasizing the importance of choosing appropriate variable names and defining their meanings. • Provide examples of problems and guide students	• Review key concepts from the core instruction lesson, including the use of variables to represent numbers and write expressions. • Provide differentiated practice problems based on students' individual needs, offering additional support for struggling students and extension activities for advanced learners. • Offer manipulatives or visual aids to support students who need additional help with abstract concepts. • Present multi-step word problems that require students to apply their understanding of using variables in real-world contexts. • Encourage students to explain their problem-solving strategies and justify	• Administer a diagnostic assessment to identify specific areas of difficulty for individual students related to using variables to represent numbers and write expressions. • Group students based on their assessment results and provide targeted instruction to address their individual needs. • Use a variety of instructional strategies, including hands-on activities and peer tutoring, to support students' understanding. • Provide one-on-one support for students who require intensive intervention, focusing on addressing their specific learning needs. • Offer additional practice opportunities and scaffolded support to help students build confidence and mastery. • Offer multiple means of

through the process of identifying variables and writing expressions to represent them. ● Provide guided practice problems where students use variables to represent numbers and write expressions for given situations. ● Walk students through each problem step-by-step, demonstrating how to choose appropriate variables and write expressions. ● Assign independent practice problems for students to complete, using variables to represent numbers and write expressions. ● Encourage students to check their work and ask questions if they encounter difficulties.	their reasoning. ● Offer multiple means of engagement by incorporating real-world examples, interactive activities, and student choice to increase motivation and engagement. ● Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations. **Universal Design for Learning (UDL)** ● Provide multiple means of representation by using visual aids, manipulatives, and verbal explanations to accommodate diverse learning styles and preferences.	engagement by incorporating real-world examples, interactive activities, and student choice to increase motivation and engagement. ● Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations. **Universal Design for Learning (UDL)** ● Provide multiple means of representation by using visual aids, manipulatives, and verbal explanations to accommodate diverse learning styles and preferences.
Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/6/25/249/249		
Previous Learning	*Current Learning*	*Future Learning*
● Students connect their previous understandings of what the equal sign is and that it shows equivalence to this cluster. The idea of equivalence is most aligned to their work in grades 4 and 5 with visual fraction models and understanding basic properties of operations to solve.	● This cluster really expands on the previous cluster of 6.EE.A.2 where students learned how to read, write and evaluate expressions in which letters stand for numbers.	● In Grade 7, students begin to formally apply the properties of operations. They will solve two step equations in the form of px + q = r and p(x + q) = r. In Grade 8, students solve linear equations in one variable that include one solution, no solution, or infinitely many solutions. They include equations that require the distributive property or combining like terms. In Grade

		8, the variable can be on both sides of the equation. In high school, students further their knowledge of solving equations with multi step equations that require the distributive property or combining like terms.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Validate and affirm students' diverse backgrounds by inviting them to share how they have encountered variables or unknown quantities in their own lives or cultural contexts. Encourage students to share examples if they feel comfortable. • Display the Cluster Standard 6.EE.B.6 on the board and explain it in simple terms to the class. Emphasize that variables are placeholders for numbers and can represent either unknown quantities or any number in a specified set, depending on the context. • Provide examples of real-world or mathematical problems where variables can be used to represent unknown quantities or quantities within a specified range. • Distribute index cards with	• Connect the concept of variable representation to real-world scenarios or contexts that students can relate to. For example, discuss how variables are used in various professions such as engineering, finance, or science to solve complex problems and make predictions. • Encourage students to make connections between mathematical concepts and their own cultural backgrounds, highlighting the diversity of perspectives and applications of variables in different contexts. • Provide additional real-world problems for students to solve independently or in groups using variable representation. • Encourage students to create their own real-world scenarios that can be solved	• Introduce and reinforce key vocabulary words related to the lesson, such as "variable," "unknown quantity," "specified set," and "expression." Provide definitions and examples and encourage students to use these words in their discussions and explanations. • Offer linguistic support for students who may be English language learners by providing translations or additional explanations in their native languages if Extension Activities: • Provide additional real-world problems for students to solve independently or in groups using variable representation. • Encourage students to create their own real-world scenarios that can be solved using variables and share them with the class. • Explore cultural perspectives

real-world or mathematical problems written on them to each student or group. Have students work together to identify the unknown quantities in the problems and represent them using variables. ● Encourage students to discuss their reasoning and strategies for choosing appropriate variables, considering the context of the problems. ● Walk around the classroom to provide support and guidance as needed and facilitate a whole-class discussion to review the variable representations identified by the students.	using variables and share them with the class. ● Explore cultural perspectives on problem-solving techniques and the use of variables in different cultural contexts or professions.	on problem-solving techniques and the use of variables in different cultural contexts or professions needed.

Suggested Student Discourse Questions

- What strategies can we use to write expressions using variables to represent quantities or relationships?
- Can you demonstrate how to simplify and evaluate expressions with variables given specific values?
- How do we determine if an expression is simplified or if it can be further simplified?
- Can you identify examples of problems where expressions with variables would be useful in finding a solution?
- How do we interpret the meaning of an expression with variables in the context of a problem?

Cross-Curricular Connections

Science:

Students can work to create, read, and evaluate expressions that result from the forces at work. Students will have to be able to create and support their argument. (MS-PS2-1, Motion and Stability: Forces and Interactions)

English:

Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

Demonstrating the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a

specific scientific or technical context relevant to grade 6-8 texts and topics.

Distinguish among facts, reasoned judgment based on research findings, and speculations in a text.

Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

Career and Skill Connections

- Architectural Drafter
- Carpenter
- Computer Engineer
- Bookkeeper
- Geologist

Grade	CCSS Domain	CCSS Cluster
6	Expressions and Equations	Reason about and solve one-variable equations and inequalities.

Cluster Standard: 6.EE.B.7

Standard	Standards for Mathematical Practice
Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 2:** Reason abstractly and quantitatively. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students focus on the meaning of an equation and use reasoning and prior knowledge to solve it. They use variables to represent numbers and write expressions when solving problems. Students learn to write inequalities in the form of x > c, x ≥ c, x < c or x ≤ c and use number line representation to show the solutions of inequalities.	• Write and solve one step addition equations (x + p= q) when x, p and q are positive • Write and solve one step multiplication equations (px=q) form when x, p and q are positive. • Model real-world situations with equations
DOK	**Blooms**
1-2	Apply

Procedural and Conceptual Understanding and Application

Procedural Skill and Fluency: Students must be able to solve these equations accurately and efficiently. This involves performing the necessary arithmetic operations, such as subtraction or division, to isolate the variable *x* and find its value when given specific values for *p* and *q*.

Conceptual Understanding: Students will explain and understand the mathematical concept of solving real world and mathematical problems of writing and solving equations connecting to prior knowledge. Develop an understanding of and be able to translate between verbal expressions and mathematical expressions.

Application: Students will be able to apply mathematical concepts of solving problems associated with writing and solving equations with nonnegative rational numbers for real-world and mathematical problems. Students will be able to create mathematical problems and problems in real world context that can be solved using equations in the correct format according to the stated standard.

Assessment Items
When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.
A fruit salad consists of blueberries, raspberries, grapes, and cherries. The fruit salad has a total of 280 pieces of fruit. There are twice as many raspberries as blueberries, three times as many grapes as cherries, and four times as many cherries as raspberries. How many cherries are there in the fruit salad? You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions
● **Translation of expressions:** Students are unable to translate between the verbal expression and the mathematical expression. ● **Applying inverse operations:** When solving equations, students can forget to apply the same operation to both sides of the equation therefore leaving it unbalanced. ● **Isolating variable procedures:** Students can struggle with the correct operation to use to eliminate the value to isolate the variable.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning		
Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
● Direct Instruction: Introduce the concept with a real-world problem (e.g., "You have $10 and buy a snack for $2. How much money do you have left?"). Modeling: Show how to set up the equation x+2=10 x+2=10 and solve for x x. ● Guided Practice: Work through several problems as a class, using different methods to solve them. **Universal Design for Learning (UDL)** ● Provide a worksheet with a variety of problems for students to solve on their	● Small Group Instruction: Pull small groups based on assessment data to work on specific skills. Use manipulatives and step-by-step guides to help students who need more practice. ● Peer Tutoring: Pair students to work together on solving equations, encouraging them to explain their thinking to each other. **Universal Design for Learning (UDL)** ● Provide additional worksheets or online practice tailored to the	● One-on-One Instruction: Provide individualized instruction for students who need more intensive support. Use manipulatives and visual aids extensively. ● Simplified Problems: Start with simpler problems and gradually increase complexity as the student gains confidence. ● Frequent Check-ins: Monitor progress closely and provide immediate feedback. **Universal Design for Learning (UDL)** ● Targeted and Intensive Support (Flexible Timing) ○ During independent

own (self-pace).	students' needs.	practice, identify students who need additional help and provide targeted or intensive support as described in the MLSS layers.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/6/25/249/249

Previous Learning	*Current Learning*	*Future Learning*
• Students connect their previous understandings of what the equal sign is and that it shows equivalence to this cluster. The idea of equivalence is most aligned to their work in grades 4 and 5 with visual fraction models and understanding basic properties of operations to solve	• This cluster really expands on the previous cluster where students learned how to read, write and evaluate expressions in which letters stand for numbers.	• In Grade 7, students begin to formally apply the properties of operations. They will solve two step equations in the form of $px + q = r$ and $p(x + q) = r$. In Grade 8, students solve linear equations in one variable that include one solution, no solution, or infinitely many solutions. They include equations that require the distributive property or combining like terms. In Grade 8, the variable can be on both sides of the equation. In high school, students further their knowledge of solving equations with multi step equations that require the distributive property or combining like terms

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Opening Discussion: Start the lesson by discussing how math is used in various cultures around the world. Ask students to share examples from their own cultures or communities where they see math being used (e.g., cooking, shopping, building). • Culturally Relevant Problems: Introduce math problems that incorporate cultural contexts familiar to the students. For example, use scenarios involving local markets, traditional recipes, or cultural events that require budgeting or planning. • Example Problem: Maria wants to buy ingredients for a traditional dish that requires 5 pounds of rice. If she already has 2 pounds, how many more pounds does she need to buy? Write and solve the equation.	• Connecting Prior Knowledge: Begin by revisiting concepts of addition and multiplication in real-life contexts that students are familiar with. Bridge these concepts to writing and solving equations. • Scaffolded Instruction: Use a gradual release model (I Do, We Do, You Do) to build confidence. Start with teacher-led examples, move to guided practice with peer support, and finally, independent practice. • Interactive Learning: Incorporate technology or manipulatives to allow students to visualize and manipulate the problems, helping them bridge abstract concepts with concrete understanding. Example Activity: Pair students to work on a scenario where they need to plan a cultural event, such as a community festival. They must budget for various items, write equations for their calculations, and solve them together.	• Vocabulary Introduction: Introduce key terms (e.g., equation, variable, solve, nonnegative, rational numbers) with visual aids and bilingual supports if applicable. • Word Walls: Create a math word wall in the classroom with definitions, examples, and translations of key terms. • Sentence Frames: Provide sentence frames to help students articulate their reasoning and solutions (e.g., "The equation I wrote is ___. To solve for x, I need to ___ because ___"). Example Vocabulary Support: Today, we will learn about 'equations.' An equation is a mathematical statement that shows two things are equal. For example, in the equation x+3=7 , x+3=7, we need to find the value of x . x.

Suggested Student Discourse Questions

- What is an inequality, and how does it differ from an equation?
- How do we determine the solutions to an inequality and represent them on a number line?
- Can you describe the difference between open and closed circles when graphing solutions to inequalities?
- What strategies can we use to determine if a value satisfies an inequality?
- How do we determine if we need to reverse the inequality symbol when solving an inequality?

Cross-Curricular Connections
Science: Students can work to create, read, and evaluate expressions that result from the forces at work. Students will have to be able to create and support their argument. (MS-PS2-1, Motion and Stability: Forces and Interactions) **English:** Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. Demonstrating the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics. Distinguish among facts, reasoned judgment based on research findings, and speculations in a text. Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

Career and Skill Connections
• Automotive Technician • Electricians • HVAC Technician • Carpenter • Industrial Engineering Technician

Grade	CCSS Domain	CCSS Cluster
6	Expressions and Equations	Reason about and solve one-variable equations and inequalities.

Cluster Standard: 6.EE.B.8

Standard	Standards for Mathematical Practice
Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 2:** Reason abstractly and quantitatively. • **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students focus on the meaning of an equation and use reasoning and prior knowledge to solve it. They use variables to represent numbers and write expressions when solving problems. Students learn to write inequalities in the form of $x > c$, $x \geq c$, $x < c$ or $x \leq c$ and use number line representation to show the solutions of inequalities.	• Represent a real-world problem with an inequality ($x > c$ or $x < c$) • Explain that an inequality can have infinite solutions and show it on a number line. • Understand the difference between $>$, $\geq$, and $<$, $\leq$ and graphing with the appropriate open or closed circle.
DOK	**Blooms**
1-2	Apply

Procedural and Conceptual Understanding and Application

Procedural Skill and Fluency: Students must be able to accurately write inequalities to represent constraints or conditions in various problems. They should also be proficient in representing the solutions of these inequalities on number line diagrams, correctly indicating the range of possible values that satisfy the inequality.

Conceptual Understanding: Students represent understanding of the inequality solutions to the real-world or mathematical problems by representation on the number line and exhibiting completion to the problems. Recognize that inequalities have infinitely many solutions; represent solutions of such inequalities on number lines. Students will be able to describe the difference between the inequality symbols and what they represent.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can hold at most eight people. Additionally, each boat can only carry 900 pounds of weight for safety reasons.

a. Let *p* represent the total number of people. Write an inequality to describe the number of people that a boat can hold. Draw a number line diagram that shows all possible solutions.
b. Let *w* represent the total weight of a group of people wishing to rent a boat. Write an inequality that describes all total weights allowed in a boat. Draw a number line diagram that shows all possible solutions.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Applying concept of open and closed circles:** Students struggle with knowing the difference between the open and closed circle and how to apply it to the line graph with the inequality.
- **Interpreting inequalities:** Students tend not to interpret the less than or greater than and the equal signs correctly.
- **Solving inequalities as solution sets:** Inequalities are solved as individual values rather than as solution sets.
- **Multiplying and dividing negative values:** Confusion occurs with not flipping the inequality when dividing or multiplying or multiplying or dividing by negative values.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Introduction (10 minutes): Present a real-world problem involving inequalities (e.g., "You need to have more than $20 to buy a concert ticket. Write an inequality to represent this situation.") • Direct Instruction (15 minutes): Explain how to write inequalities: $x>c$ x>c or $x<c$ x<c. Use a variety of examples and visual aids. Demonstrate how to represent solutions on number lines.	• Small Group Instruction (Flexible Timing): Pull small groups based on assessment data to work on specific skills. Use manipulatives and step-by-step guides to help students who need more practice. • Peer Tutoring: Pair students to work together on solving inequalities, encouraging them to explain their thinking to each other. • Additional Practice: Provide extra worksheets or online practice tailored to the	• One-on-One Instruction: Provide individualized instruction for students who need more intensive support. Use manipulatives and visual aids extensively. • Simplified Problems: Start with simpler problems and gradually increase complexity as the student gains confidence. **Universal Design for Learning (UDL)** • Frequent Check-ins: Monitor progress closely and provide immediate feedback.

• Guided Practice (20 minutes): Solve problems together as a class, with students coming up to the board to show their work. Use manipulatives and visual aids. • Provide a worksheet with a variety of problems. Circulate to assist as needed. **Universal Design for Learning (UDL)** • Review key concepts and solve a final problem together. Discuss how these skills apply to real-life situations.	students' needs. **Universal Design for Learning (UDL)** • Small Groups: Form small groups to review key concepts with students who need additional help. Use hands-on activities and peer discussions to reinforce learning.	• Language Support: Offer additional linguistic support, such as vocabulary definitions and sentence frames, for English Language Learners (ELLs). • One-on-One Support: Work individually with students who require intensive support. Break down problems into simpler steps and use visual aids extensively.
Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/6/25/249/249		
Previous Learning	*Current Learning*	*Future Learning*
• Students connect their previous understandings of what the equal sign is and that it shows equivalence to this cluster. The idea of equivalence is most aligned to their work in grades 4 and 5 with visual fraction models and understanding basic properties of operations to solve	• This cluster really expands on the previous cluster where students learned how to read, write and evaluate expressions in which letters stand for numbers.	• In Grade 7, students begin to formally apply the properties of operations. They will solve two step equations in the form of $px + q = r$ and $p(x + q) = r$. In Grade 8, students solve linear equations in one variable that include one solution, no solution, or infinitely many solutions. They include equations that require the distributive property or combining like terms. In Grade 8, the variable can be on both sides of the equation. In high school, students further their knowledge of solving equations with multi step equations that require the distributive property or combining like terms
Culturally and Linguistically Responsive Instruction		

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Opening Discussion: Start the lesson by discussing how math is used in various cultures around the world. Ask students to share examples from their own cultures or communities where they see math being used (e.g., cooking, shopping, building). • Culturally Relevant Problems: Introduce math problems that incorporate cultural contexts familiar to the students. For example, use scenarios involving local markets, traditional recipes, or cultural events that require budgeting or planning. • Example Problem: Fatima wants to buy a traditional dress that costs more than $50. She already has $30 saved. How much more money does she need to save? Write an inequality to represent this situation.	• Connecting Prior Knowledge: Begin by revisiting concepts of comparison in real-life contexts that students are familiar with, such as comparing prices or ages. Bridge these concepts to writing and solving inequalities. • Scaffolded Instruction: Use a gradual release model (I Do, We Do, You Do) to build confidence. Start with teacher-led examples, move to guided practice with peer support, and finally, independent practice. • Interactive Learning: Incorporate technology or manipulatives to allow students to visualize and manipulate the problems, helping them bridge abstract concepts with concrete understanding. Example Activity: Pair students to work on a scenario where they need to plan a cultural event, such as a community festival. They must budget for various items, write inequalities for their calculations, and solve them together.	• Vocabulary Introduction: Introduce key terms (e.g., inequality, greater than, less than, nonnegative, rational numbers) with visual aids and bilingual supports if applicable. • Word Walls: Create a math word wall in the classroom with definitions, examples, and translations of key terms. • Sentence Frames: Provide sentence frames to help students articulate their reasoning and solutions (e.g., "The inequality I wrote is ___. To solve for x, I need to ___ because ___"). Example Vocabulary Support: Today, we will learn about 'inequalities.' An inequality shows that one number is greater than or less than another number. For example, in the inequality x>50 x>50, we need to find values of x x that are greater than 50.

Suggested Student Discourse Questions
• What is an inequality, and how does it differ from an equation? • Can you explain the symbols used to represent inequalities and their meanings? • How do we interpret the solutions to an inequality in the context of a problem? • Can you think of real-life situations where inequalities would be used to model relationships between quantities?

Cross-Curricular Connections
Science: Students can work to create, read, and evaluate expressions that result from the forces at work. Students will have to be able to create and support their argument. (MS-PS2-1, Motion and Stability: Forces and Interactions) **English:** Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. Demonstrating the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics. Distinguish among facts, reasoned judgment based on research findings, and speculations in a text. Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

Career and Skill Connections
• Loan Officer • Budget Analyst • Auditors • Purchase Clerks • Business Owner

Grade	CCSS Domain	CCSS Cluster
6	Expressions and Equations	Represent and analyze quantitative relationships between dependent and independent variables.

Cluster Standard: 6.EE.C.9

Standard	Standards for Mathematical Practice
Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 2:** Reason abstractly and quantitatively. • **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 4:** Model with mathematics. • **SMP 7**: Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
The focus for this cluster is using variables to represent two quantities in a real-world problem that change in relationship to one another. Students write an equation and analyze the relationship between the dependent and independent variables using graphs and tables.	• Use variables to represent unknowns in a real-world problem and write an equation to show the relationship between two changing quantities. • Describe the variables in context of dependent and independent • Analyze the relationship between the dependent and independent variables using tables, graphs and equations
DOK	**Blooms**
1-3	Apply, Analyze

Procedural and Conceptual Understanding and Application

Procedural Skill and Fluency: Students will write and solve equations resulting from real-world situations and interpret the solution in context with speed and accuracy. Given an equation where variables represent two quantities that change in relationship to one another, create a problem in real-world context that could be represented by the equation. Explain the relationship between the dependent and independent variables and relate these to the equation.

Conceptual Understanding: Students need to understand the relationship between dependent and independent variables in real-world situations. They should grasp that the dependent variable depends on the value of the independent variable and how this relationship can be represented through an equation. For example, understanding that in the equation $d=65t$, the distance d depends on the time t when moving at a constant speed.

Application: Students are expected to apply their understanding to analyze real-world problems where two quantities change in relationship to one another. This involves using graphs, tables, and equations to explore and explain the relationship between the variables. For example, in a problem involving motion at a constant speed, students might graph the relationship between distance and time, create a table of values, and write an equation like $d=65t$ to describe the relationship. They would then analyze the graph and table to gain insights into the relationship, such as understanding how changes in time affect the distance traveled.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Stephanie is helping her band collect money to fund a field trip. The band decided to sell boxes of chocolate bars. Each bar sells for $1.50 and each box contains 20 bars. Below is a partial table of monies collected for different numbers of boxes sold.

Boxes Sold *b*	Money Collected *m*
1	$30.00
2	
3	
4	
5	$150.00
6	
7	

8	

a. Complete the table above for values of *m*.

b. Write an equation for the amount of money, *m*, that will be collected if *b* boxes of chocolate bars are sold. Which is the independent variable and which is the dependent variable?

c. Graph the equation using the ordered pairs from the table above.

d. Calculate how much money will be collected if 100 boxes of chocolate bars are sold.

e. The band collected $1530.00 from chocolate bar sales. How many boxes did they sell?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Identifying types of variables:** Confusion between identifying the independent and dependent variables.
- **Graph representation with direction:** Students may confuse what the graph represents in context. For example, moving up or down on a graph does not necessarily mean that a person is moving up or down.
- **Variable positions on the graph:** Students may reverse the independent and dependent variable in an equation, graph or table.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Present a real-world problem involving two quantities that change in relationship to one another (e.g., distance and time for a car traveling at a constant speed). Discuss the concepts of independent and dependent variables. • Explain how to use variables to represent quantities, write equations, and analyze relationships using graphs and tables. Use a variety of examples and visual aids. • Solve problems together as a class, with students coming	• Small Group Instruction (Flexible Timing): Pull small groups based on assessment data to work on specific skills. Use manipulatives and step-by-step guides to help students who need more practice. • Peer Tutoring: Pair students to work together on solving problems, encouraging them to explain their thinking to each other. • Additional Practice: Provide extra worksheets or online practice tailored to the	• One-on-One Instruction: Provide individualized instruction for students who need more intensive support. Use manipulatives and visual aids extensively. • Simplified Problems: Start with simpler problems and gradually increase complexity as the student gains confidence. • Frequent Check-ins: Monitor progress closely and provide immediate feedback. • Language Support: Offer additional linguistic support,

up to the board to show their work. Use graph paper and visual aids. ● Provide a worksheet with a variety of problems. Circulate to assist as needed. **Universal Design for Learning (UDL)** ● Review key concepts and solve a final problem together. Discuss how these skills apply to real-life situations.	students' needs. **Universal Design for Learning (UDL)** ● UDL: Small Groups: Form small groups to review key concepts with students who need additional help. Use hands-on activities and peer discussions to reinforce learning.	such as vocabulary definitions and sentence frames, for English Language Learners (ELLs). **Universal Design for Learning (UDL)** ● One-on-One Support: Work individually with students who require intensive support. Break down problems into simpler steps and use visual aids extensively.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/6/25/249/249

Previous Learning	*Current Learning*	*Future Learning*
● In Grade 5, learners are taught how to generate patterns from rules that are given to them. This will connect when students are analyzing the relationship between the dependent and independent variables in this cluster.	● The students will expand their knowledge of 6.EE.7 in this cluster by continuing practice of writing equations in real world situations. The students will expand their knowledge of 6.RP.3 by continuing to find relationships with numbers through rate reasoning.	● The students will continue using dependent and independent variables and noticing patterns throughout the rest of their mathematical career, showing up mainly in the RP clusters and as they dive into linear and non-linear relationships. In high school, they will be using this knowledge as they construct and compare linear, quadratic, and exponential models.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:
- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
● Cultural Relevance: Begin with a discussion on real-	● Connect students' prior knowledge and experiences	● Provide sentence frames to help students articulate their

world examples relevant to students' lives. For instance, talk about local markets where they might see relationships between the amount of money spent and items purchased. ● Personal Connections: Ask students to share experiences from their own lives where they have noticed one thing changing with another (e.g., how the time spent studying affects their grades, or how distance varies with the time taken when traveling).	to new mathematical concepts, facilitating a bridge to more abstract understanding: ● Prior Knowledge Activation: Begin with a brief review of basic concepts of variables and equations. Use familiar examples that students have already discussed in the Validate and Affirm section. ● Real-World Problem: Present a real-world problem that is culturally relevant to the students. For example, use a scenario involving a local community event, such as a cultural festival, where planning involves calculating distances and times for different activities. ● Example Problem: "During the community festival, a group plans to walk to the local park. If they walk at a constant speed of 3 miles per hour, how long will it take them to reach the park if it is 9 miles away?"	understanding. For example: "In this problem, the independent variable is __ because it __." "The dependent variable is __ because it __." "The equation __ shows that __ depends on __." ● Graphic Organizers: Use graphic organizers like T-charts to help students organize their thoughts and see the relationship between variables. Have students fill in the T-chart with time (t) on one side and distance (d) on the other and plot these points on a graph. ● Peer Teaching: Pair students with different linguistic strengths so they can support each other. Encourage them to explain concepts to each other using the provided language frames and vocabulary.

Suggested Student Discourse Questions

- Can you explain the role of variables in representing unknown quantities or changing values?
- What are constants, and how do they differ from variables?
- Can you explain the role of variables in representing unknown quantities or changing values?
- What are constants, and how do they differ from variables?
- How do we determine if terms are like terms or unlike terms?

Cross-Curricular Connections

Science:

Students can work to create, read, and evaluate expressions that result from the forces at work. Students will have to be able to create and support their argument. (MS-PS2-1, Motion and Stability: Forces and Interactions)

English:

following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

demonstrating the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.

distinguish among facts, reasoned judgment based on research findings, and speculations in a text.

engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

Career and Skill Connections

- Biomedical Engineer
- Aerospace Engineer
- Auditor
- Architect
- Economist

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards of Mathematical Practice
    - Procedural and Conceptual Understanding and Application
    - Sample Assessment Items
    - Common Misconceptions
    - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
    - Vertical Alignment
    - Culturally and Linguistically Responsive Instruction (CLRI)
    - Suggested Student Discourse Questions
    - Cross-Curricular and Career and Skill Connections
- A Student Discourse Guide
- Planning for a Multi-Layer System of Support (MLSS) and Universal Design for Learning (UDL) for behavioral and social and emotional supports

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Solve real-world and mathematical problems involving area, surface area, and volume.
  - 6.G.A.1
  - 6.G.A.2
  - 6.G.A.3
  - 6.G.A.4

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008).

Grade	CCSS Domain	CCSS Cluster
6	Geometry	Solve real-world and mathematical problems involving area, surface area, and volume.

Cluster Standard: 6.G.A.1

Standard	Standards for Mathematical Practice
Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems	• **SMP 2:** Reason abstractly and quantitatively.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This cluster builds on previous understanding of area and volume to deepen the understanding of volume and develop the concept of surface area. Students use knowledge and skills to solve real-world and mathematical problems and apply the concepts by manipulating nets, cubes, and other real-world materials	• Find the area of triangles and special quadrilaterals. • Decompose and compose shapes into right triangles, triangles and quadrilaterals. • Apply understanding of finding the area of triangles and quadrilaterals to finding the area of irregular shapes that are made up of these shapes. • Solve real world and mathematical problems by applying these techniques
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Procedural Skill and Fluency: Students demonstrate fluency in calculating the area of rectangles, squares, triangles, parallelograms, and other polygons and apply appropriate formulas and procedures to find the area of various shapes accurately and efficiently.

Conceptual Understanding: Students understand the concept of area as the measure of the amount of space enclosed by a two-dimensional shape, the relationship between area and the size of the shape, as well as the units used to measure area and understand the difference between area and perimeter.

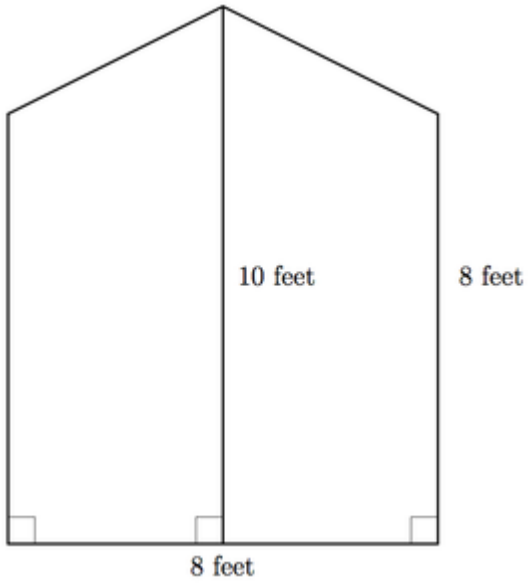
Application: Students can apply area concepts to solve real-world problems involving measurement, construction, design, and spatial reasoning and recognize situations where knowledge of area is needed and apply appropriate

strategies to find solutions.

Assessment Items

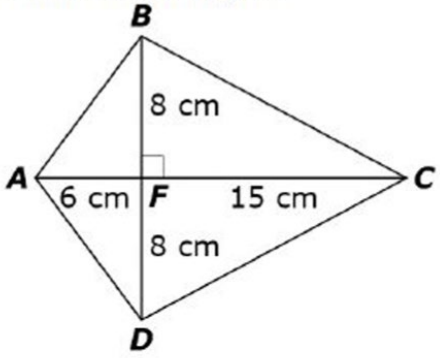
When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Jamie is planning to cover a wall with red wallpaper. The dimensions of the wall are shown below:



a. How many square feet of wallpaper are required to cover the wall?
b. Wallpaper comes in long rectangular strips which are 24 inches wide. If Jamie lays the strips of wallpaper vertically, can she cover the wall without wasting any wallpaper? Explain.
c. If Jamie lays the strips of wallpaper horizontally, can she cover the wall without wasting any wallpaper? Explain.

Example Stem 1: Consider this figure.



Enter the total area, in square centimeters, of kite *ABCD*.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Equating Area with Perimeter:** Students may mistakenly believe that area and perimeter are the same thing or that they measure similar aspects of a shape. This misconception can lead to confusion when distinguishing between the two concepts.
- **Equating Larger Perimeter with Larger Area:** Students may incorrectly assume that a shape with a larger perimeter always has a larger area. This misunderstanding overlooks the fact that the area depends on both the dimensions and shape of the figure, not just its perimeter.
- **Confusing Area with Perimeter in Word Problems:** In word problems, students may mistakenly focus on finding the perimeter of a shape when the problem is asking for its area. This confusion can lead to incorrect solutions and misinterpretation of the problem context.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Use visual aids such as diagrams, geometric shapes, and area grids to help students visualize and understand the properties and areas of different shapes. • Provide manipulatives like pattern blocks or geometric solids to allow hands-on exploration of composing and	• Provide small group instruction for students who need additional support. Focus on specific areas of difficulty and use differentiated activities to address individual learning needs. • Offer guided practice sessions where students	• Provide intensive one-on-one instruction for students who require individualized support. Tailor lessons to the student's specific learning needs, using additional explanations, examples, and practice opportunities. • Develop personalized learning plans that include targeted

decomposing shapes. ● Incorporate real-world problems that involve finding areas, such as planning a garden, designing a playground, or creating art, to make learning relevant and engaging. **Universal Design for Learning (UDL)** ● Provide multiple means of representation by using visual aids, manipulatives, and verbal explanations to accommodate diverse learning styles and preferences.	receive immediate feedback and support from the teacher or a peer mentor while working on problems related to finding areas of shapes. ● Use scaffolded activities that break down complex problems into smaller, manageable steps. Provide checklists or graphic organizers to help students organize their thinking and approach to problem-solving. **Universal Design for Learning (UDL)** ● Provide multiple means of representation by using visual aids, manipulatives, and verbal explanations to accommodate diverse learning styles and preferences.	goals, strategies, and supports for students with significant learning gaps. Collaborate with special education teachers, ELL specialists, and other support staff to address individual needs. **Universal Design for Learning (UDL)** ● Provide multiple means of representation by using visual aids, manipulatives, and verbal explanations to accommodate diverse learning styles and preferences.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/6/26

Previous Learning	*Current Learning*	*Future Learning*
● Learners build on their knowledge of area from Grade 3 where they count the area of a rectangle and connect it to their understanding of multiplication in Grade 4. Learners understand how to find the volume of right rectangular prisms with whole numbers in Grade 5. Apply the area and perimeter formulas for rectangles in real world and mathematical problems.	● Learners are flexible using the terms base and length when solving for the area of a two or three-dimensional shape. Develop the concept of surface area. Learners understand how to find the volume of right rectangular prisms using fractions in the length of the edges. ● Connects to lessons on negative integers (6.NS.8) and graphing points in all quadrants. (6.RP.3.a) ● Find distance on the coordinate plane by	● In Grade 7, learners will continue to draw, construct, and describe geometrical figures and discover relationships between them (without nets). Calculate and compare the volume of cones, cylinders, and spheres. ● Prepare for grade 8 work with transformations by working with polygons in coordinate plane. ● Learners will further their knowledge on distance in 8th grade when they start to find the lengths of diagonal lines.

	counting the units on the coordinate plane (no formula). Create polygons in quadrants I, II, III, and IV so learners can apply their knowledge of absolute value.	• Learners will use their knowledge of the Pythagorean Theorem to find distance on the coordinate plane and later use the distance formula. • In high school, learners will apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). In high school, learners will give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. • In high school, students will use the idea of nets to identify the shapes of two-dimensional cross sections of three-dimensional objects and identify three dimensional objects generated by rotations of two-dimensional objects.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Use examples of geometric shapes and their areas from various cultural contexts, such as traditional patterns in art, architecture, and design from different cultures. This approach can validate students' cultural backgrounds and show the relevance of geometric concepts in their own lives.	• Begin with what students already know about shapes and areas from their everyday experiences. Use familiar contexts to introduce new concepts, gradually bridging from concrete examples to more abstract ones. • Create problems and projects that incorporate	• Use diagrams, visual aids, and physical models to explain and reinforce key vocabulary terms like "right triangle," "quadrilateral," "polygon," "compose," "decompose," and "area." This helps all students, especially those who are English Language Learners (ELLs), better understand and retain new terms.

• Share stories and contributions of mathematicians and scientists from diverse cultures who have made significant contributions to geometry. This helps students see themselves in the subject and feel their cultural identity is respected and valued.	culturally relevant contexts. For example, students could calculate the area of various components of culturally significant buildings or art pieces. • Engage students in projects where they apply their understanding of area to real-world tasks, such as designing a community garden with triangular and rectangular plots, incorporating cultural designs.	• Provide bilingual glossaries and resources where possible. Encourage students to discuss mathematical concepts in their home languages to deepen *understanding. • Use sentence frames to help students articulate their understanding of geometric concepts (e.g., "The area of a triangle is found by..."). Maintain a word wall with key vocabulary terms and definitions in both English and students' home languages.

Suggested Student Discourse Questions

- Can you explain why the formula for finding the area of a rectangle works?
- How do you find the area of a triangle or a parallelogram? What strategies can we use?
- Can you think of real-life scenarios where understanding the area would be important? How would you use area to solve these problems?
- How does knowing how to find an area help us in fields like construction, design, or landscaping?
- What strategies can you use to break down a complex shape into simpler ones to find its area?

Cross-Curricular Connections

Science & English:

Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics.

Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).

Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 6 topics, texts, and issues, building on others' ideas and expressing their own clearly.

Career and Skill Connections

- Manufacturing Engineer Technologist
- Surveyor
- Urban and Regional Planner, Architect
- Model makers, metal and plastic

Grade	CCSS Domain	CCSS Cluster
6	Geometry	Solve real-world and mathematical problems involving area, surface area, and volume.

Cluster Standard: 6.G.A.2

Standard	Standards for Mathematical Practice
Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 3**: Construct viable arguments and critique the reasoning of others, • **SMP 6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This cluster builds on previous understanding of area and volume to deepen the understanding of volume and develop the concept of surface area. Students use knowledge and skills to solve real-world and mathematical problems and apply the concepts by manipulating nets, cubes, and other real-world materials.	• Find volume of a rectangular prism using formula (V=lwh and V=bh) and explain how this is the same as packing with unit cubes to find volume. • Apply this to using lengths that are fractional. • Solve real-world problems for volume involving fractional lengths of rectangular prisms.
DOK	**Blooms**
1-2	Apply

Procedural and Conceptual Understanding and Application

Procedural Skill and Fluency: Students demonstrate fluency in finding volume of right rectangular prisms; with prior understanding of working previously with whole numbers to working with unit fractions and multiplication of fractions. Students demonstrate fluency in using the appropriate formula for calculating the volume of right rectangular prisms accurately and efficiently.

Conceptual Understanding: Students understand the concept of volume as the amount of space an object takes up and is measured in cubic units. Students also understand the relationship between volume of three-dimensional space occupied by an object or closed surface.

Application: Students identify and apply the formulas V = lwh and V = bh to find volumes of right rectangular prisms with fractional edge lengths while solving problems involving real-world and mathematical scenarios to find solutions.

Students identify and apply the appropriate volume formula for problems of cubes of specific sizes with a fractional side length to find a solution.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Leo's recipe for banana bread won't fit in his favorite pan. The batter fills the 8.5 inch by 11 inch by 1.75 inch pan to the very top, but when it bakes it spills over the side. He has another pan that is 9 inches by 9 inches by 3 inches, and from past experience he thinks he needs about an inch between the top of the batter and the rim of the pan. Should he use this pan?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Calculating Rectangular Prism Formula as LxWxH:** Misconceptions occur when students believe that volume only occurs when length, width, and height are measurable.
- **Volume Composed of Visible Unit Cubes:** Students confuse the formula and use area formula instead of volume formula or are not able to visualize the shape of the prism.
- **Volume of Prism is Total Area of its Faces:** Students lack conceptual knowledge. Students are not able to comprehend the structure of prisms.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Introduction to the concept of volume and its importance in measuring three-dimensional space. • Review of the formula for finding the volume of a right rectangular prism (V = lwh). • Demonstration of how to find the volume of a right rectangular prism with whole number edge lengths by multiplying the length, width, and height.	• Introduction to fractional edge lengths and the concept of packing a prism with unit cubes of appropriate unit fraction edge lengths. • Guidance on using unit cubes to visually represent the volume of a prism with fractional edge lengths. • Practice exercises on finding the volume of a right rectangular prism with	• Extension activities for advanced learners, such as exploring different methods for finding volume or applying volume concepts to more complex geometric shapes. • Additional support for struggling students through one-on-one or small group instruction, using manipulatives or visual aids as needed.

Universal Design for Learning • Incorporate multiple means of representation by providing visual representations of right rectangular prisms and volume calculations, auditory explanations, and hands-on activities with unit cubes. • Offer multiple means of expression by allowing students to demonstrate their understanding through verbal explanations, written responses, or visual representations. • Provide multiple means of engagement by incorporating student interests, cultural relevance, and real-world connections into the lesson.	fractional edge lengths using unit cubes. • Encourage students to work in pairs or small groups for peer support. **Universal Design for Learning (UDL)** • Provide multiple means of representation by using visual aids, manipulatives, and verbal explanations to accommodate diverse learning styles and preferences.	**Universal Design for Learning (UDL)** • Provide multiple means of representation by using visual aids, manipulatives, and verbal explanations to accommodate diverse learning styles and preferences.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/6/26/265/265

Previous Learning	*Current Learning*	*Future Learning*
• Learners build on their knowledge of area from Grade 3 where they count the area of a rectangle and connect it to their understanding of multiplication in Grade 4. Learners understand how to find the volume of right rectangular prisms with whole numbers in Grade 5. Apply the area and perimeter formulas for rectangles in real world and mathematical problems.	• Learners are flexible using the terms base and length when solving for the area of a two or three-dimensional shape. Develop the concept of surface area. Learners understand how to find the volume of right rectangular prisms using fractions in the length of the edges. • Connects to lessons on negative integers and graphing points in all quadrants. • Find distance on the	• In Grade 7, learners will continue to draw, construct, and describe geometrical figures and discover relationships between them (without nets). Calculate and compare the volume of cones, cylinders, and spheres. • Prepare for grade 8 work with transformations by working with polygons in coordinate plane. • Learners will further their knowledge on distance in 8th grade when they start to find

	coordinate plane by counting the units on the coordinate plane (no formula). Create polygons in quadrants I, II, III, and IV so learners can apply their knowledge of absolute value.	the lengths of diagonal lines. ● Learners will use their knowledge of the Pythagorean Theorem to find distance on the coordinate plane and later use the distance formula. ● In high school, learners will apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). In high school, learners will give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. ● In high school, students will use the idea of nets to identify the shapes of two-dimensional cross sections of three-dimensional objects, and identify three dimensional objects generated by rotations of two-dimensional objects.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
● Begin by validating students' diverse cultural backgrounds and experiences related to problem-solving in mathematics. ● Affirm the importance of understanding volume	● Build on students' prior knowledge by reviewing the concept of volume and its relevance in everyday life. ● Bridge the concept to real-world scenarios by discussing how different	● Provide linguistic vocabulary support for students by introducing translated terms and visual aids related to volume computation and geometric shapes. ● Scaffold language acquisition

computation in various cultural contexts. ● Cultural Examples and Contexts: ○ Incorporate culturally relevant examples and contexts into volume computation exercises. ○ Discuss how different cultures might use volume concepts in traditional crafts, food preparation, or architecture.	cultures use volume concepts in architecture, cooking, and other cultural practices. ● Cultural Examples and Contexts: ○ Incorporate culturally relevant examples and contexts into volume computation exercises. ○ Discuss how different cultures might use volume concepts in traditional crafts, food preparation, or architecture.	through multilingual word banks or graphic organizers to facilitate comprehension. ● Cultural Examples and Contexts: ○ Incorporate culturally relevant examples and contexts into volume computation exercises. ○ Discuss how different cultures might use volume concepts in traditional crafts, food preparation, or architecture.

Suggested Student Discourse Questions

- How can unit cubes be used to determine the volume of a right prism?
- What method or methods are used to determine the volume of an object?
- How are the use of manipulatives, such as nets, cubes, and other real-world materials effective in solving problems involving volume?
- Compare your strategy of solving volume related to problems to the strategy of a classmate, were the methods chosen the same or different? Explain your findings.

Cross-Curricular Connections

Science & English:

Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics.

Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).

Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse

partners on grade 6 topics, texts, and issues, building on others' ideas and expressing their own clearly.
Career and Skill Connections
• Interior Designer • Landscaper • Neighborhood Planner • Surveyor • Manufacturing Designer

Grade	CCSS Domain	CCSS Cluster
6	Geometry	Solve real-world and mathematical problems involving area, surface area, and volume.

Cluster Standard: 6.G.A.3

Standard	Standards for Mathematical Practice
Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 6**: Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This cluster builds on previous understanding of area and volume to deepen the understanding of volume and develop the concept of surface area. Students use knowledge and skills to solve real-world and mathematical problems and apply the concepts by manipulating nets, cubes, and other real-world materials	• Draw polygons on the coordinate plane when given coordinates for vertices. • Find the side lengths of the polygons using coordinates. • Solve real-world problems by applying the use of drawing coordinates.
DOK	**Blooms**
1-2	Apply

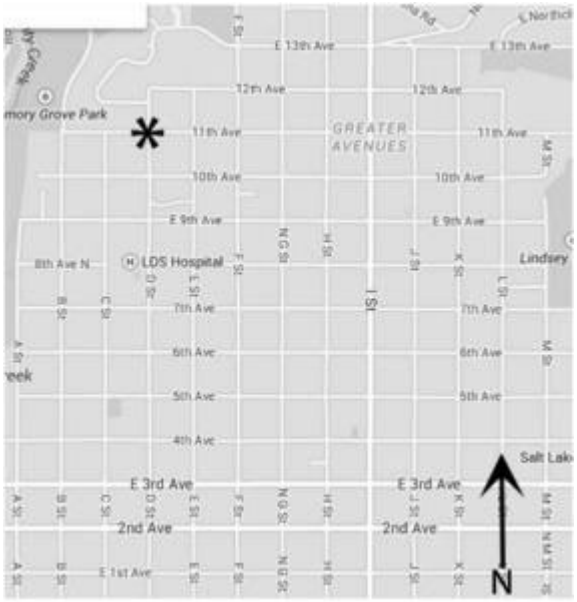
Procedural and Conceptual Understanding and Application

Procedural Skill and Fluency: Students demonstrate fluency in describing a point in space using coordinates. Students will further demonstrate fluency in how to plot points, read coordinates and find the ratio of the rise over run for slope. Students will precisely and accurately practice plotting points in a coordinate plane drawing polygons given a set of vertices.

Conceptual Understanding: Students should be encouraged to use graph paper to determine the segment side lengths as the number of units from one vertex to another, limited to vertical and horizontal segments applied to mathematical and real-world problems. Students should be able to recognize and explain the connection between finding the side length of a polygon on a coordinate plane to distance on a number line and absolute value.

Application: Students apply and practice plotting points in the coordinate plane and finding the areas of polygons. Students should create real-world problems that involve drawing a polygon in the coordinate plane and use precise mathematical language to explain the solution.

Assessment Items
When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.



Here is a map of part of Downtown Salt Lake City. You are starting at the corner of 11th Ave. and D St. (on the star).

a. If you walk East to I St., South to 7th Ave., West to D St. and then North to your starting point, how many blocks will you have walked in total? Describe the shape of your path.

b. Draw and describe in words at least two different ways that you can walk exactly 8 blocks and end up where you started.

c. Jessica said the path she took on her walk enclosed a polygon that had an area of 6 square blocks. Draw some possible shapes that her walk could have taken. Was her path necessarily rectangular?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Plotting Points in the Coordinate Plane:** Students may struggle plotting points in the coordinate plane and confuse the directions of the x and y coordinates within the coordinate plane, as well as the positive and negative directions.

- **Applying the Incorrect Formula:** Students may apply the incorrect formula for finding the area of polygons.
- **Unable to identify the correct Polygon side length:** Students may not be able to identify the correct polygon side length and the connection to the number line and absolute value.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Begin by reviewing the concept of coordinates and the Cartesian coordinate system. • Show examples of polygons and discuss their properties, including the importance of coordinates in determining the shape and size of polygons. • Introduce the process of drawing polygons in the coordinate plane using given coordinates for the vertices. • Model how to plot points on the coordinate plane and connect them to form polygons, emphasizing the importance of accurately labeling the vertices. • Demonstrate how to use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. • Provide guided practice problems where students draw polygons in the coordinate plane and find the lengths of sides using given coordinates. • Walk students through each step of the process, providing support and feedback as needed.	• Provide differentiated practice problems based on students' individual needs, offering additional support for struggling students and extension activities for advanced learners. • Offer manipulatives or visual aids for students who need additional support with spatial reasoning. • Present multi-step word problems that require students to apply their understanding of drawing polygons and finding side lengths in real-world contexts. • Encourage students to explain their problem-solving strategies and justify their reasoning. **Universal Design for Learning (UDL)** • Provide multiple means of representation by using visual aids, manipulatives, and verbal explanations to accommodate diverse learning styles and preferences. • Provide multiple means of expression by allowing students to demonstrate	• Administer a diagnostic assessment to identify specific areas of difficulty for individual students related to drawing polygons and finding side lengths using coordinates. • Group students based on their assessment results and provide targeted instruction to address their individual needs. • Use a variety of instructional strategies, including hands-on activities and peer tutoring, to support students' understanding. • Provide one-on-one support for students who require intensive intervention, focusing on addressing their specific learning needs. • Offer additional practice opportunities and scaffolded support to help students build confidence and mastery. **Universal Design for Learning (UDL)** • Provide multiple means of representation by using visual aids, manipulatives, and verbal explanations to accommodate diverse learning styles and preferences.

• Assign independent practice problems for students to complete, drawing polygons and finding side lengths using coordinates. • Encourage students to check their work and ask questions if they encounter difficulties. **Universal Design for Learning (UDL)** • Provide multiple means of representation by using visual aids, manipulatives, and verbal explanations to accommodate diverse learning styles and preferences.	their understanding using various methods, such as written explanations, verbal responses, or visual representations.	• Offer multiple means of engagement by incorporating real-world examples, interactive activities, and student choice to increase motivation and engagement.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/6/26/266/266

Previous Learning	*Current Learning*	*Future Learning*
• Learners build on their knowledge of area from Grade 3 where they count the area of a rectangle and connect it to their understanding of multiplication in Grade 4. Learners understand how to find the volume of right rectangular prisms with whole numbers in Grade 5. Apply the area and perimeter formulas for rectangles in real world and mathematical problems.	• Learners are flexible using the terms base and length when solving for the area of a two or three-dimensional shape. Develop the concept of surface area. Learners understand how to find the volume of right rectangular prisms using fractions in the length of the edges. • Connects to lessons on negative integers and graphing points in all quadrants. • Find distance on the coordinate plane by counting the units on the coordinate plane (no formula). Create polygons in	• In Grade 7, learners will continue to draw, construct, and describe geometrical figures and discover relationships between them (without nets). Calculate and compare the volume of cones, cylinders, and spheres. • Prepare for grade 8 work with transformations by working with polygons in coordinate plane. • Learners will further their knowledge on distance in 8th grade when they start to find the lengths of diagonal lines. • Learners will use their knowledge of the Pythagorean Theorem to find distance on

	quadrants I, II, III, and IV so learners can apply their knowledge of absolute value.	the coordinate plane and later use the distance formula. ● In high school, learners will apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). In high school, learners will give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. ● In high school, students will use the idea of nets to identify the shapes of two-dimensional cross sections of three-dimensional objects and identify three dimensional objects generated by rotations of two-dimensional objects.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
● Begin by validating students' diverse cultural backgrounds and experiences related to problem-solving in mathematics. ● Affirm the importance of understanding polygons and coordinate geometry in various cultural contexts. ● Incorporate culturally relevant examples and contexts into polygon drawing and side length calculations.	● Build on students' prior knowledge by reviewing the concept of polygons and coordinate geometry. ● Bridge the concept to real-world scenarios by discussing how different cultures might use polygons and coordinate geometry in architecture, art, or navigation. ● Incorporate culturally relevant examples and contexts into polygon	● Provide linguistic vocabulary support for students by introducing translated terms and visual aids related to polygons, coordinates, and geometry. ● Scaffold language acquisition through multilingual word banks or graphic organizers to facilitate comprehension. ● Cultural Examples and Contexts: ● Incorporate culturally relevant examples and contexts into

• Discuss how different cultures might use polygons and coordinate geometry in traditional crafts, patterns, or land surveying.	drawing and side length calculations. • Discuss how different cultures might use polygons and coordinate geometry in traditional crafts, patterns, or land surveying.	polygon drawing and side length calculations. • Discuss how different cultures might use polygons and coordinate geometry in traditional crafts, patterns, or land surveying.

Suggested Student Discourse Questions

- How do you compare surface area and volume?
- Make a comparison of your approach or strategy used to solve a problem related to solving volume to that of another student, how are they similar? How are they different?
- Would you need the same amount of wrapping paper to wrap a box as you would to wrap the contents of that box?
- What real-world examples can you think of that relate to the use of volume or surface area that you can apply the formulas to?

Cross-Curricular Connections

Science & English:

Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics.

Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).

Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 6 topics, texts, and issues, building on others' ideas and expressing their own clearly.

Career and Skill Connections

- Construction Worker
- Landscaper
- Interior Designer
- Computer Aided Design (CAD) engineer
- Animator

Grade	CCSS Domain	CCSS Cluster
6	Geometry	Solve real-world and mathematical problems involving area, surface area, and volume.

Cluster Standard: 6.G.A.4

Standard	Standards for Mathematical Practice
Represent three dimensional figures using nets made up of rectangles and triangles and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real world and mathematical problems.	• **SMP 3:** Construct viable arguments and critique the reasoning of others. • **SMP 4**: Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This cluster builds on previous understanding of area and volume to deepen the understanding of volume and develop the concept of surface area. Students use knowledge and skills to solve real-world and mathematical problems and apply the concepts by manipulating nets, cubes, and other real-world materials	• Create a net (using triangles and rectangles) to represent three-dimensional figures. • Use nets to find the surface area of three-dimensional figures. • Solve real-world problems by applying the use of nets of three-dimensional figures to find surface area.
DOK	**Blooms**
1-2	Apply

Procedural and Conceptual Understanding and Application

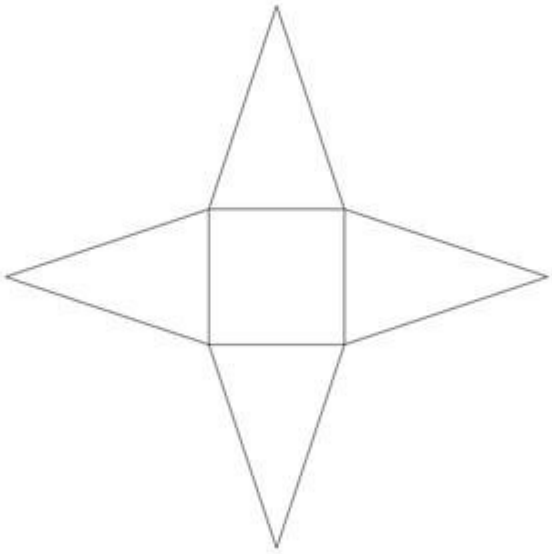
Procedural Skill and Fluency: Students demonstrate fluency in calculating the area of irregular shape. Students build fluency through drawing nets associated with three-dimensional figures and using this to find the surface area of the figure in mathematical and real-world mathematical problems.

Conceptual Understanding: Students explain connections between the area of two-dimensional figures and surface area of three-dimensional figures. Students work with visual representations and models to gain understanding that surface area is composed of the sum of each area of the figure's faces.

Application: Students apply and create a net to represent three dimensional figures and use nets to compute the surface areas of the figures in solving real-world and mathematical problems. Students solve and find the surface area of a three-dimensional figure given the net that represents that figure in the context of solving real-world and mathematical problems.

Assessment Items
When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Below is a net for a three-dimensional shape:



a. The inner quadrilateral is a square and the four triangles all have the same size and shape.
    i. What three-dimensional shape does this net make? Explain.
    ii. If the side length of the square is 2 units and the height of the triangles is 3 units, what is the surface area of this shape?

b. Draw a net for a rectangular prism whose base is a one inch by one inch square and whose faces are 3 inches by 1 inch.
    i. Is there more than one possible net for this shape? Explain.
    ii. What is the surface area of the prism?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions
• **Confusion with slant height:** Students may confuse the slant height and not recognize it for the height of the triangles in the net. Being that these are nets, students may only find one area and not the area of each individual part of the net and add them together. The concept of nets may be difficult for students to understand, specifically the translation from the 3-D figure to the net and how they coincide. This may need to be reinforced as to how a pyramid and a rectangular prism coincide to their nets. • **Confusion between formulas:** Students may have some confusion between the difference between surface area and area and using the incorrect formula when solving problems related to finding the surface area of a three-dimensional figure given its net. • **Confusion in identifying nets:** Students may become confused at understanding how to deconstruct the figure and lay out the net to allow for calculating the pieces of the net for surface area. Students must understand that surface area is the sum of the areas of all the faces of the solid, so if all areas are not laid out correctly, the surface area will not be calculated correctly.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning		
Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Begin by displaying images of various three-dimensional figures, such as cubes, rectangular prisms, and pyramids. • Ask students to discuss the characteristics of these figures and how they might be represented as two-dimensional nets. • Introduce the concept of nets as two-dimensional representations of three-dimensional figures. • Model how to identify the shapes of individual faces in a net and determine their dimensions. • Demonstrate how to calculate the surface area of a three-dimensional figure using its net, emphasizing the importance of adding the areas of all the faces.	• Review key concepts from the core instruction lesson, including identifying nets and calculating surface area. • Provide differentiated practice problems based on students' individual needs, offering additional support for struggling students and extension activities for advanced learners. • Offer manipulatives or visual aids for students who need additional support with spatial reasoning. • Present multi-step word problems that require students to apply their understanding of nets and surface area in real-world contexts. • Encourage students to explain their problem-solving strategies and justify	• Administer a diagnostic assessment to identify specific areas of difficulty for individual students related to representing three-dimensional figures and finding surface area. • Group students based on their assessment results and provide targeted instruction to address their individual needs. • Use a variety of instructional strategies, including hands-on activities and peer tutoring, to support students' understanding. • Provide one-on-one support for students who require intensive intervention, focusing on addressing their specific learning needs. • Offer additional practice opportunities and scaffolded

• Provide guided practice problems where students identify the nets of given three-dimensional figures and calculate their surface areas. • Walk students through each step of the process, providing support and feedback as needed. **Universal Design for Learning (UDL)** • Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.	their reasoning. **Universal Design for Learning (UDL)** • Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.	support to help students build confidence and mastery. **Universal Design for Learning (UDL)** • Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/6/26/267/267

Previous Learning	*Current Learning*	*Future Learning*
• Learners build on their knowledge of area from Grade 3 where they count the area of a rectangle and connect it to their understanding of multiplication in Grade 4. Learners understand how to find the volume of right rectangular prisms with whole numbers in Grade 5. Apply the area and perimeter formulas for rectangles in real world and mathematical problems.	• Learners are flexible using the terms base and length when solving for the area of a two or three-dimensional shape. Develop the concept of surface area. Learners understand how to find the volume of right rectangular prisms using fractions in the length of the edges. • Connects to lessons on negative integers and graphing points in all quadrants. • Find distance on the coordinate plane by counting the units on the	• In Grade 7, learners will continue to draw, construct, and describe geometrical figures and discover relationships between them (without nets). Calculate and compare the volume of cones, cylinders, and spheres. • Prepare for grade 8 work with transformations by working with polygons in coordinate plane. • Learners will further their knowledge on distance in 8th grade when they start to find the lengths of diagonal lines. • Learners will use their

	coordinate plane (no formula). Create polygons in quadrants I, II, III, and IV so learners can apply their knowledge of absolute value.	knowledge of the Pythagorean Theorem to find distance on the coordinate plane and later use the distance formula. ● In high school, learners will apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). In high school, learners will give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. ● In high school, students will use the idea of nets to identify the shapes of two-dimensional cross sections of three-dimensional objects, and identify three dimensional objects generated by rotations of two-dimensional objects.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
● Begin by validating students' diverse cultural backgrounds and experiences related to problem-solving in mathematics. ● Affirm the importance of understanding three-dimensional figures and surface area in various cultural contexts. ● Cultural Examples and Contexts:	● Build on students' prior knowledge by reviewing the concept of three-dimensional figures and surface area. ● Bridge the concept to real-world scenarios by discussing how different cultures might use three-dimensional figures and surface area in architecture, packaging design, or	● Provide linguistic vocabulary support for students by introducing translated terms and visual aids related to three-dimensional figures, nets, and surface area. ● Scaffold language acquisition through multilingual word banks or graphic organizers to facilitate comprehension. ● Cultural Examples and Contexts:

○ Incorporate culturally relevant examples and contexts into discussions about three-dimensional figures and surface area. ○ Discuss how different cultures might use three-dimensional figures and surface area in traditional crafts, packaging design, or architecture.	construction. ● Cultural Examples and Contexts: ○ Incorporate culturally relevant examples and contexts into discussions about three-dimensional figures and surface area. ○ Discuss how different cultures might use three-dimensional figures and surface area in traditional crafts, packaging design, or architecture.	○ Incorporate culturally relevant examples and contexts into discussions about three-dimensional figures and surface area. ○ Discuss how different cultures might use three-dimensional figures and surface area in traditional crafts, packaging design, or architecture.

Suggested Student Discourse Questions

- What is the difference between surface area and area?
- How does the area of a two-dimensional figure relate to the area of a three-dimensional figure?
- What is the process of using nets of three-dimensional figures to find the surface area of the figure in mathematical and real world mathematical problems?
- What is the process of calculating surface area when given an irregular shape; for example, a shape made exclusively of rectangles? Shapes made of triangles?

Cross-Curricular Connections

Science & English:

Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics.

Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).

Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 6 topics, texts, and issues, building on others' ideas and expressing their own clearly.

Career and Skill Connections
• Architect • Drafter • Graphic Designer • Animator • Product Designer

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards of Mathematical Practice
    - Procedural and Conceptual Understanding and Application
    - Sample Assessment Items
    - Common Misconceptions
    - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
    - Vertical Alignment
    - Culturally and Linguistically Responsive Instruction (CLRI)
    - Suggested Student Discourse Questions
    - Cross-Curricular and Career and Skill Connections
- A Student Discourse Guide
- Planning for a Multi-Layer System of Support (MLSS) and Universal Design for Learning (UDL) for behavioral and social and emotional supports

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

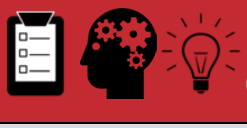
Standards Breakdown

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
  - 6.NS.A.1
- Compute fluently with multi-digit numbers and find common factors and multiples.
  - 6.NS.B.2
  - 6.NS.B.3
  - 6.NS.B.4
- Apply and extend previous understandings of numbers to the system of rational numbers.
  - 6.NS.C.5
  - 6.NS.C.6
  - 6.NS.C.7
  - 6.NS.C.8

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)

Grade	CCSS Domain	CCSS Cluster
6	The Number System	Apply and extend previous understandings of multiplication and division to divide fractions by fractions.



Cluster Standard: 6.NS.A.1

Standard	Standards for Mathematical Practice
Interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi.?*	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 6:** Attend to precision. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students will continue their previous understanding of the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to explain why the procedures for dividing fractions make sense. They use visual models and equations to divide whole numbers by fractions and fractions by fractions to solve word problems	• Describing, writing, or verbally explaining the relationship between multiplication and division of fractions. • Interpret and compute quotients of fractions using visual models and equations. • Create visual fraction models and equations to represent the problem. • Solve word problems involving division of fractions by fractions.
DOK	**Blooms**
1-2	Understand, Apply
Procedural and Conceptual Understanding and Application	
Procedural Skills and Fluency: Students demonstrate fluency in dividing fractions and mixed numbers, applying procedures to compute quotients of fractions accurately and efficiently and applying procedures to solve word	

problems involving division of fractions, including identifying the operation needed and setting up the problem correctly.

Conceptual Understanding: Students understand the concept of division of fractions as finding how many times one fraction is contained in another, the relationship between division of fractions and multiplication of fractions, including the reciprocal relationship and understand the meaning of the quotient of two fractions and its representation as a fraction or a mixed number.

Application: Students can apply division of fractions to solve real-world problems in various contexts, such as cooking, construction, and measurement., and recognize situations where division of fractions is needed and apply appropriate strategies to find solutions. This might involve creating story contexts, using visual models (such as fraction bars or area models), and writing equations to represent and solve problems.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

You are stuck in a big traffic jam on the freeway and you are wondering how long it will take to get to the next exit, which is 1 ½ miles away. You are timing your progress and find that you can travel ⅔ of a mile in one hour. If you continue to make progress at this rate, how long will it be until you reach the exit? Solve the problem with a diagram and explain your answer.

Two-thirds of a shoelace is ½ meter long. How long is the whole shoelace? Give your answer in units of meters.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Misunderstanding the Division of Fractions as Simple Subtraction or Addition:** Some students may mistakenly believe that dividing fractions involves subtracting or adding the numerators and denominators separately. For example, they may think that dividing 1/3 by 1/4 is simply subtracting 1 - 1 and 3 - 4.
- **Inverting Both Fractions for Division:** Students may incorrectly believe that to divide fractions, they need to invert both the dividend and divisor. While this is true, they might apply this rule indiscriminately to all fraction problems, regardless of whether it's appropriate.
- **Misinterpreting the Reciprocal:** Students may misunderstand the reciprocal relationship between multiplication and division. They may confuse the reciprocal of a fraction with its inverse or with the numerator and denominator being swapped.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Begin the lesson by activating students' prior knowledge of	• Differentiate instruction by providing various entry	• Provide explicit instruction on how to interpret and compute

fraction division. Use a think-pair-share activity to discuss situations where fractions are divided. ● Provide multiple means of representation by using visual aids, such as fraction bars and diagrams, to introduce the concept of dividing fractions. ● Facilitate a class discussion on the strategies used to solve fraction division problems and the relevance of these skills in everyday life. ● Encourage students to reflect on their problem-solving process and share any challenges they encountered while solving fraction division word problems. **Universal Design for Learning (UDL)** ● Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.	points for students of different learning levels. Offer scaffolded activities for students who need additional support and extension tasks for students who are ready for more challenging tasks. ● Engage students in hands-on activities where they solve real-world problems involving division of fractions by fractions. Use scenarios such as sharing food, dividing ingredients in recipes, or measuring distances. ● Give students opportunities to apply their understanding of fraction division through problem-solving tasks and real-world scenarios. ● Provide differentiated tasks based on students' readiness, interests, and learning profiles. Offer choice boards or menus with different options for demonstrating understanding. **Universal Design for Learning (UDL)** ● Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.	quotients of fractions using visual fraction models and equations. ● Teach students the relationship between multiplication and division of fractions and how to apply it to solve word problems involving fraction division. ● Offer extension activities for students who have mastered the basic concepts, such as exploring more complex fraction division problems or creating their own word problems. ● Provide enrichment opportunities for students interested in further exploring the connections between fraction division and other mathematical concepts, such as ratios and proportions. **Universal Design for Learning (UDL)** ● Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.
Vertical Alignment		
Consider using this coherence map to help guide your planning		

https://tools.achievethecore.org/coherence-map/6/27/272/272		
Previous Learning	*Current Learning*	*Future Learning*
• Connect student's 3rd and 5th grade understandings of division as an unknown factor problem. A student's ability to interpret whole number by whole number quotients and whole number by fraction quotients will be applied within this cluster. (e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.)	• Students will need to understand how to complete operations with rational numbers to help demonstrate their conceptual understanding of the distributive property	• Connect the understandings from this cluster to standards in 7th grade when students are required to demonstrate understanding of multiplication and division and of fractions to multiply and divide rational numbers. In Grade 7, learners solve real-world and mathematical problems involving the four operations with rational numbers. In HS Algebra standards, learners continue to use their understanding of division of fraction knowledge when solving more complex algebraic equations.
Culturally and Linguistically Responsive Instruction		
Consider these questions as you plan for instruction that is culturally and linguistically responsive: • How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*

• Begin by validating students' prior knowledge of fractions and division. Affirm their cultural backgrounds and experiences related to sharing food, dividing resources, and solving everyday problems. • Incorporate culturally relevant examples and scenarios into the lesson, such as sharing traditional foods from different cultures or discussing cultural practices related to food and sharing. • Conclude the lesson by reaffirming students' cultural backgrounds and experiences, and highlighting how these experiences can help them understand and solve mathematical problems. • Encourage students to continue exploring and applying fraction division concepts in real-world contexts, and to share their mathematical thinking with their families and communities.	• Build connections between students' cultural backgrounds and the mathematical concepts of fraction division. Bridge the gap between students' lived experiences and the abstract concepts of fraction division by using real-world contexts and examples. • Encourage students to share their own experiences of sharing food or resources in their communities and relate these experiences to the mathematical concept of dividing fractions.	• Provide linguistic support by using clear and concise language to explain mathematical concepts and procedures. Define key vocabulary words related to fraction division, such as quotient, numerator, denominator, and division. • Scaffold instruction by using visual aids, manipulatives, and graphic organizers to help students visualize and understand fraction division concepts. Use models such as fraction bars, diagrams, and number lines to represent fraction division problems.

Suggested Student Discourse Questions

- How does dividing fractions differ from dividing whole numbers?
- How can you use the reciprocal to solve division problems involving fractions?
- How can you determine when to divide fractions in a word problem? What are some key phrases or clues to look for?
- What did you learn from solving division of fraction problems, and how can you apply this knowledge in future situations?
- How can you tell if a fraction is in its simplest form?

Cross-Curricular Connections

English:

Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

Demonstrating the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.

Distinguish among facts, reasoned judgment based on research findings, and speculations in a text.

Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

Social Studies:

Students can determine growth in different contexts related to social studies. Students can apply their knowledge of number operations to create a claim for a question.

Students can use this idea of plotting points in a coordinate plane to adjust it to the longitude and latitude lines on a map. They can use this to track a traveling pattern and discuss it further. They can track a voyage over time.

Career and Skill Connections

- Baker
- Cook
- Computer Programmer
- Video Game Designer
- Accountant

Grade	CCSS Domain	CCSS Cluster
6	The Number System	Compute fluently with multi-digit numbers and find common factors and multiples.

Cluster Standard: 6.NS.B.2

Standard	Standards for Mathematical Practice
Fluently divide multi-digit numbers using the standard algorithm.	• **SMP 6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students will continue to build on their previous understanding of adding, subtracting, multiplying, and dividing to fluently use algorithms to solve problems. They will also work with finding the GCF to begin the early stages of factoring.	• Fluently divide multi-digit numbers.
DOK	**Blooms**
1-2	Apply

Procedural and Conceptual Understanding and Application

Procedural Skills and Fluency: Students will use prior knowledge of mathematical operations to work efficiently and accurately with solving problems computing problems involving algorithms. Compute fluently with multi-digit numbers and find common factors and multiples. Students will investigate mathematical rules of divisibility to improve fluency. This includes correctly placing the digits, performing subtraction, and carrying down digits as needed. Fluency here means that students can carry out these procedures quickly and with precision.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Use the computation shown below to find the products.

$$\begin{array}{r} 189 \\ 16\overline{)3024} \\ \underline{16} \\ 142 \\ \underline{128} \\ 144 \\ \underline{144} \\ 0 \end{array}$$

a. 189 × 16
b. 80 × 16
c. 9 × 16

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Finding GCF and LCM:** Students may have difficulty when finding the GCFs and LCMs.
- **Applying LCM and GCF:** Students may have difficulty when to apply LCM and when to apply the GCF to solve problems.
- **Identifying factors and multiples:** Students may have trouble identifying between the factors and the multiples.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Begin with a brief review of division concepts, including divisor, dividend, quotient, and remainder. Use visual aids such as manipulatives or diagrams to illustrate division processes. • Engage students in a discussion about real-life scenarios where division is used, such as dividing a pizza	• Introduce the standard algorithm for multi-digit division, starting with simple examples and gradually increasing in complexity. • Model the step-by-step process of the standard algorithm on the board or using a document camera, emphasizing the importance	• Guide students through several practice problems, providing support and feedback as needed. Offer differentiated instruction based on students' readiness levels, providing additional support for struggling students and extension activities for advanced learners.

among friends or sharing toys among siblings. **Universal Design for Learning (UDL)** ● Provide independent practice opportunities for students to apply their understanding of the standard algorithm for multi-digit division. Offer a variety of practice problems with varying levels of difficulty to cater to diverse learning needs.	of place value and alignment. **Universal Design for Learning (UDL)** ● Circulate the classroom to provide assistance and feedback to individual students, addressing any misconceptions or errors.	**Universal Design for Learning (UDL)** ● Encourage students to use the standard algorithm independently, referring back to the steps and procedures as needed.
Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/6/27/273/273		
Previous Learning	*Current Learning*	*Future Learning*
● Students will need to reflect on their previous understanding of factor pairs from 4th grade. They will connect their previous learning around multiples to finding LCMs and GCFs in this cluster. ● This cluster also connects to instruction from Grade 5 where students found whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value. These same skills will be utilized when dividing decimals	● In this cluster students use the distributive property to express a sum of whole numbers. This connects to future 6th grade learning when they explore the conceptual understanding of the distributive property in the 6.EE.A cluster.	● Students will connect their skills with the standard algorithm in order to successfully multiply and divide rational numbers. This will be connected in the standard algorithm as well as in application to real-world contexts. In high school, learners continue to use the distributive property to express a sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factor as they learn factorization.
Culturally and Linguistically Responsive Instruction		
Consider these questions as you plan for instruction that is culturally and linguistically responsive: ● How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?		

- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Begin the lesson by validating students' prior knowledge of division and affirming their abilities to solve division problems. Acknowledge the diverse backgrounds and experiences of the students in the classroom. • Incorporate culturally relevant examples of division, such as sharing resources in different cultural contexts or dividing items among family members or friends.	• Build connections between students' prior knowledge and the standard algorithm for multi-digit division. Bridge the gap between concrete understanding and abstract algorithms by using manipulatives, visual aids, and real-life examples. • Provide multiple entry points for students to access the content, including differentiated instruction based on individual learning styles and needs. Offer additional support and scaffolding for students who require it, while challenging advanced learners with extension activities.	• Provide linguistic support by breaking down complex mathematical language and providing clear explanations of division terms and procedures. Use visual aids, diagrams, and step-by-step instructions to reinforce understanding. • Scaffold instruction by gradually releasing responsibility to students, allowing them to practice and apply the standard algorithm for multi-digit division with support as needed. Encourage students to use mathematical language and explain their problem-solving strategies.

Suggested Student Discourse Questions

- Do you feel your strategy was the best one chosen or could there have been another strategy that would have made the problem easier to solve using that strategy? Explain your view.
- What is a common factor and how do we calculate a common factor?
- Talk with a classmate and compare the strategies that each of you used to solve the problem. How does your strategy compare or differentiate from the strategy of your classmate?
- Where in your personal life do you normally see the use of multiplying and dividing of fractions?

Cross-Curricular Connections

English:

- Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

- Demonstrating the meaning of symbols, key terms, and other domain specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.
- Distinguish among facts, reasoned judgment based on research findings, and speculations in a text.
- Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

Social Studies:

- Students can determine growth in different contexts.

Career and Skill Connections

- Financial Consultant
- Educator
- Accountant
- Baker
- Mechanical Engineer

Grade	CCSS Domain	CCSS Cluster
6	The Number System	Compute fluently with multi-digit numbers and find common factors and multiples.

Cluster Standard: 6.NS.B.3

Standard	Standards for Mathematical Practice
Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	• **SMP 6:** Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students will continue to build on their previous understanding of adding, subtracting, multiplying, and dividing to fluently use algorithms to solve problems. They will also work with finding the GCF to begin the early stages of factoring.	• Fluently add, subtract, multiply and divide multi-digit decimals.
DOK	**Blooms**
1-2	Apply

Procedural and Conceptual Understanding and Application

Procedural Skills and Fluency:
Students must be proficient in adding, subtracting, multiplying, and dividing multi-digit decimals using the standard algorithms for each operation. This involves:

- Addition and Subtraction: Accurately aligning decimal points and performing the operations digit by digit, while ensuring correct placement of the decimal point in the result.
- Multiplication: Multiplying as if the decimals were whole numbers, then placing the decimal point in the product by counting the total number of decimal places in the factors.
- Division: Dividing decimals by adjusting the divisor to a whole number (if necessary) and then placing the decimal point correctly in the quotient.

Fluency in these operations means that students can perform them accurately and efficiently, with confidence and speed. This is crucial for more advanced mathematical tasks and real-world problem-solving that involves decimal numbers.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Jayden has $20.56. He buys an apple for 79 cents and a granola bar for $1.76.

a. How much money did Jayden spend?
b. How much money does Jayden have now?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Decimal point positioning:** Students may misplace the decimal point when representing the product or quotient of decimals.
- **Place value recognition beyond the decimal:** Students may not understand the position of the place value beyond the decimal.
- **Value of a decimal:** Students often think that a longer decimal is a larger decimal.
- **Positioning numbers when computing addition and subtraction:** Some students confuse lining up the numbers and do not line up the decimals when making their computations of addition and subtraction.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Begin by reviewing the concept of decimals and their place value with the students. • Introduce the standard algorithms for adding, subtracting, multiplying, and dividing multi-digit decimals. • Discuss the importance of mastering these operations for real-world applications. **Universal Design for Learning (UDL)** • As students work, observe their progress and adjust support levels accordingly. NOTE: For students who demonstrate mastery of the standard algorithms, challenge them to solve real-world word problems involving multi-digit decimals. These problems can be related to topics such as money,	• Divide the class into small groups based on their readiness levels, ensuring each group has access to the necessary support materials. • Each group will work through a series of practice problems involving addition, subtraction, multiplication, and division of multi-digit decimals. • Encourage students to use the standard algorithm for each operation, referring to the place value charts as needed. **Universal Design for Learning (UDL)** • Circulate among the groups, providing guidance and support as necessary. For students in Layer 1, provide additional scaffolding and	• Bring the class back together for a brief discussion in small groups or pairs. • Review the key concepts covered during the lesson, emphasizing the importance of fluency in decimal operations. • Encourage students to continue practicing these skills independently. **Universal Design for Learning (UDL)** • Assign homework or additional practice worksheets as needed to reinforce learning. NOTE: Assess students' understanding of the standard algorithms for adding, subtracting, multiplying, and dividing multi-digit decimals through observation during the main activity and review of completed student

measurements, or data analysis.	modeling of the standard algorithm.	work. Additionally, administer a brief exit ticket or quiz to gauge individual student progress.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/6/27/274/274

Previous Learning	*Current Learning*	*Future Learning*
• Students will need to reflect on their previous understanding of factor pairs from 4th grade. They will connect their previous learning around multiples to finding LCMs and GCFs in this cluster. • This cluster also connects to instruction from Grade 5 where students found whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value. These same skills will be utilized when dividing decimals	• In this cluster students use the distributive property to express a sum of whole numbers. This connects to future 6th grade learning when they explore the conceptual understanding of the distributive property.	• Students will connect their skills with the standard algorithm in order to successfully multiply and divide rational numbers. This will be connected in the standard algorithm as well as in application to real-world contexts. In high school, learners continue to use the distributive property to express a sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factor as they learn factorization.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Begin by discussing the importance of recognizing and respecting cultural diversity in the classroom. • Introduce the concept of fluently adding, subtracting,	• Provide practice problems that incorporate culturally diverse word problems, such as scenarios involving international travel, cultural celebrations, or global	• Introduce key vocabulary words related to decimal operations, ensuring students understand the linguistic nuances of mathematical language.

multiplying, and dividing multi-digit decimals using the standard algorithm. ● Emphasize the connection between mathematical operations and real-world applications, highlighting how different cultures *use* mathematics in their daily lives. ● Share culturally relevant examples of decimal operations, such as currency conversions from different countries or recipes from diverse cultures. ● Encourage students to share their own experiences with decimal operations, validating their cultural backgrounds and perspectives.	economic trends. ● Scaffold instruction by breaking down complex problems into smaller steps, using visuals and manipulatives as needed. ● Encourage collaboration and peer support, allowing students to work together to solve problems and bridge cultural differences through shared learning experiences. ● Reinforce the importance of embracing diversity and cultural understanding in the classroom.	● Provide linguistic support by offering translations or definitions in students' home languages, if applicable. ● Encourage students to use academic language to describe their problem-solving strategies and explain their reasoning. ● Review the key concepts covered during the lesson, highlighting the cultural relevance of decimal operations. ● Invite students to reflect on how their cultural backgrounds influence their understanding of mathematics.

Suggested Student Discourse Questions

- After solving the problem, can you think of a different strategy that would have been better suited for solving the problem?
- Compare the strategy you used to solve the problem with that of a classmate, how are they different?
- Was the strategy you chose to solve the problem the only strategy you could have used? Was it the best strategy to solve that problem?
- When solving multiplication and division of decimal problems, why is it important to ensure the decimal is placed in the correct position?

Cross-Curricular Connections

English:

- Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.
- Demonstrating the meaning of symbols, key terms, and other domain specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.
- Distinguish among facts, reasoned judgment based on research findings, and speculations in a text.
- Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

Social Studies:

• Students can determine growth in different contexts.
Career and Skill Connections
• Bank Teller • Retail Store Manager • Accountant • Financial/Budget Planner • Auditor

Grade	*CCSS Domain*	*CCSS Cluster*
6	The Number System	Compute fluently with multi-digit numbers and find common factors and multiples.

Cluster Standard: 6.NS.B.4

Standard	Standards for Mathematical Practice
Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2)	• **SMP 6:** Attend to precision. • **SMP 7**: Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students will continue to build on their previous understanding of adding, subtracting, multiplying, and dividing to fluently use algorithms to solve problems. They will also work with finding the GCF to begin the early stages of factoring.	• Find the GCF of two whole numbers less than or equal to 100. • Find the LCM of two whole numbers less than or equal to 12. • Use the distributive property to express a sum of two whole numbers (1-100) with a common factor as a multiple of a sum of two whole numbers with no common factor.
DOK	**Blooms**

1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Procedural Skills and Fluency: Students calculate a common factor of two whole numbers less than or equal to 100 or a multiple of two numbers less than or equal to 12 with speed and accuracy. This involves using methods such as prime factorization, listing factors, and using the relationship between GCF and LCM. Students should also be fluent in applying the distributive property to rewrite expressions, such as transforming 36 + 8 into 4(9 + 2) by factoring out the greatest common factor. Students calculate and find the greatest common factor or the least common multiple of two numbers efficiently and with accuracy.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Nina was finding multiples of 6. She said:

18 and 42 are both multiples of 6, and when I add them, I also get a multiple of 6: 18 + 42 = 60.

Explain to Nina why adding two multiples of 6 will always result in another multiple of 6.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Difference between GCF and LCM:** Often some confusion between GCF and LCM, should teach them separately to reduce chance of confusion.
- **Multiplication fact skills:** Some students have confusion over multiplication facts and may need to reinforce skills of multiplication facts.
- **Difference between factors and multiples:** Students may confuse the concepts of factors and multiples.
- **Application of LCM and GCF:** Students may misunderstand when to apply LCM and when to apply GCF to solve a problem.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Use manipulatives or visual aids to demonstrate how to find the GCF of two numbers, guiding students through the process step by step.	• Present more challenging problems that require students to find the LCM of two numbers, encouraging them to use different	• Introduce the distributive property and explain how it can be used to express a sum of two whole numbers with a common factor as a multiple

• Provide practice problems for students to work on individually or in pairs, allowing them to apply their understanding of factors and multiples. **Universal Design for Learning (UDL)** • Circulate the classroom to provide support and guidance as needed, reinforcing key concepts and addressing any misconceptions. NOTE: Assess students' understanding through observation during class activities, review of completed worksheets, and performance on exit tickets or quizzes. Evaluate students' ability to find factors and multiples accurately, determine the GCF and LCM, and apply the distributive property to simplify expressions. Provide feedback and support to help students strengthen their skills in these areas.	strategies to solve these problems. • Have students work on additional practice problems independently, reinforcing their ability to identify factors and multiples accurately. **Universal Design for Learning (UDL)** • Offer extension activities for students who finish early, such as creating their own problems or solving word problems involving factors and multiples.	of a sum of two whole numbers with no common factor. • Provide examples of this concept and guide students through the process of applying the distributive property to simplify expressions. **Universal Design for Learning (UDL)** • Encourage students to explore different ways of using the distributive property to manipulate expressions and solve problems more efficiently. NOTE: Review the key concepts covered in the lesson, emphasizing the importance of understanding factors, multiples, GCF, LCM, and the distributive property. • Encourage students to ask questions and seek clarification on any remaining doubts or uncertainties. • Assign homework or additional practice problems to reinforce the concepts learned in class.

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/6/27/275/275		
Previous Learning	*Current Learning*	*Future Learning*
• Students will need to reflect on their previous understanding of factor pairs from 4th grade. They will connect their previous learning around multiples to finding LCMs and GCFs in this cluster.	• In this cluster students use the distributive property to express a sum of whole numbers. This connects to future 6th grade learning when they explore the conceptual understanding of the distributive property.	• Students will connect their skills with the standard algorithm in order to successfully multiply and divide rational numbers. This will be connected in the standard algorithm as well as in application to real-world

• This cluster also connects to instruction from Grade 5 where students found whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value. These same skills will be utilized when dividing decimals		contexts. In high school, learners continue to use the distributive property to express a sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factor as they learn factorization.
Culturally and Linguistically Responsive Instruction		
Consider these questions as you plan for instruction that is culturally and linguistically responsive: • How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Validate and Affirm*
• Begin by validating students' prior knowledge of factors and multiples, acknowledging that these concepts can sometimes be challenging. • Affirm students' efforts by emphasizing that everyone has the ability to learn and improve their understanding of mathematical concepts. • Create a supportive and inclusive learning environment where students feel comfortable asking questions and sharing their ideas. • Incorporate opportunities for students to explain their thinking and reasoning using academic language, promoting language development and comprehension. • UDL: Provide multiple means	• Use visual aids and manipulatives to demonstrate how to find the GCF and LCM of two numbers, providing step-by-step explanations in clear and accessible language. • Scaffold the learning process by breaking down complex problems into smaller, more manageable steps, allowing students to build their understanding gradually. • Encourage students to work collaboratively in pairs or small groups, fostering peer-to-peer support and communication. • Provide opportunities for students to practice finding GCFs and LCMs through hands-on activities and interactive exercises. • UDL: Provide multiple	• Introduce key vocabulary words related to factors, multiples, and the distributive property, such as "factor," "multiple," "greatest common factor," "least common multiple," and "distributive property." • Use vocabulary cards or visual aids to reinforce the meaning of these terms, providing definitions, examples, and illustrations as needed. • Offer language support for English language learners by providing bilingual dictionaries, sentence frames, and opportunities for vocabulary practice in context. • UDL: Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal

of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.	means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.	responses, or visual representations.

Suggested Student Discourse Questions

- What is the difference between factors and multiples?
- When do you apply the LCM to solve a problem? How do you know to use the LCM and not the CGF?
- When do you apply the GCF to solve a problem? How do you know to use the GCF and not the LCM?
- After solving problems related to finding the GCF and solving problems related to finding the LCM, how comfortable do you feel in finding and applying each of them in solving problems?

Cross-Curricular Connections

English:

- Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.
- Demonstrating the meaning of symbols, key terms, and other domain specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.
- Distinguish among facts, reasoned judgment based on research findings, and speculations in a text.
- Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

Social Studies:

- Students can determine growth in different contexts.

Career and Skill Connections

- Math Teacher
- Statistician
- Accountant
- Civil Engineer
- Economist

Grade	CCSS Domain	CCSS Cluster
6	The Number System	Apply and extend previous understandings of numbers to the system of rational numbers.

Cluster Standard: 6.NS.C.5

Standard	Standards for Mathematical Practice
Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 2:** Reason abstractly and quantitatively. • **SMP 4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students will extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical. Horizontal and vertical number lines help students move from number lines to coordinate grids. They will focus on the relationship between negative and positive numbers and the meaning of absolute value. This cluster will lay the foundation for working with rational numbers, algebraic expressions and equations, functions and the coordinate plane in seventh and eighth grade.	• Understand that positive and negative numbers are used to describe amounts having opposite values. • Represent quantities in real-world contexts and explain the meaning of 0 in each situation.
DOK	**Blooms**
2	Understand, Apply

Procedural and Conceptual Understanding and Application

Conceptual Understanding: Students will understand the meaning behind positive and negative numbers and how they are used to represent quantities. Student should explore how they are used to represent quantities that have opposite directions or values. Students will understand the meaning of absolute value and its connection to integers. Understanding the role of 0 is crucial, as it often represents a neutral point, like sea level in elevation or zero balance in a financial account.

Assessment Items
When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.
One morning the temperature is -28◦ in Anchorage, Alaska, and 65◦ in Miami, Florida. How many degrees warmer was it in Miami than in Anchorage on that morning? You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions
• **Value of negative numbers:** Students may confuse the idea that the greater the magnitude of a negative number the greater the number. • **Position of numbers on a number line:** Students may confuse the placement of rational numbers on the number line. • **Meaning of the absolute value bar:** Students may confuse the absolute value bar with parenthesis. • **Absolute value bar:** Students may confuse the absolute value bar with the number 1. • **Understanding of absolute value:** Students may think that absolute value makes things positive and not understand it is about distance from 0.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning		
Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Begin by discussing with students the concept of positive and negative numbers in real-world contexts. Provide examples such as temperature, elevation, and bank account balances. • Introduce key vocabulary terms such as "positive numbers," "negative numbers," "zero," and "opposite values." • Engage students in a brief discussion about why it's important to understand positive and negative numbers in everyday life. • Provide additional practice	• Use number line posters or digital representations to visually demonstrate positive and negative numbers. Show how positive numbers are represented to the right of 0 on the number line, while negative numbers are represented to the left. • Provide real-world scenarios and ask students to identify whether each situation involves a positive or negative quantity. Examples may include:	• Offer multiple means of representation by presenting information through visual aids, verbal explanations, and hands-on activities. • Provide multiple means of action and expression by allowing students to demonstrate their understanding through written responses, verbal explanations, or manipulative-based tasks. • UDL: Offer multiple means of engagement by incorporating real-world examples and interactive activities that appeal to different learning styles and interests. NOTES: Review the main concepts

problems for students to work on independently or in small groups, offering support and assistance as needed. NOTE: Assess students' understanding through observation during class activities, review of completed worksheets, and performance on exit tickets or quizzes. Evaluate students' ability to identify positive and negative quantities in real-world scenarios, represent them using number lines or written expressions, and explain the meaning of 0 in each situation. Provide feedback and support to help students strengthen their skills in these areas. **Universal Design for Learning (UDL)** ● Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.	~A temperature of 5 degrees above zero ~A bank account with a balance of -$50 ~A mountain with an elevation of 3,000 feet below sea level ● Guide students through the process of using positive and negative numbers to represent each scenario on a number line or through written expressions. ● Encourage students to explain the meaning of 0 in each situation. For example, in the context of temperature, 0 degrees represents the freezing point, while in a bank account balance, it represents neither a surplus nor a deficit. **Universal Design for Learning (UDL)** ● Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual	covered in the lesson, summarizing key points about positive and negative numbers and their use in real-world contexts. ● Allow students to ask any remaining questions or seek clarification on specific concepts. ● Reinforce the importance of understanding positive and negative numbers for interpreting various situations in everyday life. ● Encourage students to reflect on how they can apply their knowledge of positive and negative numbers in future situations. **Universal Design for Learning (UDL)** ● Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.

	representations.	

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/6/27/276/276		
Previous Learning	*Current Learning*	*Future Learning*
• This cluster is connected to what students previously learned in third grade, when they marked off units on a horizontal scale or number line. Students will recall that a fraction can be represented on a number line, in the space between whole numbers. They will also recall the skills from Grade 5 when they graphed points on a coordinate plane and interpreted what the points represent.	• There are connections between this cluster and the 6.EE.B cluster when learners recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams. Also, in 6.G.3, there are connections made when students use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate	• The skills from this cluster are applied in 7th grade when students make connections between their 6th grade understanding of what rational numbers are to include the addition and subtraction of integers. Students will need to represent addition and subtraction of integers on a horizontal and/or vertical number line.

Culturally and Linguistically Responsive Instruction		
Consider these questions as you plan for instruction that is culturally and linguistically responsive: • How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Begin by affirming the diverse backgrounds and experiences of students in	• Use number line posters or digital representations to	• Provide linguistic support by reviewing and reinforcing key vocabulary terms related to

relation to the concept of positive and negative numbers. ● Validate students' prior knowledge by asking them to share examples of situations where positive and negative numbers are used in everyday life. ● Affirm the importance of understanding positive and negative numbers by highlighting their relevance in various contexts, such as temperature, elevation, and financial transactions. NOTE: Review the main concepts covered in the lesson, emphasizing the importance of understanding positive and negative numbers in real-world contexts. ● Summarize key points about positive and negative numbers and their use in describing quantities with opposite directions or values. ● Encourage students to reflect on how they can apply their understanding of positive and negative numbers in their everyday lives.	visually demonstrate positive and negative numbers. Build connections between positive numbers and quantities above zero, and negative numbers and quantities below zero. ● Bridge the gap between abstract concepts and real-world applications by providing students with worksheets containing scenarios involving positive and negative quantities. Examples may include: ~A temperature of -5 degrees Celsius indicating below-freezing weather ~A bank account with a balance of $-50 representing a deficit ~A mountain with an elevation of -3,000 feet below sea level ● Guide students through the process of using positive and negative numbers to represent each scenario on a number line or through written expressions. ● Encourage students to explain the meaning of 0 in each situation. For example, in the context of temperature, 0 degrees represents the freezing point, while in a bank account balance, it represents neither a surplus nor a deficit.	positive and negative numbers, such as "positive," "negative," "zero," "opposite values," and "real-world contexts." ● Use vocabulary cards or visual aids to illustrate the meanings of these terms and provide opportunities for students to practice using them in sentences or discussions. ● Encourage students to ask questions and seek clarification on any unfamiliar terms or concepts. NOTE: Assess students' understanding through observation during class activities, review of completed worksheets, and participation in discussions. Evaluate students' ability to identify positive and negative quantities in real-world scenarios, represent them using number lines or written expressions, and explain the meaning of 0 in each situation. Provide feedback and support to help students strengthen their linguistic and conceptual understanding of positive and negative numbers.

Suggested Student Discourse Questions

- What are the similarities and differences between positive and negative rational numbers?
- Where in your personal life do you see the use of positive and/or negative numbers being used?
- In areas such as forecasting the weather or in banking, how are positive and negative numbers used or applied?
- Using a number line, are you able to identify where on the number line positive and negative numbers are located? Can you use the number line to help you solve problems involving these rational numbers?

Cross-Curricular Connections

English:

- Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.
- Demonstrating the meaning of symbols, key terms, and other domain specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.
- Distinguish among facts, reasoned judgment based on research findings, and speculations in a text.
- Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

Social Studies:

- Students can use this idea of plotting points in a coordinate plane to adjust it to the longitude and latitude lines on a map. They can use this to track a traveling pattern and discuss it further. They can track a voyage over time.

Career and Skill Connections

- Educator
- Accountant
- Financial Analyst
- Data Entry Clerk
- Mechanical Engineer

Grade	CCSS Domain	CCSS Cluster
6	The Number System	Apply and extend previous understandings of numbers to the system of rational numbers.

Cluster Standard: 6.NS.C.6

Standard	Standards for Mathematical Practice
Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. A. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite B. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. C. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	● **SMP 4**: Model with mathematics. ● **SMP 6:** Attend to precision. ● **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students will extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical. Horizontal and vertical number lines help students move from number lines to coordinate grids. They will focus on the relationship between negative and positive numbers and the meaning of absolute value. This cluster will lay the foundation for working with rational numbers, algebraic expressions and equations, functions and the coordinate plane in seventh and eighth grade.	● Explain the concept of rational numbers by understanding that a rational number is a point on a number line and extending number line diagrams to show positive and negative numbers on the line and in the coordinate plane. ● Express orally and in writing that opposite signs of a number indicate opposite places on a number line. ● Understand where positive and negative numbers in an ordered pair appear on a coordinate plane and identify quadrants.
DOK	**Blooms**

2	Understand

Procedural and Conceptual Understanding and Application

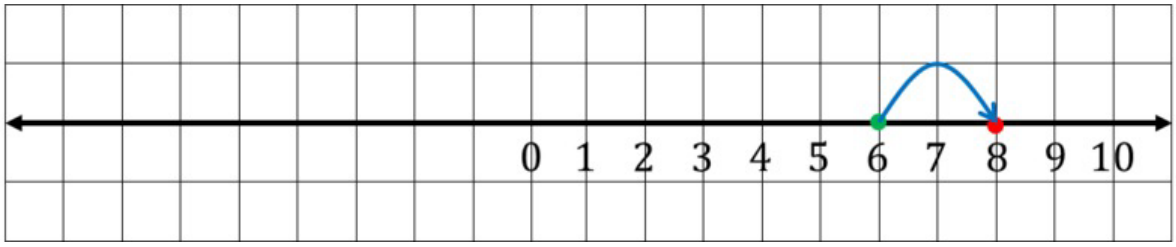
Procedural Skills and Fluency: Students fluently will find and position integers and other rational numbers on both horizontal and vertical number lines. Students with accuracy and speed will find and position pairs of integers and other rational numbers (coordinate pairs) on a coordinate plane. Fluency involves consistently identifying and reflecting points based on their coordinates, understanding how changes in signs affect the position of points in the coordinate plane.

Conceptual Understanding: Students will understand that rational numbers (positive and negative) are points on a number line and extend previous knowledge of number lines in the negative direction as well as understand how opposites are located on that line (e.g., - (-3) = 3). Students will understand that zero is a position on a number line and that it is its own opposite. Additionally, they must grasp how the signs of numbers in ordered pairs determine their location in the coordinate plane, and how reflections across the axes relate to changes in signs.

Assessment Items

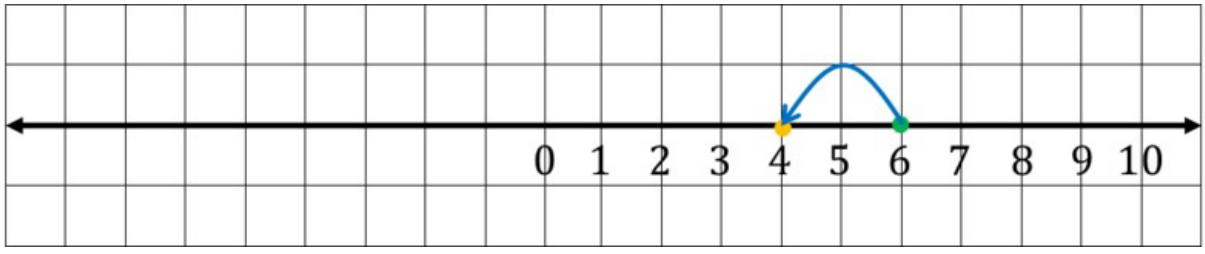
When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

a. Draw a line on graph paper. Make a tick mark in the middle of the line and label it 0. Mark and label 1, 2, 3, ... 10. Since 6+2 is 2 units to the right of 6 on the number line, we can represent 6+2 like this:



Describe the location of 3+4 on the number line in terms of 3 and 4. Draw a picture like the one above.

b. 6-2 is 2 units to the left of 6 on the number line, which we can represent like this:



Describe the location of 3-4 on the number line in terms of 3 and 4. Draw a picture like the one above.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Negative sign position:** Students believe that any symbol with a negative sign in front of it should be a negative number; for example, -p = - (-3) = 3. It is useful for students to see examples where p is a positive number even though it begins with a negative symbol.
- **Absolute value determination:** Students often believe distance can be a negative number when determining absolute value of a positive number confusing the concept with opposite/additive inverse.
- **Absolute value concepts:** Students often confuse the concept of absolute value with opposites or with the distinct locations on the number line.
- **Directions of numbers on the number line:** Students can confuse the direction of the positive and negative directions on the number line.
- **Quadrants on the coordinate plane and directions of positive and negative values:** Students can confuse the quadrants on the coordinate plane as well as the positive and negative directions within the quadrants.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
Engage students by discussing the concept of rational numbers and their representation on the number line. Encourage them to recall their previous knowledge and experiences with number lines. ● Exploring Rational Numbers on the Number Line: ○ Review the concept of the number line, emphasizing how rational numbers are represented as points on the line. ○ Demonstrate how to plot various rational numbers on the number line, including positive, negative, and fractions. ● Understanding Opposites: ○ Introduce the	● Extending to the Coordinate Plane: ○ Transition to the coordinate plane and review its components, including the x-axis and y-axis. ○ Demonstrate how to plot ordered pairs with rational number coordinates on the coordinate plane, emphasizing the significance of signs in indicating locations in quadrants. ● Reflecting Across Axes: ○ Introduce the concept of reflection across axes and its impact	Practicing with Partners: ● Divide students into pairs and provide them with practice problems involving plotting integers and rational numbers on number line diagrams and coordinate planes. ● Encourage peer collaboration and support as students work through the problems together. **Universal Design for Learning (UDL)** ● Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.

concept of opposites and their representation on the number line. ○ Explain the relationship between a number and its opposite, using examples and visual aids to illustrate (-3) and 3 as opposites. **Universal Design for Learning (UDL)** ● Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.	on the location of points in the coordinate plane. ○ Provide examples of ordered pairs that differ only by signs and demonstrate how the locations of the points are related by reflections across one or both axes. **Universal Design for Learning (UDL)** ● Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.	

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/6/27/277/277

Previous Learning	*Current Learning*	*Future Learning*
● This cluster is connected to what students previously learned in third grade, when they marked off units on a horizontal scale or number line. Students will recall that a fraction can be represented on a number line, in the space between whole numbers. They will also recall the skills from Grade 5 when they graphed points on a coordinate plane and interpreted what the points represent.	● There are connections between this cluster when learners recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams. Also, in 6.G.3, there are connections made when students use coordinates to find the length of a side joining points with the same first coordinate or the same	● The skills from this cluster are applied in 7th grade when students make connections between their 6th grade understanding of what rational numbers are to include the addition and subtraction of integers. Students will need to represent addition and subtraction of integers on a horizontal and/or vertical number line.

	second coordinate	

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Begin by affirming the students' prior knowledge of the number line and coordinate plane. • Validate their understanding of positive numbers and their positions on the number line and coordinate plane. • Introduce the concept of rational numbers and explain that they include both positive and negative integers as well as fractions and decimals. NOTE: Assess students' understanding through observation during class activities, review of completed worksheets or practice problems, and participation in discussions. Evaluate students' ability to represent rational numbers on number line diagrams and coordinate planes, recognize opposites, and interpret signs of numbers in ordered pairs. Provide feedback and support to help students strengthen their conceptual understanding of rational numbers and their applications in mathematical contexts.	• Display a number line diagram on the whiteboard or use a digital representation. Discuss how rational numbers are represented as points on the number line, including positive and negative integers, fractions, and decimals. • Model how to extend the number line to include negative number coordinates, emphasizing that numbers to the left of 0 are negative and numbers to the right are positive. • Provide integer and rational number cards to students and have them place the numbers on the number line diagram individually or in pairs. • Transition to the coordinate plane by introducing coordinate axes and quadrants. Explain how ordered pairs represent points in the coordinate plane.	• Review key vocabulary terms related to rational numbers, including "positive," "negative," "opposite," "coordinates," and "quadrants." • Use vocabulary cards or visual aids to reinforce the meanings of these terms and provide opportunities for students to practice using them in sentences or discussions. • Review the main concepts covered in the lesson, emphasizing the understanding of rational numbers as points on the number line and in the coordinate plane. • Summarize key points about representing rational numbers, recognizing opposites, and interpreting signs of numbers in ordered pairs. • Encourage students to apply their understanding of rational numbers to real-world situations and problem-solving tasks.

Suggested Student Discourse Questions

- How can you use the number line to help you solve problems involving positive and negative numbers?
- How are number lines that are horizontal and vertical different? Is there a difference?
- Thinking about vertical and horizontal number lines, is one better than the other when solving problems or identifying positive and negative numbers positions on the number line? Explain.
- Are positive and negative numbers the only values that are on the number line? Are there others?

Cross-Curricular Connections

English:

- Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.
- Demonstrating the meaning of symbols, key terms, and other domain specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.
- Distinguish among facts, reasoned judgment based on research findings, and speculations in a text.
- Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

Social Studies:

- Students can use this idea of plotting points in a coordinate plane to adjust it to the longitude and latitude lines on a map. They can use this to track a traveling pattern and discuss it further. They can track a voyage over time.

Career and Skill Connections

- Chemist
- Physicist
- Architect
- Bookkeeper
- Mathematician

Grade	CCSS Domain	CCSS Cluster
6	The Number System	Apply and extend previous understandings of numbers to the system of rational numbers.

Cluster Standard: 6.NS.C.7

Standard	Standards for Mathematical Practice
Understand ordering and absolute value of rational numbers A. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right B. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write - 3°C > -7°C to express the fact that -3°C is warmer than -7°C C. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars. D. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.	● **SMP 1:** Make sense of problems and persevere in solving them. ● **SMP 2:** Reason abstractly and quantitatively. ● **SMP 3:** Construct viable arguments and critique the reasoning of others.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students will extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical. Horizontal and vertical number lines help students move from number lines to coordinate grids. They will focus on the relationship between negative and positive numbers and the meaning of absolute value. This cluster will lay the foundation for working with rational numbers, algebraic expressions and equations, functions and the coordinate plane in seventh and eighth grade.	● Understand the absolute value of rational numbers. ● Interpret and explain the meanings behind inequality statements. ● Show understanding of rational numbers by giving them context in a real-life situation. ● Understand that absolute value is a number's distance from zero on a number line. ● Understand the difference between absolute value from order statements. ● Explain the reasoning that as a value of a negative

	rational number decreases its absolute value increases.
DOK	**Blooms**
2	Understand, Apply
Procedural and Conceptual Understanding and Application	
Procedural Skills and Fluency: Students will fluently practice writing inequality statements with the appropriate symbols with accuracy. Students will interpret statements of inequalities about the relative position of two numbers on a number line diagram with speed and accuracy. With speed and accuracy students will distinguish comparisons of absolute value from statements about order; such as recognizing that an account balance less than −30 dollars represents a debt greater than 30 dollars in a real world scenario. **Conceptual Understanding:** Students will recognize and understand that distance cannot be negative, which is why absolute values are always positive (since it is describing the distance from zero). Students will recognize and know the proper notation for absolute value. Students will be able to make connections to real world examples and the mathematical concepts relatable to the concepts within the standard.	
Assessment Items	
When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.	

The table below shows the lowest elevation above sea level in three American cities.

City	State	Elevation above sea level	Elevation below sea level
Denver	Colorado	5130	
New Orleans	Louisiana	-8	
Seattle	Washington	0	

Finish filling in the table as you think about the following statements. Decide whether each of the following statements is true or false. Explain your answer for each one.

a. New Orleans is |−8| feet below sea level.
b. New Orleans is −8 feet below sea level.
c. New Orleans is 8 feet below sea level.
d. Seattle is 0 feet above sea level.
e. Seattle is |0| feet below sea level.
f. Denver is −5130 feet below sea level.
g. Denver is |−5130| feet below sea level.
h. Denver is −|5130| feet below sea level.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Number direction on the number line:** Students find confusion in the direction of positive and negative on the number line.
- **Absolute value meaning:** Misconception that distance can be a negative number when determining the absolute value of a positive number.
- **Absolute value misrepresented:** Students may confuse the absolute value bar with parenthesis.
- **Absolute value concept; distance from zero:** Students are unable to make the connection to the meaning of absolute value in relationship to the distance a number is from zero.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Engage students by discussing the concept of ordering numbers and its importance in everyday life. ~Review the symbols used for inequalities (<, >, ≤, ≥) and their meanings. • Understanding Relative Position on the Number Line (Layer 1 - All Students): • Introduce the concept of interpreting statements of inequality as relative positions of numbers on a number line. • Demonstrate how to interpret statements such as -3 > -7 by representing them on a number line. *Real-World Contexts (Layer 1 - All Students): • Provide real-world scenarios or word problems involving rational numbers and inequalities. • Guide students in writing, interpreting, and explaining statements of order in these contexts (e.g., -3°C > -7°C to express warmer temperatures). • Extension and Differentiation: • Offer extension activities for advanced students, such as exploring complex inequalities or real-world	• Transition to discussing absolute value as the distance from zero on the number line. • Explain how absolute value represents the magnitude of a positive or negative quantity in a real-world situation. • Application to Real-World Situations (Layer 2 - Additional Support): ~ Present scenarios or word problems where absolute value is relevant (e.g., account balances, temperatures). ~ Guide students in interpreting absolute value in these contexts and explaining its significance. Extension and Differentiation: • Offer extension activities for advanced students, such as exploring complex inequalities or real-world scenarios involving multiple variables. • Provide additional support for struggling students through one-on-one or small group instruction, using manipulatives or visual aids as needed. ~Informal assessment through observation of student participation and engagement during class discussions and activities. ~Formative assessment through review of students' responses to practice problems and their ability to correctly interpret statements of order and absolute value in real-world contexts. ~Summative assessment through	• Comparisons and Distinctions (Layer 3 - Intensive Support): • Help students distinguish between comparisons of absolute value and statements about order. • Provide examples and non-examples to illustrate the difference (e.g., an account balance less than -30 dollars represents a debt greater than 30 dollars). • Collaborative Practice (Universal Design for Learning - All Students): ~Divide students into pairs or small groups to work on practice problems involving ordering and absolute value of rational numbers. ~Encourage peer collaboration and discussion to reinforce understanding. • Extension and Differentiation: ~Offer extension activities for advanced students, such as exploring complex inequalities or real-world scenarios involving multiple variables. ~Provide additional support for struggling students through one-on-one or small group instruction, using manipulatives or visual aids as needed. • Informal assessment through observation of student participation and engagement during class discussions and activities. • Formative assessment through review of students' responses to practice problems and their ability to correctly interpret statements of order and absolute value in real-world contexts. **Universal Design for Learning (UDL)**

scenarios involving multiple variables. ● Provide additional support for struggling students through one-on-one or small group instruction, using manipulatives or visual aids as needed. ● Informal assessment through observation of student participation and engagement during class discussions and activities. ● Formative assessment through review of students' responses to practice problems and their ability to correctly interpret statements of order and absolute value in real-world contexts. **Universal Design for Learning (UDL)** ● Summative assessment through quizzes or exit tickets assessing students' understanding of ordering and absolute value of rational numbers.	quizzes or exit tickets assessing students' understanding of ordering and absolute value of rational numbers. **Universal Design for Learning (UDL)** ● Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.	● Provide summative assessment through quizzes or exit tickets assessing students' understanding of ordering and absolute value of rational numbers.

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/6/27/281/281		
Previous Learning	*Current Learning*	*Future Learning*
● This cluster is connected to what students previously learned in third grade, when they marked off units on a horizontal scale or number line. Students will recall that a fraction can be represented on a number line, in the	● There are connections between this cluster when learners recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line	● The skills from this cluster are applied in 7th grade when students make connections between their 6th grade understanding of what rational numbers are to include the addition and subtraction of integers.

space between whole numbers. They will also recall the skills from Grade 5 when they graphed points on a coordinate plane and interpreted what the points represent.	diagrams. Also, there are connections made when students use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate	Students will need to represent addition and subtraction of integers on a horizontal and/or vertical number line.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Begin the lesson by validating students' diverse cultural backgrounds and experiences related to real-world situations involving rational numbers. • Affirm the importance of understanding ordering and absolute value in various cultural contexts. • Cultural Reflection and Connection (Universal Design for Learning - All Students): • Encourage students to reflect on how their cultural backgrounds influence their interpretations of ordering and absolute value in real-world situations. • Foster connections between mathematical concepts and cultural contexts to deepen understanding and appreciation for diverse perspectives.	• Build on students' prior knowledge by reviewing the concept of ordering numbers and their relative positions on a number line. • Bridge the concept to real-world scenarios by discussing how cultural factors may influence perceptions of temperature, financial transactions, or other quantitative measures. • Exploring Absolute Value in Cultural Contexts (Layer 2 - Additional Support): • Present real-life scenarios or word problems that reflect diverse cultural contexts, such as temperature measurements in different regions or financial transactions in varied cultural settings. • Guide students in interpreting absolute value as the magnitude of a	• Provide linguistic vocabulary support for students by introducing translated terms and visual aids related to ordering and absolute value. • Scaffold language acquisition through graphic organizers or multilingual word banks to facilitate comprehension. • Interpretation and Explanation (Layer 3 - Intensive Support): • Facilitate small group discussions or partner activities where students interpret and explain statements of order for rational numbers in real-world contexts. • Provide additional support for students who may require assistance in articulating their understanding through culturally relevant examples.

	positive or negative quantity within these cultural contexts.	

Suggested Student Discourse Questions

- What is the significance of absolute value in relation to positive and negative numbers?
- Are you able to determine the understanding of absolute value and how it applies to both positive and negative numbers?
- How is the number line used to identify inequalities and their solutions?
- Can you think of anywhere in your personal life that you can represent the use of inequalities? Explain.

Cross-Curricular Connections

English:

- Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.
- Demonstrating the meaning of symbols, key terms, and other domain specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.
- Distinguish among facts, reasoned judgment based on research findings, and speculations in a text.
- Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

Social Studies:

- Students can use this idea of plotting points in a coordinate plane to adjust it to the longitude and latitude lines on a map. They can use this to track a traveling pattern and discuss it further. They can track a voyage over time.

Career and Skill Connections

- Finance/ Budget Planner
- Accountant
- Economist
- Computer Programmer/ Coder
- Market Researcher

Grade	*CCSS Domain*	*CCSS Cluster*
6	The Number System	Apply and extend previous understandings of numbers to the system of rational numbers.

Cluster Standard: 6.NS.C.8

Standard	Standards for Mathematical Practice
Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 2:** Reason abstractly and quantitatively. • **SMP 4:** Model with mathematics.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students will extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical. Horizontal and vertical number lines help students move from number lines to coordinate grids. They will focus on the relationship between negative and positive numbers and the meaning of absolute value. This cluster will lay the foundation for working with rational numbers, algebraic expressions and equations, functions and the coordinate plane in seventh and eighth grade.	• Graph points in all four quadrants solving real-world problems. • Find distance between points using coordinates and absolute value.
DOK	**Blooms**
1-2	Understand

Procedural and Conceptual Understanding and Application

Procedural Skills and Fluency: Students will fluently and accurately use the coordinate plane as a tool to solve problems graphing points in the four quadrants. Students will also be expected to plot points that are positioned on the axes. Students are expected to find the distance between points on the same horizontal or same vertical line. Students will practice with speed and accuracy to determine the distance of varied points given a pair of ordered pairs.

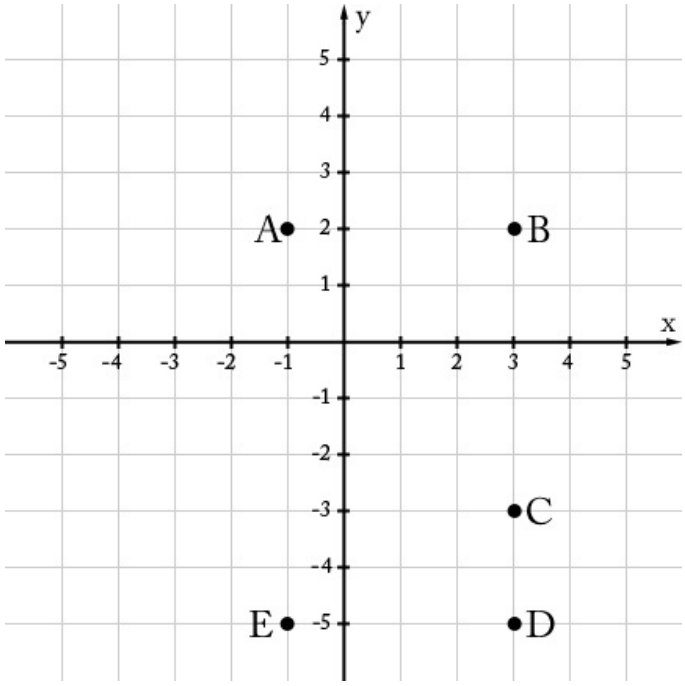
Application: Students will apply to find the distance between two points given two ordered pairs. Students can be given points on a coordinate plane and asked to solve a mathematical problem involving those points. The task lays a foundation for understanding operations on signed numbers and the use of absolute value from the coordinate points

positions. By arranging points in the four quadrants, students can identify the distances between the points by either counting to the points or using mathematical operations to identify the distances.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Some points are shown in the coordinate plane below.



a. What is the distance between points B & C?

b. What is the distance between points D & B?

c. What is the distance between points D & E?

d. Which of the points shown above are 4 units away from (−1,−3) and 2 units away from (3,−1)?

You can find the task above, as well as others aligned to this standard, here

Common Misconceptions

- **Identify coordinate plane quadrants:** Students may incorrectly identify the four quadrants.
- **Positive and Negative directions on coordinate plane:** Students may incorrectly identify the positive and negative directions within the quadrants.

- **Plotting ordered pairs:** Students may plot the 'x' an 'y' values when plotting the ordered pairs. They reverse the points when plotting them on the coordinate plane.
- **Distance between points calculation:** Students may not correctly identify the distance between the points.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***

• Begin by reviewing the concept of the coordinate plane and its four quadrants.
• Engage students by discussing real-world examples of coordinate planes, such as maps or GPS navigation systems.
• Graphing Points in All Four Quadrants (Layer 1 - All Students):
• Guide students in understanding how to graph points in all four quadrants of the coordinate plane.
  ◦ Provide examples and non-examples to illustrate the correct placement of points in each quadrant.
• Understanding Distance Calculation:
  ◦ Introduce the concept of using coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
  ◦ Demonstrate how to calculate distances using absolute value and coordinate differences.
	• Present real-world scenarios or word problems that require graphing points in the coordinate plane and calculating distances between them.
	• Provide linguistic and contextual support for students who may need assistance in understanding the scenarios.
	• Practice and Collaboration
	  ◦ Engage students in collaborative problem-solving activities where they graph points and calculate distances in the coordinate plane.
	  ◦ Provide opportunities for peer support and guidance as students work through the problems together.
	• Formative assessment through observation of students' participation and engagement during class discussions and activities.
	• Informal assessment through review of students' responses to real-world scenarios or word problems,
		• Offer extension activities for advanced students, such as exploring more complex distance calculations or investigating applications of coordinate plane graphing in advanced mathematical concepts.
		• Provide additional support for struggling students through one-on-one or small group instruction, using manipulatives or visual aids as needed.
		• Universal Design for Learning (UDL) Strategies:
		  ◦ Incorporate multiple means of representation by providing visual representations of coordinate planes and distances, auditory explanations, and hands-on activities.
		  ◦ Offer multiple means of expression by allowing students to demonstrate their understanding through verbal explanations, written responses, or visual representations.
		  ◦ Provide multiple means of engagement

• Formative assessment through observation of students' participation and engagement during class discussions and activities. • Informal assessment through review of students' responses to real-world scenarios or word problems, focusing on their ability to graph points and calculate distances accurately. **Universal Design for Learning (UDL)** • Summative assessment through quizzes or performance tasks assessing students' understanding of graphing points in the coordinate plane and calculating distances between them.	focusing on their ability to graph points and calculate distances accurately. • Summative assessment through quizzes or performance tasks assessing students' understanding of graphing points in the coordinate plane and calculating distances between them. **Universal Design for Learning (UDL)** • Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.	by incorporating student interests, cultural relevance, and real-world connections into the lesson. **Universal Design for Learning (UDL)** • Formative assessment through observation of students' participation and engagement during class discussions and activities. • Informal assessment through review of students' responses to real-world scenarios or word problems, focusing on their ability to graph points and calculate distances accurately. • Summative assessment through quizzes or performance tasks assessing students' understanding of graphing points in the coordinate plane and calculating distances between them.

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/6/27/286/286		
Previous Learning	*Current Learning*	*Future Learning*
• This cluster is connected to what students previously learned in third grade, when they marked off units on a horizontal scale or number line. Students will recall that a fraction can be represented on a number line, in the space between whole numbers. They will also recall the skills from Grade 5 when	• There are connections between this cluster when learners recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. Also, there are connections made when students use coordinates to	• The skills from this cluster are applied in 7th grade when students make connections between their 6th grade understanding of what rational numbers are to include the addition and subtraction of integers. Students will need to represent addition and subtraction of integers on a

they graphed points on a coordinate plane and interpreted what the points represent.	find the length of a side joining points with the same first coordinate or the same second coordinate	horizontal and/or vertical number line.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*

• Begin the lesson by validating students' diverse cultural backgrounds and experiences related to problem-solving in mathematics.
• Affirm the importance of understanding graphing points in all four quadrants of the coordinate plane in various cultural contexts.
• Collaborative Problem-Solving (Universal Design for Learning - All Students):
  ◦ Engage students in collaborative problem-solving activities where they apply graphing and distance calculation skills to solve real-world problems.
  ◦ Encourage peer support and collaboration, allowing students to share their cultural perspectives and problem-solving strategies.
• Cultural Reflection and
	• Build on students' prior knowledge by reviewing the concept of the coordinate plane and its quadrants.
	• Bridge the concept to real-world scenarios by discussing how different cultures might use coordinate planes in navigation, city planning, or other contexts.
	• Understanding Distance Calculation:
	  ◦ Introduce the concept of using coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
	  ◦ Demonstrate how to calculate distances using absolute value and coordinate differences, providing culturally relevant examples.
	• Collaborative Problem-
		• Provide linguistic vocabulary support for students by introducing translated terms and visual aids related to graphing points and calculating distances in the coordinate plane.
		• Scaffold language acquisition through multilingual word banks or graphic organizers to facilitate comprehension.
		• Real-World Applications (Layer 3 - Intensive Support):
		  ◦ Present real-world scenarios or word problems reflecting diverse cultural contexts that require graphing points in the coordinate plane and calculating distances between them.
		  ◦ Provide additional support for students who may require assistance in understanding the scenarios, including linguistic and

Connection: ● Encourage students to reflect on how their cultural backgrounds influence their problem-solving approaches and interpretations of mathematical concepts. ● Foster connections between mathematical concepts and cultural contexts to deepen understanding and appreciation for diverse perspectives. ● Conclude the lesson by revisiting key concepts covered, including graphing points in all four quadrants of the coordinate plane and calculating distances between points. ● Encourage students to reflect on how they can apply these skills in their own cultural contexts and everyday lives.	Solving (Universal Design for Learning - All Students): ● Engage students in collaborative problem-solving activities where they apply graphing and distance calculation skills to solve real-world problems. ● Encourage peer support and collaboration, allowing students to share their cultural perspectives and problem-solving strategies. ● Cultural Reflection and Connection: ● Encourage students to reflect on how their cultural backgrounds influence their problem-solving approaches and interpretations of mathematical concepts. ● Foster connections between mathematical concepts and cultural contexts to deepen understanding and appreciation for diverse perspectives. ● Conclude the lesson by revisiting key concepts covered, including graphing points in all four quadrants of the	contextual support. ● Collaborative Problem-Solving (Universal Design for Learning - All Students): ● Engage students in collaborative problem-solving activities where they apply graphing and distance calculation skills to solve real-world problems. ● Encourage peer support and collaboration, allowing students to share their cultural perspectives and problem-solving strategies. ● Cultural Reflection and Connection: ● Encourage students to reflect on how their cultural backgrounds influence their problem-solving approaches and interpretations of mathematical concepts. ● Foster connections between mathematical concepts and cultural contexts to deepen understanding and appreciation for diverse perspectives. ● Encourage students to reflect on how they can apply these skills in their own cultural contexts and everyday lives.

	coordinate plane and calculating distances between points. ● Encourage students to reflect on how they can apply these skills in their own cultural contexts and everyday lives.	

Suggested Student Discourse Questions

- How can you explain the meaning of infinitely many solutions in relation to recognizing that inequalities of the form $x > c$ or $x < c$ represent this concept?
- Where outside of school can you see examples of inequalities? In your home or community?
- When solving inequalities, what strategy or approach do you think is most effective in arriving at the solution?
- Was the strategy that you chose to solve plotting points in the coordinate plane and identifying the meaning of those points the best strategy? Could another have been more effective?

Cross-Curricular Connections

English:

- Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.
- Demonstrating the meaning of symbols, key terms, and other domain specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.
- Distinguish among facts, reasoned judgment based on research findings, and speculations in a text.
- Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

Social Studies:

- Students can use this idea of plotting points in a coordinate plane to adjust it to the longitude and latitude lines on a map. They can use this to track a traveling pattern and discuss it further. They can track a voyage over time.

Career and Skill Connections

- Civil Engineer
- Land Surveyor
- Mechanical Engineer
- Educator

- Data Analyst

6th Grade Ratios and Proportional Relationships Guide

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
  - Standards of Mathematical Practice
  - Procedural and Conceptual Understanding and Application
  - Sample Assessment Items
  - Common Misconceptions
  - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
  - Vertical Alignment
  - Culturally and Linguistically Responsive Instruction (CLRI)
  - Suggested Student Discourse Questions
  - Cross-Curricular and Career and Skill Connections
- A Student Discourse Guide
- Planning for a Multi-Layer System of Support (MLSS) and Universal Design for Learning (UDL) for behavioral and social and emotional supports

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Understand ratio concepts and use ratio reasoning to solve problems
  - 6.RP.A.1
  - 6.RP.A.2
  - 6.RP.A.3

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)

Grade	CCSS Domain	CCSS Cluster
6	Ratios and Proportional Relationships	Understand ratio concepts and use ratio reasoning to solve problems.

Cluster Standard: 6.RP.A.1

Standard	Standards for Mathematical Practice
Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*	• **SMP 6**: Attend to precision. • **SMP 8:** Look for and express regularity in repeated reasoning.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students are introduced to ratios, a relationship or comparison of two quantities or measures. They will represent ratios in various forms (a:b, a to b, a/b) and compare types of ratios. They will use reasoning about multiplication and division to solve ratio and rate problems about quantities. Students will learn how and where ratios and rates are used in the real world.	• Understand and explain that a ratio is a comparison of two quantities. • Describe what a ratio illustrates using ratio language. • Write a ratio relationship in the forms a:b, a to b, a/b. • Translate a ratio relationship into words. • Understand the differences between part: part and part: whole relationships.
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Conceptual Understanding: Students understand the definition of a ratio as a comparison of two quantities, the relationship between ratios and fractions, and understand the different ways ratios can be expressed (e.g., as a fraction, with a colon, or as a word). The focus here is on developing a deep understanding of the concept of ratios, rather than just performing calculations with them.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Ty took the escalator to the second floor. The escalator is 12 meters long, and he rode the escalator for 30 seconds. Which statements are true? Select all that apply.

a. He traveled 2 meters every 5 seconds.
b. Every 10 seconds he traveled 4 meters.
c. He traveled 2.5 meters per second.
d. He traveled 0.4 meters per second.
e. Every 25 seconds, he traveled 7 meters.

In a bag of marbles, 3/5 of the marbles are blue and the rest are red. If the number of red marbles is doubled and the number of blue marbles stays the same, what fraction of the marbles will be red?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Misinterpreting Ratios as Fractions:** Students may mistakenly interpret ratios as fractions or vice versa. While ratios and fractions are related, they are not the same thing.
- **Confusing Ratios with Absolute Quantities:** Students may misunderstand ratios as absolute quantities rather than relative comparisons. They might think that a ratio like "2:1" means there are exactly two of one quantity and one of another, rather than understanding it as a comparison of two quantities.
- **Misapplying Ratio Language:** Students may struggle with the language used to describe ratios, such as "part to part," "part to whole," or "out of." This can lead to confusion about how to express ratios correctly in different contexts.
- **Misunderstanding Equivalent Ratios:** Students may have difficulty recognizing equivalent ratios and understanding how to find them. They might think that two ratios with the same numbers are always equivalent, regardless of the context or units involved.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
• Provide visual representations such as diagrams, bar models, and ratio tables to help all students understand the concept of ratios visually. • Incorporate real-world examples and scenarios related to ratios, such as recipes, financial applications, or sports statistics, to make the	• Provide targeted small group instruction for students who need additional support in understanding ratios. Offer differentiated activities based on individual student needs and learning styles. • Implement peer collaboration where students work in pairs or small groups to solve ratio	• For students who require intensive support, provide one-on-one instruction tailored to their specific learning needs. Offer additional practice opportunities, reteaching sessions, and personalized learning experiences.

concept more relatable and engaging for all students. **Universal Design for Learning:** • Use diagrams, tape diagrams, ratio tables, and double number lines to visually represent ratios. These tools can help students better understand the concept of a ratio and how it compares two quantities. • Provide clear, detailed explanations with visual aids that break down the process of forming and interpreting ratios. Emphasize the language of ratios (e.g., "for every," "per") to help students articulate the relationship. • Use manipulatives like counters, cubes, or colored beads to physically create and compare ratios. This tactile approach can help students who learn best through hands-on experiences. • Use ratios in contexts that interest students, such as comparing video game stats, designing scale models, or analyzing music rhythms. This makes learning more engaging and personally meaningful.	problems together. Encourage students to explain their reasoning to one another, fostering a deeper understanding of ratios. **Universal Design for Learning:** • Encourage group work where students can solve ratio problems together, discuss their reasoning, and learn from each other's approaches. This can include activities like ratio scavenger hunts or group projects. • Provide scaffolds such as ratio tables or graphic organizers that guide students through the process of identifying and calculating ratios, helping them structure their thinking.	**Universal Design for Learning** • Incorporate videos animations and interaction simulations to explain the process of finding common denominators and performing the required operation. • Help and align personal goals for students with goals or benchmarks in existence.

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/6/28/288/288

Previous Learning	*Current Learning*	*Future Learning*
• Connect students' previous understandings of conversion tables, graphing points, and	• Connect student understandings of ratio relationships and number	• Connect student understanding of ratios and rate from Grade 6 to compute

how these ideas connect to the real world. These previous understandings will support students in their understanding of number relationships, specifically when comparing numbers. ● In Grade 4, learners were taught to multiply or divide to solve word problems involving multiplicative comparison. In Grade 5, learners had to interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b) AND interpret multiplication as scaling or resizing. These skills will need to be explicitly reviewed to support student success with this domain	relationship as they move to use variables to represent two quantities that change in relationship to one another in 6.EE.9	unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. ● These skills from this cluster are connected in Grade 7 when learners will recognize and represent proportional relationships between quantities. This includes student understanding of proportional relationships to solve multistep ratio and percent problems.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
● Begin by acknowledging the diverse backgrounds and experiences of your students. Incorporate culturally relevant examples of ratios from various cultures and communities to validate students' identities and experiences. For instance, you could explore how different cultures use ratios in cooking recipes, traditional	● Scaffold instruction by building on students' prior knowledge and experiences related to ratios. Start with concrete examples and gradually transition to more abstract concepts. Provide opportunities for students to connect their cultural backgrounds to the mathematical concept of ratios. For example, you	● Provide linguistic support to ensure that all students understand key mathematical vocabulary related to ratios. Offer clear explanations of mathematical terms and provide opportunities for students to practice using these terms in context. Use visual aids, gestures, and bilingual resources to support students' comprehension of

art forms, or cultural celebrations. This approach affirms students' cultural backgrounds and fosters a sense of belonging in the classroom.	could explore how ratios are used in traditional practices, such as weaving patterns or ancestral storytelling, and bridge these concepts to mathematical representations.	mathematical vocabulary. Additionally, encourage students to share their understanding of ratios in their home languages, providing translations or explanations as needed to scaffold their learning.

Suggested Student Discourse Questions

- Can you explain how changing one quantity in a ratio affects the relationship between the two quantities?
- Can you think of a real-world scenario where ratios would be useful for solving a problem? How would you approach solving it?
- What strategies can you use to determine if ratios are appropriate for solving a particular problem?
- How can you check if your solution to a ratio problem is reasonable?
- Can you give examples of ratios from everyday life or real-world situations?

Cross-Curricular Connections

Science:

- Students can apply this to science by creating a ratio of the model of the solar system to the actual size of the solar system. In addition, students can use their knowledge of ratios to help them interpret the ratios of time, space, and energy to determine a ratio. MS-PS3-1 (Energy), MS-ESS1-3 (Earth's Place in the Universe)
  - https://www.nextgenscience.org/pe/ms-ps3-1-energy
  - https://www.nextgenscience.org/pe/ms-ess1-3-earthsplace-universe

Social Studies:

- Students can apply the idea of ratios to social studies. They can determine ratios of populations and other types of ratios that are associated with their study of social studies

Career and Skill Connections

- Surveying and Mapping Technician
- Stock Broker
- Baker
- Retail Banker
- Mathematician

Grade	CCSS Domain	CCSS Cluster
6	Ratios and Proportional Relationships	Understand ratio concepts and use ratio reasoning to solve problems.

Cluster Standard: 6.RP.A.2

Standard	Standards for Mathematical Practice
Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0 and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1] [1]Expectations for unit rates in this grade are limited to non-complex fractions	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 4:** Model with mathematics. • **SMP 6:** Attend to precision. • **SMP 7:** Look for and make use of structure.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students are introduced to ratios, a relationship or comparison of two quantities or measures. They will represent ratios in various forms (a:b, a to b, a/b) and compare types of ratios. They will use reasoning about multiplication and division to solve ratio and rate problems about quantities. Students will learn how and where ratios and rates are used in the real world.	• Define a unit rate in relation to the concept of a ratio. • Represent units rates symbolically, in contexts, and through visuals. • Use precise language of unit rate to describe ratio relationships both orally and in writing.
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Conceptual Understanding: Students will understand the concept of a unit rate associated with a ratio and use rate language in the context of a ratio relationship, including relationships between two quantities. This includes understanding that a unit rate is the amount per one unit of another quantity and being able to express this relationship using appropriate mathematical language.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

The grocery store sells beans in bulk. The grocer's sign above the beans says, “5 pounds for $4.”

At this store, you can buy any number of pounds of beans at this same rate, and all prices include tax.
Alberto said, “The ratio of the number of dollars to the number of pounds is 4:5. That's $0.80 per pound.”
Beth said, "The sign says the ratio of the number of pounds to the number of dollars is 5:4. That's 1.25 pounds per dollar."

a. Are Alberto and Beth both correct? Explain.
b. Claude needs two pounds of beans to make soup. Show Claude how much money he will need.
c. Dora has $10 and wants to stock up on beans. Show Dora how many pounds of beans she can buy.
d. Do you prefer to answer parts (b) and (c) using Alberto's rate of $0.80 per pound, using Beth's rate of 1.25 pounds per dollar, or using another strategy? Explain.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Forming ratios:** When solving problems involving proportions students tend to struggle forming a ratio.
- **Solving ratios and rates:** Students can have difficulties solving ratio and rate problems concerning quantities and creating the understanding of part-to-part ratio compared to part-to-whole ratio.
- **Difference between ratios and fractions:** Students may struggle with understanding the difference between ratios and fractions.
- **Identifying equivalent ratios:** Students may struggle identifying equivalent ratios and how to recognize them.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
Introduce the concept of unit rates and their applications in real-world contexts. Suggested activities: • Begin with real-life scenarios, such as comparing prices of different-sized packages of items at a store. • Discuss with students WHY it’s important to compare prices based on the same unit (e.g. price per ounce) rather than just the total price. • Define unit rate as a comparison of two quantities where one of the quantities is 1 unit.	Reinforce understanding of unit rates through differentiated practice and problem-solving activities. Suggested activities: • Differentiated Practice: Provide differentiated practice worksheets with varying levels of difficulty to meet the needs of diverse learners. • Offer additional support for struggling students through guided practice or small group instruction. • Challenged advanced	Provide targeted support for students who are struggling with understanding unit rates through personalized instruction and remediation. • Administer a diagnostic assessment to identify specific areas of difficulty for individual students. Analyze assessment data to determine each student’s strengths and weaknesses related to unit rates. • Group students based on their assessment results and provide targeted instruction to address their individual needs.

• Model HOW to calculate unit rates using examples, such as miles per hour, cost per ounce, or words per minute. • Explain HOW to find unit rates by dividing the quantity of one item by its corresponding cost or measurement. **Universal Design for Learning** • Use graphs, tables, and number lines to visually demonstrate unit rates. For example, plot distance versus time to show speed as a unit rate (e.g., miles per hour). • Leverage digital tools that allow students to manipulate ratios and see how changing one quantity affects the unit rate. This can include interactive graphing software or online unit rate calculators. • Present unit rates in familiar situations, such as calculating miles per gallon, price per item, or heartbeats per minute. This helps students connect the concept to everyday experiences. • Provide step-by-step explanations with examples that break down how to find unit rates from given ratios. Use visual aids like ratio tables to clarify the process.	students with extension activities or more complex problem-solving tasks. **Universal Design for Learning** • Use manipulatives like counters, measuring tools, or scale models to physically explore unit rates. For example, students can measure the speed of toy cars or compare prices using grocery store flyers. • Allow students to demonstrate their understanding through various formats, such as creating graphs, writing explanations, making digital presentations, or building models. • Encourage students to work together on tasks involving unit rates, such as group projects where they must compare different rates or solve real-world problems using unit rates. • Provide scaffolding tools like graphic organizers or worksheets that guide students through the steps of finding and interpreting unit rates, ensuring they follow a logical process.	• Use manipulatives, visual aids, or hands-on activities to make abstract concepts more concrete for struggling students. **Universal Design for Learning** • Incorporate videos animations and interaction simulations to explain the process of finding common denominators and performing the required operation. • Help and align personal goals for students with goals or benchmarks in existence.
Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/6/28/291/291		
Previous Learning	*Current Learning*	*Future Learning*

• Connect students' previous understandings of conversion tables, graphing points, and how these ideas connect to the real world. These previous understandings will support students in their understanding of number relationships, specifically when comparing numbers. • In Grade 4, learners were taught to multiply or divide to solve word problems involving multiplicative comparison. In Grade 5, learners had to interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b) AND interpret multiplication as scaling or resizing. These skills will need to be explicitly reviewed to support student success with this domain.	• Connect student understandings of ratio relationships and number relationships as they move to use variables to represent two quantities that change in relationship to one another as seen in Grades 6.	• Connect student understanding of ratios and rates from Grade 6 to compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. • These skills from this cluster are connected in Grade 7 when learners will recognize and represent proportional relationships between quantities. This includes student understanding of proportional relationships to solve multistep ratio and percent problems.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
Cultural Validation Through Personal Narratives: 1. Begin the lesson by acknowledging the diverse cultural backgrounds and languages present in the classroom. Share a personal narrative or story from your	Provide culturally and linguistically responsive instruction to build and bridge students' understanding of ratios and rates. 1. Introduce the concept of ratios and rates using culturally relevant visuals, materials, or examples that	Provide linguistic vocabulary support in culturally and linguistically responsive instruction for understanding ratios and rates. 1. Share a brief discussion about the importance of language in mathematics and how different languages may have

own cultural background that involves the use of ratios or rates, such as a family recipe, a cultural tradition, or a personal anecdote related to sharing resources. 2. Facilitate a discussion about students' cultural backgrounds and experiences related to ratios and rates. Encourage students to share personal stories, examples, or traditions from their own cultures that involve comparing quantities, sharing resources, or calculating rates.	resonate with students' cultural backgrounds. Provide multiple representations of ratios and rates, including visual models, manipulatives, and real-life scenarios to accommodate diverse learning styles and linguistic abilities. 2. Use culturally responsive language and terminology that reflects students' cultural backgrounds and experiences, while also reinforcing academic vocabulary related to ratios and rates. 3. Scaffold students' understanding of ratios and rates by providing guided practice activities that incorporate culturally relevant contexts and examples.	unique terms or expressions for mathematical concepts like ratios and rates. 2. Review key mathematical vocabulary related to ratios and rates in English. Encourage students to share any terms or expressions they know in their native languages that relate to the concept of ratios and rates. 3. Display visual aids or multimedia resources that depict the concept of ratios and rates, using both English and students' native languages when possible. Provide opportunities for students to practice pronouncing and using the new vocabulary in context through repetition and modeling. Conduct guided practice activities where students use the newly introduced vocabulary to solve problems and engage in mathematical discussions.

Suggested Student Discourse Questions

- Can you explain how a unit rate represents the amount of one quantity per unit of another quantity?
- How can we calculate the unit rate when given a ratio or a fraction representing a rate?
- How are ratios and unit rates related? Can you describe the connection between the two concepts?
- Can you identify examples of ratios where the terms represent quantities that can be compared directly to each other?
- How do we convert a ratio to a unit rate? What steps do we take to find the unit rate?

Cross-Curricular Connections

Science:

- Students can apply this to science by creating a ratio of the model of the solar system to the actual size of the solar system. In addition, students can use their knowledge of ratios to help them interpret the ratios of time, space, and energy to determine a ratio. MS-PS3-1 (Energy), MS-ESS1-3 (Earth's Place in the Universe)
  - https://www.nextgenscience.org/pe/ms-ps3-1-energy
  - https://www.nextgenscience.org/pe/ms-ess1-3-earthsplace-universe

Social Studies:

- Students can apply the idea of ratios to social studies. They can determine ratios of populations and other types of ratios that are associated with their study of social studies

Career and Skill Connections
• Accountant • Loan Officer • Billing Clerks • Budget Analyst • Executive Chef

Grade	*CCSS Domain*	*CCSS Cluster*
6	Ratios and Proportional Relationships	Understand ratio concepts and use ratio reasoning to solve problems.

Cluster Standard: 6.RP.A.3

Standard	Standards for Mathematical Practice
Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.	• **SMP 2:** Reason abstractly and quantitatively. • **SMP 3:** Construct viable arguments and critique the reasoning of others • **SMP 4:** Model with mathematics.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations	● Create and interpret tables of equivalent ratios ● Plot values from a table on a coordinate plane ● Examine tables in order to compare ratios. ● Solve real-world unit rate problems ● Calculate the percent of a quantity as a rate per 100. ● Reason with ratios to convert, manipulate and transform units of measure
DOK	**Blooms**
1-2	Understand, Apply
Procedural and Conceptual Understanding and Application	
Procedural Skill and Fluency: Students should demonstrate proficiency in applying procedural techniques to solve problems involving ratios and rates. This includes the ability to set up and solve proportions, calculate unit rates, and perform operations with ratios. They should be able to follow step-by-step procedures to find solutions to problems and accurately execute calculations. **Conceptual Understanding:** Students will make tables with equivalent ratios and comparing ratios while solving problems involving unit pricing and constant speed. Students will also understand the concept of real-world scenario problems related to percentages of quantity involving finding the whole given a part and the percent. The student will understand the amount of something in relation to percentages and comparisons of ratios.	

Application: Students must apply their understanding and procedural skills to solve real-world problems. For instance, they might use ratios and unit rates to determine costs, speeds, or percentages in practical scenarios, and they must be able to manipulate units in these contexts. The use of diagrams and tables to reason through problems also emphasizes the application of these concepts in meaningful ways.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

- Alexis needs to paint the four exterior walls of a large rectangular barn. The length of the barn is 80 feet, the width is 50 feet, and the height is 30 feet. The paint costs $28 per gallon, and each gallon covers 420 square feet. How much will it cost Alexis to paint the barn? Explain your work.

- Callie biked 12 miles in 3 hours. Carter biked 10 miles in 2 hours. Represent each person's trip with a diagram. Explain how you can see that they are not going the same speed.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Ratio and fraction confusion:** Students often confuse ratio amounts with fractions that involve the same digits as fractions.
- **Forming Ratios:** Students tend to struggle with forming a ratio when solving problems that involve proportions.
- **Order of quantities:** When working to solve ratio problems, students may run into confusion with the order of quantities (i.e: the ratio of triangles to squares requires students to write the quantity of triangles first as the numbers are not interchangeable).

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
• Introduce the concept of ratios and proportional relationships and their applications in real-world contexts. • Begin with a real-life scenario that involves comparing quantities or rates, such as a recipe that requires adjusting ingredients based on the number of servings. Discuss	• Offer targeted small group instruction for students who require additional support. Tailor activities to address specific areas of difficulty and provide opportunities for peer collaboration and discussion. • Provide scaffolded practice opportunities with gradually increasing complexity. Use	• Provide intensive one-on-one instruction for students who need individualized support. Offer additional explanations, examples, and practice opportunities tailored to the student's learning needs. • Develop personalized learning plans that include targeted goals, strategies, and supports for students with significant

with students WHY understanding ratios and proportional relationships (rates) are important in everyday life. • Define ratios and rates and provide examples using visual aids such as tape diagrams or double number line diagrams. Teach students HOW to represent ratios and rates in different forms, such as fractions, decimals, and proportions. Model HOW to solve problems using tables of equivalent ratios or equations. **Universal Design for Learning** • Use ratio tables, double number lines, graphs, and tape diagrams to visually represent ratio and rate problems. These tools can help students better understand the relationships between quantities. • Use physical manipulatives like measuring cups, rulers, or scale models to explore ratios and rates. For instance, students can experiment with mixing colors or ingredients to understand proportions. • Incorporate online simulations or apps that allow students to experiment with ratios and rates in various contexts, providing real-time feedback and visualization. • Present problems in real-world contexts, such as recipes, speed and distance, or scaling models. This helps students connect abstract concepts to practical	guided practice sessions where students receive support and feedback as they work through problems related to unit rates. • Use checklists and graphic organizers to help students break down and organize the steps involved in finding unit rates. This can support their problem-solving process and reinforce their understanding. **Universal Design for Learning** • Use manipulatives like counters, measuring tools, or scale models to physically explore unit rates. For example, students can measure the speed of toy cars or compare prices using grocery store flyers. • Allow students to demonstrate their understanding through various formats, such as creating graphs, writing explanations, making digital presentations, or building models. • Encourage students to work together on tasks involving unit rates, such as group projects where they must compare different rates or solve real-world problems using unit rates. • Provide scaffolding tools like graphic organizers or worksheets that guide students through the steps of finding and interpreting unit rates, ensuring they follow a logical process.	learning gaps. Collaborate with special education teachers, ELL specialists, and other support staff to address individual needs. **Universal Design for Learning** • Incorporate videos animations and interaction simulations to explain the process of finding common denominators and performing the required operation. • Help and align personal goals for students with goals or benchmarks in existence.

applications. • Provide clear, detailed instructions that break down how to approach ratio and rate problems, emphasizing key concepts like unit rates, equivalent ratios, and proportional relationships.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/6/28/294/294

Previous Learning	*Current Learning*	*Future Learning*
• Connect students' previous understandings of conversion tables, graphing points, and how these ideas connect to the real world. These previous understandings will support students in their understanding of number relationships, specifically when comparing numbers. • In Grade 4, learners were taught to multiply or divide to solve word problems involving multiplicative comparison. In Grade 5, learners had to interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b) AND interpret multiplication as scaling or resizing. These skills will need to be explicitly reviewed to support student success with this domain.	• Connect student understandings of ratio relationships and number relationships as they move to use variables to represent two quantities that change in relationship to one another in the 6.EE.9 CCSS.	• Connect student understanding of ratios and rates from Grade 6 to compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. • These skills from this cluster are connected in Grade 7 when learners will recognize and represent proportional relationships between quantities. This includes student understanding of proportional relationships to solve multistep ratio and percent problems.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Begin the lesson by encouraging students to view their cultural and linguistic behaviors as an asset that adds value to the classroom and lessons. Present a personal experience related to your own cultural background involving when ratios and proportions were present or utilized; such as determining how much of an ingredient is needed when using a family recipe or making a relation to grocery shopping and cost effectiveness. • Facilitate a discussion concerning students' views on overrepresentation or underrepresentation and why they think there is underrepresentation or overrepresentation present in certain demographics. Be mindful in moderating the discussion to watch for the emergence of stereotypes that could come up and support students in breaking down personal assumptions they may be having.	• Build connections to students' prior knowledge and the new content that is being introduced, relating ratios and proportions to real-world experiences and examples they may encounter on a daily basis within their culture. • Prompt and use student thinking to promote a classroom culture to view opportunities for learning through any potential errors or mistakes made while practicing with the content introduced.	• Provide support and opportunities for students to clearly articulate their mathematical thinking through various methods; such as visually, verbally, or in writing. • Use varying teaching strategies and manipulatives to encourage communication in the students' native language. • Whenever possible and appropriate, provide support and opportunities for positive mathematical discussions through varying methods; such as whole class or group discussions.

Suggested Student Discourse Questions

- How can we use ratio and rate reasoning to solve problems involving comparisons, scaling, or proportions?
- Can you explain how ratios and rates help us make predictions, decisions, or interpretations in real-world situations?
- How do we determine which operations or strategies to use when solving problems with ratios and rates?
- How can we use ratio and rate reasoning to identify and describe proportional relationships in different contexts?

- What strategies can we use to communicate our reasoning and solutions effectively to others?

Cross-Curricular Connections

Science:

- Students can apply this to science by creating a ratio of the model of the solar system to the actual size of the solar system. In addition, students can use their knowledge of ratios to help them interpret the ratios of time, space, and energy to determine a ratio. MS-PS3-1 (Energy), MS-ESS1-3 (Earth's Place in the Universe)
  - https://www.nextgenscience.org/pe/ms-ps3-1-energy
  - https://www.nextgenscience.org/pe/ms-ess1-3-earthsplace-universe

Social Studies:

- Students can apply the idea of ratios to social studies. They can determine ratios of populations and other types of ratios that are associated with their study of social studies

Career and Skill Connections

- Construction Worker
- Software Developer
- Physicist
- Budget Analyst
- Accountant

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards of Mathematical Practice
    - Procedural and Conceptual Understanding and Application
    - Sample Assessment Items
    - Common Misconceptions
    - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
    - Vertical Alignment
    - Culturally and Linguistically Responsive Instruction (CLRI)
    - Suggested Student Discourse Questions
    - Cross-Curricular and Career and Skill Connections
- A Student Discourse Guide
- Planning for a Multi-Layer System of Support (MLSS) and Universal Design for Learning (UDL) for behavioral and social and emotional supports

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Develop understanding of statistical variability
  - 6.SP.A.1
  - 6.SP.A.2
  - 6.SP.A.3
- Summarize and describe distributions
  - 6.SP.B.4
  - 6.SP.B.5

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)

Grade	CCSS Domain	CCSS Cluster
6	Statistics and Probability	Develop understanding of statistical variability.

Cluster Standard: 6.SP.A.1

Standard	Standards for Mathematical Practice
Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 2:** Reason abstractly and quantitatively.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students will develop an understanding of statistical thinking. They will learn how to write statistical questions used to survey and collect data. They will study measures of center and variability with newly learned knowledge of mean, median, mode, and range. Students will discover that different ways to measure center produce different values and that interpreting measures of center for the same data develops the understanding of how each measure can change how the data gets interpreted	• Students will understand that their question promotes an investigation. • Students will understand the difference in quantitative (numerical) data to qualitative(categorical) data. • Students will develop a question that promotes variability in the data.
DOK	**Blooms**
1-2	Remember, Understand

Procedural and Conceptual Understanding and Application

Conceptual Understanding: Students develop an understanding of the difference between variability and central tendency, recognizing different measures of variability (e.g., range, variance, standard deviation), and understanding how variability relates to the spread or dispersion of data. This involves recognizing that a statistical question is one that anticipates variability in the data and requires analysis of that variability to answer the question. For example, understanding why "How old are the students in my school?" is a statistical question involves recognizing that the ages of students will vary, and the answer must account for this variability through data collection and analysis. This understanding is foundational for engaging with statistical reasoning and data interpretation.

Assessment Items
When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.
Zeke likes to collect buttons and he keeps them in a jar. Zeke can empty the buttons out of the jar, so he can see all of his buttons at once. Which of the following are statistical questions that someone could ask Zeke about his buttons? (A statistical question is one that anticipates an answer based on data that varies.) For each question, explain why it is or is not a statistical question. a. What is a typical number of holes for the buttons in the jar? b. How many buttons are in the jar? c. How large is the largest button in the jar? d. If Zeke grabbed a handful of buttons, what are the chances that all of the buttons in his hand are round? e. What is a typical size for the buttons in the jar? f. How are these buttons distributed according to color? Which of the following are statistical questions? a. How many days are in March? b. How old is your dog? c. On average, how old are the dogs that live on this street? d. What proportion of the students at your school like watermelons? e. Do you like watermelons? f. How many bricks are in this wall? g. What was the temperature at noon today at City Hall? You can find the task above, as well as others aligned to this standard, here.
Common Misconceptions
• **Creating Statistical Questions:** When coming up with their own statistical question, the students don't think about how many possible responses they could get. For example, when asked "what's your favorite color", I like to say, "can I choose any color I want"? "Can your classmates choose any color they want"? Students then realize that there are so many colors available that it's important to limit the number of choices made available and their question needs to be more specific. • **Confusing Variability with Average:** Students may think that variability refers to the average or typical value in a dataset. They might overlook the fact that variability actually refers to the spread or dispersion of the data points around the mean. • **Believing that High Variability always means High Values:** Some students may assume that high variability necessarily means that all values in the dataset are high. However, variability can exist in datasets with both high and low values, depending on how spread out the data points are.
Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Begin by discussing the importance of data analysis in understanding trends and making informed decisions. • Introduce the concept of Statistics and Probability (S&P) for all students. Then, explain, model, and provide concrete examples of different levels of support to meet the needs of all students to succeed. • Discuss the importance of S&P framework and how it ensures that instruction is accessible to all learners, regardless of their abilities or learning styles. • Have each group present their findings to the class, using accessible presentation formats (e.g., verbal descriptions, digital presentations with audio). **Universal Design for Learning (UDL)** • Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.	• Present a data set to the class, such as student survey responses about favorite hobbies. • Model how to organize the data using a graph or chart on the board, incorporating visual representations for students who may benefit from them. • Guide students through analyzing the data to identify trends and patterns. • Provide targeted support to students who may struggle with interpreting the data, offering additional explanations or examples as needed. **Universal Design for Learning (UDL)** • Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.	• Divide students into small groups and provide each group with a different data set to analyze. • Circulate among the groups to provide intensive support to students who need it, asking probing questions and offering scaffolding as necessary. • Encourage students to collaborate and discuss their findings with their peers, fostering peer support and learning. Application and Reflection: • Facilitate a class discussion about the different data sets and the insights gained from analyzing them. **Universal Design for Learning (UDL)** • Provide multiple means of expression by allowing students to demonstrate their understanding using various methods, such as written explanations, verbal responses, or visual representations.
Vertical Alignment		
Consider using this coherence map to help guide your planning		

https://tools.achievethecore.org/coherence-map/6/29/299/299

Previous Learning	*Current Learning*	*Future Learning*
• In Grade 5, learners made line plots to display a data set of measures in fractions of a unit.	• Mean, median, mode and range are new concepts to 6th grade students. Students will create dot plots, histograms and box plots. They will draw inferences and make comparisons between them. Mastery includes finding mean, median, mode and interquartile range.	• In Grade 7, learners build on their understanding of interpreting information about a population by using population samples. In Grade 7, learners begin to look at two separate data sets to make comparisons. In high school, learners interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Begin by discussing the importance of recognizing and valuing the diverse cultural backgrounds of all students. • Introduce the concept of cultural identity and invite students to share aspects of their own cultural heritage. • Present a culturally relevant data set (e.g., demographic data reflecting the cultural diversity of the classroom or community). • Validate and affirm students' cultural identities by	• Model how to analyze the data set using culturally responsive instructional strategies, such as making connections between students' cultural backgrounds and the data. • Facilitate a discussion about the similarities and differences observed in the data, highlighting the importance of building bridges between diverse cultures. • Encourage students to share their perspectives and	• Provide linguistic vocabulary support by introducing key terms and concepts in multiple languages represented in the classroom. • Use visual aids and gestures to enhance understanding for students who may be English language learners or speak languages other than English. • Encourage students to use their home languages to discuss the data and make connections to their cultural experiences, promoting language development and

acknowledging the significance of their contributions to the discussion. • Divide students into small groups and assign each group a different culturally relevant data set to analyze. • Circulate among the groups to provide guidance and support, ensuring that all students have opportunities to participate and contribute.	insights, fostering a sense of belonging and mutual respect in the classroom. • Have each group present their findings to the class, using culturally appropriate presentation formats (e.g., incorporating music or dance from their cultures).	cultural exchange. Encourage students to apply the culturally and linguistically responsive instruction strategies learned in class, including validating and affirming cultural identities, building bridges between cultures, and providing linguistic vocabulary support. • Facilitate a class discussion about the cultural insights gained from analyzing the data and the importance of embracing diversity in our communities.

Suggested Student Discourse Questions

- Imagine you have two datasets with the same mean but different measures of variability. What might this tell you about the distributions of the data?
- Why might it be important to understand variability when making decisions or drawing conclusions based on data?
- How might you use measures of variability to compare two different sets of data?
- How can we ensure that our interpretations of variability are accurate and meaningful?
- Can you think of any connections between measures of variability and other mathematical concepts, such as probability or data analysis?

Cross-Curricular Connections

Science:

Students can answer a question regarding their experiment by collecting data. This data can be displayed in different ways in addition to students finding the measures of center (mean and median) and describing the shape of the data. (MS-LS1-4, From Molecules to Organisms: Structures and Processes)

https://www.nextgenscience.org/pe/ms-ls1-4-moleculesorganisms-structures-and-processes

English:

Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

Demonstrate the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.

Distinguish among facts, reasoned judgment based on research findings, and speculations in a text.

Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners.
Career and Skill Connections
• Statistician • Educator • Economist • Meteorologist • Market Analyst

Grade	CCSS Domain	CCSS Cluster
6	Statistics and Probability	Develop understanding of statistical variability.

Cluster Standard: 6.SP.A.2

Standard	Standards for Mathematical Practice
Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 2:** Reason abstractly and quantitatively. • **SMP 6**: Attend to precision. • **SMP 7:** Look for and make use of structure
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students will develop an understanding of statistical thinking. They will learn how to write statistical questions used to survey and collect data. They will study measures of center and variability with newly learned knowledge of mean, median, mode, and range. Students will discover that different ways to measure center produce different values and that interpreting measures of center for the same data develops the understanding of how each measure can change how the data gets interpreted	• Describe a data set by discussing its center (e.g., mean, median), spread (e.g., range, interquartile range), and overall shape (e.g., normal, skewed). • Interpret what the center, spread, and shape of the distribution indicate about the data and the context from which it was collected. • Calculate basic descriptive statistics and use them to compare different data sets.
DOK	**Blooms**
1-2	Understand, Analyze

Procedural and Conceptual Understanding and Application

Conceptual Understanding: Students must understand that when data is collected to answer a statistical question, it forms a distribution. This distribution can be analyzed and described using three key characteristics:

- Center: Refers to a measure that represents the middle or typical value of the data set, such as the mean or median.
- Spread: Describes how much the data varies, which can be quantified using measures like range, interquartile range, or standard deviation.
- Overall Shape: Refers to the visual pattern of the data when graphed, such as whether it is symmetric, skewed, has one peak (unimodal), or multiple peaks (bimodal).

Understanding these aspects allows students to describe and interpret the distribution of data effectively, which is

crucial for making informed conclusions based on the data.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

The number of siblings for a group of sixth grade students is shown below:

1,0,2,1,6,0,2,0,1,10.

a. Make a dot plot of the data.

b. Find the mean and median of the data.

c. What does the mean tell you about the data? What about the median?

d. Which measure of average (mean or median) do you think best describes the data? Why?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Confuse median:** Students sometimes forget and calculate the median without first putting the data in numerical order.
- **Mean and median relationship:** Students often confuse that mean and median will always be the same or very close. This is not always the case, especially when there is skewed data or when there are outliers present.
- **Trouble with terminology:** Students may have trouble connecting that mean is the average, as it has previously been described this way.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Challenge students to design their own data collection projects based on topics of	• Discuss the characteristics of statistical questions, emphasizing their open-	• Provide examples of each data collection method and explain when and how they might be

interest or relevance to their lives.
- Encourage students to apply their understanding of statistical questions and data collection methods to plan and conduct their investigations independently or in small groups.
- After collecting data, guide students in organizing and analyzing their data sets using appropriate statistical techniques (e.g., tally charts, frequency tables, bar graphs).

Universal Design for Learning (UDL)
- Use graphs (histograms, box plots, dot plots) to visually represent data distributions. Visual aids help students see the center, spread, and shape of the data.
- Incorporate digital tools that allow students to manipulate data sets and immediately see how changes affect the distribution's center, spread, and shape.
- Present data from real-world contexts that students find interesting, such as sports statistics, weather patterns, or social media trends. This makes the concept of data distribution more relatable and concrete.
- Provide detailed instructions with examples to help students understand how to describe a distribution's center, spread, and shape. Use annotated graphs to highlight key features.
- Engage students in collecting their own data (e.g., measuring heights, recording

ended nature and relevance to real-world contexts.
- Model how to formulate statistical questions related to a specific topic or scenario (e.g., "What factors influence students' choice of transportation to school?").
- Guide students in brainstorming and refining their own statistical questions, encouraging creativity and critical thinking.

Universal Design for Learning (UDL)
- Allow students to demonstrate their understanding through various methods, such as creating their own graphs, writing explanations, presenting findings orally, or using digital tools.
- Use data sets that relate to students' interests and experiences, such as analyzing class survey results, examining trends in video game scores, or comparing statistics from favorite sports teams.
- Encourage students to set personal goals for understanding data distributions and provide opportunities for them to reflect on their progress, reinforcing their learning and boosting motivation.

used in different scenarios.
- Engage students in a discussion about selecting appropriate data collection methods based on the nature of their statistical questions and the available resources.
- Divide students into small groups and assign each group a statistical question to investigate.

Universal Design for Learning (UDL)
- Encourage students to set personal goals for understanding data distributions and provide opportunities for them to reflect on their progress, reinforcing their learning and boosting motivation.
- Encourage group work where students can discuss and compare data sets, analyze distributions together, and share their insights. This promotes collaborative learning and the exchange of ideas.
- Provide scaffolds such as templates or graphic organizers to help students structure their analysis of data distributions, ensuring they focus on the key aspects of center, spread, and shape.

daily temperatures) and analyzing the distribution. This hands-on approach helps them connect abstract concepts to tangible experiences.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/6/29/300/300

Previous Learning	*Current Learning*	*Future Learning*
• In Grade 5, learners made line plots to display a data set of measures in fractions of a unit.	• Mean, median, mode and range are new concepts to 6th grade students. Students will create dot plots, histograms and box plots. They will draw inferences and make comparisons between them. Mastery includes finding mean, median, mode and interquartile range.	• In Grade 7, learners build on their understanding of interpreting information about a population by using population samples. In Grade 7, learners begin to look at two separate data sets to make comparisons. In high school, learners interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Begin by affirming the students' prior knowledge of basic mathematical concepts such as addition, subtraction,	• Introduce the concept of measures of center and variation: ○ Explain that a	• Break down any complex mathematical vocabulary and provide linguistic support. Follow the steps below:

and basic arithmetic. Validate their understanding by asking questions like: ○ Can anyone tell me what we mean by the term 'numerical data set'? ○ What are some examples of numerical data sets that you encounter in everyday life? ● Connect word problems to diverse cultural backgrounds of the students.	measure of center is a single number that summarizes all the values in a data set. Emphasize that it represents the 'middle' or 'average' of the data. ○ Discuss different measures of center, such as mean, median, and mode. ○ Next, introduce the concept of measures of variation, which describe how the values in a data set vary around the center. Mention common measures of variation like range and interquartile range.	○ Provide translations or equivalents of mathematical terms in students' native languages, if applicable. ○ Use visuals, diagrams, and real-life examples to illustrate abstract concepts. ○ Encourage students to ask questions and clarify any terms or concepts they find challenging.

Suggested Student Discourse Questions

- When presented with a graphical display of data, what information are you or are you not able to determine from the data?
- Based on the context and the graphs given and displayed, what are you able to summarize in relation to the measures of center (mean, median and mode) as well as the measure of center for range?
- Based on your observations of the shape of the data set, what can you conclude or discuss about the symmetry and skew of the data distribution?
- In reference to the data distribution, can you make any further reference to clusters, peaks, or gaps? Is there any impact on the measures of center based on outliers?

Cross-Curricular Connections

Science:

Students can answer a question regarding their experiment by collecting data. This data can be displayed in different ways in addition to students finding the measures of center (mean and median) and describing the shape of the data. (MS-LS1-4, From Molecules to Organisms: Structures and Processes)

https://www.nextgenscience.org/pe/ms-ls1-4-moleculesorganisms-structures-and-processes

English:

Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

Demonstrate the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.

Distinguish among facts, reasoned judgment based on research findings, and speculations in a text.

Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners.

Career and Skill Connections

- Market Analyst
- Accountant
- Operations Analyst
- Data Scientist
- Statistician

Grade	CCSS Domain	CCSS Cluster
6	Statistics and Probability	Develop understanding of statistical variability.

Cluster Standard: 6.SP.A.3

Standard	Standards for Mathematical Practice
Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 2:** Reason abstractly and quantitatively. • **SMP 6**: Attend to precision.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students will develop an understanding of statistical thinking. They will learn how to write statistical questions used to survey and collect data. They will study measures of center and variability with newly learned knowledge of mean, median, mode, and range. Students will discover that different ways to measure center produce different values and that interpreting measures of center for the same data develops the understanding of how each measure can change how the data gets interpreted.	• Find and understand that measures of center (mean/median) summarize a set of data with a single number. • Find and understand that measures of variation (range/MAD) describe a set of data's variability with a single number.
DOK	**Blooms**
1-2	Understand, Analyze

Procedural and Conceptual Understanding and Application

Conceptual Understanding: Students need to understand that in a numerical data set:

- A measure of center (such as the mean or median) provides a summary of the entire data set by representing it with a single number that reflects a central or typical value.
- A measure of variation (such as range, interquartile range, or standard deviation) captures the extent to which the data values differ from each other, summarizing the variability within the data set with a single number.

Recognizing the distinction between these two types of measures helps students understand the different ways data can be summarized and analyzed. The measure of center gives a sense of the "average" or "middle" of the data, while the measure of variation gives insight into the spread or consistency of the data.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Statistical questions are questions that can be answered by collecting data and where we anticipate that there will be variability in that data. The data collected can be summarized in a distribution that can then be described in terms of center and in terms of spread. For some statistical questions, to answer the question you need to consider the center. For other questions you might need to consider spread.

For each of the five statistical questions below, decide if you would answer the question by considering center or considering variability in the data distribution.

Example 1: The records office at an elementary school keeps daily attendance records.

Question 1: For students at this school, what is a typical number of school days missed in the month of April?

Example 2: Suppose that third graders at your school took both a math test and a social studies test. Scores on both tests could be any number between 0 and 100.

Question 2: On average, did the students score better on the math test or the social studies test?

Question 3: Were the students' scores more consistent (more similar to one another) on the math test or on the social studies test?

Example 3: Bags of M&Ms don't all have exactly the same number of candies in each bag. Suppose you count the number of candies in each of 25 bags of plain M&Ms and in each of 25 bags of peanut M&Ms, and make two dot plots—one for the number of candies in the plain M&M bags and one for the number of candies in the peanut M&M bags.

Question 4: If you wanted to give each student in your class a bag of M&Ms and you wanted to try to make sure that each student got the same number of candies, should you give them bags of plain M&Ms or bags of peanut M&Ms?

Question 5: If you wanted to give each student in your class a bag of M&Ms and you wanted to try to give students bags with the greatest number of candies, should you give them bags of plain M&Ms or bags of peanut M&Ms?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Range Misconception:** It is a common misconception to express the range of data as low to high, such as 8 – 25, so students should be reminded that the range is a single value that describes variation in the data set.

- **Mean and median relationship:** Students often confuse that mean and median will always be the same or very close. This is not always the case, especially when there is skewed data or when there are outliers present.
- **Trouble with Terminology:** Students may confuse the terminology and the difference in the meanings of the measures of center which would cause problems in the way the methods are calculated.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Provide multiple means of representation by using visuals, manipulatives, and real-life examples to illustrate concepts. • Begin by reviewing the concept of numerical data sets and basic arithmetic operations. Use visuals and manipulatives to illustrate key concepts and engage students' interest. • Facilitate a class discussion to reflect on the significance of measures of center and variation in data analysis. **Universal Design for Learning (UDL)** • Provide clear explanations and multiple examples that define and illustrate the concepts of mean, median, range, and interquartile range. Use concrete examples to make abstract ideas more accessible. Use everyday vocabulary when introducing concepts. • Use graphs and charts, such as histograms, box plots, and dot plots, to visually demonstrate measures of center and variation. Highlighting the mean, median, range, and	• Offer opportunities for repeated practice and feedback to reinforce learning and address individual student needs. • Introduce measures of center, such as mean, median, and mode, using concrete examples and real-world scenarios. • Model how to calculate each measure using sample data sets, providing step-by-step guidance and opportunities for guided practice. **Universal Design for Learning (UDL)** • Offer differentiated instruction based on students' readiness levels, providing additional support or challenge as needed. • Use real-life examples to demonstrate the concepts of center and variation. For instance, comparing average test scores in different classes or analyzing weather data over a month helps students understand the relevance of these statistical measures. • Allow students to collect their own data (e.g.,	• Provide additional support and scaffolding for students who require it, such as guided practice and peer collaboration. • Distribute data sets or present them on the board, allowing students to practice calculating measures of center and variation independently or in small groups. • Offer feedback and support as students work through problems, addressing misconceptions and providing guidance as needed. **Universal Design for Learning (UDL)** • Allow students to choose the data sets they analyze or the statistical questions they explore. Providing options fosters a sense of ownership and keeps students motivated. • Encourage students to set personal goals related to mastering statistical measures and provide opportunities for reflection on their progress. This reinforces learning and promotes a growth mindset.

interquartile range on these graphs can help students see the relationships within the data. • Incorporate digital tools or apps that allow students to input data and visualize how changes in the data affect the mean, median, and range. These tools provide instant feedback and help students experiment with different data sets. • Use data sets that are meaningful to students, such as analyzing their own grades, sports statistics, or trends in social media. This makes learning more engaging and relevant.	measuring the heights of plants, recording daily temperatures) and calculate the mean, median, and range. This hands-on approach helps students connect the concepts to real data. • Facilitate group activities where students can work together to analyze data sets, calculate measures of center and variation, and discuss their findings. This promotes peer learning and the sharing of different strategies. • Provide tools like graphic organizers, step-by-step worksheets, or formula reference sheets to guide students through the process of calculating and interpreting the mean, median, range, and interquartile range. • Allow students to choose the data sets they analyze or the statistical questions they explore. Providing options fosters a sense of ownership and keeps students motivated.	
Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/6/29/301/301		
Previous Learning	*Current Learning*	*Future Learning*
• In Grade 5, learners made line plots to display a data set of measures in fractions of a unit.	• Mean, median, mode and range are new concepts to 6th grade students. Students will create dot plots, histograms and box plots. They will draw inferences and make	• In Grade 7, learners build on their understanding of interpreting information about a population by using population samples. In Grade 7, learners begin to look at two separate data sets to

	comparisons between them. Mastery includes finding mean, median, mode and interquartile range.	make comparisons. In high school, learners interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Validate students' cultural backgrounds and experiences by incorporating examples and contexts relevant to their diverse cultures. • Affirm students' contributions and perspectives by creating a supportive and inclusive learning environment. • Begin by acknowledging and celebrating the diverse cultural backgrounds of the students in the classroom. Encourage students to share examples of how math is used in their own cultural traditions and practices.	• Build on students' prior knowledge and experiences by connecting mathematical concepts to real-life situations and cultural contexts. • Bridge any gaps in understanding by providing multiple means of representation and scaffolding support for diverse learners. • Introduce measures of center, such as mean, median, and mode, using examples that resonate with students' cultural backgrounds. • Bridge cultural contexts by discussing how different cultures may interpret and analyze data differently, emphasizing the importance of understanding diverse perspectives.	• Provide linguistic support by using culturally relevant vocabulary and incorporating multilingual resources when necessary. • Scaffold language acquisition by offering opportunities for language practice and reinforcement throughout the lesson. • Present data sets representing diverse cultural contexts, allowing students to practice calculating measures of center and variation. • Provide linguistic support by defining and clarifying mathematical vocabulary in multiple languages, as needed, to ensure all students can fully participate in the activities.

Suggested Student Discourse Questions
• What is the difference between measures of center and measures of spread? • With use of correct mathematical language, explore and explain why and how two very different sets of data have the same median and mean but differ by their variability? • With a single number, can you recognize that a measure of center for a numerical data set summarizes all of its values? • Given a data set, can you arrange the numerical values in order and on a line plot?

Cross-Curricular Connections
Science: Students can answer a question regarding their experiment by collecting data. This data can be displayed in different ways in addition to students finding the measures of center (mean and median) and describing the shape of the data. (MS-LS1-4, From Molecules to Organisms: Structures and Processes) https://www.nextgenscience.org/pe/ms-ls1-4-moleculesorganisms-structures-and-processes **English:** Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. Demonstrate the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics. Distinguish among facts, reasoned judgment based on research findings, and speculations in a text. Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners.

Career and Skill Connections
• Financial Analyst • Data Analyst • Mathematician • Statistician • Research Scientist

Grade	CCSS Domain	CCSS Cluster
6	Statistics and Probability	Summarize and describe distributions.

Cluster Standard: 6.SP.B.4

Standard	Standards for Mathematical Practice
Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 2:** Reason abstractly and quantitatively.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students will develop an understanding of statistical thinking. They will use dot plots, histograms and box plots to draw inferences and make comparisons between data sets. Students should recognize that data distribution may not have a definite center and that interpreting those different measures of center can change how data gets interpreted.	• Understand and be able to calculate the measure of center, and the quartile ranges. • Understand when it is appropriate to use a dot plot, histogram and box plot. For example, a dot plot will show exact values for each piece of data, but a histogram will show how many pieces of data fell within a specific range. • Create and display data on number lines using dot plots, histograms and box plots
DOK	**Blooms**
1-2	Apply, Analyze

Procedural and Conceptual Understanding and Application

Procedural Skills and Fluency: Students must be able to accurately create and interpret various types of plots on a number line, including:

- **Dot Plots**: Simple plots that show individual data points above their corresponding values on a number line.
- **Histograms**: Bar graphs that group data into ranges (bins) and show the frequency of data points within each range.
- **Box Plots** (or Box-and-Whisker Plots): Plots that summarize data using five key values (minimum, first quartile, median, third quartile, and maximum) and display the spread and central tendency of the data.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Below are the 25 birth weights, in ounces, of all the Labrador Retriever puppies born at Kingston Kennels in the last six months.

13, 14, 15, 15, 16, 16, 16, 16, 17, 17, 17, 17, 17, 17, 17, 18, 18, 18, 18, 18, 18, 18, 18, 19, 20

a. Use an appropriate graph to summarize these birth weights.

b. Describe the distribution of birth weights for puppies born at Kingston Kennels in the last six months. Be sure to describe shape, center and variability.

c. What is a typical birth weight for puppies born at Kingston Kennels in the last six months? Explain why you chose this value.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Misunderstanding data or choosing the right graph:** Students may misread the data on the graph. In earlier grades, students constructed graphs but may have not got a full understanding of how to read them accurately. Students may also struggle with choosing which of the graphs to choose to represent the data appropriately.
- **Miscalculation of Measures of Center:** Students may struggle with some of the calculations of the measures of center. Some students get confused between the difference of mean and median and expect that these values will be the same or very close which is often not the case because of outliers or other contributing factors.
- **Trouble with box plots:** Students often struggle with the concepts of box plots. It is often the most difficult of the graphical displays because it represents five data measures and can be graphed horizontally or vertically which can be challenging for students. It may be helpful to spend extra time on this concept and to engage students with each of the aspects of the measures.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*

• Provide clear explanations and demonstrations of each type of plot, emphasizing key vocabulary and concepts. • Offer visual aids and manipulatives to support understanding, such as pre-made dot plot templates and histogram bars. **Universal Design for Learning (UDL)** • Provide a variety of visual examples of dot plots, histograms, and box plots. Use color-coding and annotations to highlight key features, such as the median in a box plot or the frequency in a histogram. • Utilize digital tools or apps that allow students to create and manipulate their own plots. These tools can help students see how data is distributed and how different types of plots represent data. • Offer clear, step-by-step guides on how to construct each type of plot. Include diagrams and simple explanations to ensure all students can follow along. • Present data from real-life situations that are relevant to students, such as survey results, sports statistics, or environmental data. This helps students see the practical applications of data plots. • Provide opportunities for students to collect their own data (e.g., class surveys, measurements) and create their own plots. This hands-on approach can make abstract concepts more	• Reinforce understanding through hands-on activities and interactive tasks, allowing students to create their own dot plots, histograms, and box plots. • Offer differentiated practice options to meet the diverse needs of learners, including online interactive tools and peer collaboration opportunities. **Universal Design for Learning (UDL)** • Encourage students to work in groups to analyze data and create plots. This fosters collaboration and allows students to learn from each other's ideas and strategies. • Use templates, graphic organizers, or software that guides students through the process of creating each type of plot. This support can help students who may struggle with organizing their work. • Offer students choices in the data they use or the type of plot they create. Giving students autonomy can increase their motivation and investment in the task.	• Extend learning for advanced students by introducing more complex data sets and challenging problems that require critical thinking and analysis. • Encourage students to explore real-world applications of numerical data representation, such as analyzing sports statistics or economic trends. **Universal Design for Learning (UDL)** • Encourage students to reflect on their learning process and set goals for improving their data representation skills. Reflection helps solidify understanding and fosters a growth mindset.

concrete. ● Allow students to choose how they want to display their data. They could create physical plots with manipulatives, draw plots by hand, or use digital tools.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/6/29/302/302

Previous Learning	*Current Learning*	*Future Learning*
● In Grade 5, learners made line plots to display a data set of measures in fractions of a unit. They will build upon this skill in 6th grade by summarizing increasingly complex data sets in different contexts.	● Students will create dot plots, histograms and box plots. They will draw inferences and make comparisons between them. Students will also learn mean, median, mode and interquartile range which will connect in this cluster.	● In Grade 7, students build on their understanding of interpreting information about a population by using population samples. In Grade 7, students begin to look at two separate data sets to make comparisons. In the high school standards, learners interpret differences in shape, center, and spread in the context of the data sets.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
● Begin the lesson by acknowledging the diverse cultural backgrounds and experiences of the students. ● Validate students' prior knowledge and experiences related to numerical data	● Build connections between students' cultural backgrounds and the mathematical concepts being taught, relating numerical data representation to real-world	● Provide linguistic support by using visual aids, gestures, and other nonverbal cues to clarify mathematical vocabulary and concepts. ● Scaffold learning by breaking down complex mathematical

representation, encouraging them to share examples from their own cultural contexts. ● Affirm the importance of cultural diversity in problem-solving and critical thinking, highlighting how different perspectives contribute to a richer understanding of mathematical concepts.	examples that resonate with students' experiences. ● Bridge cultural differences by incorporating culturally relevant examples and contexts into the lesson, such as using data sets related to cultural celebrations, traditional foods, or historical events that are meaningful to the students. ● Build connections between students' cultural experiences and the mathematical concepts being taught, emphasizing how numerical data representation is used in various cultural contexts.	terms into simpler language and providing multiple opportunities for students to practice using the vocabulary in context. ● Incorporate multilingual resources and materials to support students who are English language learners, such as bilingual dictionaries, translated worksheets, or language support software.

Suggested Student Discourse Questions

- With given data sets, are you able to select an appropriate display to represent that data? For example, to display data graphically in a format appropriate for the data set.
- Are you able to interpret and give meaning to the different data representations available to represent and display data sets?
- Are you able to interpret and give meaning to the varying aspects of the box and whisker plot; for example, the meaning of the five data measures and the box and whiskers?
-
- Why did you choose the graph that you did and what made it the most appropriate choice for the data set?

Cross-Curricular Connections

Science:

Students can answer a question regarding their experiment by collecting data. This data can be displayed in different ways in addition to students finding the measures of center (mean and median) and describing the shape of the data. (MS-LS1-4, From Molecules to Organisms: Structures and Processes)

https://www.nextgenscience.org/pe/ms-ls1-4-moleculesorganisms-structures-and-processes

English:

Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing

technical tasks.

Demonstrate the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.

Distinguish among facts, reasoned judgment based on research findings, and speculations in a text.

Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners.

Career and Skill Connections

- Economist
- Auditor
- Staff Engineer
- Gaming Supervisor
- Data Scientist

Grade	CCSS Domain	CCSS Cluster
6	Statistics and Probability	Summarize and describe distributions.

Cluster Standard: 6.SP.B.5

Standard	Standards for Mathematical Practice
Summarize numerical data sets in relation to their context, such as by: A. Reporting the number of observations. B. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. C. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. D. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	• **SMP 1:** Make sense of problems and persevere in solving them. • **SMP 2:** Reason abstractly and quantitatively.
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students will develop an understanding of statistical thinking. They will use dot plots, histograms and box plots to draw inferences and make comparisons between data sets. Students should recognize that data distribution may not have a definite center and that interpreting those different measures of center can change how data gets interpreted.	• Correlate the number of observations to the sample size. • Express how sample size is represented in a dot plot vs histogram, vs box plot. • Identify the initial survey question as numerical vs categorical (quantitative vs qualitative) data. • Describe the data by reading the graph's labels (units used) • Use the correct context, describe the overall pattern including any striking deviations such as outliers. • Compute the measures of center: median and/or mean. • Compute the measures of variability: interquartile range and/or mean absolute deviation. • Express how measures of center and variability change the shapes of distribution.

DOK	Blooms
2-3	Understand, Apply

Procedural and Conceptual Understanding and Application

Procedural Skills and Fluency: Students will learn and become fluent in identifying and defining measures of center that are used in statistics. With speed and accuracy students will also become fluent in calculating mean, median and range for a given set of data that is presented numerically or presented graphically. Statistical terms, such as, spread and outlier will also be defined by students accurately. Students must be able to accurately compute and report quantitative measures such as the median, mean, interquartile range, and mean absolute deviation. This includes performing calculations and summarizing data in a clear, organized manner. They should describe the data, including the number of observations and the nature of the attribute under investigation.

Conceptual Understanding: Students' understanding will be to summarize data displays by describing overall patterns in a distribution. Students will be able to communicate appropriate measures of center and spread when describing a data set. Students extend understanding of the mean, median and range considering the method used to collect the data. Students need to understand how to interpret and summarize numerical data within its specific context. This involves recognizing the relevance of different measures (like the median, mean, interquartile range) and understanding how these measures describe the data's center, spread, and overall pattern. Students should also grasp how the shape of the data distribution and the context affect the choice of summary measures.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Over a two week period, Jenna had the following number of math homework problems given each day:

20,0,7,10,1,11,0,25,15,1.

a. What is the mean number of homework problems Jenna had?

b. What is the Mean Absolute Deviation for the number of homework problems?

c. What do the mean and Mean Absolute Deviation tell you about the number of homework problems Jenna had over these two weeks?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Miscalculation of Measures of Center:** Students may struggle with some of the calculations of the measures of center. Some students get confused between the difference of mean and median and expect that these values will be the same or very close which is often not the case because of outliers or other contributing factors.
- **Trouble with Vocabulary:** Students may have issues with the vocabulary word symmetrical. They may have trouble describing data when it is not a traditional visual representation they have studied before (dot plot, histograms, etc.). In addition, students may mix up mean and median and what their purpose is for representing the data set.
- **Misunderstanding data or choosing the right graph:** Students may misread the data on the graph. In earlier grades, students constructed graphs but may have not got a full understanding of how to read them accurately. Students may also struggle with choosing which of the graphs to choose to represent the data appropriately.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
• Begin the lesson by activating students' prior knowledge on data summarization. Use a KWL chart to assess what students already know and what they want to learn about summarizing numerical data sets. • Provide multiple means of representation by using visual aids, such as charts, graphs, and diagrams, to introduce the concept of summarizing numerical data sets. • Facilitate a class discussion on the importance of choosing appropriate measures of center and variability based on the shape of the data distribution and the context in which the data were gathered. **Universal Design for Learning (UDL)** • Encourage students to reflect on their learning process and discuss any challenges they encountered while summarizing numerical data	• Differentiate instruction by providing various entry points for students of different learning levels. Offer scaffolded activities for students who need additional support and extension tasks for students who are ready for more challenging tasks. • Engage students in hands-on activities where they collect and organize numerical data sets from different contexts, such as surveys, experiments, or real-world scenarios. **Universal Design for Learning (UDL)** • Give students opportunities to apply their understanding of numerical data summarization through problem-solving tasks and real-world scenarios. • Encourage students to demonstrate their understanding in various ways, such as creating their	• Provide explicit instruction on how to summarize numerical data sets in relation to their context. Teach students how to report the number of observations, describe the nature of the attribute under investigation, and calculate quantitative measures of center and variability. • Offer targeted intervention for students who are struggling with specific concepts by providing additional examples, guided practice, and one-on-one support as needed. **Universal Design for Learning (UDL)** • Offer extension activities for students who have mastered the basic concepts, such as analyzing more complex data sets or exploring advanced statistical techniques. • Encourage students to set personal goals for mastering measures of center and variation, and provide opportunities for them to

sets. ● Provide visual representations of data sets along with calculations for measures of center (mean, median) and variation (range, interquartile range). Use color-coding and annotated graphs to highlight how these measures summarize the data. ● Use digital tools or apps that allow students to input data and see how the mean, median, range, and interquartile range are calculated and how they summarize the data. These tools often provide immediate visual feedback. ● Present data sets from real-world contexts that are relevant to students, such as class survey results, sports statistics, or environmental data. This helps students see the practical applications of these statistical measures. ● Offer detailed guides or tutorials that break down the process of calculating and interpreting measures of center and variation. Include examples and practice problems to reinforce learning. ● Allow students to collect their own data and calculate measures of center and variation. This hands-on approach makes abstract concepts more concrete and helps students connect theory to practice.	own graphs, writing explanations, or using digital tools to analyze data. They might also present their findings verbally or through multimedia presentations. ● Facilitate group work where students analyze data sets together, calculate measures, and discuss their findings. Collaborative projects help students learn from each other and refine their understanding. ● Provide scaffolding tools such as templates, graphic organizers, or worksheets to guide students through the process of summarizing data. These tools help students structure their analysis and ensure they consider all relevant aspects. ● Offer students choices in the data sets they analyze or the types of projects they undertake. Allowing students to choose topics that interest them can increase their motivation and investment in the task.	reflect on their progress and understanding. Reflection helps solidify learning and fosters a growth mindset.
Vertical Alignment		

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/6/29/303/303

Previous Learning	*Current Learning*	*Future Learning*
• In Grade 5, learners made line plots to display a data set of measures in fractions of a unit. They will build upon this skill in 6th grade by summarizing increasingly complex data sets in different contexts.	• Students will create dot plots, histograms and box plots. They will draw inferences and make comparisons between them. Students will also learn mean, median, mode and interquartile range which will connect in this cluster.	• In Grade 7, students build on their understanding of interpreting information about a population by using population samples. In Grade 7, students begin to look at two separate data sets to make comparisons. In the high school standards, learners interpret differences in shape, center, and spread in the context of the data sets.

Culturally and Linguistically Responsive Instruction

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Begin the lesson by acknowledging the diverse learning needs and backgrounds of the students. • Validate students' prior knowledge and experiences with numerical data summarization. • Affirm the importance of understanding and interpreting numerical data in various contexts. • Validate students' prior knowledge of numerical data summarization and affirm the importance of understanding data in different contexts.	• Build connections between students' prior knowledge and the new concepts being introduced, relating numerical data summarization to real-world examples and everyday experiences. • Bridge any cultural or linguistic barriers by providing multiple representations of the concepts, such as visual aids, manipulatives, and multilingual resources. • Introduce the concept of summarizing numerical data sets in relation to their context, emphasizing the	• Provide linguistic support by breaking down complex mathematical terms into simpler language and providing multiple opportunities for students to practice using the vocabulary in context. • Offer multilingual resources and materials to support students who are English language learners, ensuring they have access to the necessary language support to fully participate in the lesson. • Provide linguistic support by using simplified language and visual aids to clarify mathematical vocabulary and

	importance of understanding the nature of the attribute under investigation and how it was measured.	concepts.

Suggested Student Discourse Questions

- After summarizing your data and explaining the statistical measures, how will changes in the data affect those statistical measures?
- What is the effect of outliers on the data?
- If you add or remove data from the data set, what effect does this have on the measures of center and variability?
- Are you able to formulate any overall patterns from data sets while analyzing the information within a data set? What important aspects should you focus on?

Cross-Curricular Connections

Science:

Students can answer a question regarding their experiment by collecting data. This data can be displayed in different ways in addition to students finding the measures of center (mean and median) and describing the shape of the data. (MS-LS1-4, From Molecules to Organisms: Structures and Processes)

https://www.nextgenscience.org/pe/ms-ls1-4-moleculesorganisms-structures-and-processes

English:

Following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

Demonstrate the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.

Distinguish among facts, reasoned judgment based on research findings, and speculations in a text.

Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners.

Career and Skill Connections

- Economist
- Treasurer
- Financial Executive
- Statistician
- Comptroller

Section 3: Resources, References, and Glossary

Resources

Evidence-Based Resources	English Learner Resources	MLSS Resources	Mathematics Standard Resources
What Works Clearinghouse Best Evidence Encyclopedia Evidence for Every Students Succeeds Act Evidence in Education Lab	World-Class Instructional Design and Assessment (WIDA) Standards USCALE Language Routines for Mathematics English Language Development Standards Spanish Language Development Standards	NM Multi-Layered System of Supports (MLSS) Universal Design for Learning Guidelines Achieve the Core: Instructional Routines for Mathematics Project Zero Thinking Routines	Focus by Grade Level and Widely Applicable Prerequisites High school Coherence Map College-and Career Ready Math Shifts Fostering Math Practices: Routines for the Mathematical Practices

Planning Guidance for Multi-Layered Systems of Support: Core Instruction[9]

Core Instructional Planning must reflect and leverage scientific insights into how humans learn in order to ensure all students are ready for success, thus the following guidance for optimizing teaching and learning is grounded in the **Universal Design Learning (UDL) Framework**

Key design questions, planning actions, and potential strategies are provided below, with respect to guidance for minimizing barriers to learning and optimizing (1) universal ACCESS to learning experiences, (2) opportunities for students to BUILD their understanding of the **Learning Goal**, and (3) INTERNALIZATION of the Learning Goal.

Optimizing Universal ACCESS to Learning Experiences	
ENGAGEMENT ? How will you provide multiple options for recruiting interest?	**Recruiting Student Interest:** ? What do you anticipate in the range of student interest for this lesson? ➢ Plan for options for recruiting student interest: ❒ provide choice (e.g. sequence or timing of task completion) ❒ set personal academic goals ❒ provide contextualized examples connected to their lives ❒ support culturally relevant connections (i.e home culture) ❒ create socially relevant tasks ❒ provide novel & relevant problems to make sense of complex ideas in creative ways

[9] Adapted from: CAST (2018). *Universal Design for Learning Guidelines version 2.2*. Retrieved from http://udlguidelines.cast.org

	❒ provide time for self-reflection about content & activities ❒ create accepting and supportive classroom climate ❒ utilize **instructional routines** to involve all students
REPRESENTATION [?] How will you reduce barriers to perceiving the information presented in this lesson?	**Perception:** [?] What do you anticipate about the range in how students will perceive information presented in this lesson? ➢ Plan for different modalities and formats to reduce barriers to learning: ❒ display information in a flexible format to vary perceptual features ❒ offer alternatives for auditory information ❒ offer alternatives for visual information
ACTION & EXPRESSION [?] How will the learning for students provide a variety of methods for navigation to support access?	**Physical Action:** [?] What do you anticipate about the range in how students will physically navigate and respond to the learning experience? ➢ Plan a variety of methods for response and navigation of learning experiences by offering alternatives to: ❒ requirements for rate, timing, speed, and range of motor action with instructional materials, manipulatives, and technologies ❒ physically indicating selections ❒ interacting with materials by hand, voice, keyboard, etc.

Opportunities for Students to BUILD their Understanding

ENGAGEMENT [?] How will the learning for students provide options for sustaining effort and persistence?	**Sustaining Effort & Persistence:** [?] What do you anticipate about the range in student effort? ➢ Plan multiple methods for attending to student attention and affect by: ❒ prompting learners to explicitly formulate or restate learning goals ❒ displaying the learning goals in multiple ways ❒ using prompts or scaffolds for visualizing desired outcomes ❒ engaging assessment discussions of what constitutes excellence ❒ generating relevant examples with students that connect to their cultural background and interests ❒ providing alternatives in the math representations and scaffolds ❒ creating cooperative groups with clear goals, roles, responsibilities ❒ providing prompts to guide when and how to ask for help ❒ supporting opportunities for peer interactions and supports (e.g. peer tutors) ❒ constructing communities of learners engaged in common interests ❒ creating expectations for group work (e.g., rubrics, norms, etc.) ❒ providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge ❒ providing feedback that: ❒ emphasizes effort, improvement, and achieving a standard rather than on relative performance ❒ is frequent, timely, and specific ❒ is informative rather than comparative or competitive

	❒ models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success
REPRESENTATION ? How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners?	**Language & Symbols:** ? What do you anticipate about the range of student background experience and vocabulary? ➢ Plan multiple methods for attending to linguistic and nonlinguistic representations of mathematics to ensure universal clarity by: ❒ pre-teaching vocabulary and symbols in ways that promote connection to the learners' experience and prior knowledge ❒ graphic symbols with alternative text descriptions ❒ highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to structure ❒ embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) ❒ embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) ❒ highlighting structural relations or make them more explicit ❒ making connections to previously learned structures ❒ making relationships between elements explicit (e.g., highlighting the transition words in an argument, links between ideas, etc.) ❒ allowing the use of text-to-speech and automatic voicing with digital mathematical notation (math ml) ❒ allowing flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) ❒ clarification of notation through lists of key terms ❒ making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf ❒ linking key vocabulary words to definitions and pronunciations in both dominant and heritage languages ❒ defining domain-specific vocabulary (e.g., "map key" in social studies) using both domain-specific and common terms ❒ electronic translation tools or links to multilingual web glossaries ❒ embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc) ❒ presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) ❒ making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams
ACTION & EXPRESSION ? How will the learning provide multiple	**Expression & Communication:** ? What do you anticipate about the range in how students will express their thinking in the learning environment? ➢ Plan multiple methods for attending to the various ways in which students can express knowledge, ideas, and concepts by providing:

modalities for students to easily express knowledge, ideas, and concepts in the learning environment?	❒ options to compose in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video ❒ use of social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) ❒ flexibility in using a variety of problem solving strategies ❒ spell or grammar checkers, word prediction software ❒ text-to-speech software, human dictation, recording ❒ calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper ❒ sentence starters or sentence strips ❒ concept mapping tools ❒ Computer-Aided-Design (CAD) or mathematical notation software ❒ virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) ❒ multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches) ❒ multiple examples of novel solutions to authentic problems ❒ different approaches to motivate, guide, feedback or inform students of progress towards fluency ❒ scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) ❒ differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners)

Optimizing INTERNALIZATION of the Learning Goal

ENGAGEMENT ⍰ How will the design of the learning strategically support students to effectively cope and engage with the environment?	**Self-Regulation:** ⍰ What do you anticipate about barriers to student engagement? ➢ Plan to address barriers to engagement by promoting healthy responses and interactions, and ownership of learning goals: ❒ metacognitive approaches to frustration when doing mathematics ❒ increase length of on-task orientation through distractions ❒ frequent self-reflection and self-reinforcements ❒ address subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") ❒ offer devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the math practices for the purpose of monitoring and improving ❒ use activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely
REPRESENTATION ⍰ How will the learning support transforming accessible information into usable knowledge	**Comprehension:** ⍰ What do you anticipate about barriers to student comprehension? ➢ Plan to address barriers to comprehension by intentionally building connections to prior understandings and experiences, relating meaningful information to learning goals,

that is accessible for future learning and decision-making?	providing a process for meaning making of new learning, and applying learning to new contexts: ❒ incorporate explicit opportunities for review and practice ❒ note-taking templates, graphic organizers, concept maps ❒ scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) ❒ explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) ❒ opportunities over time to revisit key ideas and connections ❒ make explicit cross-curricular connections ❒ highlight key elements in tasks, graphics, diagrams, formulas ❒ outlines, graphic organizers, unit organizer routines, concept organizer routines, and concept mastery routines to emphasize key ideas and relationships ❒ multiple examples & non-examples ❒ cues and prompts to draw attention to critical features ❒ highlight previously learned skills that can be used to solve unfamiliar problems ❒ options for organizing and possible approaches (tables and representations for processing mathematical operations) ❒ interactive representations that guide exploration and new understandings ❒ introduce graduated scaffolds that support information processing strategies ❒ tasks with multiple entry points and optional pathways ❒ "Chunk" information into smaller elements ❒ remove unnecessary distractions unless essential to learning goal ❒ anchor instruction by linking to and activating relevant prior knowledge (e.g., using visual imagery, concept anchoring, or concept mastery routines) ❒ pre-teach critical prerequisite concepts via demonstration or representations ❒ embed new ideas in familiar ideas and contexts (e.g., use of analogy, metaphor, drama, music, film, etc.) ❒ advanced organizers (e.g., KWL methods, concept maps) ❒ bridge concepts with relevant analogies and metaphors
ACCESS ACTION & EXPRESSION ⍰ How will the learning for students support the development of executive functions to allow them to take advantage of their environment?	**Executive Functions:** ⍰ What do you anticipate about barriers to students demonstrating what they know? ➢ Plan to address barriers to demonstrating understanding by providing opportunities for students to set goals, formulate plans, use tools and processes to support organization and memory, and analyze their growth in learning and how to build from it: ❒ prompts and scaffolds to estimate effort, resources, difficulty ❒ models and examples of process and product of goal-setting ❒ guides and checklists for scaffolding goal-setting ❒ post goals, objectives, and schedules in an obvious place ❒ embed prompts to "show and explain your work" ❒ checklists and project plan templates for understanding the problem, prioritization, sequences, and schedules of steps ❒ embed coaches/mentors to demonstrate think-alouds of process ❒ guides to break long-term goals into short-term objectives ❒ graphic organizers/templates for organizing information & data ❒ embed prompts for categorizing and systematizing ❒ checklists and guides for note-taking ❒ asking questions to guide self-monitoring and reflection ❒ showing representations of progress (e.g., before and after photos, graphs/charts showing progress, process portfolios)

	❒ prompt learners to identify type of feedback or advice they seek ❒ templates to guide self-reflection on quality & completeness ❒ differentiated models of self-assessment strategies (e.g., role-playing, video reviews, peer feedback) ❒ assessment checklists, scoring rubrics, and multiple examples of annotated student work/performance examples

Planning Guidance for Culturally and Linguistically Responsive Instruction[10]

In order to ensure our students from marginalized cultures and languages view themselves as confident and competent learners and doers of mathematics within and outside of the classroom, educators must intentionally plan ways to counteract the negative or missing images and representations that exist in our curricular resources. The guiding questions below support the design of lessons that validate, affirm, build, and bridge home and school culture for learners of mathematics:

> **Validate/Affirm:** How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
>
> **Build/Bridge:** How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

In addition, Aguirre and her colleagues[11] define **mathematical identities** as the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives. Many students see themselves as "not good at math" and approach math with fear and lack of confidence. Their identity, developed through earlier years of schooling, has the potential to affect their school and career choices.

Five Equity-Based Mathematics Teaching Practices[12]

> **Go deep with mathematics.** Develop students' conceptual understanding, procedural fluency, and problem solving and reasoning.
>
> **Leverage multiple mathematical competencies.** Use students' different mathematical strengths as a resource for learning.
>
> **Affirm mathematics learners' identities.** Promote student participation and value different ways of contributing.

[10] This resource relied heavily on the work of: Hollie, S. (2011). Culturally and linguistically responsive teaching and learning. Teacher Created Materials. (see also, https://www.culturallyresponsive.org/vabb)

[11] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices. Reston, VA: National Council of Teachers of Mathematics (p. 14).

[12] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc. (p.6). (adapted from Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013) (p. 43).

Challenge spaces of marginality. Embrace student competencies, value multiple mathematical contributions, and position students as sources of expertise.

Draw on multiple resources of knowledge (mathematics, language, culture, family). Tap students' knowledge and experiences as resources for mathematics learning.

The following lesson design strategies support Culturally and Linguistically Responsive Instruction, specific examples for each cluster of standards can be found in part 2 of the document. These were adapted from the Promoting Equity section of the Taking Action series published by NCTM.[13]

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.

Mathematical Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher.

Modifying Mathematical Tasks: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)."

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics.

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages.

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we

[13] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians.

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others."

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time."

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students.

References

Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics.

Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

Cardone, T. (n.d.). Nix the Tricks. Retrieved from https://nixthetricks.com/

CAST (2018). *Universal Design for Learning Guidelines version 2.2*. Retrieved from http://udlguidelines.cast.org

Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/

English Learners Success Forum. (2020). ELSF | Resource: Analyzing Content and Language Demands. Retrieved from https://www.elsuccessforum.org/resources/math-analyzing-content-and-language-demands

Gifted and talented | NZ Maths. (n.d.). Retrieved from https://nzmaths.co.nz/gifted-and-talented

Gojak, L. M. (Ed.) (2015-2018). The Common Core Mathematics Companion: The Standards Decoded: What They Say, What They Mean, How to Teach Them (Corwin Mathematics Series) (1st ed.). Thousands Oak, CA: Corwin.

Gonzales, Lorenzo (2004). *Building personal knowledge of rational numbers through mental images, concepts, language, facts, and procedures.* Unpublished Doctoral Dissertation, New Mexico State University, College of Education, Las Cruces, New Mexico.

Hollie, S. (2011). Culturally and linguistically responsive teaching and learning. Teacher Created Materials.

Howard County Public School System: Mathematics. (n.d.). Mathematics – HCPSS. Retrieved from https://www.hcpss.org/academics/mathematics/

Illustrative Mathematics. (2014, February 14). Standards for Mathematical Practice: Commentary and Elaborations for K–5. Retrieved from http://commoncoretools.me/wp-content/uploads/2014/02/Elaborations.pdf

Illustrative Mathematics. (2014b, May 6). Standards for Mathematical Practice: Commentary and Elaborations for 6–8. Retrieved from http://commoncoretools.me/wp-content/uploads/2014/05/2014-05-06-Elaborations-6-8.pdf

Kansas State Department of Education. (2018). 2018 Kansas Mathematics Flip Books. Retrieved from https://community.ksde.org/Default.aspx?tabid=5646

Kaplinsky, R. (n.d.). SIOP Content and Language Objectives for Math. Retrieved from http://robertkaplinsky.com/wp-content/uploads/2017/04/SIOP-Content-and-Language-Objectives-for-Math.pdf

Louisiana Department of Education . (n.d.). K-12 Math Planning Resources. Retrieved from https://www.louisianabelieves.com/resources/library/k-12-math-year-long-planning

Los Angeles, CAUSA. (n.d.). VABB™. Retrieved from https://www.culturallyresponsive.org/vabb New Visions for Public Schools. (2020). Re-engagement Lessons. Retrieved from https://curriculum.newvisions.org/math/nv-math-team/curriculum-components/re-engagement-lessons/

Seeley, C. L. (2016). Math is Supposed to Make Sense. In Making sense of math: How to help every student become a mathematical thinker and problem solver. Alexandria, VA, USA: ASCD. (P. 13)

Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

Glossary[14]

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (- 3/4) = (- 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[15]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. See: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances
from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten.
For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

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[14] Glossary and tables taken from: Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/
[15] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[16] See also: median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.
Integer. A number expressible in the form a or -a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line.
Also known as a dot plot.[17]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[18] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 Ã· 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 Ã— 4/3 = 4/3 Ã— 3/4 = 1.

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[16] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

[17] Adapted from Wisconsin Department of Public Instruction, op. cit.

[18] To be more precise, this defines the arithmetic mean.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin,
selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also*: uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form *a/b* or – *a/b* for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also*: terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[19]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar
model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

[19] Adapted from Wisconsin Department of Public Instruction, op. cit.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also*: median, first quartile, interquartile range.

Table 1: Common addition and subtraction.[1]

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 =5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now?5-2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?? -2 = 3
	TOTAL UNKNOWN	ADDEND UNKNOWN	BOTH ADDENDS UNKNOWN[2]
PUT TOGETHER / TAKE APART[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5-3 = ?	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? 5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1, 5 = 2 + 3, 5 = 3 + 2
COMPARE	DIFFERENCE UKNOWN	BIGGER UNKNOWN	SMALLER UNKNOWN
	("How many more?" version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2: Common multiplication and division situations.[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	3 x ? = 18, and 18 ÷ 3 = ?	? x 6 = 18, and 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	a x b = ?	a x ? = p and p ÷ a = ?	? x b = p, and p ÷ b = ?

[1]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3: The properties of operations.

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	(a +b) + c = a + (b+c)
Commutative property of addition	a + b = b + a

Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists *-a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) x c = a x (b x c)
Commutative property of multiplication	a x b = b x a
Multiplicative identity property 1	a x 1 = 1 x a = a
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that a x $1/a$ = $1/a$ x a = 1
Distributive property of multiplication over additions	a x (b + c) = a x b + a x c

Table 4: The properties of equality.

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a.
Symmetric property of equality	If a = b, then b = a.
Transitive property of equality	If a = b and b = c , then a = c.
Addition property of equality	If a = b, then a +c = b + c.
Subtraction property of equality	If a = b then a – c = b – c.
Multiplication property of equality	If a = b, then a x c = b x c.
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c.
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality.

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b.
If a > b and b > c then a > c.
If a > b, b < a.
If a > b, then -a <-b.
If a > b, then a ± c > b ± c.
If a > b and c > 0, then a x c > b x c.
If a > b and c < 0, then a x c < b x c.
If a > b and c > 0, then a ÷ c > b ÷ c.
If a > b and c < 0, then a ÷ c < b ÷ c.