New Mexico Instructional Scope for Supporting Equitable and Culturally Responsive Mathematics Instruction

Overview

This Instructional Scope for Mathematics 3.0 was created by a cohort of New Mexico educators and the New Mexico Public Education Department.

The intention of this document is to act as a companion during the planning process alongside High Quality Instructional Materials (HQIM). A sample template is provided to show a snapshot of planning supports provided within each cluster of standards in the mathematics instructional scope.

During the creation of this document, we leveraged the work of other states, organizations, and educators from across the country and world. This work would not have been possible without all that came before it, and we wish to express our sincerest gratitude for everyone that contributed to the resources listed within our references.

To better understand the planning support provided in the mathematics instruction scope, this section provides a brief description of each planning support. This includes *what* support is provided, *why* the planning support is critical for equitable and culturally responsive mathematics instruction, and *how* to use the planning support with HQIM.

Cluster Statement

What: The New Mexico Mathematics Standards are grouped by domains, with somewhere between 4 to 10 domains per grade level. Within each domain, the standards are arranged into clusters. Cluster statements summarize groups of related standards.

Why: The New Mexico Mathematics Standards require a stronger focus on the way time and energy are spent in mathematics classrooms. Students should spend the majority of their time (65-85%) working on content within the major clusters of the grade/course. Supporting clusters and–where appropriate–additional clusters should only comprise 15-35% of the time spent in classes and be covered when they are connected to and engage students in the major work of the grade/course.

How: When planning with your HQIM, consider the time being devoted to major clusters versus additional or supporting clusters. The major work of each grade/course should be designed to provide students with strong foundations for future mathematical work, which will require more time than additional or supporting clusters. Also consider the ways your HQIM makes these connections between major clusters and additional and supporting clusters explicit for students.

Standard Text

What: Each cluster-level support document contains the text of each standard within that cluster.

Why: The cluster statement and standards are meant to be read together to understand the structure of the standards. By grouping the standards within the cluster, the connectedness of the standards is reinforced.

How: The text of the standards should ground all planning with your HQIM. Reading the standards within a cluster intentionally enables us to focus on the connections within and between the standards.

Standards for Mathematical Practice

What: The Standards for Mathematical Practice describe the expertise and habits of mind that mathematics educators at all levels should develop in their students.

Why: Equitable and excellent mathematics instruction supports students in becoming confident and competent mathematicians. By engaging with the Standards for Mathematical Practice, students engage in the practice of doing mathematics and developing mathematical habits of mind. These include the ability to think mathematically, analyze situations, understand relationships, and adapt what they know to solve a wide range of problems, including problems that do not look like any they have encountered before.

How: When planning with HQIM, it is critical to consider the connections between the content standards and the Standards for Mathematical Practice. The planning supports highlight a few practices students can engage in when learning the content of the standard. It is not necessary, or even appropriate, to engage in all of the practices every day. Instead, teachers should choose which Standards for Mathematical Practice to focus on within a given lesson, depending on the content standards being covered and the activities students will be engaging with. When teachers spend time intentionally supporting students in learning both the what (content standards) and the how (Standards for Mathematical Practice), students will have a stronger foundation of mathematical learning.

Clarification Statement

What: The clarification statement provides greater clarity for teachers in understanding the purpose of the standard.

Why: The New Mexico Mathematics Standards illustrate how progressions support student learning within each major domain of mathematics. The clarification statement provides additional context about the ways each cluster of standards supports student learning of the larger learning progression.

How: When planning with HQIM, use the clarification statement to support your understanding of how the materials use specific types of representations or change the learning sequence.

Demonstration Statements, Webb's Depth of Knowledge, Bloom's Taxonomy, and Aspect of Rigor

What: The New Mexico Mathematics Standards include one, two, or all three of the aspects of mathematical rigor: conceptual understanding, procedural skill and fluency, and application to the real world. These planning supports considers which aspect(s) of rigor are within each standard and then identifies academic skills students need to demonstrate comprehension of the standard and associated mathematical practices. The planning supports also provide information about two common classifications on cognitive complexity, Webb's Depth of Knowledge and Bloom's Taxonomy.

Why: Analyzing standards alongside the standards for mathematical practice provide a fuller picture of the mathematical competencies demanded in the standard.

How: When planning for a cluster of standards with your HQIM, a critical first step is to analyze the content and language demands of the standards and the associated Standards for Mathematical Practice. This analysis can be used to plan/design appropriate formative assessment, as well as interpret student data from formative assessments. The planning supports provide a breakdown of the standard that can serve as the basis for this sort of analysis.

Definitions of the Components of Rigor

Rigorous teaching in mathematics does not simply mean increasing the difficulty or complexity of practice problems. Incorporating rigor into classroom instruction and student learning means exploring at a greater depth, the standards and ideas with which students are grappling. There are **three** components of rigor that will be expanded upon in this document, and each is equally important to student mastery: **Conceptual Understanding, Procedural Skill and Fluency,** and **Application**.

- **Conceptual Understanding** refers to understanding mathematical concepts, operations, and relations. It is more than knowing isolated facts and methods. Students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. It also allows students to connect prior knowledge to new ideas and concepts.
- **Procedural Skill and Fluency** is the ability to apply procedures accurately, efficiently, and flexibly. It requires speed and accuracy in calculation while giving students opportunities to practice basic skills. Students' ability to solve more complex application tasks is dependent on procedural skill and fluency.
- **Application** provides valuable context for learning and the opportunity to solve problems in a relevant and a meaningful way. It is through real-world application that students learn to select an efficient method to find a solution, determine whether the solution makes sense by reasoning, and develop critical thinking skills.

A Special Note on Procedural Skill and Fluency

While speed is a component of fluency, it is not necessarily speed in producing an answer; rather, fluency can be observed by watching the speed with which a student engages with a particular problem. Furthermore, fluency does not require the most efficient strategy. The standards specify grade-level appropriate strategies or types of strategies with which students should demonstrate fluency (e.g., 1.OA.C.6 allows for students to use counting on, making ten, creating equivalent but easier or known sums, etc.). It should also be noted that teachers should expect some procedures to take longer than others (e.g., fluency with the standard algorithm for division, 6.NS.B.2, as compared to fluently adding and subtracting within 10, 1.OA.C.6).

Standards identified as targeting procedural skill and fluency do not all have an expectation of automaticity and/or rote recall. Only two standards, 2.OA.B.2 and 3.OA.C.7, have explicit expectations of students knowing facts from memory. Other standards targeting procedural skill and fluency do not require students to reach automaticity. For example, in 4.G.A.2, students do not need to reach automaticity in classifying two-dimensional figures.

*Adapted from Louisiana Department of Education

Assessment Items

What: Formative assessment is the planned, ongoing process used by teachers during learning and teaching to assess student learning in order to improve student understanding and support students in their learning. This planning support provides one or more sample items to assist teachers in their planning of formative assessments for each standard. These are intended to be used as a guide for what students should be able to complete and can be used alongside assessments provided within your HQIM. A link to additional assessment items from Illustrative Mathematics is also provided with each standard.

Why: When student thinking is made visible, the teacher can examine the progression of learning towards the goals of the standards and adjust instruction as necessary. By including students in the assessment and analysis process, teachers allow them to become strategic and goal-directed with their learning.

How: The sample items address the aspect(s) of rigor that aligns with each standard. This example can be used to discuss possible responses by students and next steps for instruction. A similar process can then be used to identify additional items from the formative assessment resources provided by your HQIM.

For additional information, see Standards Aligned Instructionally Embedded Formative Assessment Resources.

Common Misconceptions

What: This planning support identifies some of the common misconceptions students might have when engaging in learning about each mathematical topic.

Why: Students might have misconceptions based on an overgeneralization of patterns they notice or overly relying on rules rather than understanding the underlying concepts. Tips and tricks in mathematics expire over time as students move up through the grade levels. It is critical to understand some of the common misconceptions students can develop so we can address them directly with students and continue to build a strong foundation for their mathematical learning.

How: When planning with your HQIM, look for ways to directly address common misconceptions students have. This planning support provides some of the possible misconceptions, and your HQIM might include additional ones. The goal is not to avoid misconceptions–they are a natural part of the learning process. But we do want to support students in exploring their misconceptions and modifying incorrect or partial understanding.

Planning for Multi-Layered System of Support & Universal Design for Learning

What: The section on planning for Multi-Layered Systems of Supports (MLSS) and Universal Design for Learning (UDL) is designed to support teachers in planning for the needs of all students. Each section includes the three pillars of UDL, as well as options for pre-teaching and re-teaching. Some students might benefit from targeted pre-teaching and re-teaching supports, which improve students' acquisition of the knowledge and skills identified in the New Mexico Mathematics Standards. Intensive pre-teaching and re-teaching supports may also be helpful for smaller groups of students with more intensive needs. Progress monitoring should occur to assess students' responses to these additional supports.

Why: MLSS is a holistic framework that guides educators to intervene quickly when students need additional supports. The framework moves away from the "wait to fail" model and empowers teachers to use their professional judgment to make data-informed decisions regarding the students in their classrooms to ensure academic success with the grade-level expectations of the New Mexico Mathematics Standards.

How: When planning with your HQIM, use the suggestions for pre-teaching as a starting point to determine if some or all of the students in your classroom need targeted or intensive pre-teaching at the start of a unit to ensure they can access the grade-level material within the unit. The core instruction (which is grounded in the UDL Framework) and re-teach sections work together to support planning within a unit, looking for ways the materials support greater access for all students and provide options to revisit concepts based on formative assessment data. Additional information about MLSS can be found on the PED's website, and guidance and tools can be found here.

Vertical Alignment

What: The New Mexico Mathematics Standards are designed around coherent progressions of learning. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but rather an extension of previous learning. The connections to previous, current, and future learning make this coherence visible. A link to Achieve the Core's grade-level coherence maps is provided with each standard.

Why: Students build stronger foundations for learning when they see mathematics as an interconnected discipline of relationships rather than discrete skills and knowledge that are only applicable in current situations. The intentional inclusion of connections to previous, current, and future learning can support a more interconnected understanding of mathematics.

How: When planning with HQIM, use the vertical alignment supports to find ways to help students make explicit connections within their study of mathematics.

Culturally and Linguistically Responsive Instruction

What: Culturally and Linguistically Responsive Instruction (CLRI) requires educators to contribute to a positive school climate by validating and affirming students' home languages and cultures, building and bridging mathematics to previous learning (both formal and informal), and using linguistic vocabulary supports. These actions legitimize students' home culture and language, making it clear that students' culture and language are positive assets in the classroom. It is also an intentional effort to reverse negative stereotypes of non-dominant cultures and languages and must be purposeful, consistent, and authentic, as well as both proactive and reactive. By building and bridging, students learn to toggle between the behaviors and expectations of home and those of school. By focusing on creating connections between the culture and language expectations of home and the expectations at school, we help students be set up for success in school. Linguistic supports are important within this structure as well, and help students learn new, academic vocabulary in ways that allow students to bring in their prior knowledge and connect it to new experiences. Linguistic supports for helping students acquire new academic vocabulary are provided from Pathways2Careers.

Why: The mathematical identities of students are shaped by the messages they receive about their ability to do mathematics and the power of mathematics in their lives outside of school. Educators must intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages. In addition, creating connections between the cultural and linguistic behaviors of students' home culture and language supports students in creating identities as capable mathematicians within school and society.

How: When planning instruction, it is critical to consider ways to validate/affirm and build/bridge students' cultural and linguistic assets. The planning supports for each cluster provide information

and examples of how to engage with equity-based teaching practices. There may be additional support available in your HQIM to ensure all students develop strong mathematical identities.

Student Discourse Guide

What: This guide provides educators with suggested questions to engage students in discourse about mathematical concepts. Purposeful, rich classroom discourse offers students the opportunity to express their ideas and thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Ideas for supporting student discourse are also provided from Pathways2Careers.

Why: Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding.

How: When students have frequent opportunities for discussion, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences, and evolving math identities.

Cross-Curricular Connections

What: Cross-curricular connections are connections between knowledge and/or skills that can be made between two or more areas of study. These connections can be made by teachers or students.

Why: The purpose of planning cross-curricular connections in an instructional sequence is to ensure that students build connections and recognize the relevance of mathematics beyond the mathematics classroom.

How: When planning with HQIM, look for opportunities to make explicit connections to other content areas, including the examples provided for each standard.

Career and Skill Connections

What: The knowledge and/or skills students are mastering within each standard can be directly connected to various careers. These connections can be made by teachers or students and can provide motivation for students as they work through mathematical topics.

Why: Understanding and recognizing the relevance of mathematics beyond the mathematics classroom is important. This is particularly valid for students who may not plan on working in a

"mathematics" field.

How: When planning with HQIM, look for opportunities to make explicit connections to careers, including the examples provided for each standard.

Template of the New Mexico Cluster Level Planning Support for the New Mexico Mathematics Standards

Grade	*CCSS Domain*	*CCSS Cluster*
Grade level	CCSS domain	CCSS cluster statement summarizing the group of related standards
Standard and icons that indicate which aspect of rigor it aligns with		
Standard		**Standards for Mathematical Practice**
Full text of the standard.		Correlation of the standard to the Standards for Mathematical Practice to which it aligns, including a link to a descriptor of what teachers and students should be doing.
Clarification Statement		**Students Who Demonstrate Understanding Can...**
Clarifies the language of the standard.		The skills students perform to demonstrate comprehension of the standard.
DOK		**Blooms**
Correlation of the standard to Webb's Depth of Knowledge.		Correlation of the standard to Bloom's Taxonomy.
Conceptual Understanding, Procedural Skill and Fluency, and/or Application		
Highlights the aspect(s) of rigor the standard is aligned to, and descriptors for which portions of the standard fall under each aspect of rigor.		
Assessment Items		
When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.		
Provides at least one high-quality formative assessment item aligned to the standard, as well as a link to more items.		
Common Misconceptions		
Provides guidance on where student misconceptions might occur.		

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)		
Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Layer 1 ensures that all students receive strong instruction in a high-quality differentiated core curriculum that is based on the principles of UDL. This includes school-wide implementation of positive behavioral interventions and supports, data-driven instruction, targeted interventions in small group instruction, universal screening, and English Language Development (ELD) for English Learners (ELs).	Layer 2 interventions should be focused on delivering individualized and targeted support (pre-teaching and re-teaching) for students on a grade-level trajectory. The interventions must be aligned with Layer 1 skills. Students should be provided with additional time and intensity in a small-group setting.	Layer 3 interventions should be provided individually or in small groups. Students are grouped according to their skill needs. The goal is for each student to acquire academic skills that will persist and transfer when the student returns to core instruction. If needed, specialized teachers may provide specific intervention instruction based on the needs identified by the data.

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/7/30/308/308		
Previous Learning	*Current Learning*	*Future Learning*
Lists skills relevant to current learning that students should have already mastered.	Lists skills within the current learning that students will master.	Lists skills from upcoming learning that students will need to be able to master based on what they are learning now.
Culturally and Linguistically Responsive Instruction		
Consider these resources for vocabulary from Pathways2Careers: • https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf • https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf		
Consider these questions as you plan for instruction that is culturally and linguistically responsive: • How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
Provides ways to build connections between the families in the community and the mathematical content, as well as examples that connect the math to students' home lives.	Provides information on supporting students as they learn mathematics by starting with conceptual knowledge that students can make connections to based on their prior knowledge. Also provides information on helping students build positive mathematical identities.	Provides ideas and supports for helping students learn new academic vocabulary and making connections to their prior knowledge.

Suggested Student Discourse Questions
Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf
Provides questions teachers can employ to increase student discourse.
Cross-Curricular Connections
Provides various connections between the standard and the knowledge and skills that students might use in other content areas.
Career and Skill Connections
Provides various connections between the standard and future careers/skills students will need for future careers.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards for Mathematical Practice
    - Connections to procedural, conceptual understanding, and application
    - Sample assessment items
    - Common misconceptions
    - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
    - Vertical alignment
    - Culturally and Linguistically Responsive Instruction (CLRI)
    - Suggested student discourse questions
    - Cross-curricular and career/skill connections
- A Student Discourse Guide

Helpful links:

- Lesson-planning tool from Pathways 2 Careers (click here to sign up with your district email if you don't already have an account)
- Focus by Grade Level from Achieve the Core
- Coherence Map from Achieve the Core

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Use properties of operations to generate equivalent expressions
  - 7.EE.A.1
  - 7.EE.A.2
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
  - 7.EE.B.3
  - 7.EE.B.4

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)

Grade	*CCSS Domain*	*CCSS Cluster*
7	Expressions and Equations	Use properties of operations to generate equivalent expressions.

Cluster Standard: 7.EE.A.1

Standard	Standards for Mathematical Practice

7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	• **SMP 2:** Reason abstractly and quantitatively.
	  ○ Teacher and Student Actions
	• **SMP 6:** Attend to precision.
	  ○ Teacher and Student Actions

Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students apply properties of operations to add, subtract, factor and expand linear equations with rational coefficients. Students then become able to rewrite expressions in different forms to solve a multi-step problem, and explain the quantities.	• Identify properties of operations (Associative, Commutative, and Distributive). • Use properties of operations to create equivalent expressions. • Write expressions in standard or expanded form.
DOK	**Blooms**
1-2	Remember, Understand

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Explain the properties of operations.
- Relate properties of operations to strategies for adding, subtracting, factoring, and expanding linear expressions.
- Explain the quantities in an expression and in solutions.

Procedural Skill and Fluency:

- Identify and apply the properties of operations to add, subtract, factor, and expand linear expressions with rational coefficients.
- Identify the components of expressions (terms, coefficients, variables, etc).
- Solve multi-step problems by rewriting expressions in equivalent form.
- Write expressions in standard or expanded form.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Write an expression for the sequence of operations:

a. Add 3 to x, subtract the result from 1, then double what you have.

b. Add 3 to x, double what you have, then subtract 1 from the result.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Multi-step problems:** When an expression has several steps, sometimes students forget to follow the order of operations, struggle with moving from step to step, forget terms, etc.
- **Conversions:** When it is necessary to convert from a fraction to a decimal and vice versa, etc., students may struggle to rewrite those terms.
- **Combining like terms:** Students tend to combine all terms to arrive at a single term without realizing that some terms are dissimilar.
- **Distributive Property:** Students commonly overlook distributing the multiplier especially if the terms inside the parenthesis are composed of multiple terms.
- **Vocabulary:** If students are reading verbal descriptions or following directions to write an expression, they may struggle with the vocabulary (sum, product, difference, etc.) and interpreting the directions (e.g. subtract 1 from the number vs. subtract the number from 1).

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media,	**Pre-teaching** In previous classes, learners worked to extend their knowledge of creating equivalent expressions to include situations in which a knowledge of the rules of integers are needed, extend their understanding of repeated addition as multiplication, use order of operations to solve equations and inequalities with more than one step and those with negative coefficients, and use variables to represent numbers and write expressions when solving a real-world or mathematical problem with equations or expressions, including multi-step problems and problems	**Pre-teaching** Consider using standard 3.OA.B.5, which provides a foundation for work in this cluster. In 3.OA.B.5, students use properties as strategies to multiply and divide and learn to use the commutative and associative properties of multiplication with whole numbers. Also consider using standard 6.EE.B.7, which also provides a foundation for work in this cluster. In 6.EE.B.7, students solve real-world and mathematical problems in the form of $x + p = q$ and $px = q$, which are one step equations with positive rational

tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community.	with rational numbers. Students interpreted parts of an expression by using mathematical terms and viewing expressions as single entities. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as generating equivalent expressions (especially with the distributive property), comparing products, working with large numbers and numbers with units, multiplying fractions, understanding patterns in a table of values, writing and solving equations and inequalities, and writing and interpreting numerical expressions. Review and contrast the definitions of expression and equations, practice identifying parts of an expression, and review and practice the order of operations. This is the first experience students will have with variables, coefficients, and constants, and they will also be learning how to extend previous learning about exponents, the order of operations, equivalency, and like and unlike terms, so spend plenty of time covering these new ideas. Additionally, this cluster focuses on solving two-step equations and inequalities, while the 6th grade cluster focuses on one-step problems; provide time for students to struggle and to determine how to apply their previous knowledge from one-step problems rather than going straight to the procedural steps involved in a two-step problem. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of	numbers. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster moving from specific answers to generalizations for certain types of problems because properties of operations are generalized statements to help students understand the structure and pattern of expressions. Taking time to allow students to make the generalization from specific examples will help students deepen their understanding of generating equivalent expressions. Focus on conceptual understanding of solving equations and inequalities, as students might forget to keep the equation or inequality in balance when solving. Teachers can help students with this by having them use algebra tiles when solving equations and inequalities.

Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.	the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Clarify ideas such as factoring and expanding linear equations (particularly using the distributive property). Have students compare and contrast equations and inequalities.	

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/7/30/308/308		
Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners	In 7th grade, learners	In future classes, learners

• extend their knowledge of creating equivalent expressions to include situations in which a knowledge of the rules of integers are needed • extend their understanding of repeated addition as multiplication (representing 3 + 3 + 3 + 3 as 4 x 3) to simplify variable expressions (j+ j+ j+ j written as 4j) • use order of operations to solve equations and inequalities with more than one step, as well as those with negative coefficients • use variables to represent numbers and write expressions when solving a real-world or mathematical problem with equations or expressions, including multi-step problems and problems with rational numbers	• develop an understanding of operations with rational numbers when working with expressions and linear equations • apply knowledge of working with expressions and equations to solve problems involving scale drawings and informal geometric constructions • work with two- and three-dimensional shapes to solve problems involving area, surface area, and volume • use vertical angles, adjacent angles, angles on a line, and angles at a point in a multistep problem to write and solve equations for an unknown angle in a figure • develop an understanding of operations with rational numbers when working with expressions and linear equations, especially in application to problems involving area and volume • create scale drawings and geometric constructions	• solve linear equations in one variable • give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions • solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms • use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept • solve linear equations (including rational number coefficients) in one variable with one solution, infinitely many solutions, or no solutions • analyze and solve pairs of simultaneous linear equations (in one and two variables)

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*

• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can write expressions that represent situations their family and community might experience. • Students should make sure they know the real-world meaning of each part of the expression and can then create an equivalent expression and discuss what the new parts of the expression mean in reference to their family or community and the original expression. • Students can also calculate the cost of bills within a budget for a family, writing an expression or equation for each bill for the month. • Students could even create an inequality with the amount of money set aside for bills so they could determine the amount of discretionary money left after paying the bills. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.
Suggested Student Discourse Questions		
Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conv		

ersation%20Cards.pdf

- How can you justify that the two expressions/equations you have are equivalent?
- How can you assess the reasonableness of your answer?
- How can you use the properties of equality to express an expression/equation/inequality in a different but equivalent way?
- How can you prove your answer is correct?
- How can you relate the properties of operations to this real-world scenario?

Cross-Curricular Connections

Arts: Following the mathematical series of musical rhythms to learn the basic rhythms of dance. Calculating shutter speed, focal length, lighting angles, and exposure time.

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

Science: Writing number sentences for conservation of energy of a system. Collaborating with peers to define or describe an issue in society and how to evaluate solutions and running tests of solutions and change designs as needed. Constructing scientific arguments for how uneven distributions of earth's mineral, energy, groundwater resources are the result of past and current geoscience processes (e.g. metal ores, volcanic activity, soil weathering, rock deposits, and mining by humans). Solving chemistry problems. Understanding the movements of the planets. Solving equations when writing computer programs and figuring out algorithms. Comparing linear relationships and systems of equations in scientific data.

Career and Skill Connections

- Advertising
- Analysis
- Anthropology
- Archeology
- Architecture
- Arts
- Astronomy
- Atmospheric science
- Aviation
- Banking/finance
- Biology
- Bookkeeping
- Botany
- Business
- Carpentry
- Chemistry
- Choreography
- Computer programming
- Conservation science
- Construction
- Economist
- Education
- Electrician
- Engineering
- Environmental restoration
- Epidemiology
- Event planning
- Floral design
- Food science
- Forensics
- Forestry
- Fundraising
- Geology
- Health science
- HVAC
- Information technology
- Insurance
- Landscaping
- Law enforcement
- Machinist
- Mechanics
- Medicine
- Microbiology
- Mining
- Physical therapy
- Physics
- Plumbing
- Policy analysis
- Ranching/farming
- Sales
- Sociology
- Software development
- Soil science
- Statistics
- Technician
- Technology
- Transportation
- Travel agent
- Veterinary
- Video game design

• Counseling • Culinary arts • Ecology	• Management • Marketing	• Web development • Zoology

Grade	*CCSS Domain*	*CCSS Cluster*
7	Expressions and Equations	Use properties of operations to generate equivalent expressions.

Cluster Standard: 7.EE.A.2

Standard	Standards for Mathematical Practice
7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05".	● **SMP 5:** Use appropriate tools strategically. ○ Teacher and Student Actions ● **SMP 7:** Look for and make use of structure. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students apply properties of operations to add, subtract, factor and expand linear equations with rational coefficients. Students then become able to rewrite expressions in different forms to solve a multi-step problem and explain the quantities.	● Use properties to create equivalent expressions. ● Rewrite an expression in different forms. ● Demonstrate how quantities in an equation are related. ● Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide. ● Solve real-life and mathematical problems.
DOK	**Blooms**
1-2	Remember, Understand

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.
- Explain and relate properties of operations to strategies for adding, subtracting, factoring, and expanding linear expressions
- Explain the quantities in an expression and in solutions.
- Analyzing and interpreting problem contexts to identify quantities and relationships to be used in writing and rewriting expressions.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Malia is at an amusement park. She bought 14 tickets, and each ride requires 2 tickets.

1. Write an expression that gives the number of tickets Malia has left in terms of x, the number of rides she has already gone on. Find at least one other expression that is equivalent to it.

2. The expression *14 - 2x* represents the number of tickets Malia has left after she has gone on x rides. How can each of the following numbers and expressions be interpreted in terms of tickets and rides?
    a. 14
    b. -2
    c. -2x

3. *2(7 − x)* also represents the number of tickets Malia has left after she has gone on x rides. How can each of the following numbers and expressions be interpreted in terms of tickets and rides?
    a. 2
    b. 7
    c. (7 - x)

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Multi-step problems:** When an expression has several steps, sometimes students forget to follow the order of operations, struggle with moving from step to step, forget terms, etc.
- **Conversions:** When it is necessary to convert from a fraction to a decimal and vice versa, etc., students may struggle to rewrite those terms.
- **Combining like terms:** Students tend to combine all terms to arrive at a single term without realizing that some terms are dissimilar.
- **Distributive Property:** Students commonly overlook distributing the multiplier especially if the terms inside the parenthesis are composed of multiple terms.
- **Vocabulary:** If students are reading verbal descriptions or following directions to write an expression, they may struggle with the vocabulary (sum, product, difference, etc.) and interpreting the directions (e.g. subtract 1 from the number vs. subtract the number from 1).
- **Terms:** Students might be unsure how to relate the terms in an expression to their meaning in a real-world problem.
- **Units:** If a real-world problem involves units, students might struggle to keep track of those units and report their answers in the correct unit (rounded to the correct number of decimal places if needed).

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Representation Teachers can reduce barriers and	**Pre-teaching** In previous classes, learners worked	**Pre-teaching** Consider using standard 3.OA.B.5,

leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships.

Engagement

Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which

to extend their knowledge of creating equivalent expressions to include situations in which a knowledge of the rules of integers are needed, extend their understanding of repeated addition as multiplication, use order of operations to solve equations and inequalities with more than one step and those with negative coefficients, and use variables to represent numbers and write expressions when solving a real-world or mathematical problem with equations or expressions, including multi-step problems and problems with rational numbers. Students interpreted parts of an expression by using mathematical terms and viewing expressions as single entities.

Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as generating equivalent expressions (especially with the distributive property), comparing products, working with large numbers and numbers with units, multiplying fractions, understanding patterns in a table of values, writing and solving equations and inequalities, and writing and interpreting numerical expressions. Review and contrast the definitions of expression and equations, practice identifying parts of an expression, and review and practice the order of operations. This is the first experience students will have with variables, coefficients, and constants, and they will also be learning how to extend previous learning about exponents, the order of operations, equivalency, and like and unlike terms, so spend plenty of time covering these new ideas.

which provides a foundation for work in this cluster. In 3.OA.B.5, students use properties as strategies to multiply and divide and learn to use the commutative and associative properties of multiplication with whole numbers.

Also consider using standard 6.EE.B.7, which also provides a foundation for work in this cluster. In 6.EE.B.7, students solve real-world and mathematical problems in the form of $x + p = q$ and $px = q$, which are one step equations with positive rational numbers.

If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

Re-teaching

Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster moving from specific answers to generalizations for certain types of problems because properties of operations are generalized statements to help students understand the structure and pattern of expressions. Taking time to allow students to make the generalization from specific examples will help students deepen their

strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post	Additionally, this cluster focuses on solving two-step equations and inequalities, while the 6th grade cluster focuses on one-step problems; provide time for students to struggle and to determine how to apply their previous knowledge from one-step problems rather than going straight to the procedural steps involved in a two-step problem. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Clarify ideas such as factoring and expanding linear equations (particularly using the distributive property). Have students compare and contrast equations and inequalities.	understanding of generating equivalent expressions. Focus on conceptual understanding of solving equations and inequalities, as students might forget to keep the equation or inequality in balance when solving. Teachers can help students with this by having them use algebra tiles when solving equations and inequalities.

visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/7/30/309/309

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● extend their knowledge of creating equivalent expressions to include situations in which a knowledge of the rules of integers are needed ● extend their understanding of repeated addition as multiplication (representing 3 + 3 + 3 + 3 as 4 x 3) to simplify variable expressions (j+ j+ j+ j written as 4j) ● use order of operations to solve equations and inequalities with more than one step, as well as those with negative coefficients ● use variables to represent numbers and write expressions when solving a real-world or mathematical problem with equations or expressions, including multi-step problems and problems with rational numbers	In 7th grade, learners ● develop an understanding of operations with rational numbers when working with expressions and linear equations ● apply knowledge of working with expressions and equations to solve problems involving scale drawings and informal geometric constructions ● work with two- and three-dimensional shapes to solve problems involving area, surface area, and volume ● use vertical angles, adjacent angles, angles on a line, and angles at a point in a multistep problem to write and solve equations for an unknown angle in a figure ● develop an understanding of operations with rational numbers when working with expressions and linear equations, especially in application to problems involving area and volume ● create scale drawings and geometric constructions	In future classes, learners ● solve linear equations in one variable ● give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions ● solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms ● use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept ● solve linear equations (including rational number coefficients) in one variable with one solution, infinitely many solutions, or no solutions ● analyze and solve pairs of simultaneous linear equations (in one and two variables)

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can write expressions that represent situations their family and community might experience. • Students should make sure they know the real-world meaning of each part of the expression and can then create an equivalent expression and discuss what the new parts of the expression mean in reference to their family or community and the original expression. • Students can also calculate the cost of bills within a budget for a family, writing an expression or equation for each bill for the month. Students could even create an inequality with the amount of money set aside for bills so they could determine the amount of discretionary money left after paying the bills. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

• Consider inviting community members to talk with students about the math they use in their careers or crafts.	low expectations and low achievement.	

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- How does the meaning of an equation change when it is rewritten in an equivalent form?
- How can you use the distributive property in relation to factoring an expression?
- How can you use factoring in relation to the distributive property when working with an expression?
- How can you justify that the two expressions/equations you have are equivalent?
- How can you assess the reasonableness of your answer?
- How can you write equivalent expressions for a real-world situation? What information is important to consider?
- How can you use the properties of equality to express an expression/equation/inequality in a different but equivalent way?
- How can you prove your answer is correct?
- How can you relate the properties of operations to this real-world scenario?

Cross-Curricular Connections

Arts: Following the mathematical series of musical rhythms to learn the basic rhythms of dance. Calculating shutter speed, focal length, lighting angles, and exposure time.

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

Science: Writing number sentences for conservation of energy of a system. Collaborating with peers to define or describe an issue in society and how to evaluate solutions and running tests of solutions and change designs as needed. Constructing scientific arguments for how uneven distributions of earth's mineral, energy, groundwater resources are the result of past and current geoscience processes (e.g. metal ores, volcanic activity, soil weathering, rock deposits, and mining by humans). Solving chemistry problems. Understanding the movements of the planets. Solving equations when writing computer programs and figuring out algorithms. Comparing linear relationships and systems of equations in scientific data.

Career and Skill Connections

• Advertising • Analysis • Anthropology • Archeology • Architecture • Arts	• Economist • Education • Electrician • Engineering • Environmental restoration • Epidemiology	• Mechanics • Medicine • Microbiology • Mining • Physical therapy • Physics

• Astronomy • Atmospheric science • Aviation • Banking/finance • Biology • Bookkeeping • Botany • Business • Carpentry • Chemistry • Choreography • Computer programming • Conservation science • Construction • Counseling • Culinary arts • Ecology	• Event planning • Floral design • Food science • Forensics • Forestry • Fundraising • Geology • Health science • HVAC • Information technology • Insurance • Landscaping • Law enforcement • Machinist • Management • Marketing	• Plumbing • Policy analysis • Ranching/farming • Sales • Sociology • Software development • Soil science • Statistics • Technician • Technology • Transportation • Travel agent • Veterinary • Video game design • Web development • Zoology

Grade	CCSS Domain	CCSS Cluster
7	Expressions and Equations	Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Cluster Standard: 7.EE.B.3

Standard	Standards for Mathematical Practice

7.EE.B.3: Solve multi-step real life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1 /10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3 /4 inches long in the center of a door that is 27 1 /2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.	• **SMP 5**: Use appropriate tools strategically.
	  ○ Teacher and Student Actions
	• **SMP 8:** Look for and express regularity in repeated reasoning.
	  ○ Teacher and Student Actions

Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students apply properties of operations to add, subtract, factor and expand linear equations with rational coefficients. Students then become able to rewrite expressions in different forms to solve a multi-step problem and explain the quantities.	• Solve multi-step real life and mathematical problems that include positive and negative rational numbers. • Convert between fractions, decimals, and percentages. • Use properties of operations as needed to solve the problems. • Justify the reasonableness of their answers using estimation.
DOK	**Blooms**
2	Remember, Understand
Procedural and Conceptual Understanding and Application	
Conceptual Understanding:	

- Understand available tools and use them strategically to solve multi-step problems.
- Assess the reasonableness of answers.
- Understand estimation strategies.

Procedural Skill and Fluency:

- Solve multi-step mathematical problems posed with positive and negative rational numbers in any form while using tools strategically.
- Perform calculations using addition, subtraction, multiplication, and division on numbers in any form, converting between forms as needed.
- Convert between fractions, decimals, and percentages
- Assess the reasonableness of answers.

Application:

- Solving multi-step real life problems posed with positive and negative rational numbers in any form while using tools strategically.
- Use appropriate tools to model, illustrate, and solve multistep problems.
- Reflect on and justify the accuracy of answers in a real-world setting.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Katie and Margarita have $20.00 each to spend at Students' Choice bookstore, where all students receive a 20% discount. They both want to purchase a copy of the same book which normally sells for $22.50 plus 10% sales tax. To check if she has enough to purchase the book, Katie takes 20% of $22.50 and subtracts that amount from the normal price. She takes 10% of the discounted selling price and adds it back to find the purchase amount. Margarita takes 80% of the normal purchase price and then computes 110% of the reduced price.

1. Is Katie correct? Justify your answer. Does she have enough money to purchase the book?

2. Is Margarita correct? Justify your answer. Does she have enough money to purchase the book?

3. Explain how their processes are similar, how they are different, and why their answers are either both correct, both incorrect, or one is correct and one is incorrect.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Multi-step problems:** When an expression has several steps, sometimes students forget to follow the order of operations, struggle with moving from step to step, forget terms, etc.
- **Conversions:** When it is necessary to convert from a fraction to a decimal and vice versa, etc., students may struggle to rewrite those terms.
- **Combining like terms:** Students tend to combine all terms to arrive at a single term without realizing that some terms are dissimilar.
- **Distributive Property:** Students commonly overlook distributing the multiplier especially if the terms inside

the parenthesis are composed of multiple terms.

- **Vocabulary:** If students are reading verbal descriptions or following directions to write an expression, they may struggle with the vocabulary (sum, product, difference, etc.) and interpreting the directions (e.g. subtract 1 from the number vs. subtract the number from 1).
- **Terms:** Students might be unsure how to relate the terms in an expression to their meaning in a real-world problem.
- **Units:** If a real-world problem involves units, students might struggle to keep track of those units and report their answers in the correct unit (rounded to the correct number of decimal places if needed).
- **Negative terms in context:** Many problems use negative numbers to represent debt, depth below ground level, etc. Students might struggle to understand this because it is a convention that does not make sense in context (owing $30 is still 30, not -30) even if it makes problems easier to work with. Using visuals, number lines, etc. can help students understand the idea of negative numbers in context.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to	**Pre-teaching** In previous classes, learners worked to extend their knowledge of creating equivalent expressions to include situations in which a knowledge of the rules of integers are needed, extend their understanding of repeated addition as multiplication, use order of operations to solve equations and inequalities with more than one step and those with negative coefficients, and use variables to represent numbers and write expressions when solving a real-world or mathematical problem with equations or expressions, including multi-step problems and problems with rational numbers. Students interpreted parts of an expression by using mathematical terms and viewing expressions as single entities. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as generating equivalent expressions (especially with the distributive property),	**Pre-teaching** Consider using standard 3.OA.B.5, which provides a foundation for work in this cluster. In 3.OA.B.5, students use properties as strategies to multiply and divide and learn to use the commutative and associative properties of multiplication with whole numbers. Also consider using standard 6.EE.B.7, which also provides a foundation for work in this cluster. In 6.EE.B.7, students solve real-world and mathematical problems in the form of $x + p = q$ and $px = q$, which are one step equations with positive rational numbers. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships.

Engagement

Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals.

Action and Expression

Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate

comparing products, working with large numbers and numbers with units, multiplying fractions, understanding patterns in a table of values, writing and solving equations and inequalities, and writing and interpreting numerical expressions. Review and contrast the definitions of expression and equations, practice identifying parts of an expression, and review and practice the order of operations. This is the first experience students will have with variables, coefficients, and constants, and they will also be learning how to extend previous learning about exponents, the order of operations, equivalency, and like and unlike terms, so spend plenty of time covering these new ideas. Additionally, this cluster focuses on solving two-step equations and inequalities, while the 6th grade cluster focuses on one-step problems; provide time for students to struggle and to determine how to apply their previous knowledge from one-step problems rather than going straight to the procedural steps involved in a two-step problem.

If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

Re-teaching

Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and

Re-teaching

Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster moving from specific answers to generalizations for certain types of problems because properties of operations are generalized statements to help students understand the structure and pattern of expressions. Taking time to allow students to make the generalization from specific examples will help students deepen their understanding of generating equivalent expressions. Focus on conceptual understanding of solving equations and inequalities, as students might forget to keep the equation or inequality in balance when solving. Teachers can help students with this by having them use algebra tiles when solving equations and inequalities.

and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.	examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Clarify ideas such as factoring and expanding linear equations (particularly using the distributive property). Have students compare and contrast equations and inequalities.	

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/7/30/310/310

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● extend their knowledge of creating equivalent expressions to include situations in which a knowledge of the rules of integers are needed ● extend their understanding of repeated addition as multiplication (representing 3 + 3 + 3 + 3 as 4 x 3) to simplify variable expressions (j+ j+ j+ j written as 4j) ● use order of operations to	In 7th grade, learners ● develop an understanding of operations with rational numbers when working with expressions and linear equations ● apply knowledge of working with expressions and equations to solve problems involving scale drawings and informal geometric constructions ● work with two- and three-dimensional shapes to solve	In future classes, learners ● solve linear equations in one variable ● give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions ● solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and

solve equations and inequalities with more than one step, as well as those with negative coefficients • use variables to represent numbers and write expressions when solving a real-world or mathematical problem with equations or expressions, including multi-step problems and problems with rational numbers	problems involving area, surface area, and volume • use vertical angles, adjacent angles, angles on a line, and angles at a point in a multistep problem to write and solve equations for an unknown angle in a figure • develop an understanding of operations with rational numbers when working with expressions and linear equations, especially in application to problems involving area and volume • create scale drawings and geometric constructions	collecting like terms • use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept • solve linear equations (including rational number coefficients) in one variable with one solution, infinitely many solutions, or no solutions • analyze and solve pairs of simultaneous linear equations (in one and two variables)

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can write expressions that represent situations their family and community might experience. • Students should make sure	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for

they know the real-world meaning of each part of the expression and can then create an equivalent expression and discuss what the new parts of the expression mean in reference to their family or community and the original expression. • Students can also calculate the cost of bills within a budget for a family, writing an expression or equation for each bill for the month. • Students could even create an inequality with the amount of money set aside for bills so they could determine the amount of discretionary money left after paying the bills. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- How are positive and negative rational numbers related?
- How can you solve problems with algebraic expressions?
- How do you write and solve a multi-step equation?
- How can you apply the properties of application in solving a multi-step equation?
- How can you prove your answer is correct?
- What are the steps in solving a word problem?
- How can you relate the properties of operations to this real world scenario?
- How can you justify that the two expressions/equations you have are equivalent?
- How can you assess the reasonableness of your answer?
- How can you use the properties of equality to express an equation in a different but equivalent way?
- Is there another strategy you can use to solve this equation?

- Would this two-step equation yield the same solution if you reversed the order of the operations when solving it?
- How do we use variables to represent unknown quantities in mathematical problems to construct and solve simple equations and inequalities?

Cross-Curricular Connections

Arts: Following the mathematical series of musical rhythms to learn the basic rhythms of dance. Calculating shutter speed, focal length, lighting angles, and exposure time.

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

Science: Writing number sentences for conservation of energy of a system. Collaborating with peers to define or describe an issue in society and how to evaluate solutions and running tests of solutions and change designs as needed. Constructing scientific arguments for how uneven distributions of earth's mineral, energy, groundwater resources are the result of past and current geoscience processes (e.g. metal ores, volcanic activity, soil weathering, rock deposits, and mining by humans). Solving chemistry problems. Understanding the movements of the planets. Solving equations when writing computer programs and figuring out algorithms. Comparing linear relationships and systems of equations in scientific data.

Career and Skill Connections

- Advertising
- Analysis
- Anthropology
- Archeology
- Architecture
- Arts
- Astronomy
- Atmospheric science
- Aviation
- Banking/finance
- Biology
- Bookkeeping
- Botany
- Business
- Carpentry
- Chemistry
- Choreography
- Computer programming
- Conservation science
- Construction
- Counseling
- Culinary arts
- Ecology
- Economist
- Education
- Electrician
- Engineering
- Environmental restoration
- Epidemiology
- Event planning
- Floral design
- Food science
- Forensics
- Forestry
- Fundraising
- Geology
- Health science
- HVAC
- Information technology
- Insurance
- Landscaping
- Law enforcement
- Machinist
- Management
- Marketing
- Mechanics
- Medicine
- Microbiology
- Mining
- Physical therapy
- Physics
- Plumbing
- Policy analysis
- Ranching/farming
- Sales
- Sociology
- Software development
- Soil science
- Statistics
- Technician
- Technology
- Transportation
- Travel agent
- Veterinary
- Video game design
- Web development
- Zoology

Grade	CCSS Domain	CCSS Cluster
7	Expressions and Equations	Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Cluster Standard: 7.EE.B.4

Standard	Standards for Mathematical Practice
7.EE.B.4 Use variables to represent quantities in a real world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities. ● 7.EE.B.4.A: Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? ● 7.EE.B.4.B: Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make and describe the solutions.	● **SMP 1:** Make sense of problems and persevere in solving them. ○ Teacher and Student Actions ● **SMP 3:** Construct viable arguments and critique the reasoning of others. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students apply properties of operations to add, subtract, factor and expand linear equations with rational coefficients. Students then become able to rewrite expressions in different forms to solve a multi-step problem, and explain the quantities.	● Write equations in the appropriate form. ● Solve and graph inequalities. ● Apply the inequality and the solution in the context of the problem.
DOK	**Blooms**
2	Remember, Understand

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Construct simple equations and inequalities to solve problems by reasoning about the quantities.
- Understand the use of symbols to represent the unknown or changing quantities in a problem.
- Understand and interpret situations in which an equation should be used and when an inequality should be used.

Procedural Skill and Fluency:

- Use variables to represent quantities in a mathematical problem.
- Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers.
- Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers.
- Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach, such as in perimeter problems with one side length unknown.
- Construct equations or inequalities to solve problems.
- Use the properties of operations to solve problems.

Application:

- Use variables to model and solve real-world problems in equations and inequalities.
- Graph the solution set of an inequality and interpret it in the context of the problem.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

1. The taxi fare in Albuquerque is $2.40 for the first ½ mile and additional mileage charged at the rate $0.20 for each additional 0.1 mile. You plan to give the driver a $2 tip. How many miles can you ride for $10? Create an equation describing the situation and explain any variables you use.

2. Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can only carry 1200 pounds of people and gear for safety reasons. Assume the average weight of a person is 150 pounds. Each group will require 200 lbs of gear for the boat plus 10 lbs of gear for each person.

    a. Create an inequality describing the restrictions on the number of people possible in a rented boat and explain any variables you use. Graph the solution set.

    b. Several groups of people wish to rent a boat. Group 1 has 4 people. Group 2 has 5 people. Group 3 has 8 people. Which of the groups, if any, can safely rent a boat? Show your work or justify your answer.

    c. What is the maximum number of people that may rent a boat?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Multi-step problems:** When an expression has several steps, sometimes students forget to follow the order of operations, struggle with moving from step to step, forget terms, etc.
- **Conversions:** When it is necessary to convert from a fraction to a decimal and vice versa, etc., students may struggle to rewrite those terms.
- **Combining like terms:** Students tend to combine all terms to arrive at a single term without realizing that some terms are dissimilar.
- **Distributive Property:** Students commonly overlook distributing the multiplier especially if the terms inside the parenthesis are composed of multiple terms.
- **Vocabulary:** If students are reading verbal descriptions or following directions to write an expression, they may struggle with the vocabulary (sum, product, difference, etc.) and interpreting the directions (e.g. subtract 1 from the number vs. subtract the number from 1).
- **Terms:** Students might be unsure how to relate the terms in an expression to their meaning in a real-world problem.
- **Units:** If a real-world problem involves units, students might struggle to keep track of those units and report their answers in the correct unit (rounded to the correct number of decimal places if needed).
- **Inequality Symbol:** Students may be confused about which direction to write the inequality symbol and which symbol to use, or if given an inequality may not understand its meaning in the context of the problem.
- **Negative terms in context:** Many problems use negative numbers to represent debt, depth below ground level, etc. Students might struggle to understand this because it is a convention that does not make sense in context (owing $30 is still 30, not -30) even if it makes problems easier to work with. Using visuals, number lines, etc. can help students understand the idea of negative numbers in context.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and	**Pre-teaching** In previous classes, learners worked to extend their knowledge of creating equivalent expressions to include situations in which a knowledge of the rules of integers are needed, extend their understanding of repeated addition as multiplication, use order of operations to solve equations and inequalities with more than one step and those with negative coefficients, and use variables to represent numbers and write expressions when solving a real-world or mathematical problem with equations or expressions, including multi-step problems and problems with rational numbers. Students interpreted parts of an expression by using	**Pre-teaching** Consider using standard 3.OA.B.5, which provides a foundation for work in this cluster. In 3.OA.B.5, students use properties as strategies to multiply and divide and learn to use the commutative and associative properties of multiplication with whole numbers. Also consider using standard 6.EE.B.7, which also provides a foundation for work in this cluster. In 6.EE.B.7, students solve real-world and mathematical problems in the form of $x + p = q$ and $px = q$, which are one step equations with positive rational numbers.

highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort	mathematical terms and viewing expressions as single entities. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as generating equivalent expressions (especially with the distributive property), comparing products, working with large numbers and numbers with units, multiplying fractions, understanding patterns in a table of values, writing and solving equations and inequalities, and writing and interpreting numerical expressions. Review and contrast the definitions of expression and equations, practice identifying parts of an expression, and review and practice the order of operations. This is the first experience students will have with variables, coefficients, and constants, and they will also be learning how to extend previous learning about exponents, the order of operations, equivalency, and like and unlike terms, so spend plenty of time covering these new ideas. Additionally, this cluster focuses on solving two-step equations and inequalities, while the 6th grade cluster focuses on one-step problems; provide time for students to struggle and to determine how to apply their previous knowledge from one-step problems rather than going straight to the procedural steps involved in a two-step problem. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend	If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster moving from specific answers to generalizations for certain types of problems because properties of operations are generalized statements to help students understand the structure and pattern of expressions. Taking time to allow students to make the generalization from specific examples will help students deepen their understanding of generating equivalent expressions. Focus on conceptual understanding of solving equations and inequalities, as students might forget to keep the equation or inequality in balance when solving. Teachers can help students with this by having them use algebra tiles when solving equations and inequalities.

and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.	most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Clarify ideas such as factoring and expanding linear equations (particularly using the distributive property). Have students compare and contrast equations and inequalities.	

Vertical Alignment		
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Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● extend their knowledge of creating equivalent	In 7th grade, learners ● develop an understanding of operations with rational	In future classes, learners ● solve linear equations in one variable

expressions to include situations in which a knowledge of the rules of integers are needed • extend their understanding of repeated addition as multiplication (representing 3 + 3 + 3 + 3 as 4 x 3) to simplify variable expressions (j+ j+ j+ j written as 4j) • use order of operations to solve equations and inequalities with more than one step, as well as those with negative coefficients • use variables to represent numbers and write expressions when solving a real-world or mathematical problem with equations or expressions, including multi-step problems and problems with rational numbers	numbers when working with expressions and linear equations • apply knowledge of working with expressions and equations to solve problems involving scale drawings and informal geometric constructions • work with two- and three-dimensional shapes to solve problems involving area, surface area, and volume • use vertical angles, adjacent angles, angles on a line, and angles at a point in a multistep problem to write and solve equations for an unknown angle in a figure • develop an understanding of operations with rational numbers when working with expressions and linear equations, especially in application to problems involving area and volume • create scale drawings and geometric constructions	• give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions • solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms • use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept • solve linear equations (including rational number coefficients) in one variable with one solution, infinitely many solutions, or no solutions • analyze and solve pairs of simultaneous linear equations (in one and two variables)

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- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and	• Instruction should begin with conceptual understanding	• Scaffold tasks and amplify language so students can

communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can write expressions that represent situations their family and community might experience. • Students should make sure they know the real-world meaning of each part of the expression and can then create an equivalent expression and discuss what the new parts of the expression mean in reference to their family or community and the original expression. • Students can also calculate the cost of bills within a budget for a family, writing an expression or equation for each bill for the month. • Students could even create an inequality with the amount of money set aside for bills so they could determine the amount of discretionary money left after paying the bills. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.
Suggested Student Discourse Questions		
Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf		

- What is the same about solving equations and inequalities?
- What is different about solving equations and inequalities?
- What are the steps in solving a word problem?
- How can you relate the properties of operations to this real-world scenario?
- Solve this problem algebraically and then arithmetically. What connections do you see? What differences?
- How can you justify that the two expressions/equations you have are equivalent?
- How can you assess the reasonableness of your answer?
- How can you write equivalent expressions for a real-world situation? What information is important to consider?
- How can you use the properties of equality to express an equation in a different but equivalent way?
- Is there another strategy you can use to solve this equation?
- Would this two-step equation yield the same solution if you reversed the order of the operations when solving it?
- How do we use variables to represent unknown quantities in mathematical problems to construct and solve simple equations and inequalities?
- How should we deal with negative coefficients when solving inequalities? Why do we have to follow this process?
- How can you prove your answer is correct?

Cross-Curricular Connections

Arts: Following the mathematical series of musical rhythms to learn the basic rhythms of dance. Calculating shutter speed, focal length, lighting angles, and exposure time.

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

Science: Writing number sentences for conservation of energy of a system. Collaborating with peers to define or describe an issue in society and how to evaluate solutions and running tests of solutions and change designs as needed. Constructing scientific arguments for how uneven distributions of earth's mineral, energy, groundwater resources are the result of past and current geoscience processes (e.g. metal ores, volcanic activity, soil weathering, rock deposits, and mining by humans). Solving chemistry problems. Understanding the movements of the planets. Solving equations when writing computer programs and figuring out algorithms. Comparing linear relationships and systems of equations in scientific data.

Career and Skill Connections

- Advertising
- Analysis
- Anthropology
- Archeology
- Architecture
- Arts
- Astronomy
- Atmospheric science
- Aviation
- Banking/finance
- Economist
- Education
- Electrician
- Engineering
- Environmental restoration
- Epidemiology
- Event planning
- Floral design
- Food science
- Forensics
- Mechanics
- Medicine
- Microbiology
- Mining
- Physical therapy
- Physics
- Plumbing
- Policy analysis
- Ranching/farming
- Sales

• Biology • Bookkeeping • Botany • Business • Carpentry • Chemistry • Choreography • Computer programming • Conservation science • Construction • Counseling • Culinary arts • Ecology	• Forestry • Fundraising • Geology • Health science • HVAC • Information technology • Insurance • Landscaping • Law enforcement • Machinist • Management • Marketing	• Sociology • Software development • Soil science • Statistics • Technician • Technology • Transportation • Travel agent • Veterinary • Video game design • Web development • Zoology

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards for Mathematical Practice
    - Connections to procedural, conceptual understanding, and application
    - Sample assessment items
    - Common misconceptions
    - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
    - Vertical alignment
    - Culturally and Linguistically Responsive Instruction (CLRI)
    - Suggested student discourse questions
    - Cross-curricular and career/skill connections
- A Student Discourse Guide

Helpful links:

- Lesson-planning tool from Pathways 2 Careers (click here to sign up with your district email if you don't already have an account)
- Focus by Grade Level from Achieve the Core
- Coherence Map from Achieve the Core

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Draw construct and describe geometrical figures and describe the relationships between them
  - 7.G.A.1
  - 7.G.A.2
  - 7.G.A.3
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume
  - 7.G.B.4
  - 7.G.B.5
  - 7.G.B.6

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)

Grade	CCSS Domain	CCSS Cluster
7	Geometry	Draw, construct and describe geometrical figures and describe the relationships between them.

Cluster Standard: 7.G.A.1

Standard	Standards for Mathematical Practice

7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	• **SMP 2**: Reason abstractly and quantitatively.
	  ○ Teacher and Student Actions
	• **SMP 4:** Model with mathematics.
	  ○ Teacher and Student Actions

Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students work to draw and construct geometric shapes, particularly triangles from given angle and side measurements. Students find relationships and connections between a 3D figure and slicing it into a plane figure. Students use scale drawings to find the actual lengths from scale drawing or redrawing a scale drawing to another scale.	• Solve problems involving scale drawings. • Calculate length and area from scale drawings. • Reproduce a scale drawing at a different scale.
DOK	**Blooms**
1-2	Remember, Understand

Procedural and Conceptual Understanding and Application

Procedural Skill and Fluency:

- Solve problems involving scale drawings of geometric figures.
- Calculate actual lengths and areas from a scale drawing.
- Slice a 3D figure into a plane figure.

Application:

- Draw/construct geometric shapes, particularly triangles, from given angle and side measurements.
- Find relationships and connections between a 3D figure and slicing it into a plane figure.
- Reproduce a scale drawing at a different scale.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Mariko has an 80:1 scale-drawing of the floor plan of her house. On the floor plan, the dimensions of her rectangular living room are $1\frac{7}{8}$ inches by $2\frac{1}{2}$ inches.

1. What is the area of her real living room in square feet?

2. If the scale was 40:2 instead, what would the dimensions of the living room be on the floor plan? Find the dimensions and make a scale drawing below.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Proportions and ratios:** Students may not understand ratios/proportions and may struggle to use them to solve problems.
- **Converting between units:** Students commonly struggle with conversions and might not be able to determine if their answers are reasonable when converting to larger or smaller units of measure.
- **Area:** Students may struggle to understand what area is and struggle to remember how to calculate it.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply	**Pre-teaching** In previous classes, learners worked to classify two dimensional figures in a hierarchy based on properties, understand and solve ratios and rates, generate equivalent ratios, use ratios and rates to solve problems, calculate perimeter and area of two dimensional figures and find volume of three dimensional figures, explore the characteristics of a right rectangular prism and rectangular pyramid, find the area of rectangles, special quadrilaterals, triangles, and polygons, explore volume, finding the volume of rectangular prisms, and find surface area using nets. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key	**Pre-teaching** Consider using standard 5.NF.B.4, which provides a foundation for work in this cluster. In 5.NF.B.4, students develop procedural fluency while performing operations with fractions. Also consider using standard 6.G.A.1, which also provides a foundation for work in this cluster. In 6.G.A.1, students solve problems involving area and volume when provided a context. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-

background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals.	concepts and skills such as connecting geometric shapes in math to those used in cultural art and design, connecting the student's home life to the mathematical principles they are learning at school. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Allow students to explore the key features of commonly used shapes, then work on scaling given shapes before moving into constructing the shapes by hand. Allow students to use visuals and manipulatives. Review formulas and what the various pieces in them mean and how they connect to the shapes they describe.	level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster moving away from generating specific answers and instead making generalizations for certain types of problems. Teachers can support students by giving them practice using various geometric tools.

Action and Expression Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/7/31/316/316		
Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● classify two dimensional figures in a hierarchy based on properties ● understand and solve ratios and rates, generate equivalent ratios, and use	In 7th grade, learners ● write and solve equations related to similar figures, scale drawings, and the missing angle measures of triangles ● work with similar figures	In future classes, learners ● connect their previous understanding of similar figures with the properties of translations, rotations, reflections and dilations ● build on their

ratios and rates to solve problems • calculate perimeter and area of two dimensional figures and find volume of three dimensional figures • explore the characteristics of a right rectangular prism and rectangular pyramid • learn how to find the area of rectangles, special quadrilaterals, triangles, and polygons • explore volume, finding the volume of rectangular prisms • find surface area using nets	• study direct variation and proportional reasoning • determine missing information about particular geometric figures	experimentation with triangles and start to make informal arguments about their properties, such as angle sum, exterior angles of a triangle, and angles created when parallel lines are cut by a transversal line • build on knowledge of triangle side lengths, which leads to the investigation of the Pythagorean Theorem and its converse • find the volume of cones, cylinders, and spheres

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about geometric shapes that are used in cultural art and design. • Students can also talk with their families about the type	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and

of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- What is the relationship between the ratios of side lengths and areas of geometric figures in scale drawings?
- How can we apply proportions when you draw a scale model?
- How do you apply scale drawing or similar figures in the real world?
- What is the difference and similarities between scale drawing and scale factors?
- How can you use the scale drawing to find the actual length and area of this shape?

Cross-Curricular Connections

Arts: Making/interpreting geometric and architectural drawings. Crafting art pieces based on understanding basic geometric formulas. Using a 3-D glass shape to create sand art and calculating the amount of sand needed to create their art piece.

Literature: Using linear, logical thinking to write more clearly and logically.

Science: Modeling the solar system at scale. Solving equations when writing computer programs and figuring out algorithms.

Career and Skill Connections		
• Architecture • Arts • Atmospheric science • Aviation • CAD Programmer • Carpentry • Cartography • Chemistry • Computer programming • Construction • Criminal investigation • Culinary arts • Education • Electrician • Engineering • Fashion design • Fencing • FIlm editing	• Film/show set design • Firefighting • Floor laying • Forestry • Geology • Graphic design • Historian • Illustrator • Industrial design • Interior design/decoration • Landscaping • Machinist • Maintenance • Management • Masonry • Mechanical drafting • Mechanics • Metal fabrication/metalworking	• Model-making • Optometry • Photography • Physical therapy • Plumbing • Publishing • Ranching/farming • Real estate • Robotics • Roofing • Special effects animation • Surveying • Technician • Technologist • Urban/regional planning • Veterinary • Web design

Grade	*CCSS Domain*	*CCSS Cluster*
7	Geometry	Draw, construct and describe geometrical figures and describe the relationships between them.

Cluster Standard: 7.G.A.2

Standard	Standards for Mathematical Practice
7.G.A.2: Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	● **SMP 5:** Use appropriate tools strategically. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students work to draw and construct geometric shapes, particularly triangles from given angle and side measurements. Students find relationships and connections between a 3D figure and slicing it into a plane figure. Students use scale drawings to find the actual lengths from scale drawing or redrawing a scale drawing to another scale.	● Draw a geometric figure with given conditions. ● Explain why a set of given conditions does (or does not) produce the desired figure. ● Measure side lengths and angle measures with given tools.
DOK	**Blooms**
1-2	Remember, Understand

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Understand the characteristics and properties of geometric shapes, specifically triangles, including angles and side lengths
- Understand the process of constructing triangles based on given conditions
- Comprehend the process of scaling (up or down) of a geometric figure
- Familiarize the use of tools in measuring side lengths and angle measures
- Understand the relationship of a 3D figure and the plane figure made after slicing it.
- Understand and explain why a set of given conditions does (or does not) produce the desired figure.

Procedural Skill and Fluency:

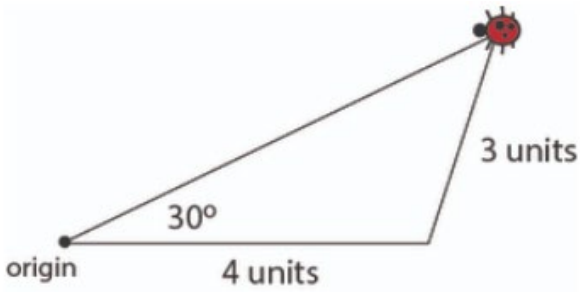
- Draw a geometric figure with a given condition
- Verify that a set of given conditions does (or does not) produce the desired figure.

- Measure side lengths and angle measures with given tools.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Starting at the origin, a ladybug walked 4 units east. Then she walked a distance of 3 units in an unknown direction. At that time she was 30 degrees to the north of her original walking direction. The diagram here shows one possibility for the ladybug's final location.



1. Find a different final location that is also consistent with the given information, and draw the ladybug there.

2. Are there any more possibilities other than the diagram you were given and the one you just came up with? Explain why or why not.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Proportions and ratios:** Students may not understand ratios/proportions and may struggle to use them to scale problems..
- **Assumptions on 3 sides and angles:** Students may think that if they are given three sides and three angles, it always forms a triangle regardless of the lengths and angle measures.
- **Angle Measures:** Students may misinterpret angle measures or incorrectly measure angles, leading to inaccuracies in triangle construction.
- **Converting between units:** Students commonly struggle with conversions and might not be able to determine if their answers are reasonable when converting to larger or smaller units of measure.
- **Area:** Students may struggle to understand what area is and struggle to remember how to calculate it.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc.	**Pre-teaching** In previous classes, learners worked to classify two dimensional figures in a hierarchy based on properties, understand and solve ratios and rates, generate equivalent ratios, use ratios and rates to solve problems, calculate perimeter and area of two	**Pre-teaching** Consider using standard 5.NF.B.4, which provides a foundation for work in this cluster. In 5.NF.B.4, students develop procedural fluency while performing operations with fractions. Also consider using standard 6.G.A.1,

Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into	dimensional figures and find volume of three dimensional figures, explore the characteristics of a right rectangular prism and rectangular pyramid, find the area of rectangles, special quadrilaterals, triangles, and polygons, explore volume, finding the volume of rectangular prisms, and find surface area using nets. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as connecting geometric shapes in math to those used in cultural art and design, connecting the student's home life to the mathematical principles they are learning at school. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Allow students to explore the key features of commonly used shapes, then work on scaling given shapes before moving into constructing the shapes	which also provides a foundation for work in this cluster. In 6.G.A.1, students solve problems involving area and volume when provided a context. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster moving away from generating specific answers and instead making generalizations for certain types of problems. Teachers can support students by giving them practice using various geometric tools.

more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and	by hand. Allow students to use visuals and manipulatives. Review formulas and what the various pieces in them mean and how they connect to the shapes they describe.	

schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/7/31/317/317

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● classify two dimensional figures in a hierarchy based on properties ● understand and solve ratios and rates, generate equivalent ratios, and use ratios and rates to solve problems ● calculate perimeter and area of two-dimensional figures and find volume of three-dimensional figures ● explore the characteristics of a right rectangular prism and rectangular pyramid ● learn how to find the area of rectangles, special quadrilaterals, triangles, and polygons ● explore volume, finding the volume of rectangular prisms ● find surface area using nets	In 7th grade, learners ● write and solve equations related to similar figures, scale drawings, and the missing angle measures of triangles ● work with similar figures ● study direct variation and proportional reasoning ● determine missing information about particular geometric figures	In future classes, learners ● connect their previous understanding of similar figures with the properties of translations, rotations, reflections and dilations ● build on their experimentation with triangles and start to make informal arguments about their properties, such as angle sum, exterior angles of a triangle, and angles created when parallel lines are cut by a transversal line ● build on knowledge of triangle side lengths, which leads to the investigation of the Pythagorean Theorem and its converse ● find the volume of cones, cylinders, and spheres

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:
- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about geometric shapes that are used in cultural art and design. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.
Suggested Student Discourse Questions		
Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf		

- What are the key components of the given shape? What can be different between different shapes of this same kind?
- What are the key components of a triangle? What can be different between triangles?
- Compare your drawing with your partners. Do they have to be the exact same drawing?
- Is there more than one correct way to draw the figure with these conditions? Why or why not?

Cross-Curricular Connections

Arts: Making/interpreting geometric and architectural drawings. Crafting art pieces based on understanding basic geometric formulas. Using a 3-D glass shape to create sand art and calculating the amount of sand needed to create their art piece.

Literature: Using linear, logical thinking to write more clearly and logically.

Science: Modeling the solar system at scale. Solving equations when writing computer programs and figuring out algorithms.

Career and Skill Connections

- Architecture
- Arts
- Atmospheric science
- Aviation
- CAD Programmer
- Carpentry
- Cartography
- Chemistry
- Computer programming
- Construction
- Criminal investigation
- Culinary arts
- Education
- Electrician
- Engineering
- Fashion design
- Fencing
- FIlm editing
- Film/show set design
- Firefighting
- Floor laying
- Forestry
- Geology
- Graphic design
- Historian
- Illustrator
- Industrial design
- Interior design/decoration
- Landscaping
- Machinist
- Maintenance
- Management
- Masonry
- Mechanical drafting
- Mechanics
- Metal fabrication/metalworking
- Model-making
- Optometry
- Photography
- Physical therapy
- Plumbing
- Publishing
- Ranching/farming
- Real estate
- Robotics
- Roofing
- Special effects animation
- Surveying
- Technician
- Technologist
- Urban/regional planning
- Veterinary
- Web design

Grade	CCSS Domain	CCSS Cluster
7	Geometry	Draw, construct and describe geometrical figures and describe the relationships between them.

Cluster Standard: 7.G.A.3

Standard	Standards for Mathematical Practice
7.G.A.3: Describe the two-dimensional figures that result from slicing three dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	● **SMP 6**: Attend to precision. ○ Teacher and Student Actions ● **SMP 8**: Look for and express regularity in repeated reasoning. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students work to draw and construct geometric shapes, particularly triangles from given angle and side measurements. Students find relationships and connections between a 3D figure and slicing it into a plane figure. Students use scale drawings to find the actual lengths from scale drawing or redrawing a scale drawing to another scale.	● Identify the two-dimensional cross-sections that are formed by slicing three-dimensional figures. ● Describe the resulting face shape from cuts made parallel and perpendicular to the bases of right rectangular prisms and pyramids.
DOK	**Blooms**
1-3	Understand, Apply, Analyze

Procedural and Conceptual Understanding and Application

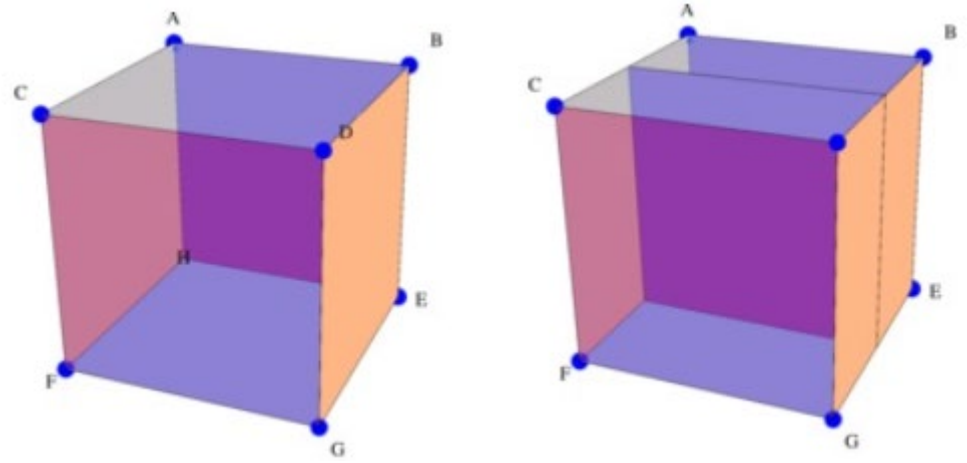
Conceptual Understanding:

- Understand the two-dimensional cross-sections that are formed by slicing three-dimensional figures.
- Recognize and describe resulting two-dimensional plane figures that result from slicing three-dimensional figures.
- Understand and describe the properties and attributes of the resulting plane figure from cuts made parallel and perpendicular to the bases of right rectangular prisms and pyramids.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Imagine you are a ninja that can slice solid objects straight through. You have a solid cube in front of you. You are curious about what 2-dimensional shapes are formed when you slice the cube. For example, if you make a slice through the center of the cube that is parallel to one of the faces, the cross section is a square:



For each of the following slices, describe using precise mathematical language the shape of the cross section. Then draw a diagram showing the cross section of the cube.

1. A slice containing edge AC and edge EG
2. A slice containing the vertices C, B, and G.
3. A slice containing the vertex A, the midpoint of edge EG, and the midpoint of edge FG.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Slice shape:** Students tend to assume that the resulting plane figure will be uniform regardless of the orientation and position.
- **Scale Drawing:** Students without a solid grasp of measurement units such as those for area will have difficulty with this standard, as will students who need more help with proportional reasoning. Use the opportunity to measure the classroom or other hands-on measurements to reinforce measurement units for those students.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language,	**Pre-teaching** In previous classes, learners worked to classify two dimensional figures in a hierarchy based on properties, understand and solve ratios and rates, generate equivalent ratios, use	**Pre-teaching** Consider using standard 5.NF.B.4, which provides a foundation for work in this cluster. In 5.NF.B.4, students develop procedural fluency while performing operations with fractions.

different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible	ratios and rates to solve problems, calculate perimeter and area of two dimensional figures and find volume of three dimensional figures, explore the characteristics of a right rectangular prism and rectangular pyramid, find the area of rectangles, special quadrilaterals, triangles, and polygons, explore volume, finding the volume of rectangular prisms, and find surface area using nets. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as connecting geometric shapes in math to those used in cultural art and design, connecting the student's home life to the mathematical principles they are learning at school. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Allow students to explore the key features of commonly used shapes, then work	Also consider using standard 6.G.A.1, which also provides a foundation for work in this cluster. In 6.G.A.1, students solve problems involving area and volume when provided a context. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster moving away from generating specific answers and instead making generalizations for certain types of problems. Teachers can support students by giving them practice using various geometric tools.

timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to	on scaling given shapes before moving into constructing the shapes by hand. Allow students to use visuals and manipulatives. Review formulas and what the various pieces in them mean and how they connect to the shapes they describe.	

monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning:
https://tools.achievethecore.org/coherence-map/7/31/318/318

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners • classify two dimensional figures in a hierarchy based on properties • understand and solve ratios and rates, generate equivalent ratios, and use ratios and rates to solve problems • calculate perimeter and area of two dimensional figures and find volume of three dimensional figures • explore the characteristics of a right rectangular prism and rectangular pyramid • learn how to find the area of rectangles, special quadrilaterals, triangles, and polygons • explore volume, finding the volume of rectangular prisms • find surface area using nets	In 7th grade, learners • write and solve equations related to similar figures, scale drawings, and the missing angle measures of triangles • work with similar figures • study direct variation and proportional reasoning • determine missing information about particular geometric figures	In future classes, learners • connect their previous understanding of similar figures with the properties of translations, rotations, reflections and dilations • build on their experimentation with triangles and start to make informal arguments about their properties, such as angle sum, exterior angles of a triangle, and angles created when parallel lines are cut by a transversal line • build on knowledge of triangle side lengths, which leads to the investigation of the Pythagorean Theorem and its converse • find the volume of cones, cylinders, and spheres

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:
- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating

mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about geometric shapes that are used in cultural art and design. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.
Suggested Student Discourse Questions		
Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conv		

ersation%20Cards.pdf
• How can you make the slice more real so that it's easier to see? • How do we solve real-world and mathematical problems involving areas of two-dimensional objects composed of triangles, quadrilaterals, and polygons using area and surface area?
Cross-Curricular Connections
Arts: Making/interpreting geometric and architectural drawings. Crafting art pieces based on understanding basic geometric formulas. Using a 3-D glass shape to create sand art and calculating the amount of sand needed to create their art piece. **Literature:** Using linear, logical thinking to write more clearly and logically. **Science:** Modeling the solar system at scale. Solving equations when writing computer programs and figuring out algorithms.
Career and Skill Connections

• Architecture • Arts • Atmospheric science • Aviation • CAD Programmer • Carpentry • Cartography • Chemistry • Computer programming • Construction • Criminal investigation • Culinary arts • Education • Electrician • Engineering • Fashion design • Fencing • FIlm editing	• Film/show set design • Firefighting • Floor laying • Forestry • Geology • Graphic design • Historian • Illustrator • Industrial design • Interior design/decoration • Landscaping • Machinist • Maintenance • Management • Masonry • Mechanical drafting • Mechanics • Metal fabrication/metalworking	• Model-making • Optometry • Photography • Physical therapy • Plumbing • Publishing • Ranching/farming • Real estate • Robotics • Roofing • Special effects animation • Surveying • Technician • Technologist • Urban/regional planning • Veterinary • Web design

Grade	CCSS Domain	CCSS Cluster
7	Geometry	Solve real-life and mathematical problems involving angle measure, area, surface area, and volume

Cluster Standard: 7.G.B.4

Standard	Standards for Mathematical Practice
7.G.B.4: Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	• **SMP 1:** Make sense of problems and persevere in solving them. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students work on geometric problem solving. Students use basic information such as area, surface area, and volume formulas and facts about types of angles (supplementary, complementary, vertical, and adjacent) to solve real-world problems.	• Explain the relationships between radius and diameter. • Explain that the ratio of circumference to diameter can be expressed as pi. • Apply formulas to determine area, circumference, diameter, and radius of a circle to solve real-world problems. • Solve real world problems involving circumference and area of a circle.
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Understand the relationship between the circumference and area of a circle.
- Understand and explain the relationship between a circle's radius and its diameter.
- Explain the ratio of circumference to diameter can be expressed as pi.

Procedural Skill and Fluency:

- Solve geometric problems involving area, surface area, and volume.
- Find the radius, circumference, and diameter of a circle.
- Verify the formula of surface area of the solid with the net of the solid

Application:

- Apply formulas to determine the area, circumference, diameter and radius of a circle to solve real-world scenarios.
- Draw, construct and describe geometrical figures and its relationship

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Juan wants to know the cross-sectional area of a circular pipe. He measures the diameter which he finds, to the nearest millimeter, to be 5 centimeters.

1. How large is the possible error in Juan's measurement of the diameter of the circle? Explain.
2. As a percentage of the diameter, how large is the possible error in Juan's measurement?
3. What is the area of the circular pipe?
4. As a percentage, how large is the possible error in your measurement for the area of the circle?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Formulas:** Students may choose the wrong formula for calculations. The formulas for the area of a circle and the circumference of a circle are often confused by students. Teaching students to memorize these formulas without any understanding of how they relate to a circle increases the chance for confusion. In cases with complex shapes, students must truly understand the formulas for circles in order to find partial perimeters and/or areas.
- **Unit of measurement:** Students tend to forget to write the unit of measurement of the given geometric shapes.
- **Scale Drawing:** Students without a solid grasp of measurement units such as those for area will have difficulty with this standard, as will students who need more help with proportional reasoning. Use the opportunity to measure the classroom or other hands-on measurements to reinforce measurement units for those students.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

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- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating

mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about geometric shapes that are used in cultural art and design. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.
Suggested Student Discourse Questions		
Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conv		

ersation%20Cards.pdf

- How do we find the circumference and area of a circle using the relationship between them?
- What are real life situations that use the area and circumference of a circle?
- How does changing the radius of a circle affect its area and circumference?
- Can you think of any strategies to help remember the formulas for the area and circumference of a circle?

Cross-Curricular Connections

Arts: Making/interpreting geometric and architectural drawings. Crafting art pieces based on understanding basic geometric formulas. Using a 3-D glass shape to create sand art and calculating the amount of sand needed to create their art piece.

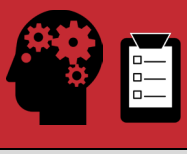
Literature: Using linear, logical thinking to write more clearly and logically.

Science: Modeling the solar system at scale. Solving equations when writing computer programs and figuring out algorithms.

Career and Skill Connections

- Architecture
- Arts
- Atmospheric science
- Aviation
- CAD Programmer
- Carpentry
- Cartography
- Chemistry
- Computer programming
- Construction
- Criminal investigation
- Culinary arts
- Education
- Electrician
- Engineering
- Fashion design
- Fencing
- Film editing
- Film/show set design
- Firefighting
- Floor laying
- Forestry
- Geology
- Graphic design
- Historian
- Illustrator
- Industrial design
- Interior design/decoration
- Landscaping
- Machinist
- Maintenance
- Management
- Masonry
- Mechanical drafting
- Mechanics
- Metal fabrication/metalworking
- Model-making
- Optometry
- Photography
- Physical therapy
- Plumbing
- Publishing
- Ranching/farming
- Real estate
- Robotics
- Roofing
- Special effects animation
- Surveying
- Technician
- Technologist
- Urban/regional planning
- Veterinary
- Web design

Grade	CCSS Domain	CCSS Cluster
7	Geometry	Solve real-life and mathematical problems involving angle measure, area, surface area, and volume



Cluster Standard: 7.G.B.5

Standard	Standards for Mathematical Practice
7.G.B.5: Use facts about supplementary, complementary, vertical, and adjacent angles in a multistep problem to write and solve simple equations for an unknown angle in a figure.	● **SMP 1:** Make sense of problems and persevere in solving them. ○ Teacher and Student Actions ● **SMP 4:** Model with mathematics. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students work on geometric problem solving. Students use basic information such as area, surface area, and volume formulas and facts about types of angles (supplementary, complementary, vertical, and adjacent) to solve real-world problems.	● Use understandings of angles (supplementary, complementary, vertical, adjacent) and deductive reasoning to write and solve equations. ● Write and solve equations based on a diagram of intersecting lines with some known angle measures. ● Justify angle measurements using facts about complementary, supplementary, vertical and/or adjacent angles.
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Understand and describe the differences of supplementary, complementary and vertical angles.
- Understand that real-world problems can be solved using geometric properties.
- Recognize equations based on diagrams and measurements of angles.
- Justify angle measurements using facts about complementary, supplementary, vertical and/or adjacent angles.

Procedural Skill and Fluency:

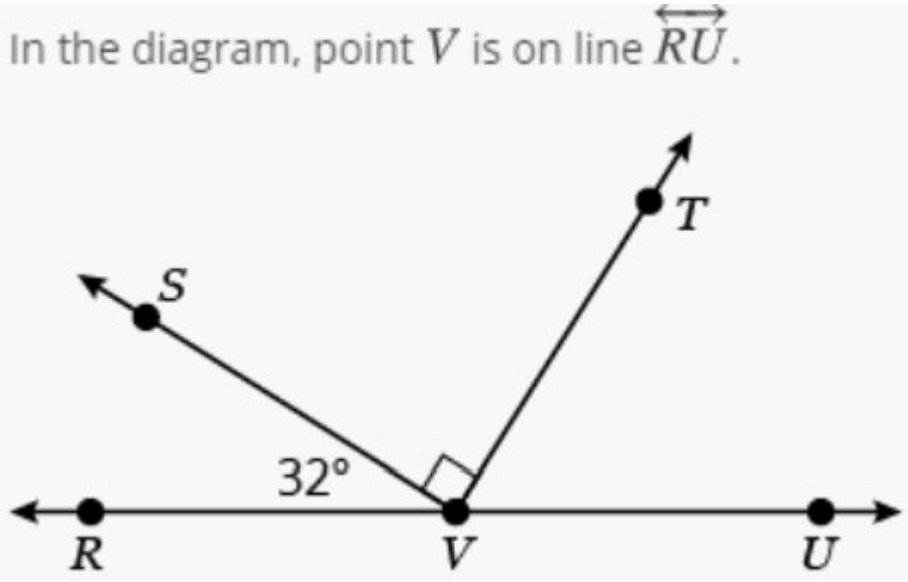
- Solve multi-step problems involving complementary, supplementary, vertical and/or adjacent angles.
- Find the unknown angle in a given equation.

- Draw supplementary, complementary, vertical, and adjacent angles.
- Recognize the types of angles.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Use the diagram to find the measure of angle *TVU*. Explain your reasoning/show your work.



You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Diagrams:** Students may struggle to interpret figures.
- **Measurement:** Students may struggle to measure accurately and/or choose the wrong tools when taking measurements.
- **Vocabulary:** Students may struggle to understand and remember all angle types.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating	**Pre-teaching** In previous classes, learners worked to classify two dimensional figures in a hierarchy based on properties, understand and solve ratios and	**Pre-teaching** Consider using standard 5.NF.B.4, which provides a foundation for work in this cluster. In 5.NF.B.4, students develop procedural fluency while

displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they	rates, generate equivalent ratios, use ratios and rates to solve problems, calculate perimeter and area of two dimensional figures and find volume of three dimensional figures, explore the characteristics of a right rectangular prism and rectangular pyramid, find the area of rectangles, special quadrilaterals, triangles, and polygons, explore volume, finding the volume of rectangular prisms, and find surface area using nets. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as connecting geometric shapes in math to those used in cultural art and design, connecting the student's home life to the mathematical principles they are learning at school. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Allow students to explore the key features	performing operations with fractions. Also consider using standard 6.G.A.1, which also provides a foundation for work in this cluster. In 6.G.A.1, students solve problems involving area and volume when provided a context. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster moving away from generating specific answers and instead making generalizations for certain types of problems. Teachers can support students by giving them practice using various geometric tools.

complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-	of commonly used shapes, then work on scaling given shapes before moving into constructing the shapes by hand. Allow students to use visuals and manipulatives. Review formulas and what the various pieces in them mean and how they connect to the shapes they describe.	

assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/7/31/322/322

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners • classify two dimensional figures in a hierarchy based on properties • understand and solve ratios and rates, generate equivalent ratios, and use ratios and rates to solve problems • calculate perimeter and area of two dimensional figures and find volume of three dimensional figures • explore the characteristics of a right rectangular prism and rectangular pyramid • learn how to find the area of rectangles, special quadrilaterals, triangles, and polygons • explore volume, finding the volume of rectangular prisms • find surface area using nets	In 7th grade, learners • write and solve equations related to similar figures, scale drawings, and the missing angle measures of triangles • work with similar figures • study direct variation and proportional reasoning • determine missing information about particular geometric figures	In future classes, learners • connect their previous understanding of similar figures with the properties of translations, rotations, reflections and dilations • build on their experimentation with triangles and start to make informal arguments about their properties, such as angle sum, exterior angles of a triangle, and angles created when parallel lines are cut by a transversal line • build on knowledge of triangle side lengths, which leads to the investigation of the Pythagorean Theorem and its converse • find the volume of cones, cylinders, and spheres

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:
- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture

and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about geometric shapes that are used in cultural art and design. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.
Suggested Student Discourse Questions		
Consider this resource for student discourse from Pathways2Careers:		

https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- Can you explain the difference between supplementary and complementary angles?
- How would you identify vertical angles in a figure?
- How can you use the properties of supplementary angles to solve for an unknown angle in a figure?
- What strategy would you use to write and solve equations for finding unknown angles in a figure?
- What are different ways to partition this two-dimension object to find the area? How can you verify that the different strategies yield the same result?
- How do we solve real-world and mathematical problems involving areas of two-dimensional objects composed of triangles, quadrilaterals, and polygons using area and surface area?

Cross-Curricular Connections

Arts: Making/interpreting geometric and architectural drawings. Crafting art pieces based on understanding basic geometric formulas. Using a 3-D glass shape to create sand art and calculating the amount of sand needed to create their art piece.

Literature: Using linear, logical thinking to write more clearly and logically.

Science: Modeling the solar system at scale. Solving equations when writing computer programs and figuring out algorithms.

Career and Skill Connections

- Architecture
- Arts
- Atmospheric science
- Aviation
- CAD Programmer
- Carpentry
- Cartography
- Chemistry
- Computer programming
- Construction
- Criminal investigation
- Culinary arts
- Education
- Electrician
- Engineering
- Fashion design
- Fencing
- FIlm editing
- Film/show set design
- Firefighting
- Floor laying
- Forestry
- Geology
- Graphic design
- Historian
- Illustrator
- Industrial design
- Interior design/decoration
- Landscaping
- Machinist
- Maintenance
- Management
- Masonry
- Mechanical drafting
- Mechanics
- Metal fabrication/metalworking
- Model-making
- Optometry
- Photography
- Physical therapy
- Plumbing
- Publishing
- Ranching/farming
- Real estate
- Robotics
- Roofing
- Special effects animation
- Surveying
- Technician
- Technologist
- Urban/regional planning
- Veterinary
- Web design

Grade	CCSS Domain	CCSS Cluster
7	Geometry	Solve real-life and mathematical problems involving angle measure, area, surface area, and volume

Cluster Standard: 7.G.B.6

Standard	Standards for Mathematical Practice

7.G.B.6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	• **SMP 1:** Make sense of problems and persevere in solving them.
	  ○ Teacher and Student Actions
	• **SMP 4:** Model with mathematics.
	  ○ Teacher and Student Actions

Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students work on geometric problem solving. Students use basic information such as area, surface area, and volume formulas and facts about types of angles (supplementary, complementary, vertical, and adjacent) to solve real-world problems.	• Use understandings of angles (supplementary, complementary, vertical, adjacent) and deductive reasoning to write and solve equations. • Write and solve equations based on a diagram of intersecting lines with some known angle measures. • Justify angle measurements using facts about complementary, supplementary, vertical and/or adjacent angles.
DOK	**Blooms**
1-2	Apply

Procedural and Conceptual Understanding and Application

Procedural Skill and Fluency:

- Write and solve equations based on a diagram of intersecting lines with some known angle measures.
- Draw geometric figures and types of angles.
- Compute the surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Application:

- Apply formula to solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects
- Construct a creative design using geometric figures and angles on graphing paper.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

The 7th graders at Sunview Middle School were helping to renovate a playground for the kindergartners at a nearby elementary school. City regulations require that the sand underneath the swings be at least 15 inches deep. The sand under both swing sets was only 12 inches deep when they started. The rectangular area under the small swing set measures 9 feet by 12 feet and required 40 bags of sand to increase the depth by 3 inches. How many bags of sand will the students need to cover the rectangular area under the large swing set if it is 1.5 times as long and 1.5 times as wide as the area under the small swing set?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Diagrams**: Students may struggle to interpret diagrams or draw accurate diagrams of verbal explanations.
- **Formulas:** Students may choose the wrong formula for calculations. The formulas for the area of a circle and the circumference of a circle are often confused by students. Teaching students to memorize these formulas without any understanding of how they relate to a circle increases the chance for confusion. In cases with complex shapes, students must truly understand the formulas for circles in order to find partial perimeters and/or areas.
- **Area:** Students may struggle to understand what area is and struggle to remember how to calculate it.
- **Volume:** Students may struggle to understand what volume is and struggle to remember how to calculate it.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media,	**Pre-teaching** In previous classes, learners worked to classify two dimensional figures in a hierarchy based on properties, understand and solve ratios and rates, generate equivalent ratios, use ratios and rates to solve problems, calculate perimeter and area of two dimensional figures and find volume of three dimensional figures, explore the characteristics of a right rectangular prism and rectangular pyramid, find the area of rectangles, special quadrilaterals, triangles, and polygons, explore volume, finding the volume of rectangular prisms, and find surface area using nets.	**Pre-teaching** Consider using standard 5.NF.B.4, which provides a foundation for work in this cluster. In 5.NF.B.4, students develop procedural fluency while performing operations with fractions. Also consider using standard 6.G.A.1, which also provides a foundation for work in this cluster. In 6.G.A.1, students solve problems involving area and volume when provided a context. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-

tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community.	Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as connecting geometric shapes in math to those used in cultural art and design, connecting the student's home life to the mathematical principles they are learning at school. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Allow students to explore the key features of commonly used shapes, then work on scaling given shapes before moving into constructing the shapes by hand. Allow students to use visuals and manipulatives. Review formulas and what the various pieces in them mean and how they connect to the shapes they describe.	teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster moving away from generating specific answers and instead making generalizations for certain types of problems. Teachers can support students by giving them practice using various geometric tools.

Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/7/31/323/323		
Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners	In 7th grade, learners	In future classes, learners

• classify two dimensional figures in a hierarchy based on properties • understand and solve ratios and rates, generate equivalent ratios, and use ratios and rates to solve problems • calculate perimeter and area of two-dimensional figures and find volume of three dimensional figures • explore the characteristics of a right rectangular prism and rectangular pyramid • learn how to find the area of rectangles, special quadrilaterals, triangles, and polygons • explore volume, finding the volume of rectangular prisms • find surface area using nets	• write and solve equations related to similar figures, scale drawings, and the missing angle measures of triangles • work with similar figures • study direct variation and proportional reasoning • determine missing information about particular geometric figures	• connect their previous understanding of similar figures with the properties of translations, rotations, reflections and dilations • build on their experimentation with triangles and start to make informal arguments about their properties, such as angle sum, exterior angles of a triangle, and angles created when parallel lines are cut by a transversal line • build on knowledge of triangle side lengths, which leads to the investigation of the Pythagorean Theorem and its converse • find the volume of cones, cylinders, and spheres

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example,	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their

students can learn about geometric shapes that are used in cultural art and design. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.
Suggested Student Discourse Questions		
Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf		
• Can you explain the difference between volume and surface area? • How do you verify the formula of surface area of solid with the net of the solid? • What are different ways to partition this two-dimension object to find the area? How can you verify that the different strategies yield the same result? • How do we solve real-world and mathematical problems involving areas of two-dimensional objects composed of triangles, quadrilaterals, and polygons using area and surface area? • Explain each part of the formula of surface area and what each part represents on the net of the solid.		
Cross-Curricular Connections		

Arts: Making/interpreting geometric and architectural drawings. Crafting art pieces based on understanding basic geometric formulas. Using a 3-D glass shape to create sand art and calculating the amount of sand needed to create their art piece.

Literature: Using linear, logical thinking to write more clearly and logically.

Science: Modeling the solar system at scale. Solving equations when writing computer programs and figuring out algorithms.

Career and Skill Connections

- Architecture
- Arts
- Atmospheric science
- Aviation
- CAD Programmer
- Carpentry
- Cartography
- Chemistry
- Computer programming
- Construction
- Criminal investigation
- Culinary arts
- Education
- Electrician
- Engineering
- Fashion design
- Fencing
- FIlm editing
- Film/show set design
- Firefighting
- Floor laying
- Forestry
- Geology
- Graphic design
- Historian
- Illustrator
- Industrial design
- Interior design/decoration
- Landscaping
- Machinist
- Maintenance
- Management
- Masonry
- Mechanical drafting
- Mechanics
- Metal fabrication/metalworking
- Model-making
- Optometry
- Photography
- Physical therapy
- Plumbing
- Publishing
- Ranching/farming
- Real estate
- Robotics
- Roofing
- Special effects animation
- Surveying
- Technician
- Technologist
- Urban/regional planning
- Veterinary
- Web design

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards for Mathematical Practice
    - Connections to procedural, conceptual understanding, and application
    - Sample assessment items
    - Common misconceptions
    - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
    - Vertical alignment
    - Culturally and Linguistically Responsive Instruction (CLRI)
    - Suggested student discourse questions
    - Cross-curricular and career/skill connections
- A Student Discourse Guide

Helpful links:

- Lesson-planning tool from Pathways 2 Careers (click here to sign up with your district email if you don't already have an account)
- Focus by Grade Level from Achieve the Core
- Coherence Map from Achieve the Core

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
  - 7.NS.A.1
  - 7.NS.A.2
  - 7.NS.A.3

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)

Grade	*CCSS Domain*	*CCSS Cluster*
7	The Number System	Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Cluster Standard: 7.NS.A.1

Standard	Standards for Mathematical Practice
7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. ● 7.NS.A.1.A: Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. ● 7.NS.A.1.B: Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. ● 7.NS.A.1.C: Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts. ● 7.NS.A.1.D: Apply properties of operations as strategies to add and subtract rational numbers.	● **SMP 5:** Use appropriate tools strategically. ○ Teacher and Student Actions ● **SMP 8:** Look for and express regularity in repeated reasoning. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students learn to add and subtract rational numbers. As students begin this work visual representations are critical; they become less necessary as students become more fluent with these operations. In sixth grade, students found the distance of horizontal and vertical segments on the coordinate plane. In seventh grade, students build on this understanding to recognize subtraction as finding the distance between two numbers	● Solve numerical addition and subtraction equations by using the properties of operations. ● Define and apply the commutative, associative, and additive identity properties to rational numbers. ● Formulate rules for integer operations. ● Expressively (orally and in writing) express understanding of “positive”, “negative”, “additive

on a number line. This standard allows for adding and subtracting negative fractions and decimals and interpreting solutions in a given context. Students should learn to use the terms “rational numbers”, “additive inverse”, and “integers” with increasing precision.	inverse”, and “zero”. ● Model combining positive and negative numbers and provide a rationale for their solutions. ● Apply mathematics to real world examples of positive and negative numbers.
DOK	**Blooms**
1-2	Understand

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Understand and explain how quantities of the same size but opposite signs cancel each other out.
- Understand and represent addition of rational numbers on a number line.
- Understand and represent subtraction as adding the additive inverse of rational numbers on a number line.
- Model combining positive and negative numbers and provide a rationale for their solutions.
- Apply mathematics to real world examples of positive and negative numbers.
- Express (orally and in writing) understanding of “positive”, “negative”, “additive inverse”, and “zero”.

Procedural Skill and Fluency:

- Solve numerical addition and subtraction equations by using the properties of operations.
- Define and apply the commutative, associative, and additive identity properties to rational numbers.
- Formulate rules for integer operations.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don’t have local instructional materials available.

1. Aakash, Bao Ying, Chris and Donna all live on the same street as their school, which runs from east to west. Aakash lives 512 blocks to the west. Bao Ying lives 414 blocks to the east. Chris lives 234 blocks to the west. Donna lives 612 blocks to the east.

    a. Draw a picture that represents the positions of their houses along the street.

    b. Find how far is each house from every other house?

    c. Represent the relative position of the houses on a number line, with the school at zero, points to the west represented by negative numbers, and points to the east represented by positive numbers.

    d. How can you see the answers to part (b) on the number line? Using the numbers (some of which are positive and some negative) that label the positions of houses on the number line, represent these distances using sums or differences.

2. At the beginning of the month, Evan had $24 in his account at the school bookstore. Use a variable to represent the unknown quantity in each transaction below and write an equation to represent it. Then represent each transaction on a number line. What is the unknown quantity in each case?

    a. First he bought some notebooks and pens that cost $16.

    b. Then he deposited some more money and his account balance was $28.

    c. Then he bought a book for English class that cost $34.

    d. Then he deposited exactly enough money so that he paid off his debt to the bookstore.

3. Explain why it makes sense to use a negative number to represent Evan's account balance when he owes money.

4. Explain how you found the answer to part d. Represent your solution with a number line.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Integer operations:** Students may have trouble conceptualizing integer operations, especially subtraction since it is not commutative. They also may find difficulties with conceptualization when expressing that subtraction is the same as adding the inverse.
- **Number lines:** Students may struggle to use the number lines, especially when representing numbers as distance from 0.
- **Signed numbers:** Students may have trouble representing some quantities as positive or negative, especially in real-world scenarios, unless using a visual model or number line. They may also struggle with finding and interpreting additive inverse pairs that result in 0. Students may especially struggle when they have to write their answer in words or explain verbally, but doing so will allow the teacher to catch misconceptions. Many problems use negative numbers to represent debt, depth below ground level, etc. Students might struggle to understand this because it is a convention that does not make sense in context (owing $30 is still 30, not -30) even if it makes problems easier to work with. Using visuals, number lines, etc. can help students understand the idea of negative numbers in context.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes,	**Pre-teaching** In previous classes, learners worked to understand that positive and negative numbers are used together to describe quantities having opposite directions or values, solve problems involving dividing fractions	**Pre-teaching** Consider using standard 6.NS.C.5, which provides a foundation for work in this cluster. In 6.NS.C.5, students apply and extend previous understanding of operations on fractions (addition, subtraction,

diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more	by fractions, and use the order of operations to solve problems. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as operations on fractions (addition, subtraction, multiplication, and division). If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Allow students to practice working with operations on fractions.	multiplication, and division). In grade 6 the number line is extended to include negative numbers and students use negative numbers in contexts where it is natural to describe both the magnitude of the quantity, e.g. vertical distance from sea level in meters, and the direction of the quantity, e.g. above or below sea level. Review number lines and both the mathematical and contextual meanings that negative numbers can represent. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster connecting operations with fractions to rational numbers.

manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self- assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/7/32/325/325		
Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners • understand that positive and negative numbers are used together to describe quantities having opposite directions or values • solve problems involving dividing fractions by fractions • use the order of operations to solve problems	In 7th grade, learners • apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers • solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form • use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities	In future classes, learners • understand that there are numbers that are not rational and approximate them by rational numbers • use square root and cube root symbols to represent solutions to equations
Culturally and Linguistically Responsive Instruction		
Consider these resources for vocabulary from Pathways2Careers: • https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf • https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf		
Consider these questions as you plan for instruction that is culturally and linguistically responsive: • How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist.

school to create stronger home to school connections for students. For example, students can consider fractions used for cooking. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	• Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- What happens when we add two positive numbers? What happens when we subtract two positive numbers? Can you draw a picture to represent each situation? Can you show it on a number line?
- What happens when we add two negative numbers? What happens when we subtract two negative numbers? Can you draw a picture to represent each situation? Can you show it on a number line?
- Can you rewrite an additional problem with subtraction? What about rewriting a subtraction problem with addition?

- What do the lines on your number line mean? What does the 0 mean?
- How can we predict that the sum of two integers is positive, negative or zero?
- How is subtraction the same as adding the inverse (additive property)?
- Can you make a connection between the commutative property and how we live in our community?
- What is the difference between the opposite of a number and the absolute value of a number?

Cross-Curricular Connections

Arts: Following the mathematical series of musical rhythms to learn the basic rhythms of dance.

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Using linear, logical thinking to write more clearly and logically.

Science: Representing collected data in different forms.

Career and Skill Connections

● Archeology	● Culinary arts	● Management
● Arts	● Education	● Mechanics
● Banking/finance	● Electrician	● Metalworking
● Business	● Engineering	● Video game design
● Computer programming	● Information technology	● Web design

Grade	CCSS Domain	CCSS Cluster
7	The Number System	Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Cluster Standard: 7.NS.A.2

Standard	Standards for Mathematical Practice
7.NS.A.2: Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. ● 7.NS.A.2.A: Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. ● 7.NS.A.2.B: Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then –(p/q) = (–p)/q = p/(– q). Interpret quotients of rational numbers by describing real world contexts. ● 7.NS.A.2.C: Apply properties of operations as strategies to multiply and divide rational numbers. ● 7.NS.A.2.D: Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	● **SMP 5:** Use appropriate tools strategically. ○ Teacher and Student Actions ● **SMP 6**: Attend to precision. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students learn to add and subtract rational numbers. As students begin this work visual representations are critical; they become less necessary as students become more fluent with these operations. In sixth grade, students found the distance of horizontal and vertical segments on the coordinate plane. In seventh grade,	● Discover the rules for multiplying rational numbers Refer to the negative (-) sign correctly as "negative" or ``the opposite of" to make sense of real-world context. ● Conclude that properties of the operations for multiplication are still applicable to rational

students build on this understanding to recognize subtraction as finding the distance between two numbers on a number line. This standard allows for adding and subtracting negative fractions and decimals and interpreting solutions in a given context. Students should learn to use the terms “rational numbers”, “additive inverse”, and “integers” with increasing precision.	numbers. ● Use reasoning to determine that division by zero is undefined. ● Discover that division as the inverse of multiplication still applies to rational numbers. ● Generalize rules for division with signed numbers from examples. Use and articulate notations interchangeably p ÷(-q) is the same as p⁄-q. Interpret a rational quotient ● Clarify their own understanding of the relationship between multiplications and division of rational numbers through writing. ● Develop fluency through practice with multiplication and division of rational numbers. ● Use properties of the operations to explain the solutions to real world problems. ● Clarify and explain their understanding of properties of operations using mathematical discourse. ● Use math vocabulary appropriately. ● Use long division to convert rational numbers in fraction form to decimal form ● Explain why and how they know a long division quotient will repeat. ● Sort the decimal form of a rational number into two types: terminating or repeating.
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Understand that the rules in multiplying and dividing fractions in rational numbers are applicable to rational numbers.
- Recognize that properties of the operations for multiplication are applicable to rational numbers.
- Understand that integers can be divided (except by zero) and every quotient of integers (with non-zero divisor) is a rational number.
- Understand that rational numbers divided by zero are undefined.
- Interpret quotients of rational numbers by describing real world contexts.
- Understand the process of converting rational numbers into decimal through long division.
- Understand that the decimal form of a rational number terminates in 0s or eventually repeats.
- Explain why a long division quotient will repeat.
- Use properties of the operations to explain the solutions to real world problems.
- Clarify and explain their understanding of properties of operations using mathematical discourse.

Procedural Skill and Fluency:

- Multiply and divide rational numbers
- Multiply and divide rational numbers in the real-world context.
- Use math vocabulary appropriately.
- Use long division to convert rational numbers in fraction form to decimal form.
- Sort the decimal form of a rational number into two types: terminating or repeating.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

1. A water well drilling rig has dug to a height of –60 feet after one full day of continuous use.

a. Assuming the rig drilled at a constant rate, what was the height of the drill after 15 hours?

b. If the rig has been running constantly and is currently at a height of –143.6 feet, for how long has the rig been running?

2. Each expression below gives the rate of change of temperature in degrees Celsius over a certain period of time. Interpret each expression below in terms of the temperature change.

a. $-\frac{2\ degrees}{3\ hours}$ b. $\frac{-2\ degrees}{3\ hours}$ c. $\frac{2\ degrees}{-3\ hours}$

3. Consider the rational number $\frac{29}{13}$

a. Use long division to find the repeating decimal that represents $\frac{29}{13}$

b. Take the number obtained by including only the first two digits after the decimal point, and multiply that by 13.

c. Take the number obtained by including only the first four digits after the decimal point, and multiply that by 13.

d. Take the number obtained by including only the first six digits after the decimal point, and multiply that by 13.

e. What do you notice about the product of 13 and decimal approximations of $\frac{29}{13}$ as more and more digits are included after the decimal point?

f. How does what you observed in Part (e) help make sense of what it means for $\frac{29}{13}$ to be equal to the repeating decimal expression you found in the Part (a)?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Inverse relationship:** Students may assume that multiplication and division are unrelated operations. They may not recognize that division is the inverse of multiplication and vice versa.
- **Negative signs:** Students may be confused about the concept of multiplying and dividing negative rational numbers, particularly when dealing with negatives and/or different signs.
- **Fraction and decimal operations**: Students may struggle when applying fraction and decimal operations to rational numbers. This may include difficulty in simplifying fractions and converting fractions to decimals and seeing how decimals and fractions are connected. Students may struggle with what a repeating decimal represents and the fact that when we use decimals they are often rounded.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to	**Pre-teaching** In previous classes, learners worked to understand that positive and negative numbers are used together to describe quantities having opposite directions or values, solve problems involving dividing fractions by fractions, and use the order of operations to solve problems. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as operations on fractions (addition, subtraction, multiplication, and division). If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at	**Pre-teaching** Consider using standard 6.NS.C.5, which provides a foundation for work in this cluster. In 6.NS.C.5, students apply and extend previous understanding of operations on fractions (addition, subtraction, multiplication, and division). In grade 6 the number line is extended to include negative numbers and students use negative numbers in contexts where it is natural to describe both the magnitude of the quantity, e.g. vertical distance from sea level in meters, and the direction of the quantity, e.g. above or below sea level. Review number lines and both the mathematical and contextual meanings that negative numbers can represent. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching**

emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives,	revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Allow students to practice working with operations on fractions.	Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster connecting operations with fractions to rational numbers.

drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/7/32/325/336

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● understand that positive and negative numbers are used together to describe quantities having opposite directions or values ● solve problems involving dividing fractions by fractions ● use the order of operations to solve problems	In 7th grade, learners ● apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers ● solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form ● use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities	In future classes, learners ● understand that there are numbers that are not rational and approximate them by rational numbers ● use square root and cube root symbols to represent solutions to equations

Culturally and Linguistically Responsive Instruction		
Consider these resources for vocabulary from Pathways2Careers: • https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf • https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf		
Consider these questions as you plan for instruction that is culturally and linguistically responsive: • How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can consider fractions used for cooking. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

	also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- How can you convert rational numbers to decimal numbers?
- What happens when we multiply positive numbers? What happens when we divide two positive numbers? Can you draw a picture to represent each situation? Can you show it on a number line?
- What happens when we multiply two negative numbers? What happens when we divide two negative numbers? Can you draw a picture to represent each situation? Can you show it on a number line?
- What happens when we multiply one positive and one negative number? What happens when we divide one positive and one negative number? Can you draw a picture to represent each situation? Can you show it on a number line?
- How does multiplying integers relate to dividing integers and vice versa?
- What do the lines on your number line mean? What does the 0 mean?
- How can we predict that the sum of two integers is positive, negative or zero?
- Can you make a connection between the commutative property and how we live in our community?
- What is the difference between the opposite of a number and the absolute value of a number?

Cross-Curricular Connections

Arts: Following the mathematical series of musical rhythms to learn the basic rhythms of dance.

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Using linear, logical thinking to write more clearly and logically.

Science: Representing collected data in different forms.

Career and Skill Connections

• Archeology • Arts • Banking/finance • Business • Computer programming	• Culinary arts • Education • Electrician • Engineering • Information technology	• Management • Mechanics • Metalworking • Video game design • Web design

Grade	*CCSS Domain*	*CCSS Cluster*
7	The Number System	Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Cluster Standard: 7.NS.A.3

Standard	Standards for Mathematical Practice

7.NS.A.3: Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions).	• **SMP 2**: Reason abstractly and quantitatively.
	  ○ Teacher and Student Actions
	• **SMP 3**: Construct viable arguments and critique the reasoning of others.
	  ○ Teacher and Student Actions
	• **SMP 4**: Model with mathematics.
	  ○ Teacher and Student Actions

Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students learn to add and subtract rational numbers. As students begin this work visual representations are critical; they become less necessary as students become more fluent with these operations. In sixth grade, students found the distance of horizontal and vertical segments on the coordinate plane. In seventh grade, students build on this understanding to recognize subtraction as finding the distance between two numbers on a number line. This standard allows for adding and subtracting negative fractions and decimals and interpreting solutions in given context. Students should learn to use the terms “rational numbers”, “additive inverse”, and “integers” with increasing precision	• Apply operations with rational numbers to problems that involve the order of operations. • Solve mathematical problems that use the four operations with rational numbers. • Compute with complex fractions.
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Procedural Skill and Fluency:

- Solve mathematical problems involving the four operations with rational numbers.
- Add and subtract rational numbers, including negative fractions and decimals.

Application:

- Solve real-world problems involving the four operations with rational numbers.
- Add and subtract rational numbers, including negative fractions and decimals, and interpret solutions in the context of the problem.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

1. The three seventh grade classes at Sunview Middle School collected the most box tops for a school fundraiser, and so they won a $600 prize to share among them. Mr. Aceves' class collected 3,760 box tops, Mrs. Baca's class collected 2,301, and Mr. Canyon's class collected 1,855. How should they divide the money so that each class gets the same fraction of the prize money as the fraction of the box tops that they collected?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Operations on rational numbers:** Students may assume that the rules of operations in whole numbers are the same for rational numbers.
- **Complex fractions:** Students may struggle to understand and simplify complex fractions.
- **Simplifying solutions**: Students usually forget to simplify solutions when solving rational numbers.
- **Signed numbers:** Students may have trouble representing some quantities as positive or negative, especially in real-world scenarios, unless using a visual model or number line. They may also struggle with finding and interpreting additive inverse pairs that result in 0. Students may especially struggle when they have to write their answer in words or explain verbally, but doing so will allow the teacher to catch misconceptions. Many problems use negative numbers to represent debt, depth below ground level, etc. Students might struggle to understand this because it is a convention that does not make sense in context (owing $30 is still 30, not -30) even if it makes problems easier to work with. Using visuals, number lines, etc. can help students understand the idea of negative numbers in context.
- **Inverse relationship:** Students may assume that multiplication and division are unrelated operations. They may not recognize that division is the inverse of multiplication and vice versa.
- **Negative signs:** Students may be confused about the concept of multiplying and dividing negative rational numbers, particularly when dealing with negatives and/or different signs.
- **Fraction and decimal operations**: Students may struggle when applying fraction and decimal operations to rational numbers. This may include difficulty in simplifying fractions and converting fractions to decimals and seeing how decimals and fractions are connected. Students may struggle with what a repeating decimal represents and the fact that when we use decimals they are often rounded.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Representation Teachers can reduce barriers and	**Pre-teaching** In previous classes, learners worked	**Pre-teaching** Consider using standard 6.NS.C.5,

leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide	to understand that positive and negative numbers are used together to describe quantities having opposite directions or values, solve problems involving dividing fractions by fractions, and use the order of operations to solve problems. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as operations on fractions (addition, subtraction, multiplication, and division). If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Allow students to practice working with operations on fractions.	which provides a foundation for work in this cluster. In 6.NS.C.5, students apply and extend previous understanding of operations on fractions (addition, subtraction, multiplication, and division). In grade 6 the number line is extended to include negative numbers and students use negative numbers in contexts where it is natural to describe both the magnitude of the quantity, e.g. vertical distance from sea level in meters, and the direction of the quantity, e.g. above or below sea level. Review number lines and both the mathematical and contextual meanings that negative numbers can represent. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster connecting operations with fractions to rational numbers.

which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **<u>Action and Expression</u>** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills		

in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/7/32/325/349

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● understand that positive and negative numbers are used together to describe quantities having opposite directions or values ● solve problems involving dividing fractions by fractions ● use the order of operations to solve problems	In 7th grade, learners ● apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers ● solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form ● use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities	In future classes, learners ● understand that there are numbers that are not rational and approximate them by rational numbers ● use square root and cube root symbols to represent solutions to equations

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:
- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can consider fractions used for cooking. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.
Suggested Student Discourse Questions		
Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf		

- How can you convert rational numbers to decimal numbers?
- What happens when we add two positive numbers? What happens when we subtract two positive numbers? Can you draw a picture to represent each situation? Can you show it on a number line?
- What happens when we add two negative numbers? What happens when we subtract two negative numbers? Can you draw a picture to represent each situation? Can you show it on a number line?
- What happens when we multiply positive numbers? What happens when we divide two positive numbers? Can you draw a picture to represent each situation? Can you show it on a number line?
- What happens when we multiply two negative numbers? What happens when we divide two negative numbers? Can you draw a picture to represent each situation? Can you show it on a number line?
- What happens when we multiply one positive and one negative number? What happens when we divide one positive and one negative number? Can you draw a picture to represent each situation? Can you show it on a number line?
- How does multiplying integers relate to dividing integers and vice versa?
- What do the lines on your number line mean? What does the 0 mean?
- How can we predict that the sum of two integers is positive, negative or zero?
- What is the difference between the opposite of a number and the absolute value of a number?
- How can you tell a word problem involves a complex fraction?

Cross-Curricular Connections

Arts: Following the mathematical series of musical rhythms to learn the basic rhythms of dance.

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Using linear, logical thinking to write more clearly and logically.

Science: Representing collected data in different forms.

Career and Skill Connections

• Archeology • Arts • Banking/finance • Business • Computer programming	• Culinary arts • Education • Electrician • Engineering • Information technology	• Management • Mechanics • Metalworking • Video game design • Web design

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards for Mathematical Practice
    - Connections to procedural, conceptual understanding, and application
    - Sample assessment items
    - Common misconceptions
    - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
    - Vertical alignment
    - Culturally and Linguistically Responsive Instruction (CLRI)
    - Suggested student discourse questions
    - Cross-curricular and career/skill connections
- A Student Discourse Guide

Helpful links:

- Lesson-planning tool from Pathways 2 Careers (click here to sign up with your district email if you don't already have an account)
- Focus by Grade Level from Achieve the Core
- Coherence Map from Achieve the Core

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Analyze proportional relationships and use them to solve real-world and mathematical problems
  - 7.RP.A.1
  - 7.RP.A.2
  - 7.RP.A.3

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)

Grade	CCSS Domain	CCSS Cluster
7	Ratios and Proportional Relationships	Analyze proportional relationships and use them to solve real-world and mathematical problems.

Cluster Standard: 7.RP.A.1

Standard	Standards for Mathematical Practice

7.RP.A.1: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½ / ¼ miles per hour, equivalently 2 miles per hour.	• **SMP 1:** Make sense of problems and persevere in solving them.
	  ○ Teacher and Student Actions
	• **SMP 6:** Attend to precision.
	  ○ Teacher and Student Actions

Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students will continue their work with ratios to analyze proportions and proportional relationships.	• Discover that the structure of computing unit rates with whole numbers is the same concept as unit rates with ratios of fractions. • Compute unit rates in real world problems that involve complex fractions. • In writing, explain the errors that can be made when computing unit rates with complex fractions.
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Procedural Skill and Fluency:

- Compute unit rates in real world problems with complex fractions, including fractions involving length, area, and other quantities (with the same or different units)
- Work with ratios to analyze proportions and proportional relationships
- Explain errors that can be made when computing unit rates with complex fractions

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Angel and Jayden were at track practice. The track is $\frac{2}{5}$ kilometers around. Angel ran 1 lap in 2 minutes. Jayden ran 3 laps in 5 minutes.

1. How many minutes does it take Angel to run one kilometer? What about Jayden?
2. How far does Angel run in one minute? What about Jayden?
3. Who is running faster? Explain your reasoning.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Direct versus proportional division:** Students might use direct instead of proportional division, mistakenly thinking that if it takes 2 people 4 hours to do a certain task, it would take 1 person 2 hours rather than 8 hours.
- **Vocabulary:** Words such as sale, discount, and tax will commonly be used (among others), and students may or may not have the background knowledge to understand what these terms mean.
- **Graphing:** When using a graph and locating the unit rate, students might have difficulty identifying which variable (the x or the y) is the unit rate.
- **Ratios:** Students are likely to use different strategies which the teacher can then use to help students understand the connection between, for example, making a table and strategically scaling a ratio.
- **Fractions, decimals, and percents:** Students may not understand fractions, particularly what the numerator and denominator mean. Students may struggle with the division of fractions and conversion to decimals and vice versa. They might struggle to see the connection between fractions, decimals, and ratios. Students may not understand percentages conceptually and may struggle to understand that the percent decrease from a larger to a smaller value is not equal to the percent increase from a smaller to a larger value.
- **Units:** If units in a problem differ, such as miles per hour and kilometers per hour, students may struggle to compare the two quantities. Not all information about the units may be given in all tasks, and students must be able to discern between mass vs. volume measurements, for example.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary,	**Pre-teaching** In previous classes, learners worked to understand, represent, compare, and reason with ratios. Students learned to reason about ratios by using equivalent ratios, tables of equivalent ratios, bar diagrams, and double-number-line diagrams. They also were introduced to a special type	**Pre-teaching** Consider using standard 6.RP.A.2, which provides a foundation for work in this cluster. In 6.RP.A.2, students analyze proportional relationships and use them to solve real-world and mathematical problems. Teachers can help students develop the concept of unit rates and help

mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in	of ratio called a rate. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as unit rates and using rate language, dividing fractions by other fractions, finding percents of numbers, reading and interpreting written descriptions of real-world problems involving ratios, and translating between written descriptions and the numerical expressions that represent those descriptions. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Remind students that they can find equivalent ratios using multiplication or division.	students see that when you have a context that can be modeled with a ratio and associated unit rate, there is almost always another ratio with its associated unit rate (the only exception is when one of the quantities is zero). Encourage students to flexibly choose either unit rate depending on the question at hand. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster analyzing proportional relationships and using them to solve real-world and mathematical problems. Offer opportunities to understand and explore different strategies and make sure students understand the difference between a rate and unit rate, specifically that a unit rate is one of many representations of equivalent ratios they can find.

with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/7/33/352/352		
Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners • understand, represent, compare, and reason with ratios	In 7th grade, learners • solve for unit rates, proportional reasoning and percent problems	In future classes, learners • understand problems and find the solution in a multi-step problem • choose and interpret units consistently in formulas • choose and interpret the scale and origin in graphs and data displays
Culturally and Linguistically Responsive Instruction		
Consider these resources for vocabulary from Pathways2Careers: • https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf • https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf		
Consider these questions as you plan for instruction that is culturally and linguistically responsive: • How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and

less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- How can we assess the reasonableness of answers using estimation?
- Where do we see the use of unit rates in our day to day life?
- What would be the meaning of the unit rates if the numerator and denominator are reversed (reciprocal of the unit rate)?
- How would you describe the information given by the unit rate?
- Can you compare quantities if they are measured using different units?

Cross-Curricular Connections

Arts: Following the mathematical series of musical rhythms to learn the basic rhythms of dance. Calculating shutter speed, focal length, lighting angles, and exposure time.

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

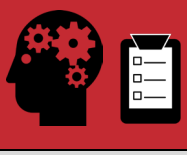
Science: Evaluate design solutions for maintaining biodiversity and probability of surviving and reproducing in a specific environment. Reading scientific charts and graphs. Understanding the movements of the planets. Analyzing scientific studies. Solving equations when writing computer programs and figuring out algorithms. Comparing linear relationships and systems of equations in scientific data.

Social studies: Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations
have different average life spans.

Career and Skill Connections

- Aeronautics
- Analysis
- Architecture
- Arts
- Astronomy
- Aviation
- Banking/finance
- Business
- Carpentry
- Cartography
- Construction
- Culinary arts
- Editing
- Education
- Electrician
- Engineering
- Equipment operation
- Event planning
- Food science
- Health/beauty
- Hydrology
- Insurance
- Lifeguard
- Machinist
- Manufacturing
- Marine biology
- Materials science
- Mechanics
- Medicine
- Meteorology
- Physical therapy
- Physicist
- Production
- Publishing
- Sales
- Surveying
- Transportation

Grade	CCSS Domain	CCSS Cluster
7	Ratios and Proportional Relationships	Analyze proportional relationships and use them to solve real-world and mathematical problems.



Cluster Standard: 7.RP.A.2

Standard	Standards for Mathematical Practice
7.RP.A.2: Recognize and represent proportional relationships between quantities. ● 7.RP.A.2.A: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. ● 7.RP.A.2.B: Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. ● 7.RP.A.2.C: Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. ● 7.RP.A.2.D: Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.	● **SMP 5:** Use appropriate tools strategically. ○ Teacher and Student Actions ● **SMP 7:** Look for and make use of structure. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students expand their knowledge of unit rates to include computations with complex fractions. They recognize and represent proportional relationships in equations, in tables, and on graphs.	● Sort real-world examples from non-examples. ● Create their own examples to demonstrate they understand the concept of proportional relationships. ● Communicate (orally/writing) that a proportion is a statement of two equivalent ratios. ● Model proportional relationships- concrete, visual, abstract (verbal [sentence], table, graph, equation). ● Prove or disprove proportional relationships between two points.

	• Determine appropriate representation of a proportional relationship. • Fluently assess and solve problems from various representations. • Model proportional relationships in several different ways. • Connect that the unit rate is the pattern or numerical coefficient (k or m) of the equation y=kx + b or y = mx + b. • Translate a proportional relationship from verbal, table, graph, equation. • Determine the unit rate from verbal, tables, graphs, equations, diagrams. • Model proportional relationships in equation form. • Justify in writing the reasoning used to create an • equation. • Explain the meaning of a point on a graph in context. • Discover that graphed proportional relationships are straight lines
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Understand and explain the definition of a proportional relationship between quantities.
- Understand how a proportional relationship is represented in tables, graphs, equations, diagrams, and verbal descriptions
- Understand that a proportional relationship can be represented with an equation in real world examples, such as the equation t = pn, where t represents total cost, n represents the number of items purchased, and p represents the constant price, and be able to justify reasoning.
- Understand and explain the meaning of points on the graph of a proportional relationship, including the significance of the points (0,0) and (1,r), where r is the unit rate.
- Create examples of proportional relationships and of non-proportional relationships.
- Model with proportional relationships in several different ways.
- Understand that the unit rate is the pattern or numerical coefficient (k or m) of the equation y = kx + b or y = mx + b.

Procedural Skill and Fluency:

- Decide whether two quantities are in a proportional relationship by testing for equivalent ratios in tables, graphing on a coordinate plane, etc.
- Identify the constant of proportionality in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

- Represent proportional relationships by writing equations, such as the equation t = pn, where t represents total cost, n represents the number of items purchased, and p represents the constant price.
- Illustrate point (x,y) on the graph of a proportional relationship.
- Fluently assess and solve problems from various representations.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

1. A textbook has the following definition for two quantities to be directly proportional: We say that y is directly proportional to x if y=kx for some constant k. For homework, students were asked to restate the definition in their own words and to give an example for the concept. Below are some of their answers. Discuss each statement and example. Translate the statements and examples into equations to help you decide if they are correct.

    a. Marcus: This means that both quantities are the same. When one increases the other increases by the same amount. An example of this would be the amount of air in a balloon and the volume of a balloon.

    b. Sadie: Two quantities are proportional if one change is accompanied by a change in the other. For example the radius of a circle is proportional to the area.

    c. Ben: When two quantities are directly proportional it means that if one quantity goes up by a certain percentage, the other quantity goes up by the same percentage as well. An example could be as gas prices go up in cost, food prices go up in cost.

    d. Jessica: When two quantities are proportional, it means that as one quantity increases the other will also increase and the ratio of the quantities is the same for all values. An example could be the circumference of a circle and its diameter, the ratio of the values would equal π.

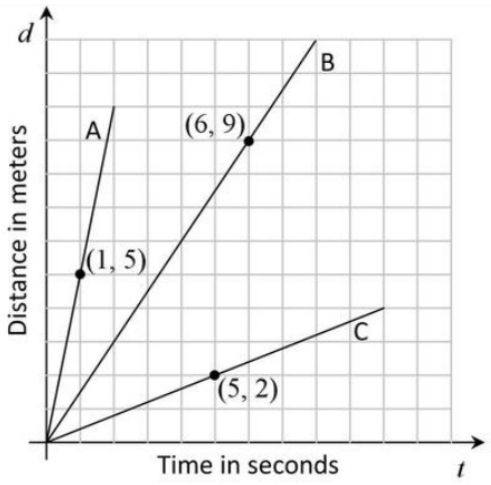
2. BeatStreet, TunesTown, and MusicMind are music companies. BeatStreet offers to buy 1.5 million shares of TunesTown for $561 million. At the same time, MusicMind offers to buy 1.5 million shares of TunesTown at $373 per share. Who would get the better deal, BeatStreet or MusicMind? What is the total price difference?

3. Carli's class built some solar-powered robots. They raced the robots in the parking lot of the school. The graphs shown to the right are all line segments that show the distance d, in meters, that each of three robots traveled after t seconds. Each graph has a point labeled.

    a. What does the point tell you about how far that robot has traveled?

    b. Carli said that the ratio between the number of seconds each robot travels and the number of meters it has traveled is constant. Is she correct? Explain.

    c. How fast is each robot traveling? How did you compute this from the graph?



You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Direct versus proportional division:** Students might use direct instead of proportional division, mistakenly thinking that if it takes 2 people 4 hours to do a certain task, it would take 1 person 2 hours rather than 8 hours.
- **Vocabulary:** Words such as sale, discount, and tax will commonly be used (among others), and students may or may not have the background knowledge to understand what these terms mean.
- **Graphing:** When using a graph and locating the unit rate, students might have difficulty identifying which variable (the x or the y) is the unit rate. They may also struggle to interpret the meaning of graphs.
- **Ratios:** Students are likely to use different strategies which the teacher can then use to help students understand the connection between, for example, making a table and strategically scaling a ratio.
- **Proportional relationships:** Students might have been exposed to ratios and have practice setting them up, but they might not understand different aspects of ratios and how they can be represented in different forms (tables, graphs, verbally, etc.).
- **Fractions, decimals, and percents:** Students may not understand fractions, particularly what the numerator and denominator mean. Students may struggle with the division of fractions and conversion to decimals and vice versa. They might struggle to see the connection between fractions, decimals, and ratios. Students may not understand percentages conceptually and may struggle to understand that the percent decrease from a larger to a smaller value is not equal to the percent increase from a smaller to a larger value.
- **Units:** If units in a problem differ, such as miles per hour and kilometers per hour, students may struggle to compare the two quantities. Not all information about the units may be given in all tasks, and students must be able to discern between mass vs. volume measurements, for example.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*

Representation
Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships.

Engagement
Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to

Pre-teaching
In previous classes, learners worked to understand, represent, compare, and reason with ratios. Students learned to reason about ratios by using equivalent ratios, tables of equivalent ratios, bar diagrams, and double-number-line diagrams. They also were introduced to a special type of ratio called a rate.

Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as unit rates and using rate language, dividing fractions by other fractions, finding percents of numbers, reading and interpreting written descriptions of real-world problems involving ratios, and translating between written descriptions and the numerical expressions that represent those descriptions.

If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

Re-teaching
Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient

Pre-teaching
Consider using standard 6.RP.A.2, which provides a foundation for work in this cluster. In 6.RP.A.2, students analyze proportional relationships and use them to solve real-world and mathematical problems. Teachers can help students develop the concept of unit rates and help students see that when you have a context that can be modeled with a ratio and associated unit rate, there is almost always another ratio with its associated unit rate (the only exception is when one of the quantities is zero). Encourage students to flexibly choose either unit rate depending on the question at hand.

If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

Re-teaching
Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster analyzing proportional relationships and using them to solve real-world and mathematical problems. Offer opportunities to understand and explore different strategies and make sure students understand the difference between a rate and unit rate, specifically that a unit rate is one of many

their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support	ways of solving problems. Remind students that they can find equivalent ratios using multiplication or division.	representations of equivalent ratios they can find.

independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/7/33/352/355

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners • understand, represent, compare, and reason with ratios	In 7th grade, learners • solve for unit rates, proportional reasoning and percent problems	In future classes, learners • understand problems and find the solution in a multi-step problem • choose and interpret units consistently in formulas • choose and interpret the scale and origin in graphs and data displays

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:
- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this	• Instruction should begin with conceptual understanding that allows students to contribute their informal	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates

mathematics exists outside of school to create stronger home to school connections for students. For example, students can examine proportional relationships in different recipes and can determine that when making their favorite recipe they might have to double or triple the ingredients based on the number of servings the recipe yields vs. the number of servings needed. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- What are some ways we can tell whether two ratios are proportional?
- When can you use the concept of proportional ratios in your daily life?
- How important is unit rate in proportional ratios?
- What would the graph of proportional ratios look like?
- Can you write a proportional ratio using different units?

- Can you give an example of a problem that uses proportional relationship?
- Can you share a strategy you use to solve problems with proportional relationships?

Cross-Curricular Connections

Arts: Following the mathematical series of musical rhythms to learn the basic rhythms of dance. Calculating shutter speed, focal length, lighting angles, and exposure time.

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

Science: Evaluate design solutions for maintaining biodiversity and probability of surviving and reproducing in a specific environment. Reading scientific charts and graphs. Understanding the movements of the planets. Analyzing scientific studies. Solving equations when writing computer programs and figuring out algorithms. Comparing linear relationships and systems of equations in scientific data.

Social studies: Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations
have different average life spans.

Career and Skill Connections

- Aeronautics
- Analysis
- Architecture
- Arts
- Astronomy
- Aviation
- Banking/finance
- Business
- Carpentry
- Cartography
- Construction
- Culinary arts
- Editing
- Education
- Electrician
- Engineering
- Equipment operation
- Event planning
- Food science
- Health/beauty
- Hydrology
- Insurance
- Lifeguard
- Machinist
- Manufacturing
- Marine biology
- Materials science
- Mechanics
- Medicine
- Meteorology
- Physical therapy
- Physicist
- Production
- Publishing
- Sales
- Surveying
- Transportation

Grade	CCSS Domain	CCSS Cluster
7	Ratios and Proportional Relationships	Analyze proportional relationships and use them to solve real-world and mathematical problems.

Cluster Standard: 7.RP.A.3

Standard	Standards for Mathematical Practice

7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, and percent increase and decrease, percent error.	• **SMP 3:** Construct viable arguments and critique the reasoning of others.
	  ○ Teacher and Student Actions
	• **SMP 8:** Look for and express regularity in repeated reasoning.
	  ○ Teacher and Student Actions

Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students use proportional reasoning to solve multi-step ratio and percent problems involving real world scenarios (percent change, sales tax, simple interest, etc.).	• Explore and connect vocabulary terms with real world examples. • Explain how they are used in each situation. • Solve problems proportional problems using cross multiplication. • Solve percent error and percent increase/decrease problems. • Explain how formulas for percent error and increase/decrease are similar.
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Procedural Skill and Fluency:

- Use proportional relationships to solve multistep ratio and percent problems.
- Perform calculations involving percentages, such as converting percent to decimals, finding percentages, calculating percent increase or decrease, and determining percent error.
- Solve problems involving proportions using cross-multiplication.

Application:

- Use proportional reasoning to solve multistep ratio and percent involving real world problems involving simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and

decrease, and percent error.
- Connect vocabulary terms with real world examples involving simple interest, tax, markups and markdowns, gratuities and commission, fees, percent increase and decrease, and percent error, and explain how they are used in each situation.
- Explain how the formulas for percent error and percent increase/decrease are similar.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

The sales team at an electronics store sold 48 computers last month. The manager at the store wants to encourage the sales team to sell more computers and is going to give all the sales team members a bonus if the number of computers sold increases by 30% in the next month. How many computers must the sales team sell to receive the bonus? Explain your reasoning.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Direct versus proportional division:** Students might use direct instead of proportional division, mistakenly thinking that if it takes 2 people 4 hours to do a certain task, it would take 1 person 2 hours rather than 8 hours.
- **Vocabulary:** Words such as sale, discount, and tax will commonly be used (among others), and students may or may not have the background knowledge to understand what these terms mean.
- **Graphing:** When using a graph and locating the unit rate, students might have difficulty identifying which variable (the x or the y) is the unit rate. They may also struggle to interpret the meaning of graphs.
- **Ratios:** Students are likely to use different strategies which the teacher can then use to help students understand the connection between, for example, making a table and strategically scaling a ratio.
- **Proportional relationships:** Students might have been exposed to ratios and have practice setting them up, but they might not understand different aspects of ratios and how they can be represented in different forms (tables, graphs, verbally, etc.).
- **Measurement:** Students may make errors in their own measurements as well as understanding measurements and should be given the chance to look for sources of error. This is particularly true if students are using a formula that uses π.
- **Fractions, decimals, and percents:** Students may not understand fractions, particularly what the numerator and denominator mean. Students may struggle with the division of fractions and conversion to decimals and vice versa. They might struggle to see the connection between fractions, decimals, and ratios. Students may not understand percentages conceptually and may struggle to understand that the percent decrease from a larger to a smaller value is not equal to the percent increase from a smaller to a larger value.
- **Units:** If units in a problem differ, such as miles per hour and kilometers per hour, students may struggle to compare the two quantities. Not all information about the units may be given in all tasks, and students must be able to discern between mass vs. volume measurements, for example.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more	**Pre-teaching** In previous classes, learners worked to understand, represent, compare, and reason with ratios. Students learned to reason about ratios by using equivalent ratios, tables of equivalent ratios, bar diagrams, and double-number-line diagrams. They also were introduced to a special type of ratio called a rate. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as unit rates and using rate language, dividing fractions by other fractions, finding percents of numbers, reading and interpreting written descriptions of real-world problems involving ratios, and translating between written descriptions and the numerical expressions that represent those descriptions. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with	**Pre-teaching** Consider using standard 6.RP.A.2, which provides a foundation for work in this cluster. In 6.RP.A.2, students analyze proportional relationships and use them to solve real-world and mathematical problems. Teachers can help students develop the concept of unit rates and help students see that when you have a context that can be modeled with a ratio and associated unit rate, there is almost always another ratio with its associated unit rate (the only exception is when one of the quantities is zero). Encourage students to flexibly choose either unit rate depending on the question at hand. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster analyzing proportional relationships and using them to solve real-world and mathematical problems. Offer

independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with	common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Remind students that they can find equivalent ratios using multiplication or division.	opportunities to understand and explore different strategies and make sure students understand the difference between a rate and unit rate, specifically that a unit rate is one of many representations of equivalent ratios they can find.

graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/7/33/352/363

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- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:
- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning	• Instruction should begin with	• Scaffold tasks and amplify

from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can examine proportional relationships in different recipes and can determine that when making their favorite recipe they might have to double or triple the ingredients based on the number of servings the recipe yields vs. the number of servings needed. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.
Suggested Student Discourse Questions		
Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf		
• How can we assess the reasonableness of answers using estimation? • Can you compare quantities if they are measured using different units?		

- How can proportional relationships be applied to solve multistep ratio and percent problems?

Cross-Curricular Connections

Arts: Following the mathematical series of musical rhythms to learn the basic rhythms of dance. Calculating shutter speed, focal length, lighting angles, and exposure time.

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

Science: Evaluate design solutions for maintaining biodiversity and probability of surviving and reproducing in a specific environment. Reading scientific charts and graphs. Understanding the movements of the planets. Analyzing scientific studies. Solving equations when writing computer programs and figuring out algorithms. Comparing linear relationships and systems of equations in scientific data.

Social studies: Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations
have different average life spans.

Career and Skill Connections

- Aeronautics
- Analysis
- Architecture
- Arts
- Astronomy
- Aviation
- Banking/finance
- Business
- Carpentry
- Cartography
- Construction
- Culinary arts
- Editing
- Education
- Electrician
- Engineering
- Equipment operation
- Event planning
- Food science
- Health/beauty
- Hydrology
- Insurance
- Lifeguard
- Machinist
- Manufacturing
- Marine biology
- Materials science
- Mechanics
- Medicine
- Meteorology
- Physical therapy
- Physicist
- Production
- Publishing
- Sales
- Surveying
- Transportation

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards for Mathematical Practice
    - Connections to procedural, conceptual understanding, and application
    - Sample assessment items
    - Common misconceptions
    - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
    - Vertical alignment
    - Culturally and Linguistically Responsive Instruction (CLRI)
    - Suggested student discourse questions
    - Cross-curricular and career/skill connections
- A Student Discourse Guide

Helpful links:

- Lesson-planning tool from Pathways 2 Careers (click here to sign up with your district email if you don't already have an account)
- Focus by Grade Level from Achieve the Core
- Coherence Map from Achieve the Core

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Use random sampling to draw inferences about a population.
  - 7.SP.A.1
  - 7.SP.A.2
- Draw informal comparative inferences about two populations.
  - 7.SP.B.3
  - 7.SP.B.4
- Investigate chance processes and develop, use, and evaluate probability models.
  - 7.SP.C.5
  - 7.SP.C.6
  - 7.SP.C.7
  - 7.SP.C.8

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)

Grade	CCSS Domain	CCSS Cluster
7	Statistics and Probability	Use random sampling to draw inferences about a population.

Cluster Standard: 7.SP.A.1

Standard	Standards for Mathematical Practice

7.SP.A.1: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	• **SMP 2**: Reason abstractly and quantitatively.
	  ○ Teacher and Student Actions
	• **SMP 3:** Construct viable arguments and critique the reasoning of others.
	  ○ Teacher and Student Actions
	• **SMP 4:** Model with mathematics.
	  ○ Teacher and Student Actions
	• **SMP 5:** Use appropriate tools strategically.
	  ○ Teacher and Student Actions

Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students learn about sampling populations and that a sampling must be representative of the population in order to make valid inferences and generalizations. To measure variation and estimates or predictions about a characteristic, students must conduct multiple samples of the same size from populations with unknown characteristics.	• Critique examples of sampling as statistical tools using precise mathematical vocabulary; random sampling, population, and valid generalization. • Design random samplings to collect the data given statistical questions. • Defend the samplings as random.
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Understand that statistics can be used to gain information about a population by examining a sample of the population
- Understand that generalizations about a population from a sample are valid only if the sample is representative of that population and know that estimates or predictions are tested by conducting multiple samples of the same size from populations with unknown characteristics.
- Understand that random sampling tends to produce representative samples and support valid inferences.
- Understand variation and know that variation is measured by conducting multiple samples of the same size

from populations with unknown characteristics.
- Critique examples of sampling as statistical tools, using precise mathematical vocabulary.
- Design and defend random sampling tools.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

In a poll of Mr. Briggs's math class, 67% of the students say that math is their favorite academic subject. The editor of the school paper is in the class, and he wants to write an article for the paper saying that math is the most popular subject at the school. Explain why this is not a valid conclusion and suggest a way to gather better data to determine what subject is most popular.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Vocabulary:** There is a lot of new vocabulary that students might struggle to internalize, especially if they are not learning those terms in context and are only being asked to memorize their meanings.
- **Bias:** Students will likely come in with a lot of prior knowledge and biases, and many will struggle to release those ideas and instead focus only on the information at hand.
- **Random sampling:** Many students will struggle with the idea of random sampling and it may be necessary to physically demonstrate a random vs. a non-random sampling to eliminate misconceptions.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + Universal*	*Layer 2* *Core + Targeted*	*Layer 3* *Core + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media,	**Pre-teaching** In previous classes, learners worked to develop an understanding of graphs, mean, median, mode, Mean Absolute Deviation (M.A.D.), and interquartile range (IQR), recognize there will be variability in the data of a statistical question and will account for it in the answers, understand a data set has a distribution which can be described by its center, spread, and overall shape, summarize numerical data sets by reporting the number of observations along with describing the nature of the attribute under investigation and how it was measured and its units, approximate	**Pre-teaching** Consider using standard 6.SP.A.1, which provides a foundation for work in this cluster. In 6.SP.A.1, students are introduced to a statistical question and the variability present in data. Also consider using standard 7.SP.A.2, which also provides a foundation for work in this cluster. In 7.SP.A.2, students draw informal comparative inferences about two populations. Also consider using standard 6.RP.A.3, which also provides a foundation for work in this cluster. In

tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community.	the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, predict the approximate relative frequency given the probability, and use ratio and rate reasoning to solve real-world and mathematical problems. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as making informal comparative inferences about one (from earlier grades) and connecting that to the new work involving two populations. This will be the first time students learn about random sampling and evaluating probability models, so make connections to earlier work as possible. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Ensure students can explain what a random	6.RP.A.3, students work with ratio concepts and use ratio reasoning to solve problems, which will help in solving probability models. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster reviewing random sampling to draw inferences about populations. Give real-world examples and allow students to try different strategies, such as using lists, tables, tree diagrams, and simulations.

Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.	sample is and give them opportunities to describe and generate random samples in the context of a problem. Review probability, its meaning, and how to calculate it.	

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/7/34/364/364		
Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners	In 7th grade, learners	In future classes, learners

• develop an understanding of graphs, mean, median, mode, Mean Absolute Deviation (M.A.D.), and interquartile range (IQR) • recognize there will be variability in the data of a statistical question and will account for it in the answers. • understand a data set has a distribution which can be described by its center, spread, and overall shape • summarize numerical data sets by reporting the number of observations along with describing the nature of the attribute under investigation and how it was measured and its units • approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency • predict the approximate relative frequency given the probability • use ratio and rate reasoning to solve real-world and mathematical problems	• use random sampling to draw inferences about a population • recognize and represent proportional relationships between quantities • use proportional relationships to solve multistep ratio and percent problems	• represent data with plots on the real number line (dot plots, histograms, and box plots • use statistics appropriate to the shape and context of the data distribution to compare measures of center (median, mean) and spread (IQR, standard deviation) of two or more different data sets • interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points • construct and interpret a two-way table summarizing data on two categorical variables collected from the same subject • recognize the purposes of and differences among sample surveys, experiments, and observational studies and explain how randomization relates to each • find the conditional probability of A given B as the fraction of B's outcomes that also belong to A • interpret conditional probabilities in terms of a model's context

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture

and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.
Suggested Student Discourse Questions		

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- What is a population? What is a population sample? Why is it important to use a sample to learn about a population?
- Why is it important to use random sampling?
- Can you give and explain a situation where a sample is not a representative of a population?

Cross-Curricular Connections

Literature: Using linear, logical thinking to write more clearly and logically.

Science: Examining biological characteristics of a sample. Making an argument about the growth and development of organisms. Developing models and identifying components and describing relationships between components. Modeling data and describing how the data can be used to create designs through testing and modification. Identifying limitations of models. Engaging in the Engineering Design Process. Modeling genetic information and sexual reproduction results and using Punnett squares. Creating scatterplots of bivariate data. Using simulations to generate data that can be used to modify a proposed object, tool, or process. Conducting experiments in connection with NGSS science standards, collecting bivariate data, representing that data in a two-way table, and hypothesizing correlations between the two variables. Reading scientific charts and graphs. Analyzing scientific studies.

Social Studies: Sampling populations and analyzing data. Studying trends in areas such as species populations, the stock market, or gross domestic product. Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations have different average life spans. Understanding statistical information on populations.

Career and Skill Connections

- Advertising
- Aeronautics
- Agriculture
- Analysis
- Aviation
- Banking/finance
- Botanist
- Coaching
- Counseling
- Data science
- Ecology
- Economist
- Education
- Engineering
- Gardening
- Health science
- Information technology
- Law
- Machinist
- Management
- Mechanic
- Medicine
- Park ranger
- Political science
- Psychology
- Ranching/farming
- Sales
- Statistics
- Technician
- Transportation
- Urban planning
- Veterinary

Grade	CCSS Domain	CCSS Cluster
7	Statistics and Probability	Use random sampling to draw inferences about a population.

Cluster Standard: 7.SP.A.2

Standard	Standards for Mathematical Practice
7.SP.A.2: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.	● **SMP 2**: Reason abstractly and quantitatively. ○ Teacher and Student Actions ● **SMP 3**: Construct viable arguments and critique the reasoning of others. ○ Teacher and Student Actions ● **SMP 4**: Model with mathematics. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students learn about sampling populations and that a sampling must be representative of the population in order to make valid inferences and generalizations. To measure variation and make estimates or predictions about a characteristic, students must conduct multiple samples of the same size from populations with unknown characteristics.	● Draw valid inferences and generalizations from random samplings of populations ● Justify their inferences and generalizations as valid using appropriate vocabulary ● Explain the variability in multiple random samples and gauge how far off an estimate may be.
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Identify sampling populations in order to make valid inferences and generalizations.
- Explain the variability in multiple random samples and gauge how far off an estimate may be.
- Understand variation and know that variation is measured by conducting multiple samples of the same size from populations with unknown characteristics.
- Critique examples of sampling as statistical tools, using precise mathematical vocabulary.

Procedural Skills and Fluency:

- Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions and gauge how far off the estimate or prediction might be.

Application:

- Use data from a random sample to draw inferences about a population with an unknown characteristic of interest.
- Draw valid inferences and generalizations from random samplings of populations.

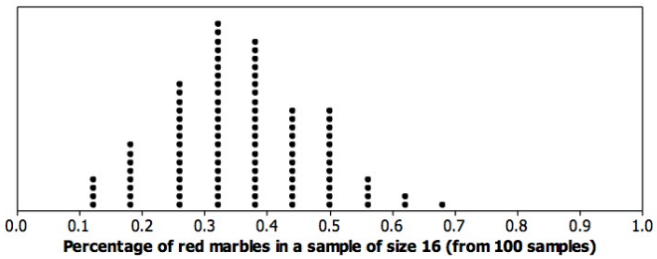
Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

A hotel holds a Valentine's Day contest where guests are invited to estimate the percentage of red marbles in a huge clear jar containing both red marbles and white marbles. There are 11,000 total marbles in the jar: 3696 are red, 7304 are white. The actual percentage of red marbles in the entire jar (33.6% = $\frac{3696}{11000}$) is known to some members of the hotel staff. Any guest who makes an estimate that is within 9 percentage points of the true percentage of red marbles in the jar wins a prize, so any estimate from 24.6% to 42.6% will be considered a winner. To help with the estimating, a guest is allowed to take a random sample of 16 marbles from the jar in order to come up with an estimate.

One of the hotel employees who does not know that the true percentage of red marbles in the jar is 33.6% is asked to record the results of the first 100 random samples. A table and dotplot of the results appears below.

Percentage of red marbles in the sample of size 16	Number of times the percentage was obtained
12.50%	4
18.75%	8
25.00%	15
31.25%	22
37.50%	20
43.75%	12
50.00%	12
56.25%	4
62.50%	2
68.75%	1
Total:	100



For example, 15 of the random samples had exactly 25.00% red marbles; only 2 of the random samples had exactly 62.50% red marbles, and so on.

1. Assuming that each of the 100 guests who took a random sample used their random sample's red marble percentage to estimate the whole jar's red marble percentage. Based on the table above, how many of these guests would be "winners"?

2. How many of the 100 guests obtained a sample that was more than half red marbles?

3. Should we be concerned that none of the samples had a red marble percentage of exactly 33.6% even though that value is the true red marble percentage for the whole jar? Explain briefly why a guest can't obtain a sample red marble percentage of 33.6% for a random sample of size 16.

4. Recall that the hotel employee who made the table and dot plot above didn't know that the real percentage of red marbles in the entire jar was 33.6%. If another person thought that half of the marbles in the jar were red, explain briefly how the hotel employee could use the dot plot and table results to challenge this person's claim. Specifically, what aspects of the table and dot plot would encourage the employee to challenge the claim?

5. Design a simulation that takes a large number of samples of size 16 from a population in which 65% of the members of the population have a particular characteristic. For each sample of size 16, compute the percentage of red items in the sample. Record these percentages, and then summarize all of your sample percentages using a table and dot plot similar to those shown above. In what ways is your dot plot similar to the dotplot used in this task? In what ways does it differ?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Vocabulary:** There is a lot of new vocabulary that students might struggle to internalize, especially if they are not learning those terms in context and are only being asked to memorize their meanings.
- **Bias:** Students will likely come in with a lot of prior knowledge and biases, and many will struggle to release those ideas and instead focus only on the information at hand.
- **Random sampling:** Many students will struggle with the idea of random sampling and it may be necessary to physically demonstrate a random vs. a non-random sampling to eliminate misconceptions.
- **Multiple samples:** Students may struggle to conduct multiple samples and choose one correct sample to make inferences about. If a simulation is created using software, a program, etc., students will need more guidance about how that sample is generated in the simulation.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + Universal*	*Layer 2* *Core + Targeted*	*Layer 3* *Core + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language,	**Pre-teaching** In previous classes, learners worked to develop an understanding of graphs, mean, median, mode, Mean Absolute Deviation (M.A.D.), and interquartile range (IQR), recognize	**Pre-teaching** Consider using standard 6.SP.A.1, which provides a foundation for work in this cluster. In 6.SP.A.1, students are introduced to a statistical question and the variability present in

different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible	there will be variability in the data of a statistical question and will account for it in the answers, understand a data set has a distribution which can be described by its center, spread, and overall shape, summarize numerical data sets by reporting the number of observations along with describing the nature of the attribute under investigation and how it was measured and its units, approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, predict the approximate relative frequency given the probability, and use ratio and rate reasoning to solve real-world and mathematical problems. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as making informal comparative inferences about one (from earlier grades) and connecting that to the new work involving two populations. This will be the first time students learn about random sampling and evaluating probability models, so make connections to earlier work as possible. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching**	data. Also consider using standard 7.SP.A.2, which also provides a foundation for work in this cluster. In 7.SP.A.2, students draw informal comparative inferences about two populations. Also consider using standard 6.RP.A.3, which also provides a foundation for work in this cluster. In 6.RP.A.3, students work with ratio concepts and use ratio reasoning to solve problems, which will help in solving probability models. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster reviewing random sampling to draw inferences about populations. Give real-world examples and allow students to try different strategies, such as using lists, tables, tree diagrams, and simulations.

timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to	Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Ensure students can explain what a random sample is and give them opportunities to describe and generate random samples in the context of a problem. Review probability, its meaning, and how to calculate it.	

monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/7/34/364/366

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners • develop an understanding of graphs, mean, median, mode, Mean Absolute Deviation (M.A.D.), and interquartile range (IQR) • recognize there will be variability in the data of a statistical question and will account for it in the answers. • understand a data set has a distribution which can be described by its center, spread, and overall shape • summarize numerical data sets by reporting the number of observations along with describing the nature of the attribute under investigation and how it was measured and its units • approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency • predict the approximate relative frequency given the probability • use ratio and rate reasoning to solve real-world and mathematical problems	In 7th grade, learners • use random sampling to draw inferences about a population • recognize and represent proportional relationships between quantities • use proportional relationships to solve multistep ratio and percent problems	In future classes, learners • represent data with plots on the real number line (dot plots, histograms, and box plots • use statistics appropriate to the shape and context of the data distribution to compare measures of center (median, mean) and spread (IQR, standard deviation) of two or more different data sets • interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points • construct and interpret a two-way table summarizing data on two categorical variables collected from the same subject • recognize the purposes of and differences among sample surveys, experiments, and observational studies and explain how randomization relates to each • find the conditional probability of A given B as the fraction of B's outcomes that also belong to A • interpret conditional probabilities in terms of a model's context

Culturally and Linguistically Responsive Instruction

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Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

	encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- What is a population? What is a population sample? Why is it important to use a sample to learn about a population?
- Why is it important to use random sampling?
- Can you give and explain a situation where a sample is not a representative of a population?
- How would you use data from a random sample to make a prediction about a population characteristic that you don't know? How can you determine how close your prediction is?

Cross-Curricular Connections

Literature: Using linear, logical thinking to write more clearly and logically.

Science: Examining biological characteristics of a sample. Making an argument about the growth and development of organisms. Developing models and identifying components and describing relationships between components. Modeling data and describing how the data can be used to create designs through testing and modification. Identifying limitations of models. Engaging in the Engineering Design Process. Modeling genetic information and sexual reproduction results and using Punnett squares. Creating scatterplots of bivariate data. Using simulations to generate data that can be used to modify a proposed object, tool, or process. Conducting experiments in connection with NGSS science standards, collecting bivariate data, representing that data in a two-way table, and hypothesizing correlations between the two variables. Reading scientific charts and graphs. Analyzing scientific studies.

Social Studies: Sampling populations and analyzing data. Studying trends in areas such as species populations, the stock market, or gross domestic product. Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations have different average life spans. Understanding statistical information on populations.

Career and Skill Connections

• Advertising • Aeronautics • Agriculture • Analysis • Aviation • Banking/finance • Botanist • Coaching • Counseling • Data science • Ecology	• Economist • Education • Engineering • Gardening • Health science • Information technology • Law • Machinist • Management • Mechanic • Medicine	• Park ranger • Political science • Psychology • Ranching/farming • Sales • Statistics • Technician • Transportation • Urban planning • Veterinary

Grade	CCSS Domain	CCSS Cluster
7	Statistics and Probability	Draw informal comparative inferences about two populations.

Cluster Standard: 7.SP.B.3

Standard	Standards for Mathematical Practice

7.SP.B.3: Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.	• **SMP 5:** Use appropriate tools strategically.
	  ○ Teacher and Student Actions
	• **SMP 8:** Look for and express regularity in repeated reasoning.
	  ○ Teacher and Student Actions

Clarification Statement	**Students Who Demonstrate Understanding Can...**
In this cluster students draw valid, comparable inferences about two populations using measures of center (mean, median) and measures of variability (mean absolute deviation, interquartile range).	• Find measures of center and measures of variation for two or more data sets. • Compare two data sets for variability by comparing graphs. • Make inferences about data sets by comparing their statistical measures. • Model and compare two real-world data sets by measuring the difference between centers and expressing it multiple of a measure of variability.
DOK	**Blooms**
1-2	Understand, Evaluate

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Draw valid comparable inferences about two populations using measures of center (mean, median) and measures of variability (mean absolute deviation, interquartile range).
- Understand the various measures of center and measures of variability and when and how they can be used.
- Make inferences about data sets by comparing their statistical measures.

- Model and compare two real-world data sets by measuring the difference between centers and expressing it multiple of a measure of variability.

Procedural Skill and Fluency:

- Assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.
- Find measures of center and measures of variability for two or more data sets.

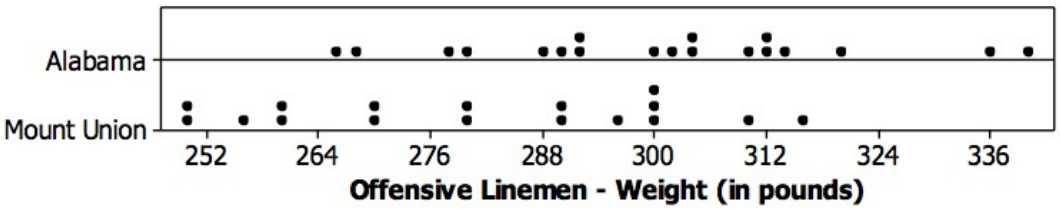
Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

College football teams are grouped with similar teams into "divisions" (and in some cases, "subdivisions") based on many factors such as game attendance, level of competition, athletic department resources, and so on. Schools from the Football Bowl Subdivision (FBS, formerly known as Division 1-A) are typically much larger schools than schools of any other division in terms of enrollment and revenue. "Division III" is a division of schools with typically smaller enrollment and resources. One particular position on a football team is called "offensive lineman," and it is generally believed that the offensive linemen of FBS schools are heavier on average than the offensive linemen of Division III schools. For the 2012 season, the University of Mount Union Purple Raiders football team won the Division III National Football Championship while the University of Alabama Crimson Tide football team won the FBS National Championship. Below are the weights of the offensive linemen for both teams from that season.

Alabama
277
265
292
303
303
320
300
313
267
288
311
280
302
335
310
290
312
340
292

Mount Union
250
250
290
260
270
270
310
290
280
315
280
295
300
300
260
255
300



1. Based on visual inspection of the dotplots, which group appears to have the larger average weight? Does one group seem to have greater variability in its weights than the other, or do the two groups look similar in that regard?

2. Compute the mean and mean absolute deviation (MAD) for each group. Do your measures support your answers in part (a)?

3. Choose from the following to fill in the blank: "The average Alabama offensive lineman's weight is about _____ than the average Mount Union offensive lineman's weight."

a. 20 pounds lighter b. 15 pounds lighter c. 15 pounds heavier d. 20 pounds heavier

4. "This difference in average weights is approximately _____ of either team."

a. About half of the MAD b. Slightly more than 1 MAD c. Twice the MAD

5. The offensive linemen on the Alabama team are not a random sample from all FBS offensive linemen. Similarly, the offensive linemen on the Mount Union Team are not a random sample from all Division III offensive linemen. However, for purposes of this task, suppose that these two groups can be regarded as random samples of offensive linemen from their respective divisions/subdivisions. If these were random samples, would you think that offensive linemen from FBS schools are typically heavier than offensive linemen from Division III schools? Explain your decision using answers to the previous questions and/or additional analysis.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Vocabulary:** There is a lot of new vocabulary that students might struggle to internalize, especially if they are not learning those terms in context and are only being asked to memorize their meanings.
- **Bias:** Students will likely come in with a lot of prior knowledge and biases, and many will struggle to release those ideas and instead focus only on the information at hand.
- **Random sampling:** Many students will struggle with the idea of random sampling, and it may be necessary to physically demonstrate a random vs. a non-random sampling to eliminate misconceptions.
- **Multiple samples:** Students may struggle to conduct multiple samples and choose one correct sample to make inferences about. If a simulation is created using software, a program, etc., students will need more guidance about how that sample is generated in the simulation.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning		
Layer 1 ***Core Instruction + Universal***	***Layer 2*** ***Core + Targeted***	***Layer 3*** ***Core + Targeted + Intensive***
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and	**Pre-teaching** In previous classes, learners worked to develop an understanding of graphs, mean, median, mode, Mean Absolute Deviation (M.A.D.), and interquartile range (IQR), recognize there will be variability in the data of a statistical question and will account for it in the answers, understand a data set has a distribution which can be described by its center, spread, and overall shape, summarize numerical data sets by reporting the number of observations along with describing the nature of the attribute under investigation and how it was measured and its units, approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, predict the approximate relative frequency given the probability, and use ratio and rate reasoning to solve real-world and mathematical problems. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as making informal comparative inferences about one (from earlier grades) and connecting that to the new work involving two populations. This will be the first-time students learn about random sampling and evaluating probability models, so make connections to earlier work as possible. If students have unfinished learning	**Pre-teaching** Consider using standard 6.SP.A.1, which provides a foundation for work in this cluster. In 6.SP.A.1, students are introduced to a statistical question and the variability present in data. Also consider using standard 7.SP.A.2, which also provides a foundation for work in this cluster. In 7.SP.A.2, students draw informal comparative inferences about two populations. Also consider using standard 6.RP.A.3, which also provides a foundation for work in this cluster. In 6.RP.A.3, students work with ratio concepts and use ratio reasoning to solve problems, which will help in solving probability models. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster reviewing random

materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut	leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Ensure students can explain what a random sample is and give them opportunities to describe and generate random samples in the context of a problem. Review probability, its meaning, and how to calculate it.	sampling to draw inferences about populations. Give real-world examples and allow students to try different strategies, such as using lists, tables, tree diagrams, and simulations.

materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/7/34/370/370

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● develop an understanding of graphs, mean, median, mode, Mean Absolute Deviation (M.A.D.), and interquartile range (IQR) ● recognize there will be variability in the data of a statistical question and will account for it in the answers. ● understand a data set has a distribution which can be described by its center, spread, and overall shape ● summarize numerical data sets by reporting the number of observations along with describing the nature of the attribute under investigation and how it was measured and its units ● approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency	In 7th grade, learners ● use random sampling to draw inferences about a population ● recognize and represent proportional relationships between quantities ● use proportional relationships to solve multistep ratio and percent problems	In future classes, learners ● represent data with plots on the real number line (dot plots, histograms, and box plots ● use statistics appropriate to the shape and context of the data distribution to compare measures of center (median, mean) and spread (IQR, standard deviation) of two or more different data sets ● interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points ● construct and interpret a two-way table summarizing data on two categorical variables collected from the same subject ● recognize the purposes of and differences among sample surveys, experiments, and observational studies and

• predict the approximate relative frequency given the probability • use ratio and rate reasoning to solve real-world and mathematical problems		explain how randomization relates to each • find the conditional probability of A given B as the fraction of B's outcomes that also belong to A • interpret conditional probabilities in terms of a model's context

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible.

their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	• Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- What does it mean when two data distributions have a similar overlap? What about when they do not?
- How can we measure the difference between the centers of two data distributions?
- How can comparing the visual overlap of two distributions help us understand the data better?
- What does the measure of variability mean in the context of a data distribution? Why is this important to consider?

Cross-Curricular Connections

Literature: Using linear, logical thinking to write more clearly and logically.

Science: Examining biological characteristics of a sample. Making an argument about the growth and development of organisms. Developing models and identifying components and describing relationships between components. Modeling data and describing how the data can be used to create designs through testing and modification. Identifying limitations of models. Engaging in the Engineering Design Process. Modeling genetic information and sexual reproduction results and using Punnett squares. Creating scatterplots of bivariate data. Using simulations to generate data that can be used to modify a proposed object, tool, or process. Conducting experiments in connection with NGSS science standards, collecting bivariate data, representing that data in a two-way table, and hypothesizing correlations between the two variables. Reading scientific charts and graphs. Analyzing scientific studies.

Social Studies: Sampling populations and analyzing data. Studying trends in areas such as species populations, the stock market, or gross domestic product. Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations have different average life spans. Understanding statistical information on populations.

Career and Skill Connections		
• Advertising • Aeronautics • Agriculture • Analysis • Aviation • Banking/finance • Botanist • Coaching • Counseling • Data science • Ecology	• Economist • Education • Engineering • Gardening • Health science • Information technology • Law • Machinist • Management • Mechanic • Medicine	• Park ranger • Political science • Psychology • Ranching/farming • Sales • Statistics • Technician • Transportation • Urban planning • Veterinary

Grade	CCSS Domain	CCSS Cluster
7	Statistics and Probability	Draw informal comparative inferences about two populations.

Cluster Standard: 7.SP.B.4

Standard	Standards for Mathematical Practice
7.SP.B.4: Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.	● **SMP 1:** Make sense of problems and persevere in solving them. ○ Teacher and Student Actions ● **SMP 6**: Attend to precision. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
In this cluster students draw valid, comparable inferences about two populations using measures of center (mean, median) and measures of variability (mean absolute deviation, interquartile range).	● Draw valid comparative inferences about two populations. ● Select the appropriate measure(s) of center (mean and median) or variability (MAD and IQR) when comparing two sets of data and justify that selection.
DOK	**Blooms**
1-2	Understand, Evaluate

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Understand the differences in the measures of center (mean and median) and variability (mean absolute deviation and interquartile range) and use those measures to draw inferences about populations.
- Compare the two populations using measures of center (mean, median) and measures of variability (mean absolute deviation, interquartile range).

Procedural Skill and Fluency:

- Find the measures of center (mean and median) and variability (mean absolute deviation and interquartile range).

Application:

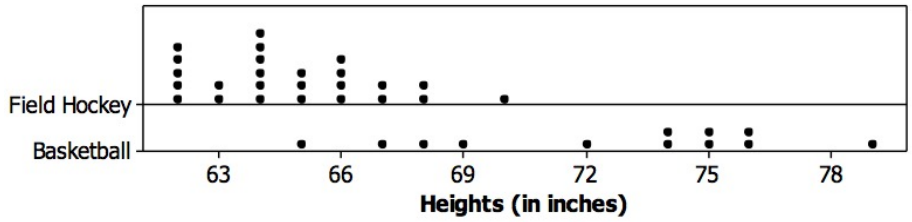
- Compare and draw valid comparative inferences about two populations.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Below are the heights of the players on the University of Maryland women's basketball team for the 2012-2013 season and the heights of the players on the women's field hockey team for the 2012 season. Note: it is typical for a women's field hockey team to have more players than a women's basketball team would.

Field Hockey Player Heights (inches)	Basketball Player Heights (inches)
66	75
64	65
66	76
63	75
67	76
62	72
62	67
64	69
64	74
64	68
65	74
66	79
65	
64	
63	
62	
62	
68	
68	
66	
70	
67	
65	
62	
64	



1. Based on visual inspection of the dot plots, which group appears to have the larger average height? Which group appears to have the greater variability in the heights?

2. Compute the mean and mean absolute deviation (MAD) for each group. Do these values support your answers in #1?

3. How many of the 12 basketball players are shorter than the tallest field hockey player?

4. Imagine that an athlete from one of the two teams told you she needs to go to practice. You estimate that she is about 65 inches tall. If you had to pick, would you think that she was a field hockey player or that she was a basketball player? Explain your reasoning.

5. The women on the Maryland field hockey team are not a random sample of all female college field hockey players. Similarly, the women on the Maryland basketball team are not a random sample of all female college basketball players. However, for purposes of this task, suppose that these two groups can be regarded as random samples of all female college field hockey players and all female college basketball players, respectively. If these were random samples, would you think that female college basketball players are typically taller than female college field hockey players? Explain your decision using answers to the previous questions and/or additional analysis.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Vocabulary:** There is a lot of new vocabulary that students might struggle to internalize, especially if they are not learning those terms in context and are only being asked to memorize their meanings.
- **Bias:** Students will likely come in with a lot of prior knowledge and biases, and many will struggle to release those ideas and instead focus only on the information at hand.
- **Statistical measures:** Students may get confused about which measure (mean or median and mean absolute deviation or interquartile range) should be used for a given problem.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + Universal*	*Layer 2* *Core + Targeted*	*Layer 3* *Core + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc.	**Pre-teaching** In previous classes, learners worked to develop an understanding of graphs, mean, median, mode, Mean Absolute Deviation (M.A.D.), and interquartile range (IQR), recognize there will be variability in the data of a statistical question and will account	**Pre-teaching** Consider using standard 6.SP.A.1, which provides a foundation for work in this cluster. In 6.SP.A.1, students are introduced to a statistical question and the variability present in data.

Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into	for it in the answers, understand a data set has a distribution which can be described by its center, spread, and overall shape, summarize numerical data sets by reporting the number of observations along with describing the nature of the attribute under investigation and how it was measured and its units, approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, predict the approximate relative frequency given the probability, and use ratio and rate reasoning to solve real-world and mathematical problems. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as making informal comparative inferences about one (from earlier grades) and connecting that to the new work involving two populations. This will be the first time students learn about random sampling and evaluating probability models, so make connections to earlier work as possible. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address	Also consider using standard 7.SP.A.2, which also provides a foundation for work in this cluster. In 7.SP.A.2, students draw informal comparative inferences about two populations. Also consider using standard 6.RP.A.3, which also provides a foundation for work in this cluster. In 6.RP.A.3, students work with ratio concepts and use ratio reasoning to solve problems, which will help in solving probability models. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster reviewing random sampling to draw inferences about populations. Give real-world examples and allow students to try different strategies, such as using lists, tables, tree diagrams, and simulations.

more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and	misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Ensure students can explain what a random sample is and give them opportunities to describe and generate random samples in the context of a problem. Review probability, its meaning, and how to calculate it.	

schedules.		
Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/7/34/370/371		
Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● develop an understanding of graphs, mean, median, mode, Mean Absolute Deviation (M.A.D.), and interquartile range (IQR) ● recognize there will be variability in the data of a statistical question and will account for it in the answers. ● understand a data set has a distribution which can be described by its center, spread, and overall shape ● summarize numerical data sets by reporting the number of observations along with describing the nature of the attribute under investigation and how it was measured and its units ● approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency ● predict the approximate relative frequency given the probability ● use ratio and rate reasoning to solve real-world and mathematical problems	In 7th grade, learners ● use random sampling to draw inferences about a population ● recognize and represent proportional relationships between quantities ● use proportional relationships to solve multistep ratio and percent problems	In future classes, learners ● represent data with plots on the real number line (dot plots, histograms, and box plots ● use statistics appropriate to the shape and context of the data distribution to compare measures of center (median, mean) and spread (IQR, standard deviation) of two or more different data sets ● interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points ● construct and interpret a two-way table summarizing data on two categorical variables collected from the same subject ● recognize the purposes of and differences among sample surveys, experiments, and observational studies and explain how randomization relates to each ● find the conditional probability of A given B as the fraction of B's outcomes that also belong to A ● interpret conditional probabilities in terms of a model's context
Culturally and Linguistically Responsive Instruction		

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
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Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

	encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- What does it mean when two data distributions have a similar overlap? What about when they do not?
- How can we measure the difference between the centers of two data distributions?
- How can comparing the visual overlap of two distributions help us understand the data better?
- What does the measure of variability mean in the context of a data distribution? Why is this important to consider?

Cross-Curricular Connections

Literature: Using linear, logical thinking to write more clearly and logically.

Science: Examining biological characteristics of a sample. Making an argument about the growth and development of organisms. Developing models and identifying components and describing relationships between components. Modeling data and describing how the data can be used to create designs through testing and modification. Identifying limitations of models. Engaging in the Engineering Design Process. Modeling genetic information and sexual reproduction results and using Punnett squares. Creating scatterplots of bivariate data. Using simulations to generate data that can be used to modify a proposed object, tool, or process. Conducting experiments in connection with NGSS science standards, collecting bivariate data, representing that data in a two-way table, and hypothesizing correlations between the two variables. Reading scientific charts and graphs. Analyzing scientific studies.

Social Studies: Sampling populations and analyzing data. Studying trends in areas such as species populations, the stock market, or gross domestic product. Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations have different average life spans. Understanding statistical information on populations.

Career and Skill Connections

• Advertising • Aeronautics • Agriculture • Analysis • Aviation • Banking/finance • Botanist • Coaching	• Economist • Education • Engineering • Gardening • Health science • Information technology • Law • Machinist	• Park ranger • Political science • Psychology • Ranching/farming • Sales • Statistics • Technician • Transportation

● Counseling ● Data science ● Ecology	● Management ● Mechanic ● Medicine	● Urban planning ● Veterinary

Grade	CCSS Domain	CCSS Cluster
7	Statistics and Probability	Investigate chance processes and develop, use, and evaluate probability models.

Cluster Standard: 7.SP.C.5

Standard	Standards for Mathematical Practice
7.SP.C.5: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely or likely, and a probability near 1 indicates a likely event.	● **SMP 2**: Reason abstractly and quantitatively. ○ Teacher and Student Actions ● **SMP 7**: Look for and make use of structure. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This cluster focuses on probability and is the first-time students encounter this topic formally. Students learn the likelihood of chance events and approximate probabilities. They investigate chance using probability models they develop. The cluster begins with single events and builds up to finding the probability of compound events using tree diagrams, lists, tables, and simulations.	● In writing, express the likelihood of a chance event with a probability range from 0 to 1. ● Recognize that the probability of any single event can be expressed with the terms impossible, unlikely, equally likely, likely, or certain. ● Express probability as a fraction, decimal or percent.
DOK	**Blooms**
1	Understand

Procedural and Conceptual Understanding and Application

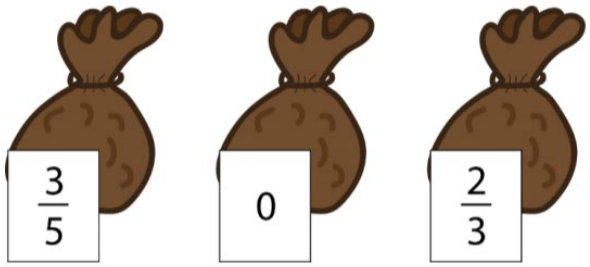
Conceptual Understanding:

- Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.
- Recognize and differentiate between events that are impossible, unlikely, equally likely, likely, or certain based on their probabilities.
- Develop probability models to investigate chance.
- Understand when and how to use tree diagrams, lists, tables, and simulations to find the probability of compound events.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

At a school party, there are three bags on a table.



The teacher says, "I have put some red marbles and white marbles into bags. Choose a bag, reach in without looking, and take out a marble. If the marble is red, you win a prize! The sign in front of each bag tells the probability of getting a red marble."

1. What does it mean that the middle bag has a probability of 0?

2. To have the greatest chance of winning a prize, which of the three bags should you choose? Answer by selecting one of the boxes below.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Vocabulary:** There is a lot of new vocabulary that students might struggle to internalize, especially if they are not learning those terms in context and are only being asked to memorize their meanings.
- **Bias:** Students will likely come in with a lot of prior knowledge and biases, and many will struggle to release those ideas and instead focus only on the information at hand.
- **Probability** Students may confuse an unlikely event (probability near 0) with an impossible event (probability of 0). Students might also struggle with the idea of probability being expressed as decimals less than one that can also be expressed as fractions and percents.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + Universal*	*Layer 2* *Core + Targeted*	*Layer 3* *Core + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating	**Pre-teaching** In previous classes, learners worked to develop an understanding of graphs, mean, median, mode, Mean Absolute Deviation (M.A.D.), and	**Pre-teaching** Consider using standard 6.SP.A.1, which provides a foundation for work in this cluster. In 6.SP.A.1, students are introduced to a statistical

displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they	interquartile range (IQR), recognize there will be variability in the data of a statistical question and will account for it in the answers, understand a data set has a distribution which can be described by its center, spread, and overall shape, summarize numerical data sets by reporting the number of observations along with describing the nature of the attribute under investigation and how it was measured and its units, approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, predict the approximate relative frequency given the probability, and use ratio and rate reasoning to solve real-world and mathematical problems. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as making informal comparative inferences about one (from earlier grades) and connecting that to the new work involving two populations. This will be the first-time students learn about random sampling and evaluating probability models, so make connections to earlier work as possible. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.	question and the variability present in data. Also consider using standard 7.SP.A.2, which also provides a foundation for work in this cluster. In 7.SP.A.2, students draw informal comparative inferences about two populations. Also consider using standard 6.RP.A.3, which also provides a foundation for work in this cluster. In 6.RP.A.3, students work with ratio concepts and use ratio reasoning to solve problems, which will help in solving probability models. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster reviewing random sampling to draw inferences about populations. Give real-world examples and allow students to try different strategies, such as using lists, tables, tree diagrams, and simulations.

complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-	**Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Ensure students can explain what a random sample is and give them opportunities to describe and generate random samples in the context of a problem. Review probability, its meaning, and how to calculate it.	

assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/7/34/373/373

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● develop an understanding of graphs, mean, median, mode, Mean Absolute Deviation (M.A.D.), and interquartile range (IQR) ● recognize there will be variability in the data of a statistical question and will account for it in the answers. ● understand a data set has a distribution which can be described by its center, spread, and overall shape ● summarize numerical data sets by reporting the number of observations along with describing the nature of the attribute under investigation and how it was measured and its units ● approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency ● predict the approximate relative frequency given the probability ● use ratio and rate reasoning to solve real-world and mathematical problems	In 7th grade, learners ● use random sampling to draw inferences about a population ● recognize and represent proportional relationships between quantities ● use proportional relationships to solve multistep ratio and percent problems	In future classes, learners ● represent data with plots on the real number line (dot plots, histograms, and box plots ● use statistics appropriate to the shape and context of the data distribution to compare measures of center (median, mean) and spread (IQR, standard deviation) of two or more different data sets ● interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points ● construct and interpret a two-way table summarizing data on two categorical variables collected from the same subject ● recognize the purposes of and differences among sample surveys, experiments, and observational studies and explain how randomization relates to each ● find the conditional probability of A given B as the fraction of B's outcomes that also belong to A ● interpret conditional probabilities in terms of a model's context

Culturally and Linguistically Responsive Instruction		
Consider these resources for vocabulary from Pathways2Careers: • https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf • https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf		
Consider these questions as you plan for instruction that is culturally and linguistically responsive: • How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics.	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

	• Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- What is probability? What does it represent? Can we use any value for probability? What about negative numbers?
- What does it mean if an event is unlikely? Likely? Neither unlikely or likely?

Cross-Curricular Connections

Literature: Using linear, logical thinking to write more clearly and logically.

Science: Examining biological characteristics of a sample. Making an argument about the growth and development of organisms. Developing models and identifying components and describing relationships between components. Modeling data and describing how the data can be used to create designs through testing and modification. Identifying limitations of models. Engaging in the Engineering Design Process. Modeling genetic information and sexual reproduction results and using Punnett squares. Creating scatterplots of bivariate data. Using simulations to generate data that can be used to modify a proposed object, tool, or process. Conducting experiments in connection with NGSS science standards, collecting bivariate data, representing that data in a two-way table, and hypothesizing correlations between the two variables. Reading scientific charts and graphs. Analyzing scientific studies.

Social Studies: Sampling populations and analyzing data. Studying trends in areas such as species populations, the stock market, or gross domestic product. Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations have different average life spans. Understanding statistical information on populations.

Career and Skill Connections

• Advertising • Aeronautics • Agriculture • Analysis • Aviation • Banking/finance • Botanist • Coaching	• Economist • Education • Engineering • Gardening • Health science • Information technology • Law • Machinist	• Park ranger • Political science • Psychology • Ranching/farming • Sales • Statistics • Technician • Transportation

● Counseling ● Data science ● Ecology	● Management ● Mechanic ● Medicine	● Urban planning ● Veterinary

Grade	CCSS Domain	CCSS Cluster
7	Statistics and Probability	Investigate chance processes and develop, use, and evaluate probability models.

Cluster Standard: 7.SP.C.6

Standard	Standards for Mathematical Practice

7.SP.C.6: Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.	• **SMP 4**: Model with mathematics.
	  ○ Teacher and Student Actions
	• **SMP 8:** Look for and express regularity in repeated reasoning.
	  ○ Teacher and Student Actions

Clarification Statement	**Students Who Demonstrate Understanding Can...**
This cluster focuses on probability and is the first-time students encounter this topic formally. Students learn the likelihood of chance events and approximate probabilities. They investigate chance using probability models they develop. The cluster begins with single events and builds up to finding the probability of compound events using tree diagrams, lists, tables, and simulations.	• Collect data on chance events (hands-on events such as spinning a spinner and simulations) and approximate the relative frequency of an event given the probability. • Students recognize that as the number of trials increases, the relative frequency approaches the probability. • Explain the difference between relative frequency and theoretical probability using appropriate language. • Determine the sample space for a probability model.
DOK	**Blooms**
2-3	Understand, Apply

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Understand chance events and develop probability models to investigate chance.
- Understand that as the number of trials increases, the relative frequency approaches the probability of an event.
- Understand how to predict the approximate relative frequency of an event given its probability

- Understand and explain the difference between relative frequency and theoretical probability using appropriate language

Procedural Skill and Fluency:

- Collect data on a chance process that produces a chance event and analyze it to approximate the relative frequency given the probability.
- Calculate the approximate relative frequency of an event given its probability.
- Determine the sample space for a probability model.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Each of the 20 students in Mr. Anderson's class flipped a coin ten times and recorded how many times it came out heads.

1. How many heads do you think you will see out of ten tosses?

2. Would it surprise you to see 4 heads out of ten tosses? Explain why or why not.

3. Here are the results for the twenty students in Mr. Anderson's class. Use this data to estimate the probability of observing 4, 5 or 6 heads in ten tosses of the coin. (It might help to organize the data in a table or in a dot plot first.)

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Number of heads	3	5	4	6	4	8	5	4	9	5	3	4	7	5	8	6	3	6	5	7

4. How many heads do you think you will see out of 100 tosses? 200? 300?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Vocabulary:** There is a lot of new vocabulary that students might struggle to internalize, especially if they are not learning those terms in context and are only being asked to memorize their meanings.
- **Bias:** Students will likely come in with a lot of prior knowledge and biases, and many will struggle to release those ideas and instead focus only on the information at hand.
- **Probability** Students may confuse an unlikely event (probability near 0) with an impossible event (probability of 0). Students might also struggle with the idea of probability being expressed as decimals less than one that can also be expressed as fractions and percents.
- **Long-run frequencies:** Students might not understand why many trials are necessary to obtain probability. They may also assume that the actual outcomes of trials will exactly match the predicted relative frequency and may struggle to make predictions about future events based on probability.

- **Relative frequency:** Students may want to express this as probability and not understand that probability helps determine the approximate relative frequency.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + Universal*	*Layer 2* *Core + Targeted*	*Layer 3* *Core + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in	**Pre-teaching** In previous classes, learners worked to develop an understanding of graphs, mean, median, mode, Mean Absolute Deviation (M.A.D.), and interquartile range (IQR), recognize there will be variability in the data of a statistical question and will account for it in the answers, understand a data set has a distribution which can be described by its center, spread, and overall shape, summarize numerical data sets by reporting the number of observations along with describing the nature of the attribute under investigation and how it was measured and its units, approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, predict the approximate relative frequency given the probability, and use ratio and rate reasoning to solve real-world and mathematical problems. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as making informal comparative inferences about one (from earlier grades) and connecting that to the new work involving two populations. This will be the first-time students learn about random sampling and evaluating probability models, so make connections to earlier work as	**Pre-teaching** Consider using standard 6.SP.A.1, which provides a foundation for work in this cluster. In 6.SP.A.1, students are introduced to a statistical question and the variability present in data. Also consider using standard 7.SP.A.2, which also provides a foundation for work in this cluster. In 7.SP.A.2, students draw informal comparative inferences about two populations. Also consider using standard 6.RP.A.3, which also provides a foundation for work in this cluster. In 6.RP.A.3, students work with ratio concepts and use ratio reasoning to solve problems, which will help in solving probability models. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made.

which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure	possible. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Ensure students can explain what a random sample is and give them opportunities to describe and generate random samples in the context of a problem. Review probability, its meaning, and how to calculate it.	Students may benefit from intensive extra time during and after work within this cluster reviewing random sampling to draw inferences about populations. Give real-world examples and allow students to try different strategies, such as using lists, tables, tree diagrams, and simulations.

students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/7/34/375/375

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● develop an understanding of graphs, mean, median, mode, Mean Absolute Deviation (M.A.D.), and interquartile range (IQR) ● recognize there will be variability in the data of a statistical question and will account for it in the answers. ● understand a data set has a distribution which can be described by its center, spread, and overall shape ● summarize numerical data sets by reporting the number of observations along with describing the nature of the attribute under investigation and how it was measured and its units ● approximate the probability of a chance event by collecting data on the chance	In 7th grade, learners ● use random sampling to draw inferences about a population ● recognize and represent proportional relationships between quantities ● use proportional relationships to solve multistep ratio and percent problems	In future classes, learners ● represent data with plots on the real number line (dot plots, histograms, and box plots ● use statistics appropriate to the shape and context of the data distribution to compare measures of center (median, mean) and spread (IQR, standard deviation) of two or more different data sets ● interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points ● construct and interpret a two-way table summarizing data on two categorical variables collected from the same subject ● recognize the purposes of

process that produces it and observing its long-run relative frequency • predict the approximate relative frequency given the probability • use ratio and rate reasoning to solve real-world and mathematical problems		and differences among sample surveys, experiments, and observational studies and explain how randomization relates to each • find the conditional probability of A given B as the fraction of B's outcomes that also belong to A • interpret conditional probabilities in terms of a model's context

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive

towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- What is probability? What does it represent? Can we use any value for probability? What about negative numbers?
- What does it mean if an event is unlikely? Likely? Neither unlikely or likely?
- Why do we assess probability based on long-run observations? What would happen if we did not?
- Are probabilities always exactly accurate? Why or why not?

Cross-Curricular Connections

Literature: Using linear, logical thinking to write more clearly and logically.

Science: Examining biological characteristics of a sample. Making an argument about the growth and development of organisms. Developing models and identifying components and describing relationships between components. Modeling data and describing how the data can be used to create designs through testing and modification. Identifying limitations of models. Engaging in the Engineering Design Process. Modeling genetic information and sexual reproduction results and using Punnett squares. Creating scatterplots of bivariate data. Using simulations to generate data that can be used to modify a proposed object, tool, or process. Conducting experiments in connection with NGSS science standards, collecting bivariate data, representing that data in a two-way table, and hypothesizing correlations between the two variables. Reading scientific charts and graphs. Analyzing scientific studies.

Social Studies: Sampling populations and analyzing data. Studying trends in areas such as species populations, the

stock market, or gross domestic product. Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations have different average life spans. Understanding statistical information on populations.

Career and Skill Connections

- Advertising
- Aeronautics
- Agriculture
- Analysis
- Aviation
- Banking/finance
- Botanist
- Coaching
- Counseling
- Data science
- Ecology
- Economist
- Education
- Engineering
- Gardening
- Health science
- Information technology
- Law
- Machinist
- Management
- Mechanic
- Medicine
- Park ranger
- Political science
- Psychology
- Ranching/farming
- Sales
- Statistics
- Technician
- Transportation
- Urban planning
- Veterinary

Grade	CCSS Domain	CCSS Cluster
7	Statistics and Probability	Investigate chance processes and develop, use, and evaluate probability models.

Cluster Standard: 7.SP.C.7

Standard	Standards for Mathematical Practice
7.SP.C.7: Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy ● 7.SP.C.7.A: Develop a uniform probability model by assigning equal probability to all outcomes and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected ● 7.SP.C.7.B: Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?	● **SMP 3**: Construct viable arguments and critique the reasoning of others. ○ Teacher and Student Actions ● **SMP 6**: Attend to precision. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This cluster focuses on probability and is the first-time students encounter this topic formally. Students learn the likelihood of chance events and approximate probabilities. They investigate chance using probability models they develop. The cluster begins with single events and builds up to finding the probability of compound events using tree diagrams, lists, tables, and simulations.	● Calculate the probability of a (simple) event as a fraction, decimal, or percent. ● Determine the probability of events by developing uniform and non-uniform probability models (theoretical probability). ● Compare the models to the observed frequency and explain their reasoning for any discrepancies between the model and the observed frequency using appropriate vocabulary. ● Develop their understanding of probability by making predictions, comparing the predictions, replicating experiments, and comparing results.

DOK	Blooms
2-3	Apply, Analyze, Evaluate

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- If the agreement between probability and observed frequencies is not good, understand and explain possible sources of the discrepancy.
- Understand the difference between probability models that are uniform and those that are not, as well as instances to use each type of model.
- Compare probabilities from a model to observed frequencies and explain any discrepancies between the model and the observed frequency using appropriate vocabulary.
- Develop understanding of probability by making predictions, comparing the predictions, replicating experiments, and comparing results.

Procedural Skill and Fluency:

- Develop uniform probability models and use them to calculate the probabilities of events.
- Develop probability models by observing frequencies in data generated from a chance process and use them to calculate the probabilities of events.
- Calculate the probability of a simple event as a fraction, decimal, or percent.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Look at the shirt you are wearing today and determine how many buttons it has. Then complete the following table for all the members of your class.

	No Buttons	One or Two Buttons	Three or Four Buttons	More Than Four Buttons
Male				
Female				

Suppose each student writes his or her name on an index card, and one card is selected randomly.

1. What is the probability that the student whose card is selected is wearing a shirt with no buttons?

2. What is the probability that the student whose card is selected is female and is wearing a shirt with two or fewer buttons?

3. Imagine you were going to roll two dice 10 times. What fraction of the 10 rolls would you predict would result in at least one six?

4. Make a list of all the different possible outcomes that might be observed when two dice are rolled. (Hint: There are 36 different possible outcomes.) What fraction of the 36 possible outcomes result in at least one six?

5. Now roll two dice 10 times. After each roll, note whether any sixes were observed and record your results in the table below.

Roll	Any Sixes? (Y/N)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

6. What fraction of the 10 rolls resulted in at least one six? How does this compare to you prediction in #3? Explain why they are the same or different.

7. Combine your results with those of your classmates. What fraction of all the rolls in the class resulted in at least one six? How does this compare to you prediction in #3? Explain why they are the same or different.

8. Suppose you and your classmates were able to roll the two dice many thousands of times. What fraction of the time would you expect to roll at least one six?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Vocabulary:** There is a lot of new vocabulary that students might struggle to internalize, especially if they are not learning those terms in context and are only being asked to memorize their meanings.
- **Bias:** Students will likely come in with a lot of prior knowledge and biases, and many will struggle to release those ideas and instead focus only on the information at hand.
- **Probability** Students may confuse an unlikely event (probability near 0) with an impossible event (probability of 0). Students might also struggle with the idea of probability being expressed as decimals less than one that can also be expressed as fractions and percents. Students might be confused about the difference between observed frequencies and probability.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + Universal*	*Layer 2* *Core + Targeted*	*Layer 3* *Core + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or	**Pre-teaching** In previous classes, learners worked to develop an understanding of graphs, mean, median, mode, Mean Absolute Deviation (M.A.D.), and interquartile range (IQR), recognize there will be variability in the data of a statistical question and will account for it in the answers, understand a data set has a distribution which can be described by its center, spread, and overall shape, summarize numerical data sets by reporting the number of observations along with describing the nature of the attribute under investigation and how it was measured and its units, approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, predict the approximate relative frequency given the probability, and use ratio and rate reasoning to solve real-world and mathematical problems. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as making informal comparative inferences about one (from earlier grades) and	**Pre-teaching** Consider using standard 6.SP.A.1, which provides a foundation for work in this cluster. In 6.SP.A.1, students are introduced to a statistical question and the variability present in data. Also consider using standard 7.SP.A.2, which also provides a foundation for work in this cluster. In 7.SP.A.2, students draw informal comparative inferences about two populations. Also consider using standard 6.RP.A.3, which also provides a foundation for work in this cluster. In 6.RP.A.3, students work with ratio concepts and use ratio reasoning to solve problems, which will help in solving probability models. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of

representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives,	connecting that to the new work involving two populations. This will be the first time students learn about random sampling and evaluating probability models, so make connections to earlier work as possible. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Ensure students can explain what a random sample is and give them opportunities to describe and generate random samples in the context of a problem. Review probability, its meaning, and how to calculate it.	lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster reviewing random sampling to draw inferences about populations. Give real-world examples and allow students to try different strategies, such as using lists, tables, tree diagrams, and simulations.

drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/7/34/375/377

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● develop an understanding of graphs, mean, median, mode, Mean Absolute Deviation (M.A.D.), and interquartile range (IQR) ● recognize there will be variability in the data of a statistical question and will account for it in the answers. ● understand a data set has a distribution which can be described by its center, spread, and overall shape ● summarize numerical data sets by reporting the number of observations along with describing the nature of the	In 7th grade, learners ● use random sampling to draw inferences about a population ● recognize and represent proportional relationships between quantities ● use proportional relationships to solve multistep ratio and percent problems	In future classes, learners ● represent data with plots on the real number line (dot plots, histograms, and box plots ● use statistics appropriate to the shape and context of the data distribution to compare measures of center (median, mean) and spread (IQR, standard deviation) of two or more different data sets ● interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points

attribute under investigation and how it was measured and its units • approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency • predict the approximate relative frequency given the probability • use ratio and rate reasoning to solve real-world and mathematical problems		• construct and interpret a two-way table summarizing data on two categorical variables collected from the same subject • recognize the purposes of and differences among sample surveys, experiments, and observational studies and explain how randomization relates to each • find the conditional probability of A given B as the fraction of B's outcomes that also belong to A • interpret conditional probabilities in terms of a model's context

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such

whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- What does it mean if an event is unlikely? Likely? Neither unlikely or likely?
- Why do we assess probability based on long-run observations? What would happen if we did not?
- Are probabilities always exactly accurate? Why or why not?
- How do we find probability when all outcomes are equally likely?

Cross-Curricular Connections

Literature: Using linear, logical thinking to write more clearly and logically.

Science: Examining biological characteristics of a sample. Making an argument about the growth and development of organisms. Developing models and identifying components and describing relationships between components. Modeling data and describing how the data can be used to create designs through testing and modification. Identifying limitations of models. Engaging in the Engineering Design Process. Modeling genetic information and sexual reproduction results and using Punnett squares. Creating scatterplots of bivariate data. Using simulations to

generate data that can be used to modify a proposed object, tool, or process. Conducting experiments in connection with NGSS science standards, collecting bivariate data, representing that data in a two-way table, and hypothesizing correlations between the two variables. Reading scientific charts and graphs. Analyzing scientific studies.

Social Studies: Sampling populations and analyzing data. Studying trends in areas such as species populations, the stock market, or gross domestic product. Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations have different average life spans. Understanding statistical information on populations.

Career and Skill Connections

• Advertising • Aeronautics • Agriculture • Analysis • Aviation • Banking/finance • Botanist • Coaching • Counseling • Data science • Ecology	• Economist • Education • Engineering • Gardening • Health science • Information technology • Law • Machinist • Management • Mechanic • Medicine	• Park ranger • Political science • Psychology • Ranching/farming • Sales • Statistics • Technician • Transportation • Urban planning • Veterinary

Grade	CCSS Domain	CCSS Cluster
7	Statistics and Probability	Investigate chance processes and develop, use, and evaluate probability models.

Cluster Standard: 7.SP.C.8

Standard	Standards for Mathematical Practice
7.SP.C.8: Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. ● 7.SP.C.8.A: Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. ● 7.SP.C.8.B: Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. ● 7.SP.C.8.C: Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?	● **SMP 1:** Make sense of problems and persevere in solving them. ○ Teacher and Student Actions ● **SMP 6**: Attend to precision. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
This cluster focuses on probability and is the first-time students encounter this topic formally. Students learn the likelihood of chance events and approximate probabilities. They investigate chance using probability models they develop. The cluster begins with single events and builds up to finding the probability of compound events using tree diagrams, lists, tables, and simulations.	● Understand similarities and differences between compound events and simple events. ● Find the sample space of a compound event. ● Create organized lists, tables, tree diagrams, and simulations to find the probability of a compound event. ● Represent the probability of a compound event as a fraction, decimal, or percent. ● Design and use a simulation (using a random number table, calculator, dice, cards, or other manipulatives) to generate frequencies of compound events. ● Justify their selection of a particular situation and

	explain how it models a compound event.
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs
- Understand the similarities and differences between compound events and simple events.
- Describe scenarios that represent dependent and independent events for compound events.

Procedural Skill and Fluency:

- Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
- Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams, identifying the outcomes in the sample space which compose the event.
- Find the sample space of a compound event.
- Represent the probability of a compound event as a fraction, decimal, or percent.

Application:

- Design and use a simulation (using a random number table, calculator, dice, cards, or other manipulatives) to generate frequencies of compound events.
- Justify the selection of a particular situation and explain how it models a compound event.
- Calculate the probabilities of compound events in various contexts, such as games, sports, and everyday situations.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

1. Angie, Bridget, Carlos, and Diego are seated at random around a square table, one person to a side. What is the theoretical probability that Angie and Carlos are seated opposite each other?

2. Many games use dice which are six-sided and fair (meaning each face on the die is equally likely to land face up). Many games also use the sum of two dice rolled at the same time to determine movement of game pieces. However, not all dice are six-sided. Imagine a game in which two fair four-sided (tetrahedral) dice are rolled simultaneously. These dice are in the shape of a pyramid, and when a die is rolled, the outcome is determined by the side that lands face down. Suppose that for these two dice, the possible values (corresponding to the four sides of the die) that can be obtained from each die are as follows:

   Die #1: 1, 2, 3, or 4 Die #2: 2, 4, 6, or 8

A certain game determines the movement of players' game pieces based on the SUM of the numbers on the face down sides when two dice are rolled. There are 10 distinct sum values that can occur, and some of those sums occur more often than others.

a. Using an organized list, table, tree diagram, or method of your choosing, develop a list of all 16 possible outcomes (for example, Die #1 = 1 and Die #2 = 2 for a sum of 3; Die #1 = 1 and Die #2 = 4 for a sum of 5; and so on).

b. From your work in part a, determine the 10 **distinct sum values** that are possible and calculate the probability of obtaining each sum value. Note: as mentioned above, some values will occur more frequently than others.

c. Using your work in part b, answer the following questions:

   I. What is the probability of obtaining a sum of 5?

   II. What is the probability of obtaining a sum that is more than 5?

   III. What is the probability of obtaining a sum that is at most 5?

   IV. What is the probability of obtaining a sum that is at least 5?

   V. What is the probability of obtaining a sum that is no less than 5?

3. Now consider the case where the DIFFERENCE in the numbers on the face down sides when two dice are rolled is important to the game. Unless the two die values are the same (in which case the difference is 0), the difference for purposes of this game will always be computed as the larger number value rolled minus the smaller number value rolled. In this way, the difference value for any roll of the two dice will always be positive or 0.

a. Using an organized list, table, tree diagram, or method of your choosing, develop a list of all 16 possible outcomes (for example, Die #1 = 1 and Die #2 = 2 for a difference of 1; Die #1 = 1 and Die #2 = 4 for a difference of 3; and so on).

b. From your work in part a, determine the 8 distinct difference values that are possible and calculate the probability of obtaining each difference value. Note: as mentioned above, some values will occur more frequently than others.

c. Using your work in part eb answer the following questions:

   I. What is the probability of obtaining a difference of 5?

   II. What is the probability of obtaining a difference that is more than 5?

   III. What is the probability of obtaining a difference that is less than or equal to 5?

4. Suppose each box of a popular brand of cereal contains a pen as a prize. The pens come in four colors, blue, red, green and yellow. Each color of pen is equally likely to appear in any box of cereal. Design and carry out a simulation to help you answer each of the following questions.

a. What is the probability of having to buy at least five boxes of cereal to get a blue pen? What is the mean (average) number of boxes you would have to buy to get a blue pen if you repeated the process many times?

b. What is the probability of having to buy at least ten boxes of cereal to get a full set of pens (all four colors)? What is the mean (average) number of boxes you would have to buy to get a full set of pens if you repeated the process many times?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Dependent and independent:** Students often believe that all compound events are independent and do not understand the difference between independent and dependent events.
- **Ignoring Sample Space:** Students might not consider all possible outcomes, leading to incorrect probability calculations.
- **Compound events:** Students might struggle to understand how to break compound events up into smaller pieces that can be modeled with tree diagrams, lists, tables, and simulations. Additionally, they may have trouble constructing these diagrams, lists, tables, and simulations in terms of organization and the chance that when creating lists randomly they will miss one or more outcomes.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + Universal*	*Layer 2* *Core + Targeted*	*Layer 3* *Core + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other	**Pre-teaching** In previous classes, learners worked to develop an understanding of graphs, mean, median, mode, Mean Absolute Deviation (M.A.D.), and interquartile range (IQR), recognize there will be variability in the data of a statistical question and will account for it in the answers, understand a data set has a distribution which can be described by its center, spread, and overall shape, summarize numerical data sets by reporting the number of observations along with describing the nature of the attribute under investigation and how it was measured and its units, approximate the probability of a chance event by	**Pre-teaching** Consider using standard 6.SP.A.1, which provides a foundation for work in this cluster. In 6.SP.A.1, students are introduced to a statistical question and the variability present in data. Also consider using standard 7.SP.A.2, which also provides a foundation for work in this cluster. In 7.SP.A.2, students draw informal comparative inferences about two populations. Also consider using standard 6.RP.A.3, which also provides a foundation for work in this cluster. In 6.RP.A.3, students work with ratio

mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps	collecting data on the chance process that produces it and observing its long-run relative frequency, predict the approximate relative frequency given the probability, and use ratio and rate reasoning to solve real-world and mathematical problems. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as making informal comparative inferences about one (from earlier grades) and connecting that to the new work involving two populations. This will be the first-time students learn about random sampling and evaluating probability models, so make connections to earlier work as possible. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Ensure students can explain what a random sample is and give them	concepts and use ratio reasoning to solve problems, which will help in solving probability models. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster reviewing random sampling to draw inferences about populations. Give real-world examples and allow students to try different strategies, such as using lists, tables, tree diagrams, and simulations.

students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.	opportunities to describe and generate random samples in the context of a problem. Review probability, its meaning, and how to calculate it.	

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/7/34/375/384		
Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● develop an understanding of	In 7th grade, learners ● use random sampling to draw	In future classes, learners ● represent data with plots on

graphs, mean, median, mode, Mean Absolute Deviation (M.A.D.), and interquartile range (IQR) • recognize there will be variability in the data of a statistical question and will account for it in the answers. • understand a data set has a distribution which can be described by its center, spread, and overall shape • summarize numerical data sets by reporting the number of observations along with describing the nature of the attribute under investigation and how it was measured and its units • approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency • predict the approximate relative frequency given the probability • use ratio and rate reasoning to solve real-world and mathematical problems	inferences about a population • recognize and represent proportional relationships between quantities • use proportional relationships to solve multistep ratio and percent problems	the real number line (dot plots, histograms, and box plots • use statistics appropriate to the shape and context of the data distribution to compare measures of center (median, mean) and spread (IQR, standard deviation) of two or more different data sets • interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points • construct and interpret a two-way table summarizing data on two categorical variables collected from the same subject • recognize the purposes of and differences among sample surveys, experiments, and observational studies and explain how randomization relates to each • find the conditional probability of A given B as the fraction of B's outcomes that also belong to A • interpret conditional probabilities in terms of a model's context

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating

mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.
Suggested Student Discourse Questions		
Consider this resource for student discourse from Pathways2Careers:		

https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- What does it mean if an event is unlikely? Likely? Neither unlikely or likely?
- Why do we assess probability based on long-run observations? What would happen if we did not?
- Are probabilities always exactly accurate? Why or why not?
- What is a compound event? How do we find probability when there are compound events?
- How can we represent outcomes of compound events in lists? What about tables? Tree diagrams? Simulations? Which do you prefer? Why?

Cross-Curricular Connections

Literature: Using linear, logical thinking to write more clearly and logically.

Science: Examining biological characteristics of a sample. Making an argument about the growth and development of organisms. Developing models and identifying components and describing relationships between components. Modeling data and describing how the data can be used to create designs through testing and modification. Identifying limitations of models. Engaging in the Engineering Design Process. Modeling genetic information and sexual reproduction results and using Punnett squares. Creating scatterplots of bivariate data. Using simulations to generate data that can be used to modify a proposed object, tool, or process. Conducting experiments in connection with NGSS science standards, collecting bivariate data, representing that data in a two-way table, and hypothesizing correlations between the two variables. Reading scientific charts and graphs. Analyzing scientific studies.

Social Studies: Sampling populations and analyzing data. Studying trends in areas such as species populations, the stock market, or gross domestic product. Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations have different average life spans. Understanding statistical information on populations.

Career and Skill Connections

- Advertising
- Aeronautics
- Agriculture
- Analysis
- Aviation
- Banking/finance
- Botanist
- Coaching
- Counseling
- Data science
- Ecology
- Economist
- Education
- Engineering
- Gardening
- Health science
- Information technology
- Law
- Machinist
- Management
- Mechanic
- Medicine
- Park ranger
- Political science
- Psychology
- Ranching/farming
- Sales
- Statistics
- Technician
- Transportation
- Urban planning
- Veterinary

Section 3: Resources, References, and Glossary

Resources

Evidence-Based Resources	English Learner Resources	MLSS Resources	Mathematics Standard Resources
What Works Clearinghouse Best Evidence Encyclopedia Evidence for Every Students Succeeds Act Evidence in Education Lab	World-Class Instructional Design and Assessment (WIDA) Standards USCALE Language Routines for Mathematics English Language Development Standards Spanish Language Development Standards	NM Multi-Layered System of Supports (MLSS) Universal Design for Learning Guidelines Achieve the Core: Instructional Routines for Mathematics Project Zero Thinking Routines	Focus by Grade Level and Widely Applicable Prerequisites High school Coherence Map College-and Career Ready Math Shifts Fostering Math Practices: Routines for the Mathematical Practices

Planning Guidance for Multi-Layered Systems of Support: Core Instruction[9]

Core Instructional Planning must reflect and leverage scientific insights into how humans learn in order to ensure all students are ready for success, thus the following guidance for optimizing teaching and learning is grounded in the **Universal Design Learning (UDL) Framework**

Key design questions, planning actions, and potential strategies are provided below, with respect to guidance for minimizing barriers to learning and optimizing (1) universal ACCESS to learning experiences, (2) opportunities for students to BUILD their understanding of the **Learning Goal**, and (3) INTERNALIZATION of the Learning Goal.

Optimizing Universal ACCESS to Learning Experiences	
ENGAGEMENT ? How will you provide multiple options for recruiting interest?	**Recruiting Student Interest:** ? What do you anticipate in the range of student interest for this lesson? ➢ Plan for options for recruiting student interest: ❒ provide choice (e.g. sequence or timing of task completion) ❒ set personal academic goals ❒ provide contextualized examples connected to their lives ❒ support culturally relevant connections (i.e home culture) ❒ create socially relevant tasks ❒ provide novel & relevant problems to make sense of complex ideas in creative ways

[9] Adapted from: CAST (2018). *Universal Design for Learning Guidelines version 2.2*. Retrieved from http://udlguidelines.cast.org

	❒ provide time for self-reflection about content & activities ❒ create accepting and supportive classroom climate ❒ utilize **instructional routines** to involve all students
REPRESENTATION [?] How will you reduce barriers to perceiving the information presented in this lesson?	**Perception:** [?] What do you anticipate about the range in how students will perceive information presented in this lesson? ➢ Plan for different modalities and formats to reduce barriers to learning: ❒ display information in a flexible format to vary perceptual features ❒ offer alternatives for auditory information ❒ offer alternatives for visual information
ACTION & EXPRESSION [?] How will the learning for students provide a variety of methods for navigation to support access?	**Physical Action:** [?] What do you anticipate about the range in how students will physically navigate and respond to the learning experience? ➢ Plan a variety of methods for response and navigation of learning experiences by offering alternatives to: ❒ requirements for rate, timing, speed, and range of motor action with instructional materials, manipulatives, and technologies ❒ physically indicating selections ❒ interacting with materials by hand, voice, keyboard, etc.

Opportunities for Students to BUILD their Understanding

ENGAGEMENT [?] How will the learning for students provide options for sustaining effort and persistence?	**Sustaining Effort & Persistence:** [?] What do you anticipate about the range in student effort? ➢ Plan multiple methods for attending to student attention and affect by: ❒ prompting learners to explicitly formulate or restate learning goals ❒ displaying the learning goals in multiple ways ❒ using prompts or scaffolds for visualizing desired outcomes ❒ engaging assessment discussions of what constitutes excellence ❒ generating relevant examples with students that connect to their cultural background and interests ❒ providing alternatives in the math representations and scaffolds ❒ creating cooperative groups with clear goals, roles, responsibilities ❒ providing prompts to guide when and how to ask for help ❒ supporting opportunities for peer interactions and supports (e.g. peer tutors) ❒ constructing communities of learners engaged in common interests ❒ creating expectations for group work (e.g., rubrics, norms, etc.) ❒ providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge ❒ providing feedback that: ❒ emphasizes effort, improvement, and achieving a standard rather than on relative performance ❒ is frequent, timely, and specific ❒ is informative rather than comparative or competitive

	❒ models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success
REPRESENTATION ? How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners?	**Language & Symbols:** ? What do you anticipate about the range of student background experience and vocabulary? ➢ Plan multiple methods for attending to linguistic and nonlinguistic representations of mathematics to ensure universal clarity by: ❒ pre-teaching vocabulary and symbols in ways that promote connection to the learners' experience and prior knowledge ❒ graphic symbols with alternative text descriptions ❒ highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to structure ❒ embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) ❒ embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) ❒ highlighting structural relations or make them more explicit ❒ making connections to previously learned structures ❒ making relationships between elements explicit (e.g., highlighting the transition words in an argument, links between ideas, etc.) ❒ allowing the use of text-to-speech and automatic voicing with digital mathematical notation (math ml) ❒ allowing flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) ❒ clarification of notation through lists of key terms ❒ making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf ❒ linking key vocabulary words to definitions and pronunciations in both dominant and heritage languages ❒ defining domain-specific vocabulary (e.g., "map key" in social studies) using both domain-specific and common terms ❒ electronic translation tools or links to multilingual web glossaries ❒ embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc) ❒ presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) ❒ making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams
ACTION & EXPRESSION ? How will the learning provide multiple	**Expression & Communication:** ? What do you anticipate about the range in how students will express their thinking in the learning environment? ➢ Plan multiple methods for attending to the various ways in which students can express knowledge, ideas, and concepts by providing:

modalities for students to easily express knowledge, ideas, and concepts in the learning environment?	❒ options to compose in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video ❒ use of social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) ❒ flexibility in using a variety of problem solving strategies ❒ spell or grammar checkers, word prediction software ❒ text-to-speech software, human dictation, recording ❒ calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper ❒ sentence starters or sentence strips ❒ concept mapping tools ❒ Computer-Aided-Design (CAD) or mathematical notation software ❒ virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) ❒ multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches) ❒ multiple examples of novel solutions to authentic problems ❒ different approaches to motivate, guide, feedback or inform students of progress towards fluency ❒ scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) ❒ differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners)

Optimizing INTERNALIZATION of the Learning Goal

ENGAGEMENT ⍰ How will the design of the learning strategically support students to effectively cope and engage with the environment?	**Self-Regulation:** ⍰ What do you anticipate about barriers to student engagement? ➢ Plan to address barriers to engagement by promoting healthy responses and interactions, and ownership of learning goals: ❒ metacognitive approaches to frustration when doing mathematics ❒ increase length of on-task orientation through distractions ❒ frequent self-reflection and self-reinforcements ❒ address subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") ❒ offer devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the math practices for the purpose of monitoring and improving ❒ use activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely
REPRESENTATION ⍰ How will the learning support transforming accessible information into usable knowledge	**Comprehension:** ⍰ What do you anticipate about barriers to student comprehension? ➢ Plan to address barriers to comprehension by intentionally building connections to prior understandings and experiences, relating meaningful information to learning goals,

that is accessible for future learning and decision-making?	providing a process for meaning making of new learning, and applying learning to new contexts: ❒ incorporate explicit opportunities for review and practice ❒ note-taking templates, graphic organizers, concept maps ❒ scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) ❒ explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) ❒ opportunities over time to revisit key ideas and connections ❒ make explicit cross-curricular connections ❒ highlight key elements in tasks, graphics, diagrams, formulas ❒ outlines, graphic organizers, unit organizer routines, concept organizer routines, and concept mastery routines to emphasize key ideas and relationships ❒ multiple examples & non-examples ❒ cues and prompts to draw attention to critical features ❒ highlight previously learned skills that can be used to solve unfamiliar problems ❒ options for organizing and possible approaches (tables and representations for processing mathematical operations) ❒ interactive representations that guide exploration and new understandings ❒ introduce graduated scaffolds that support information processing strategies ❒ tasks with multiple entry points and optional pathways ❒ "Chunk" information into smaller elements ❒ remove unnecessary distractions unless essential to learning goal ❒ anchor instruction by linking to and activating relevant prior knowledge (e.g., using visual imagery, concept anchoring, or concept mastery routines) ❒ pre-teach critical prerequisite concepts via demonstration or representations ❒ embed new ideas in familiar ideas and contexts (e.g., use of analogy, metaphor, drama, music, film, etc.) ❒ advanced organizers (e.g., KWL methods, concept maps) ❒ bridge concepts with relevant analogies and metaphors
ACCESS ACTION & EXPRESSION ⍰ How will the learning for students support the development of executive functions to allow them to take advantage of their environment?	**Executive Functions:** ⍰ What do you anticipate about barriers to students demonstrating what they know? ➢ Plan to address barriers to demonstrating understanding by providing opportunities for students to set goals, formulate plans, use tools and processes to support organization and memory, and analyze their growth in learning and how to build from it: ❒ prompts and scaffolds to estimate effort, resources, difficulty ❒ models and examples of process and product of goal-setting ❒ guides and checklists for scaffolding goal-setting ❒ post goals, objectives, and schedules in an obvious place ❒ embed prompts to "show and explain your work" ❒ checklists and project plan templates for understanding the problem, prioritization, sequences, and schedules of steps ❒ embed coaches/mentors to demonstrate think-alouds of process ❒ guides to break long-term goals into short-term objectives ❒ graphic organizers/templates for organizing information & data ❒ embed prompts for categorizing and systematizing ❒ checklists and guides for note-taking ❒ asking questions to guide self-monitoring and reflection ❒ showing representations of progress (e.g., before and after photos, graphs/charts showing progress, process portfolios)

	❒ prompt learners to identify type of feedback or advice they seek ❒ templates to guide self-reflection on quality & completeness ❒ differentiated models of self-assessment strategies (e.g., role-playing, video reviews, peer feedback) ❒ assessment checklists, scoring rubrics, and multiple examples of annotated student work/performance examples

Planning Guidance for Culturally and Linguistically Responsive Instruction[10]

In order to ensure our students from marginalized cultures and languages view themselves as confident and competent learners and doers of mathematics within and outside of the classroom, educators must intentionally plan ways to counteract the negative or missing images and representations that exist in our curricular resources. The guiding questions below support the design of lessons that validate, affirm, build, and bridge home and school culture for learners of mathematics:

> **Validate/Affirm:** How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
>
> **Build/Bridge:** How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

In addition, Aguirre and her colleagues[11] define **mathematical identities** as the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives. Many students see themselves as "not good at math" and approach math with fear and lack of confidence. Their identity, developed through earlier years of schooling, has the potential to affect their school and career choices.

Five Equity-Based Mathematics Teaching Practices[12]

> **Go deep with mathematics.** Develop students' conceptual understanding, procedural fluency, and problem solving and reasoning.
>
> **Leverage multiple mathematical competencies.** Use students' different mathematical strengths as a resource for learning.
>
> **Affirm mathematics learners' identities.** Promote student participation and value different ways of contributing.

[10] This resource relied heavily on the work of: Hollie, S. (2011). Culturally and linguistically responsive teaching and learning. Teacher Created Materials. (see also, https://www.culturallyresponsive.org/vabb)

[11] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices. Reston, VA: National Council of Teachers of Mathematics (p. 14).

[12] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc. (p.6). (adapted from Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013) (p. 43).

Challenge spaces of marginality. Embrace student competencies, value multiple mathematical contributions, and position students as sources of expertise.

Draw on multiple resources of knowledge (mathematics, language, culture, family). Tap students' knowledge and experiences as resources for mathematics learning.

The following lesson design strategies support Culturally and Linguistically Responsive Instruction, specific examples for each cluster of standards can be found in part 2 of the document. These were adapted from the Promoting Equity section of the Taking Action series published by NCTM.[13]

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.

Mathematical Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher.

Modifying Mathematical Tasks: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)."

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics.

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages.

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we

[13] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians.

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others."

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time."

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students.

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Glossary[14]

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (- 3/4) = (- 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[15]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. See: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances
from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten.
For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

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[14] Glossary and tables taken from: Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/
[15] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[16] See also: median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.
Integer. A number expressible in the form a or -a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line.
Also known as a dot plot.[17]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[18] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 Ã· 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 Ã— 4/3 = 4/3 Ã— 3/4 = 1.

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[16] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

[17] Adapted from Wisconsin Department of Public Instruction, op. cit.

[18] To be more precise, this defines the arithmetic mean.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin,
selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also*: uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form *a/b* or – *a/b* for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also*: terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[19]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar
model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

[19] Adapted from Wisconsin Department of Public Instruction, op. cit.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also*: median, first quartile, interquartile range.

Table 1: Common addition and subtraction.[1]

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 =5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now?5-2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?? -2 = 3
	TOTAL UNKNOWN	**ADDEND UNKNOWN**	**BOTH ADDENDS UNKNOWN[2]**
PUT TOGETHER / TAKE APART[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5-3 = ?	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? 5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1, 5 = 2 + 3, 5 = 3 + 2
COMPARE	**DIFFERENCE UKNOWN**	**BIGGER UNKNOWN**	**SMALLER UNKNOWN**
	("How many more?" version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2: Common multiplication and division situations.[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	3 x ? = 18, and 18 ÷ 3 = ?	? x 6 = 18, and 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	a x b = ?	a x ? = p and p ÷ a = ?	? x b = p, and p ÷ b = ?

[1]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3: The properties of operations.

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	(a +b) + c = a + (b+c)
Commutative property of addition	a + b = b + a

Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists *-a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) x c = a x (b x c)
Commutative property of multiplication	a x b = b x a
Multiplicative identity property 1	a x 1 = 1 x a = a
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that a x $1/a$ = $1/a$ x a = 1
Distributive property of multiplication over additions	a x (b + c) = a x b + a x c

Table 4: The properties of equality.

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a.
Symmetric property of equality	If a = b, then b = a.
Transitive property of equality	If a = b and b = c , then a = c.
Addition property of equality	If a = b, then a +c = b + c.
Subtraction property of equality	If a = b then a – c = b – c.
Multiplication property of equality	If a = b, then a x c = b x c.
Division property of equality	If a = b and $c \neq 0$, then a ÷ c = b ÷ c.
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality.

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b.
If a > b and b > c then a > c.
If a > b, b < a.
If a > b, then -a <-b.
If a > b, then a ± c > b ± c.
If a > b and c > 0, then a x c > b x c.
If a > b and c < 0, then a x c < b x c.
If a > b and c > 0, then a ÷ c > b ÷ c.
If a > b and c < 0, then a ÷ c < b ÷ c.