New Mexico Instructional Scope for Supporting Equitable and Culturally Responsive Mathematics Instruction

Overview

This Instructional Scope for Mathematics 3.0 was created by a cohort of New Mexico educators and the New Mexico Public Education Department.

The intention of this document is to act as a companion during the planning process alongside High Quality Instructional Materials (HQIM). A sample template is provided to show a snapshot of planning supports provided within each cluster of standards in the mathematics instructional scope.

During the creation of this document, we leveraged the work of other states, organizations, and educators from across the country and world. This work would not have been possible without all that came before it, and we wish to express our sincerest gratitude for everyone that contributed to the resources listed within our references.

To better understand the planning support provided in the mathematics instruction scope, this section provides a brief description of each planning support. This includes *what* support is provided, *why* the planning support is critical for equitable and culturally responsive mathematics instruction, and *how* to use the planning support with HQIM.

Cluster Statement

What: The New Mexico Mathematics Standards are grouped by domains, with somewhere between 4 to 10 domains per grade level. Within each domain, the standards are arranged into clusters. Cluster statements summarize groups of related standards.

Why: The New Mexico Mathematics Standards require a stronger focus on the way time and energy are spent in mathematics classrooms. Students should spend the majority of their time (65-85%) working on content within the major clusters of the grade/course. Supporting clusters and–where appropriate–additional clusters should only comprise 15-35% of the time spent in classes and be covered when they are connected to and engage students in the major work of the grade/course.

How: When planning with your HQIM, consider the time being devoted to major clusters versus additional or supporting clusters. The major work of each grade/course should be designed to provide students with strong foundations for future mathematical work, which will require more time than additional or supporting clusters. Also consider the ways your HQIM makes these connections between major clusters and additional and supporting clusters explicit for students.

Standard Text

What: Each cluster-level support document contains the text of each standard within that cluster.

Why: The cluster statement and standards are meant to be read together to understand the structure of the standards. By grouping the standards within the cluster, the connectedness of the standards is reinforced.

How: The text of the standards should ground all planning with your HQIM. Reading the standards within a cluster intentionally enables us to focus on the connections within and between the standards.

Standards for Mathematical Practice

What: The Standards for Mathematical Practice describe the expertise and habits of mind that mathematics educators at all levels should develop in their students.

Why: Equitable and excellent mathematics instruction supports students in becoming confident and competent mathematicians. By engaging with the Standards for Mathematical Practice, students engage in the practice of doing mathematics and developing mathematical habits of mind. These include the ability to think mathematically, analyze situations, understand relationships, and adapt what they know to solve a wide range of problems, including problems that do not look like any they have encountered before.

How: When planning with HQIM, it is critical to consider the connections between the content standards and the Standards for Mathematical Practice. The planning supports highlight a few practices students can engage in when learning the content of the standard. It is not necessary, or even appropriate, to engage in all of the practices every day. Instead, teachers should choose which Standards for Mathematical Practice to focus on within a given lesson, depending on the content standards being covered and the activities students will be engaging with. When teachers spend time intentionally supporting students in learning both the what (content standards) and the how (Standards for Mathematical Practice), students will have a stronger foundation of mathematical learning.

Clarification Statement

What: The clarification statement provides greater clarity for teachers in understanding the purpose of the standard.

Why: The New Mexico Mathematics Standards illustrate how progressions support student learning within each major domain of mathematics. The clarification statement provides additional context about the ways each cluster of standards supports student learning of the larger learning progression.

How: When planning with HQIM, use the clarification statement to support your understanding of how the materials use specific types of representations or change the learning sequence.

Demonstration Statements, Webb's Depth of Knowledge, Bloom's Taxonomy, and Aspect of Rigor

What: The New Mexico Mathematics Standards include one, two, or all three of the aspects of mathematical rigor: conceptual understanding, procedural skill and fluency, and application to the real world. These planning supports considers which aspect(s) of rigor are within each standard and then identifies academic skills students need to demonstrate comprehension of the standard and associated mathematical practices. The planning supports also provide information about two common classifications on cognitive complexity, Webb's Depth of Knowledge and Bloom's Taxonomy.

Why: Analyzing standards alongside the standards for mathematical practice provide a fuller picture of the mathematical competencies demanded in the standard.

How: When planning for a cluster of standards with your HQIM, a critical first step is to analyze the content and language demands of the standards and the associated Standards for Mathematical Practice. This analysis can be used to plan/design appropriate formative assessment, as well as interpret student data from formative assessments. The planning supports provide a breakdown of the standard that can serve as the basis for this sort of analysis.

Definitions of the Components of Rigor

Rigorous teaching in mathematics does not simply mean increasing the difficulty or complexity of practice problems. Incorporating rigor into classroom instruction and student learning means exploring at a greater depth, the standards and ideas with which students are grappling. There are **three** components of rigor that will be expanded upon in this document, and each is equally important to student mastery: **Conceptual Understanding, Procedural Skill and Fluency,** and **Application**.

- **Conceptual Understanding** refers to understanding mathematical concepts, operations, and relations. It is more than knowing isolated facts and methods. Students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. It also allows students to connect prior knowledge to new ideas and concepts.
- **Procedural Skill and Fluency** is the ability to apply procedures accurately, efficiently, and flexibly. It requires speed and accuracy in calculation while giving students opportunities to practice basic skills. Students' ability to solve more complex application tasks is dependent on procedural skill and fluency.
- **Application** provides valuable context for learning and the opportunity to solve problems in a relevant and a meaningful way. It is through real-world application that students learn to select an efficient method to find a solution, determine whether the solution makes sense by reasoning, and develop critical thinking skills.

A Special Note on Procedural Skill and Fluency

While speed is a component of fluency, it is not necessarily speed in producing an answer; rather, fluency can be observed by watching the speed with which a student engages with a particular problem. Furthermore, fluency does not require the most efficient strategy. The standards specify grade-level appropriate strategies or types of strategies with which students should demonstrate fluency (e.g., 1.OA.C.6 allows for students to use counting on, making ten, creating equivalent but easier or known sums, etc.). It should also be noted that teachers should expect some procedures to take longer than others (e.g., fluency with the standard algorithm for division, 6.NS.B.2, as compared to fluently adding and subtracting within 10, 1.OA.C.6).

Standards identified as targeting procedural skill and fluency do not all have an expectation of automaticity and/or rote recall. Only two standards, 2.OA.B.2 and 3.OA.C.7, have explicit expectations of students knowing facts from memory. Other standards targeting procedural skill and fluency do not require students to reach automaticity. For example, in 4.G.A.2, students do not need to reach automaticity in classifying two-dimensional figures.

*Adapted from Louisiana Department of Education

Assessment Items

What: Formative assessment is the planned, ongoing process used by teachers during learning and teaching to assess student learning in order to improve student understanding and support students in their learning. This planning support provides one or more sample items to assist teachers in their planning of formative assessments for each standard. These are intended to be used as a guide for what students should be able to complete and can be used alongside assessments provided within your HQIM. A link to additional assessment items from Illustrative Mathematics is also provided with each standard.

Why: When student thinking is made visible, the teacher can examine the progression of learning towards the goals of the standards and adjust instruction as necessary. By including students in the assessment and analysis process, teachers allow them to become strategic and goal-directed with their learning.

How: The sample items address the aspect(s) of rigor that aligns with each standard. This example can be used to discuss possible responses by students and next steps for instruction. A similar process can then be used to identify additional items from the formative assessment resources provided by your HQIM.

For additional information, see Standards Aligned Instructionally Embedded Formative Assessment Resources.

Common Misconceptions

What: This planning support identifies some of the common misconceptions students might have when engaging in learning about each mathematical topic.

Why: Students might have misconceptions based on an overgeneralization of patterns they notice or overly relying on rules rather than understanding the underlying concepts. Tips and tricks in mathematics expire over time as students move up through the grade levels. It is critical to understand some of the common misconceptions students can develop so we can address them directly with students and continue to build a strong foundation for their mathematical learning.

How: When planning with your HQIM, look for ways to directly address common misconceptions students have. This planning support provides some of the possible misconceptions, and your HQIM might include additional ones. The goal is not to avoid misconceptions–they are a natural part of the learning process. But we do want to support students in exploring their misconceptions and modifying incorrect or partial understanding.

Planning for Multi-Layered System of Support & Universal Design for Learning

What: The section on planning for Multi-Layered Systems of Supports (MLSS) and Universal Design for Learning (UDL) is designed to support teachers in planning for the needs of all students. Each section includes the three pillars of UDL, as well as options for pre-teaching and re-teaching. Some students might benefit from targeted pre-teaching and re-teaching supports, which improve students' acquisition of the knowledge and skills identified in the New Mexico Mathematics Standards. Intensive pre-teaching and re-teaching supports may also be helpful for smaller groups of students with more intensive needs. Progress monitoring should occur to assess students' responses to these additional supports.

Why: MLSS is a holistic framework that guides educators to intervene quickly when students need additional supports. The framework moves away from the "wait to fail" model and empowers teachers to use their professional judgment to make data-informed decisions regarding the students in their classrooms to ensure academic success with the grade-level expectations of the New Mexico Mathematics Standards.

How: When planning with your HQIM, use the suggestions for pre-teaching as a starting point to determine if some or all of the students in your classroom need targeted or intensive pre-teaching at the start of a unit to ensure they can access the grade-level material within the unit. The core instruction (which is grounded in the UDL Framework) and re-teach sections work together to support planning within a unit, looking for ways the materials support greater access for all students and provide options to revisit concepts based on formative assessment data. Additional information about MLSS can be found on the PED's website, and guidance and tools can be found here.

Vertical Alignment

What: The New Mexico Mathematics Standards are designed around coherent progressions of learning. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but rather an extension of previous learning. The connections to previous, current, and future learning make this coherence visible. A link to Achieve the Core's grade-level coherence maps is provided with each standard.

Why: Students build stronger foundations for learning when they see mathematics as an interconnected discipline of relationships rather than discrete skills and knowledge that are only applicable in current situations. The intentional inclusion of connections to previous, current, and future learning can support a more interconnected understanding of mathematics.

How: When planning with HQIM, use the vertical alignment supports to find ways to help students make explicit connections within their study of mathematics.

Culturally and Linguistically Responsive Instruction

What: Culturally and Linguistically Responsive Instruction (CLRI) requires educators to contribute to a positive school climate by validating and affirming students' home languages and cultures, building and bridging mathematics to previous learning (both formal and informal), and using linguistic vocabulary supports. These actions legitimize students' home culture and language, making it clear that students' culture and language are positive assets in the classroom. It is also an intentional effort to reverse negative stereotypes of non-dominant cultures and languages and must be purposeful, consistent, and authentic, as well as both proactive and reactive. By building and bridging, students learn to toggle between the behaviors and expectations of home and those of school. By focusing on creating connections between the culture and language expectations of home and the expectations at school, we help students be set up for success in school. Linguistic supports are important within this structure as well, and help students learn new, academic vocabulary in ways that allow students to bring in their prior knowledge and connect it to new experiences. Linguistic supports for helping students acquire new academic vocabulary are provided from Pathways2Careers.

Why: The mathematical identities of students are shaped by the messages they receive about their ability to do mathematics and the power of mathematics in their lives outside of school. Educators must intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages. In addition, creating connections between the cultural and linguistic behaviors of students' home culture and language supports students in creating identities as capable mathematicians within school and society.

How: When planning instruction, it is critical to consider ways to validate/affirm and build/bridge students' cultural and linguistic assets. The planning supports for each cluster provide information

and examples of how to engage with equity-based teaching practices. There may be additional support available in your HQIM to ensure all students develop strong mathematical identities.

Student Discourse Guide

What: This guide provides educators with suggested questions to engage students in discourse about mathematical concepts. Purposeful, rich classroom discourse offers students the opportunity to express their ideas and thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Ideas for supporting student discourse are also provided from Pathways2Careers.

Why: Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding.

How: When students have frequent opportunities for discussion, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences, and evolving math identities.

Cross-Curricular Connections

What: Cross-curricular connections are connections between knowledge and/or skills that can be made between two or more areas of study. These connections can be made by teachers or students.

Why: The purpose of planning cross-curricular connections in an instructional sequence is to ensure that students build connections and recognize the relevance of mathematics beyond the mathematics classroom.

How: When planning with HQIM, look for opportunities to make explicit connections to other content areas, including the examples provided for each standard.

Career and Skill Connections

What: The knowledge and/or skills students are mastering within each standard can be directly connected to various careers. These connections can be made by teachers or students and can provide motivation for students as they work through mathematical topics.

Why: Understanding and recognizing the relevance of mathematics beyond the mathematics classroom is important. This is particularly valid for students who may not plan on working in a

"mathematics" field.

How: When planning with HQIM, look for opportunities to make explicit connections to careers, including the examples provided for each standard.

Template of the New Mexico Cluster Level Planning Support for the New Mexico Mathematics Standards

Grade	*CCSS Domain*	*CCSS Cluster*
Grade level	CCSS domain	CCSS cluster statement summarizing the group of related standards
Standard and icons that indicate which aspect of rigor it aligns with		
Standard		**Standards for Mathematical Practice**
Full text of the standard.		Correlation of the standard to the Standards for Mathematical Practice to which it aligns, including a link to a descriptor of what teachers and students should be doing.
Clarification Statement		**Students Who Demonstrate Understanding Can...**
Clarifies the language of the standard.		The skills students perform to demonstrate comprehension of the standard.
DOK		**Blooms**
Correlation of the standard to Webb's Depth of Knowledge.		Correlation of the standard to Bloom's Taxonomy.
Conceptual Understanding, Procedural Skill and Fluency, and/or Application		
Highlights the aspect(s) of rigor the standard is aligned to, and descriptors for which portions of the standard fall under each aspect of rigor.		
Assessment Items		
When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.		
Provides at least one high-quality formative assessment item aligned to the standard, as well as a link to more items.		
Common Misconceptions		
Provides guidance on where student misconceptions might occur.		

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)		
Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Layer 1 ensures that all students receive strong instruction in a high-quality differentiated core curriculum that is based on the principles of UDL. This includes school-wide implementation of positive behavioral interventions and supports, data-driven instruction, targeted interventions in small group instruction, universal screening, and English Language Development (ELD) for English Learners (ELs).	Layer 2 interventions should be focused on delivering individualized and targeted support (pre-teaching and re-teaching) for students on a grade-level trajectory. The interventions must be aligned with Layer 1 skills. Students should be provided with additional time and intensity in a small-group setting.	Layer 3 interventions should be provided individually or in small groups. Students are grouped according to their skill needs. The goal is for each student to acquire academic skills that will persist and transfer when the student returns to core instruction. If needed, specialized teachers may provide specific intervention instruction based on the needs identified by the data.

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/7/30/308/308		
Previous Learning	*Current Learning*	*Future Learning*
Lists skills relevant to current learning that students should have already mastered.	Lists skills within the current learning that students will master.	Lists skills from upcoming learning that students will need to be able to master based on what they are learning now.
Culturally and Linguistically Responsive Instruction		
Consider these resources for vocabulary from Pathways2Careers: • https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf • https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf		
Consider these questions as you plan for instruction that is culturally and linguistically responsive: • How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
Provides ways to build connections between the families in the community and the mathematical content, as well as examples that connect the math to students' home lives.	Provides information on supporting students as they learn mathematics by starting with conceptual knowledge that students can make connections to based on their prior knowledge. Also provides information on helping students build positive mathematical identities.	Provides ideas and supports for helping students learn new academic vocabulary and making connections to their prior knowledge.

Suggested Student Discourse Questions
Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf
Provides questions teachers can employ to increase student discourse.
Cross-Curricular Connections
Provides various connections between the standard and the knowledge and skills that students might use in other content areas.
Career and Skill Connections
Provides various connections between the standard and future careers/skills students will need for future careers.

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards for Mathematical Practice
    - Connections to procedural, conceptual understanding, and application
    - Sample assessment items
    - Common misconceptions
    - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
    - Vertical alignment
    - Culturally and Linguistically Responsive Instruction (CLRI)
    - Suggested student discourse questions
    - Cross-curricular and career/skill connections
- A Student Discourse Guide

Helpful links:

- Lesson-planning tool from Pathways 2 Careers (click here to sign up with your district email if you don't already have an account)
- Focus by Grade Level from Achieve the Core
- Coherence Map from Achieve the Core

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Work with radicals and integer exponents.
  - 8.EE.A.1
  - 8.EE.A.2
  - 8.EE.A.3
  - 8.EE.A.4
- Understand the connections between proportional relationships, lines, and linear equations.
  - 8.EE.B.5
  - 8.EE.B.6
- Analyze and solve linear equations and pairs of simultaneous linear equations.
  - 8.EE.C.7
  - 8.EE.C.8

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)

Grade	CCSS Domain	CCSS Cluster
8	Expressions and Equations	Work with radicals and integer exponents.

Cluster Standard: 8.EE.A.1

Standard	Standards for Mathematical Practice

8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = 1/27$.	• **SMP 2:** Reason abstractly and quantitatively.
	  ○ Teacher and Student Actions
	• **SMP 5**: Use appropriate tools strategically.
	  ○ Teacher and Student Actions
	• **SMP 6:** Attend to precision.
	  ○ Teacher and Student Actions

Clarification Statement	**Students Who Demonstrate Understanding Can...**
In this cluster, students explore the properties of exponents, radicals, and scientific notation.	• Calculate integer exponents by understanding their properties. • Generate equivalent expressions using the single properties of integer exponents and combinations of the properties.
DOK	**Blooms**
1	Apply

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Understand the basic properties of integer exponents, such as Product of Powers, Quotient of Powers, Power of a Power, and Power of a Product.
- Understand that applying the properties of exponents allows expressions to be rewritten in equivalent forms.

Procedural Skill and Fluency:

- Apply the properties of integer exponents to calculate integer exponents and rewrite expressions in equivalent numerical forms.
- Simplify expressions involving integer exponents.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Marco and Seth are lab partners studying bacterial growth. They were surprised to find that the population of the bacteria doubled every hour.

1. The table shows that there were 2,000 bacteria at the beginning of the experiment. What was the size of the population after 1 hour? After 2, 3 and 4 hours? Enter this information into the table:

Hours into study	0	1	2	3	4
Population (thousands)	2				

2. If you know the size of the population at a certain time, how do you find the population one hour later?

3. Marco said he thought that they could use the equation $P = 2t + 2$ to find the population at time t. Seth said he thought that they could use the equation $P = 2 \cdot 2^t$. Decide whether either of these equations produces the correct populations for t=1,2,3,4.

4. Assuming the population doubled every hour before the study began, what was the population of the bacteria 1 hour before the students started their study? What about 3 hours before?

5. If you know the size of the population at a certain time, how do you find the population one hour earlier?

6. What number would you use to represent the time 1 hour before the study started? 2 hours before? 3 hours before? Finish filling in the table if you haven't already.

7. Now use Seth's equation to find the population of the bacteria 1 hour before the study started. Use the equation to find the population of the bacteria 3 hours before. Do these values produce results consistent with the arithmetic you did earlier?

8. Use the context to explain why it makes sense that $2^{n-1} = (\frac{1}{2})^n = \frac{1}{2^n}$. That is, describe why, based on the population growth, it makes sense to define 2 raised to a negative integer exponent as repeated multiplication by 12.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Rules for exponents:** Students may confuse the rules for exponents, especially if they are taught to memorize them rather than understand them conceptually.
- **Power of a power:** Students may think that finding a power of a power involves adding exponents.
- **Negative exponents:** Students may not understand why negative exponents can be represented by a fraction with the term and negative exponents in the denominator.
- **Negative terms in context:** Many problems use negative numbers to represent debt, depth below ground level, etc. Students might struggle to understand this because it is a convention that does not make sense in context (owing $30 is still 30, not -30) even if it makes problems easier to work with. Using visuals, number

lines, etc. can help students understand the idea of negative numbers in context.

- **0 and 1 as exponents:** Students often think that a number raised to the power of 0 is 0 and a number raised to the power of 1 is 1. Tables can help students see patterns in the ways different powers affect numbers, especially when a problem involves a real-world context.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement**	**Pre-teaching** In previous classes, learners worked to develop understanding of the powers of 10 and the placement of the decimal when multiplying or dividing by powers of 10, write and evaluate numerical expressions involving whole number exponents, use ratio, rate reasoning, and unit rates, compute unit rates and recognize and represent proportional relationships, use variables to write expressions and equations and apply the properties of operations to generate equivalent expressions, and solve equations, including those that involve real world problems. Students worked with linear equations, isolating a single variable using inverse operations. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as evaluating numbers to a given exponent, evaluating expressions written in scientific notation, writing and solving expressions and equations in one variable, finding area to solve multi-step problems, interpreting graphs (especially comparing rates), calculating rates, using ratio reasoning to solve problems, generating equivalent expressions (especially using the distributive	**Pre-teaching** Consider using standard 6.EE.A.1, which provides a foundation for work in this cluster. In 6.EE.A.1, students write and evaluate numerical expressions involving whole-number exponents. Also consider using standard 6.EE.B.5, which also provides a foundation for work in this cluster. In 6.EE.B.5, students learn that solving an equation or inequality is a process of determining which value(s) (if any) make the equation or inequality true. Also consider using standard 7.RP.A.2, which also provides a foundation for work in this cluster. In 7.RP.A.2, students recognize and represent proportional relationships between quantities. Students must understand how to write and evaluate numerical expressions using exponents and need to understand the concept exponents as representing repeated multiplication or division. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing

Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals.

Action and Expression
Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support

property), and comparing products. The language of this 8th grade cluster is completely new, but the skills needed for success began in 6th grade. Previously, slope is referred to as rate, unit rate, and the constant of proportionality, and the constant of proportionality is structured in the form $y = kx$, so shifting students away from this and to the form $y = bx$ or even $y = mx + b$ will require a shift in language and terminology.

If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

Re-teaching
Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Clarify different representations for students by allowing them to compare and contrast tables, graphs, ordered pairs, and equations. Take time to practice solving linear equations, especially when working with rational coefficients.

their current grade-level content.

Re-teaching
Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster exploring different strategies for working with radicals, square and cube roots, numbers in scientific notation, and exponents. Give students opportunities to compare numbers written differently to gain an understanding of relative sizes written in forms they are unfamiliar with.

discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/35/394/394

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● develop understanding of the powers of 10 and the placement of the decimal when multiplying or dividing by powers of 10 ● write and evaluate numerical expressions involving whole number exponents ● use ratio, rate reasoning, and unit rates ● compute unit rates and recognize and represent proportional relationships ● use variables to write expressions and equations and apply the properties of operations to generate equivalent expressions ● solve equations, including those that involve real world	In 8th grade, learners ● use squares and square roots and cubes and cube roots when working with irrational numbers and volume ● compare properties of functions given a table, a graph, or an equation ● use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept ● use equations to graph linear and proportional relationships	In future classes, learners ● use properties of exponents to rewrite expressions and extend their knowledge of integer exponents to rational exponents ● understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line) ● create, solve, and rewrite equations, inequalities, and systems of equations (include equations arising from linear, exponential, and quadratic functions) ● construct a viable argument to justify a solution method

problems		
Culturally and Linguistically Responsive Instruction		
Consider these resources for vocabulary from Pathways2Careers: • https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf • https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf		
Consider these questions as you plan for instruction that is culturally and linguistically responsive: • How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

crafts.	achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- Can a number be expressed in only one way? Can you explain or give an example of how a number can be expressed in more than one way? How can you be sure multiple expressions are equivalent?
- How can we use the process of division to simplify when we have the same bases? Do you think this is the only strategy, or can you find another strategy?
- What are some real-life examples of using square roots, cube roots, and scientific notation to write expressions to communicate mathematical thinking?
- What does it mean to say “squared” or “cubed”? What is the inverse of each of those?
- What relationship do a negative exponent and a fraction have?

Cross-Curricular Connections

Arts: Following the mathematical series of musical rhythms to learn the basic rhythms of dance. Calculating shutter speed, focal length, lighting angles, and exposure time.

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

Science: Representing the large distances of planets from the sun. Comparing rates and relationships in scientific data. Comparing linear relationships and systems of equations in scientific data. Writing number sentences for conservation of energy of a system. Collaborating with peers to define or describe an issue in society and how to evaluate solutions and running tests of solutions and change designs as needed. Constructing scientific arguments for how uneven distributions of earth’s mineral, energy, groundwater resources are the result of past and current geoscience processes (e.g. metal ores, volcanic activity, soil weathering, rock deposits, and mining by humans). Solving chemistry problems. Understanding the movements of the planets. Solving equations when writing computer programs and figuring out algorithms. Comparing linear relationships and systems of equations in scientific data.

Career and Skill Connections

• Advertising	• Economist	• Mechanics

• Analysis • Anthropology • Archeology • Architecture • Arts • Astronomy • Atmospheric science • Aviation • Banking/finance • Biology • Bookkeeping • Botany • Business • Carpentry • Chemistry • Choreography • Computer programming • Conservation science • Construction • Counseling • Culinary arts • Ecology	• Education • Electrician • Engineering • Environmental restoration • Epidemiology • Event planning • Floral design • Food science • Forensics • Forestry • Fundraising • Geology • Health science • HVAC • Information technology • Insurance • Landscaping • Law enforcement • Machinist • Management • Marketing	• Medicine • Microbiology • Mining • Physical therapy • Physics • Plumbing • Policy analysis • Ranching/farming • Sales • Sociology • Software development • Soil science • Statistics • Technician • Technology • Transportation • Travel agent • Veterinary • Video game design • Web development • Zoology

Grade	CCSS Domain	CCSS Cluster
8	Expressions and Equations	Work with radicals and integer exponents.

Cluster Standard: 8.EE.A.2

Standard	Standards for Mathematical Practice
8.EE.A.2: Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.	● **SMP 8:** Look for and express regularity in repeated reasoning. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
In this cluster, students explore the properties of exponents, radicals, and scientific notation.	● Calculate a square root of a perfect square number or cube root of a perfect cube root number. ● Use the square root and cube root symbol in an equation $x^2 = p$ or $x^3 = p$. ● Explain that the square root of 2 is an irrational number.
DOK	**Blooms**
1-2	Application

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Know and explain why $\sqrt{2}$ is irrational.
- Understand the properties of exponents, radicals, and scientific notation.

Procedural Skill and Fluency:

- Represent solutions to equations like $x^2 = p$ and $x^3 = p$ for positive rational numbers p with square and rube root symbols.
- Evaluate square roots of small perfect squares and cube roots of small perfect cubes.
- Use the properties of exponents, radicals, and scientific notation.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

1. What values for x make each equation true?

a. $x^2 + 9 = 16$ b. $x^3 - 9 = 16$ c. $x^2 - 5 = 31$ d. $x^3 + 14 = 41$

2. Determine if the following numbers are rational or irrational. Explain your thinking.

a. -7 b. $\sqrt{2}$ c. 5 d. $-\sqrt{16}$ e. $\sqrt[3]{27}$ f. $-\sqrt[3]{8}$

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Squared/cubed numbers and square/cube roots:** Students may divide by 2 or 3 instead of finding the square root or cube, respectively. Students might also fail to recognize the relationship between square numbers and area or between cube numbers and volume.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply	**Pre-teaching** In previous classes, learners worked to develop understanding of the powers of 10 and the placement of the decimal when multiplying or dividing by powers of 10, write and evaluate numerical expressions involving whole number exponents, use ratio, rate reasoning, and unit rates, compute unit rates and recognize and represent proportional relationships, use variables to write expressions and equations and apply the properties of operations to generate equivalent expressions, and solve equations, including those that involve real world problems. Students worked with linear equations, isolating a single variable using inverse operations. Students might benefit from opportunities to review vocabulary	**Pre-teaching** Consider using standard 6.EE.A.1, which provides a foundation for work in this cluster. In 6.EE.A.1, students write and evaluate numerical expressions involving whole-number exponents. Also consider using standard 6.EE.B.5, which also provides a foundation for work in this cluster. In 6.EE.B.5, students learn that solving an equation or inequality is a process of determining which value(s) (if any) make the equation or inequality true. Also consider using standard 7.RP.A.2, which also provides a foundation for work in this cluster. In 7.RP.A.2, students recognize and represent proportional relationships between quantities.

background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships.

Engagement

Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals.

terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as evaluating numbers to a given exponent, evaluating expressions written in scientific notation, writing and solving expressions and equations in one variable, finding area to solve multi-step problems, interpreting graphs (especially comparing rates), calculating rates, using ratio reasoning to solve problems, generating equivalent expressions (especially using the distributive property), and comparing products. The language of this 8th grade cluster is completely new, but the skills needed for success began in 6th grade. Previously, slope is referred to as rate, unit rate, and the constant of proportionality, and the constant of proportionality is structured in the form $y = kx$, so shifting students away from this and to the form $y = bx$ or even $y = mx + b$ will require a shift in language and terminology.

If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

Re-teaching

Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain

Students must understand how to write and evaluate numerical expressions using exponents and need to understand the concept exponents as representing repeated multiplication or division.

If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

Re-teaching

Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster exploring different strategies for working with radicals, square and cube roots, numbers in scientific notation, and exponents. Give students opportunities to compare numbers written differently to gain an understanding of relative sizes written in forms they are unfamiliar with.

Action and Expression Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.	mistakes and find new, more efficient ways of solving problems. Clarify different representations for students by allowing them to compare and contrast tables, graphs, ordered pairs, and equations. Take time to practice solving linear equations, especially when working with rational coefficients.	

Vertical Alignment		
Consider using this coherence map to help guide your planning https://tools.achievethecore.org/coherence-map/8/35/396/396		
Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● develop understanding of the powers of 10 and the placement of the decimal when multiplying or dividing by powers of 10	In 8th grade, learners ● use squares and square roots and cubes and cube roots when working with irrational numbers and volume ● compare properties of	In future classes, learners ● use properties of exponents to rewrite expressions and extend their knowledge of integer exponents to rational exponents

• write and evaluate numerical expressions involving whole number exponents • use ratio, rate reasoning, and unit rates • compute unit rates and recognize and represent proportional relationships • use variables to write expressions and equations and apply the properties of operations to generate equivalent expressions • solve equations, including those that involve real world problems	functions given a table, a graph, or an equation • use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept • use equations to graph linear and proportional relationships	• understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line) • create, solve, and rewrite equations, inequalities, and systems of equations (include equations arising from linear, exponential, and quadratic functions) • construct a viable argument to justify a solution method

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and

less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- Can a number be expressed in only one way? Can you explain or give an example of how a number can be expressed in more than one way? How can you be sure multiple expressions are equivalent?
- What are some real-life examples of using square roots, cube roots, and scientific notation to write expressions to communicate mathematical thinking?
- What does it mean to say "squared" or "cubed"? What is the inverse of each of those?
- What relationship do a negative exponent and a fraction have?

Cross-Curricular Connections

Arts: Following the mathematical series of musical rhythms to learn the basic rhythms of dance. Calculating shutter speed, focal length, lighting angles, and exposure time.

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

Science: Representing the large distances of planets from the sun. Comparing rates and relationships in scientific data. Comparing linear relationships and systems of equations in scientific data. Writing number sentences for conservation of energy of a system. Collaborating with peers to define or describe an issue in society and how to evaluate solutions and running tests of solutions and change designs as needed. Constructing scientific arguments for how uneven distributions of earth's mineral, energy, groundwater resources are the result of past and current geoscience processes (e.g. metal ores, volcanic activity, soil weathering, rock deposits, and mining by humans). Solving chemistry problems. Understanding the movements of the planets. Solving equations when writing computer programs and figuring out algorithms. Comparing linear relationships and systems of equations in scientific data.

Career and Skill Connections

- Advertising
- Analysis
- Anthropology
- Archeology
- Architecture
- Arts
- Astronomy
- Atmospheric science
- Aviation
- Banking/finance
- Biology
- Bookkeeping
- Botany
- Business
- Carpentry
- Chemistry
- Choreography
- Computer programming
- Conservation science
- Construction
- Counseling
- Culinary arts
- Ecology
- Economist
- Education
- Electrician
- Engineering
- Environmental restoration
- Epidemiology
- Event planning
- Floral design
- Food science
- Forensics
- Forestry
- Fundraising
- Geology
- Health science
- HVAC
- Information technology
- Insurance
- Landscaping
- Law enforcement
- Machinist
- Management
- Marketing
- Mechanics
- Medicine
- Microbiology
- Mining
- Physical therapy
- Physics
- Plumbing
- Policy analysis
- Ranching/farming
- Sales
- Sociology
- Software development
- Soil science
- Statistics
- Technician
- Technology
- Transportation
- Travel agent
- Veterinary
- Video game design
- Web development
- Zoology

Grade	CCSS Domain	CCSS Cluster
8	Expressions and Equations	Work with radicals and integer exponents.

Cluster Standard: 8.EE.A.3

Standard	Standards for Mathematical Practice
8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9 , and determine that the world population is more than 20 times larger.	● **SMP 2:** Reason abstractly and quantitatively. ○ Teacher and Student Actions ● **SMP 5:** Use appropriate tools strategically. ○ Teacher and Student Actions ● **SMP 6:** Attend to precision. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
In this cluster, students explore the properties of exponents, radicals, and scientific notation.	● Explain the benefits of scientific notation. ● Write very small or very big numbers in 'scientific notation. ● Understand that some numbers written in scientific notation are estimates. ● Compare very small or very big numbers written in scientific notation to determine which is larger or smaller and by how much.
DOK	**Blooms**
1-2	Application, Analysis

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Understand the properties of exponents, radicals, and scientific notation.
- Understand and explain the benefits and drawbacks of using scientific notation.
- Understand the uses of scientific notation and how to use numbers in scientific notation to estimate very large or very small quantities.

Procedural Skill and Fluency:

- Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

- Use the properties of exponents, radicals, and scientific notation.
- Write very small or very big numbers in scientific notation.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

An ant has a mass of approximately $4x10^{-3}$ grams and an elephant has a mass of approximately 8 metric tons.

1. How many ants does it take to have the same mass as an elephant? Write your answer in scientific notation and explain how many times more an elephant weighs than an ant.

2. An ant is $1x10^{-1}$ centimeters long. If you put all these ants from your answer to part (a) in a line (front to back), how long would the line be? Find two cities in the United States that are a similar distance apart to illustrate this length.

   Note: 1 kilogram = 1000 grams, 1 metric ton = 1000 kilograms, 1 meter = 100 centimeters, 1 kilometer = 1000 meters

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Rules for exponents:** Students may confuse the rules for exponents, especially if they are taught to memorize them rather than understand them conceptually.
- **Power of a power:** Students may think that finding a power of a power involves adding exponents.
- **Negative exponents:** Students may not understand why negative exponents can be represented by a fraction with the term and negative exponents in the denominator.
- **0 and 1 as exponents:** Students often think that a number raised to the power of 0 is 0 and a number raised to the power of 1 is 1. Tables can help students see patterns in the ways different powers affect numbers, especially when a problem involves a real-world context.
- **Negative terms in context:** Many problems use negative numbers to represent debt, depth below ground level, etc. Students might struggle to understand this because it is a convention that does not make sense in context (owing $30 is still 30, not -30) even if it makes problems easier to work with. Using visuals, number lines, etc. can help students understand the idea of negative numbers in context.
- **Multi-step problems:** When an expression has several steps, sometimes students forget to follow the order of operations, struggle with moving from step to step, forget terms, etc.
- **Scientific notation:** Students may forget that correct scientific notation requires that the first factor be written with only one digit to the left of the decimal, and they may be confused about when to use positive vs. negative exponents, as well as what the sign of the exponent means when they are writing numbers in standard form from scientific notation. Some students may not fully understand why scientific notation is used and may see it as just another way to write numbers without understanding its significance in representing very large or very small quantities.
- **Comparing numbers:** Students may struggle to understand which number should be divided when expressing how many times as much one number is than the other.
- **Units:** In numbers involving units of mass, students might struggle with conversions. In problems where

students must represent numbers with units, they may struggle to choose the correct units to use, or which units will be the easiest to work with given the context of the problem.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in	**Pre-teaching** In previous classes, learners worked to develop understanding of the powers of 10 and the placement of the decimal when multiplying or dividing by powers of 10, write and evaluate numerical expressions involving whole number exponents, use ratio, rate reasoning, and unit rates, compute unit rates and recognize and represent proportional relationships, use variables to write expressions and equations and apply the properties of operations to generate equivalent expressions, and solve equations, including those that involve real world problems. Students worked with linear equations, isolating a single variable using inverse operations. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as evaluating numbers to a given exponent, evaluating expressions written in scientific notation, writing and solving expressions and equations in one variable, finding area to solve multi-step problems, interpreting graphs (especially comparing rates), calculating rates, using ratio reasoning to solve problems, generating equivalent expressions (especially using the distributive property), and comparing products. The language of this 8th grade cluster	**Pre-teaching** Consider using standard 6.EE.A.1, which provides a foundation for work in this cluster. In 6.EE.A.1, students write and evaluate numerical expressions involving whole-number exponents. Also consider using standard 6.EE.B.5, which also provides a foundation for work in this cluster. In 6.EE.B.5, students learn that solving an equation or inequality is a process of determining which value(s) (if any) make the equation or inequality true. Also consider using standard 7.RP.A.2, which also provides a foundation for work in this cluster. In 7.RP.A.2, students recognize and represent proportional relationships between quantities. Students must understand how to write and evaluate numerical expressions using exponents and need to understand the concept exponents as representing repeated multiplication or division. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure	is completely new, but the skills needed for success began in 6th grade. Previously, slope is referred to as rate, unit rate, and the constant of proportionality, and the constant of proportionality is structured in the form $y = kx$, so shifting students away from this and to the form $y = bx$ or even $y = mx + b$ will require a shift in language and terminology. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Clarify different representations for students by allowing them to compare and contrast tables, graphs, ordered pairs, and equations. Take time to practice solving linear equations, especially when working with rational coefficients.	**Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster exploring different strategies for working with radicals, square and cube roots, numbers in scientific notation, and exponents. Give students opportunities to compare numbers written differently to gain an understanding of relative sizes written in forms they are unfamiliar with.

students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/35/394/397

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners • develop understanding of the powers of 10 and the placement of the decimal when multiplying or dividing by powers of 10 • write and evaluate numerical expressions involving whole number exponents • use ratio, rate reasoning, and unit rates • compute unit rates and recognize and represent proportional relationships • use variables to write expressions and equations and apply the properties of operations to generate equivalent expressions • solve equations, including those that involve real world problems	In 8th grade, learners • use squares and square roots and cubes and cube roots when working with irrational numbers and volume • compare properties of functions given a table, a graph, or an equation • use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept • use equations to graph linear and proportional relationships	In future classes, learners • use properties of exponents to rewrite expressions and extend their knowledge of integer exponents to rational exponents • understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line) • create, solve, and rewrite equations, inequalities, and systems of equations (include equations arising from linear, exponential, and quadratic functions) • construct a viable argument to justify a solution method

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

	encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- Can a number be expressed in only one way? Can you explain or give an example of how a number can be expressed in more than one way? How can you be sure multiple expressions are equivalent?
- What are some real-life examples of using square roots, cube roots, and scientific notation to write expressions to communicate mathematical thinking?
- What relationship do a negative exponent and a fraction have?
- What does each part of a number written in scientific notation represent?
- How can we compare these numbers written in scientific notation?

Cross-Curricular Connections

Arts: Following the mathematical series of musical rhythms to learn the basic rhythms of dance. Calculating shutter speed, focal length, lighting angles, and exposure time.

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

Science: Representing the large distances of planets from the sun. Comparing rates and relationships in scientific data. Comparing linear relationships and systems of equations in scientific data. Writing number sentences for conservation of energy of a system. Collaborating with peers to define or describe an issue in society and how to evaluate solutions and running tests of solutions and change designs as needed. Constructing scientific arguments for how uneven distributions of earth's mineral, energy, groundwater resources are the result of past and current geoscience processes (e.g. metal ores, volcanic activity, soil weathering, rock deposits, and mining by humans). Solving chemistry problems. Understanding the movements of the planets. Solving equations when writing computer programs and figuring out algorithms. Comparing linear relationships and systems of equations in scientific data.

Career and Skill Connections

• Advertising • Analysis • Anthropology • Archeology • Architecture	• Economist • Education • Electrician • Engineering • Environmental restoration	• Mechanics • Medicine • Microbiology • Mining • Physical therapy

• Arts • Astronomy • Atmospheric science • Aviation • Banking/finance • Biology • Bookkeeping • Botany • Business • Carpentry • Chemistry • Choreography • Computer programming • Conservation science • Construction • Counseling • Culinary arts • Ecology	• Epidemiology • Event planning • Floral design • Food science • Forensics • Forestry • Fundraising • Geology • Health science • HVAC • Information technology • Insurance • Landscaping • Law enforcement • Machinist • Management • Marketing	• Physics • Plumbing • Policy analysis • Ranching/farming • Sales • Sociology • Software development • Soil science • Statistics • Technician • Technology • Transportation • Travel agent • Veterinary • Video game design • Web development • Zoology

Grade	CCSS Domain	CCSS Cluster
8	Expressions and Equations	Work with radicals and integer exponents.

Cluster Standard: 8.EE.A.4

Standard	Standards for Mathematical Practice
8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	● **SMP 2:** Reason abstractly and quantitatively. ○ Teacher and Student Actions ● **SMP 4**: Model with mathematics. ○ Teacher and Student Actions ● **SMP 6:** Attend to precision. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
In this cluster, students explore the properties of exponents, radicals, and scientific notation.	● Add, subtract, multiply or divide numbers written in scientific notation. ● Assess the appropriate size for measurement written in scientific notation.
DOK	**Blooms**
1-3	Evaluation, Application

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Interpret scientific notation that has been generated by technology.
- Understand the properties of exponents, radicals, and scientific notation.
- Assess the appropriate size for measurement written in scientific notation.
- Interpret scientific notation and relate it to the actual magnitude of a quantity.
- Understand the concept of scale and use scale to choose units of measurements that make sense in the context of the problem.

Procedural Skill and Fluency:

- Perform arithmetic operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used.
- Use scientific notation and choose units of appropriate size for measurements of very large or very small

quantities.
- Use the properties of exponents, radicals, and scientific notation.
- Add, subtract, multiply, and divide numbers written in scientific notation.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

This headline appeared in a newspaper: Every day 7% of Americans eat at Giantburger restaurants

1. Decide whether this headline is true using the following information, making sure to explain your reasons and show clearly how you figured it out.

- There are about $8x10^3$ Giantburger restaurants in America.
- Each restaurant serves on average $2.5x10^3$ people every day.
- There are about $3x10^8$ Americans.

2. Perform the following operations.

a. $3.1x10^3 + 4.3x10^3$ b. $5.57x10^4 - 2.4x10^4$ c. $-2.74x10^{-1} + 6.4x10^2$

3. Match each of the following with the appropriate unit of measure.

a. Annual seafloor spreading	Meters
b. Mountain height	Nanometers
c. Distance between planets	Kilometers
d. Diameter of an atom	Millimeters

4. Write each of the units of measure above in scientific notation in the form $x10^?$ by replacing the "?" in the exponent.

a. Meters b. Nanometers c. Kilometers d. Millimeters

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Rules for exponents:** Students may confuse the rules for exponents, especially if they are taught to memorize them rather than understand them conceptually.
- **Power of a power:** Students may think that finding a power of a power involves adding exponents.
- **Negative exponents:** Students may not understand why negative exponents can be represented by a fraction with the term and negative exponents in the denominator.
- **0 and 1 as exponents:** Students often think that a number raised to the power of 0 is 0 and a number raised to the power of 1 is 1. Tables can help students see patterns in the ways different powers affect numbers, especially when a problem involves a real-world context.

- **Negative terms in context:** Many problems use negative numbers to represent debt, depth below ground level, etc. Students might struggle to understand this because it is a convention that does not make sense in context (owing $30 is still 30, not -30) even if it makes problems easier to work with. Using visuals, number lines, etc. can help students understand the idea of negative numbers in context.
- **Multi-step problems:** When an expression has several steps, sometimes students forget to follow the order of operations, struggle with moving from step to step, forget terms, etc.
- **Scientific notation:** Students may forget that correct scientific notation requires that the first factor be written with only one digit to the left of the decimal, and they may be confused about when to use positive vs. negative exponents, as well as what the sign of the exponent means when they are writing numbers in standard form from scientific notation. Some students may not fully understand why scientific notation is used and may see it as just another way to write numbers without understanding its significance in representing very large or very small quantities.
- **Units:** In numbers involving units of mass, students might struggle with conversions. In problems where students must represent numbers with units, they may struggle to choose the correct units to use, or which units will be the easiest to work with given the context of the problem.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming	**Pre-teaching** In previous classes, learners worked to develop understanding of the powers of 10 and the placement of the decimal when multiplying or dividing by powers of 10, write and evaluate numerical expressions involving whole number exponents, use ratio, rate reasoning, and unit rates, compute unit rates and recognize and represent proportional relationships, use variables to write expressions and equations and apply the properties of operations to generate equivalent expressions, and solve equations, including those that involve real world problems. Students worked with linear equations, isolating a single variable using inverse operations. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as evaluating	**Pre-teaching** Consider using standard 6.EE.A.1, which provides a foundation for work in this cluster. In 6.EE.A.1, students write and evaluate numerical expressions involving whole-number exponents. Also consider using standard 6.EE.B.5, which also provides a foundation for work in this cluster. In 6.EE.B.5, students learn that solving an equation or inequality is a process of determining which value(s) (if any) make the equation or inequality true. Also consider using standard 7.RP.A.2, which also provides a foundation for work in this cluster. In 7.RP.A.2, students recognize and represent proportional relationships between quantities. Students must understand how to write and evaluate numerical expressions using exponents and need to understand the concept

connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships.

Engagement

Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals.

Action and Expression

Throughout the curriculum, students should be invited to share both their understanding and their reasoning

numbers to a given exponent, evaluating expressions written in scientific notation, writing and solving expressions and equations in one variable, finding area to solve multi-step problems, interpreting graphs (especially comparing rates), calculating rates, using ratio reasoning to solve problems, generating equivalent expressions (especially using the distributive property), and comparing products. The language of this 8th grade cluster is completely new, but the skills needed for success began in 6th grade. Previously, slope is referred to as rate, unit rate, and the constant of proportionality, and the constant of proportionality is structured in the form $y = kx$, so shifting students away from this and to the form $y = bx$ or even $y = mx + b$ will require a shift in language and terminology.

If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

Re-teaching

Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Clarify different representations for students by allowing them to compare and

exponents as representing repeated multiplication or division.

If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

Re-teaching

Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster exploring different strategies for working with radicals, square and cube roots, numbers in scientific notation, and exponents. Give students opportunities to compare numbers written differently to gain an understanding of relative sizes written in forms they are unfamiliar with.

about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.	contrast tables, graphs, ordered pairs, and equations. Take time to practice solving linear equations, especially when working with rational coefficients.	

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/35/394/399

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● develop understanding of the powers of 10 and the placement of the decimal when multiplying or dividing by powers of 10 ● write and evaluate numerical expressions involving whole number exponents ● use ratio, rate reasoning, and	In 8th grade, learners ● use squares and square roots and cubes and cube roots when working with irrational numbers and volume ● compare properties of functions given a table, a graph, or an equation ● use the equation of a linear model to solve problems in	In future classes, learners ● use properties of exponents to rewrite expressions and extend their knowledge of integer exponents to rational exponents ● understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate

unit rates • compute unit rates and recognize and represent proportional relationships • use variables to write expressions and equations and apply the properties of operations to generate equivalent expressions • solve equations, including those that involve real world problems	the context of bivariate measurement data, interpreting the slope and intercept • use equations to graph linear and proportional relationships	plane, often forming a curve (which could be a line) • create, solve, and rewrite equations, inequalities, and systems of equations (include equations arising from linear, exponential, and quadratic functions) • construct a viable argument to justify a solution method

Culturally and Linguistically Responsive Instruction

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- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive

towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- Can a number be expressed in only one way? Can you explain or give an example of how a number can be expressed in more than one way? How can you be sure multiple expressions are equivalent?
- What are some real-life examples of using square roots, cube roots, and scientific notation to write expressions to communicate mathematical thinking?
- What relationship do a negative exponent and a fraction have?
- What does each part of a number written in scientific notation represent?
- How can we compare these numbers written in scientific notation?
- What parts of a number written in scientific notation do we monitor when adding? Subtracting? Multiplying? Dividing?

Cross-Curricular Connections

Arts: Following the mathematical series of musical rhythms to learn the basic rhythms of dance. Calculating shutter speed, focal length, lighting angles, and exposure time.

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

Science: Representing the large distances of planets from the sun. Comparing rates and relationships in scientific data. Comparing linear relationships and systems of equations in scientific data. Writing number sentences for conservation of energy of a system. Collaborating with peers to define or describe an issue in society and how to evaluate solutions and running tests of solutions and change designs as needed. Constructing scientific arguments for how uneven distributions of earth's mineral, energy, groundwater resources are the result of past and current geoscience processes (e.g. metal ores, volcanic activity, soil weathering, rock deposits, and mining by humans). Solving chemistry problems. Understanding the movements of the planets. Solving equations when writing computer programs and figuring out algorithms. Comparing linear relationships and systems of equations in scientific data.

Career and Skill Connections

- Advertising
- Analysis
- Anthropology
- Archeology
- Architecture
- Arts
- Astronomy
- Atmospheric science
- Aviation
- Banking/finance
- Biology
- Bookkeeping
- Botany
- Business
- Carpentry
- Chemistry
- Choreography
- Computer programming
- Conservation science
- Construction
- Counseling
- Culinary arts
- Ecology
- Economist
- Education
- Electrician
- Engineering
- Environmental restoration
- Epidemiology
- Event planning
- Floral design
- Food science
- Forensics
- Forestry
- Fundraising
- Geology
- Health science
- HVAC
- Information technology
- Insurance
- Landscaping
- Law enforcement
- Machinist
- Management
- Marketing
- Mechanics
- Medicine
- Microbiology
- Mining
- Physical therapy
- Physics
- Plumbing
- Policy analysis
- Ranching/farming
- Sales
- Sociology
- Software development
- Soil science
- Statistics
- Technician
- Technology
- Transportation
- Travel agent
- Veterinary
- Video game design
- Web development
- Zoology

Grade	CCSS Domain	CCSS Cluster
8	Expressions and Equations	Understand the connections between proportional relationships, lines, and linear equations.

Cluster Standard: 8.EE.B.5

Standard	Standards for Mathematical Practice

8.EE.B.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.	• **SMP 2:** Reason abstractly and quantitatively.
	  ○ Teacher and Student Actions
	• **SMP 3:** Construct viable arguments and critique the reasoning of others.
	  ○ Teacher and Student Actions
	• **SMP 4**: Model with mathematics.
	  ○ Teacher and Student Actions
	• **SMP 5:** Use appropriate tools strategically.
	  ○ Teacher and Student Actions

Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students connect slope to unit rates, tables, lines, and equations. Students will also connect similar triangles to slope.	• Graph proportional relationships. • Interpret the unit rate as the slope of the graph. • Compare two proportional relationships whether it is table, graph or equation.
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Connect similar triangles to slope.
- Interpret the unit rate as the slope of the graph.
- Understand that in proportional relationships, the ratio of the two quantities remains constant.

Procedural Skill and Fluency:

- Graph proportional relationships, interpreting the unit rate as the slope of the graph.
- Find slope from unit rates, tables, lines, and equations, using similar triangles as one method of doing so.
- Translate proportional relationships between graphs, tables, and equations.

Application:

- Compare two different proportional relationships represented in different ways.
- Make connections between slope and unit rates, tables, lines, and equations.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Nia and Trey both had a sore throat so their mom told them to gargle with warm salt water. Nia mixed 1 teaspoon salt with 3 cups of water, and Trey mixed ½ teaspoon salt with 1½ cups of water. Nia tasted Trey's salt water and said, "I added more salt so I expected that mine would be more salty, but they taste the same."

1. Explain why the salt water mixtures taste the same.
2. What is the unit rate of salt to water?
3. Find an equation that relates s, the number of teaspoons of salt, with w, the number of cups of water, for both of these mixtures.
4. Draw the graph of your equations from part b.
5. Your graphs in part c should be linear. What does the slope mean in the context of the problem?

Kell works at an after-school program at an elementary school. The table shown here shows how much money he earned every day last week. Mariko has a job mowing lawns that pays $7 per hour.

Time worked	1.5 hours	2.5 hours	4 hours
Money earned	$12.60	$21.00	$33.60

6. Who would make more money for working 10 hours? Justify your answer.
7. Draw a graph that represents y, the amount of money Kell would make for working x hours, assuming he made the same hourly rate he was making last week.
8. Using the same coordinate axes, draw a graph that represents y, the amount of money Mariko would make for working x hours.
9. How can you see who makes more per hour just by looking at the graphs? Explain.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Unit rate:** Students may make mistakes if they estimate the unit rate from a graph instead of calculating the rate from data or an equation, or if they find a single unit rate instead of comparing unit rates. Students might also struggle to understand negative unit rates and assume that a negative unit rate is slower/smaller.

- **Slope:** Students might calculate the slope incorrectly if they confuse the x- and y-axes, if they divide x by y, or if they use x_1 and y_1 together and x_2 and y_2 together instead of x_1 and x_2 together and y_1 and y_2 together. Students might also struggle to understand negative slopes and assume that a negative slope is slower/smaller.
- **Terms:** Students may confuse the slope and y-intercept.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships.	**Pre-teaching** In previous classes, learners worked to develop understanding of the powers of 10 and the placement of the decimal when multiplying or dividing by powers of 10, write and evaluate numerical expressions involving whole number exponents, use ratio, rate reasoning, and unit rates, compute unit rates and recognize and represent proportional relationships, use variables to write expressions and equations and apply the properties of operations to generate equivalent expressions, and solve equations, including those that involve real world problems. Students worked with linear equations, isolating a single variable using inverse operations. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as evaluating numbers to a given exponent, evaluating expressions written in scientific notation, writing and solving expressions and equations in one variable, finding area to solve multi-step problems, interpreting graphs (especially comparing rates), calculating rates, using ratio reasoning to solve problems, generating equivalent expressions	**Pre-teaching** Consider using standard 6.EE.A.1, which provides a foundation for work in this cluster. In 6.EE.A.1, students write and evaluate numerical expressions involving whole-number exponents. Also consider using standard 6.EE.B.5, which also provides a foundation for work in this cluster. In 6.EE.B.5, students learn that solving an equation or inequality is a process of determining which value(s) (if any) make the equation or inequality true. Also consider using standard 7.RP.A.2, which also provides a foundation for work in this cluster. In 7.RP.A.2, students recognize and represent proportional relationships between quantities. Students must understand how to write and evaluate numerical expressions using exponents and need to understand the concept exponents as representing repeated multiplication or division. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should

Engagement
Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals.

Action and Expression
Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or (especially using the distributive property), and comparing products. The language of this 8th grade cluster is completely new, but the skills needed for success began in 6th grade. Previously, slope is referred to as rate, unit rate, and the constant of proportionality, and the constant of proportionality is structured in the form $y = kx$, so shifting students away from this and to the form $y = bx$ or even $y = mx + b$ will require a shift in language and terminology.

If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

Re-teaching
Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Clarify different representations for students by allowing them to compare and contrast tables, graphs, ordered pairs, and equations. Take time to practice solving linear equations, especially when working with rational coefficients.

spend most of their time accessing their current grade-level content.

Re-teaching
Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster exploring different strategies for working with radicals, square and cube roots, numbers in scientific notation, and exponents. Give students opportunities to compare numbers written differently to gain an understanding of relative sizes written in forms they are unfamiliar with.

responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/35/404/404

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● develop understanding of the powers of 10 and the placement of the decimal when multiplying or dividing by powers of 10 ● write and evaluate numerical expressions involving whole number exponents ● use ratio, rate reasoning, and unit rates ● compute unit rates and recognize and represent proportional relationships ● use variables to write expressions and equations and apply the properties of operations to generate equivalent expressions ● solve equations, including	In 8th grade, learners ● use squares and square roots and cubes and cube roots when working with irrational numbers and volume ● compare properties of functions given a table, a graph, or an equation ● use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept ● use equations to graph linear and proportional relationships	In future classes, learners ● use properties of exponents to rewrite expressions and extend their knowledge of integer exponents to rational exponents ● understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line) ● create, solve, and rewrite equations, inequalities, and systems of equations (include equations arising from linear, exponential, and quadratic functions) ● construct a viable argument to justify a solution method

those that involve real world problems		

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

use in their careers or crafts.	mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	

Suggested Student Discourse Questions

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- How can we determine if a graph is proportional?
- How do we graph an equation? Where do we start? Why?
- What do proportional relationships look like when written in equation form? In a table? In a graph? When described? In real life?
- How can we extend strategies for proportional graphs to all linear equations?
- Why would someone need to know the slope of something in everyday life? Give an example.
- What can you infer about the connections between proportional relationships, lines, and linear equations?

Cross-Curricular Connections

Arts: Following the mathematical series of musical rhythms to learn the basic rhythms of dance. Calculating shutter speed, focal length, lighting angles, and exposure time.

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

Science: Representing the large distances of planets from the sun. Comparing rates and relationships in scientific data. Comparing linear relationships and systems of equations in scientific data. Writing number sentences for conservation of energy of a system. Collaborating with peers to define or describe an issue in society and how to evaluate solutions and running tests of solutions and change designs as needed. Constructing scientific arguments for how uneven distributions of earth's mineral, energy, groundwater resources are the result of past and current geoscience processes (e.g. metal ores, volcanic activity, soil weathering, rock deposits, and mining by humans). Solving chemistry problems. Understanding the movements of the planets. Solving equations when writing computer programs and figuring out algorithms. Comparing linear relationships and systems of equations in scientific data.

Career and Skill Connections

• Advertising	• Economist	• Mechanics

• Analysis • Anthropology • Archeology • Architecture • Arts • Astronomy • Atmospheric science • Aviation • Banking/finance • Biology • Bookkeeping • Botany • Business • Carpentry • Chemistry • Choreography • Computer programming • Conservation science • Construction • Counseling • Culinary arts • Ecology	• Education • Electrician • Engineering • Environmental restoration • Epidemiology • Event planning • Floral design • Food science • Forensics • Forestry • Fundraising • Geology • Health science • HVAC • Information technology • Insurance • Landscaping • Law enforcement • Machinist • Management • Marketing	• Medicine • Microbiology • Mining • Physical therapy • Physics • Plumbing • Policy analysis • Ranching/farming • Sales • Sociology • Software development • Soil science • Statistics • Technician • Technology • Transportation • Travel agent • Veterinary • Video game design • Web development • Zoology

Grade	CCSS Domain	CCSS Cluster
8	Expressions and Equations	Understand the connections between proportional relationships, lines, and linear equations.

Cluster Standard: 8.EE.B.6

Standard	Standards for Mathematical Practice

8.EE.B.6: Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.	• **SMP 2:** Reason abstractly and quantitatively.
	  ○ Teacher and Student Actions
	• **SMP 3:** Construct viable arguments and critique the reasoning of others.
	  ○ Teacher and Student Actions
	• **SMP 4**: Model with mathematics.
	  ○ Teacher and Student Actions
	• **SMP 5:** Use appropriate tools strategically.
	  ○ Teacher and Student Actions

Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students connect slope to unit rates, tables, lines, and equations. Students will also connect similar triangles to slope.	• Identify the Y-intercept of the graph and understand the meaning of the y-intercept in a real-world problem situation. • Use similar triangles to explain why the slope m is the same between any two distinct points on a nonvertical line in the coordinate plane. • Graph a line from an equation in the form of y= mx + b, understand what m is (slope) and the b (y intercept). • Discover the equation y = mx for a line through the origin (proportional) and the equation y = mx + b for a line intercepting the vertical axis at b.
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

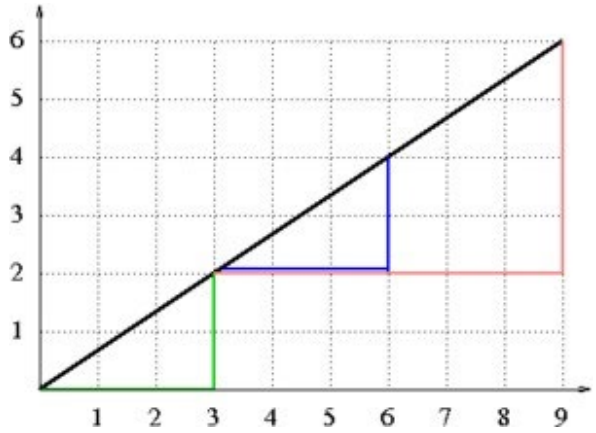
- Connect similar triangles to slope and use them to explain why the slope *m* is the same between any two distinct points on a nonvertical line in the coordinate plane.

- Derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.
- Make connections between slope and unit rates, tables, lines, and equations.
- Graph a line from an equation in the form of y= mx + b and understand and interpret what slope *m* and *b* y intercept mean.
- Identify the y-intercept of the graph and understand its meaning in a real-world problem.

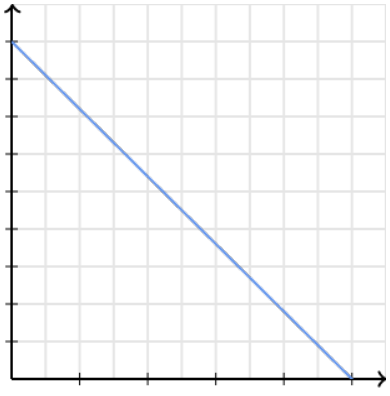
Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Eva, Carl, and Maria are computing the slope between pairs of points on a line. Eva finds the slope between the points (0,0) and (3,2). Carl finds the slope between the points (3,2) and (6,4). Maria finds the slope between the points (3,2) and (9,6). They have each drawn a triangle to help with their calculations (shown on the graph).



1. Which student has drawn which triangle?
2. Finish the slope calculation for each student. How can the differences in the x- and y-values be interpreted geometrically in the pictures they have drawn?
3. Consider any two points $(x_1, y_1,)$ and $(x_2, y_2,)$ on the line shown above. Draw a triangle like the triangles drawn by Eva, Carl, and Maria. What is the slope between these two points? Why should this slope be the same as the slopes calculated by the three students?



4. What is the equation of the line shown on the graph?
5. Write another equation for the graph shown here.
6. How does this graph compare to the graph above?
7. How does the equation for this graph compare to the equation for the graph above?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Slope:** Students might calculate the slope incorrectly if they confuse the x- and y-axes, if they divide x by y, or if they use x_1 and y_1 together and x_2 and y_2 together instead of x_1 and x_2 together and y_1 and y_2 together. Students might struggle to grasp conceptually that slope is the ratio of vertical change to horizontal change and that this ratio remains constant for a line, as well as that slope can be positive, negative, zero, or undefined.
- **Vertical and horizontal lines:** Students may confuse the slope of a horizontal line (0) with that of a vertical line (undefined), and especially struggle to understand what undefined slope means.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

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Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key	**Pre-teaching** In previous classes, learners worked to develop understanding of the powers of 10 and the placement of the decimal when multiplying or dividing by powers of 10, write and evaluate numerical expressions involving whole number exponents, use ratio, rate reasoning, and unit rates, compute unit rates and recognize and represent proportional relationships, use variables to write expressions and equations and apply the properties of operations to generate equivalent expressions, and solve equations, including those that involve real world problems. Students worked with linear equations, isolating a single variable using inverse operations. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as evaluating numbers to a given exponent, evaluating expressions written in scientific notation, writing and solving expressions and equations in one variable, finding area to solve multi-step problems, interpreting graphs (especially comparing rates), calculating rates, using ratio	**Pre-teaching** Consider using standard 6.EE.A.1, which provides a foundation for work in this cluster. In 6.EE.A.1, students write and evaluate numerical expressions involving whole-number exponents. Also consider using standard 6.EE.B.5, which also provides a foundation for work in this cluster. In 6.EE.B.5, students learn that solving an equation or inequality is a process of determining which value(s) (if any) make the equation or inequality true. Also consider using standard 7.RP.A.2, which also provides a foundation for work in this cluster. In 7.RP.A.2, students recognize and represent proportional relationships between quantities. Students must understand how to write and evaluate numerical expressions using exponents and need to understand the concept exponents as representing repeated multiplication or division. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are

ideas and relationships.

Engagement
Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals.

Action and Expression
Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide

reasoning to solve problems, generating equivalent expressions (especially using the distributive property), and comparing products. The language of this 8th grade cluster is completely new, but the skills needed for success began in 6th grade. Previously, slope is referred to as rate, unit rate, and the constant of proportionality, and the constant of proportionality is structured in the form $y = kx$, so shifting students away from this and to the form $y = bx$ or even $y = mx + b$ will require a shift in language and terminology.

If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

Re-teaching
Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Clarify different representations for students by allowing them to compare and contrast tables, graphs, ordered pairs, and equations. Take time to practice solving linear equations, especially when working with rational coefficients.

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independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

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https://tools.achievethecore.org/coherence-map/8/35/406/406

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● develop understanding of the powers of 10 and the placement of the decimal when multiplying or dividing by powers of 10 ● write and evaluate numerical expressions involving whole number exponents ● use ratio, rate reasoning, and unit rates ● compute unit rates and recognize and represent proportional relationships ● use variables to write expressions and equations and apply the properties of operations to generate	In 8th grade, learners ● use squares and square roots and cubes and cube roots when working with irrational numbers and volume ● compare properties of functions given a table, a graph, or an equation ● use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept ● use equations to graph linear and proportional relationships	In future classes, learners ● use properties of exponents to rewrite expressions and extend their knowledge of integer exponents to rational exponents ● understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line) ● create, solve, and rewrite equations, inequalities, and systems of equations (include equations arising from linear, exponential, and quadratic functions)

equivalent expressions • solve equations, including those that involve real world problems		• construct a viable argument to justify a solution method

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- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

members to talk with students about the math they use in their careers or crafts.	that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	

Suggested Student Discourse Questions

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- How do we graph an equation? Where do we start? Why?
- What do linear relationships look like when written in equation form? In a table? In a graph? When described? In real life?
- Why would someone need to know the slope of something in everyday life? Give an example.
- What can you infer about the connections between proportional relationships, lines, and linear equations?
- What does each piece in the equation $y = mx$ represent? What about the equation $y = mx + b$?

Cross-Curricular Connections

Arts: Following the mathematical series of musical rhythms to learn the basic rhythms of dance. Calculating shutter speed, focal length, lighting angles, and exposure time.

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

Science: Representing the large distances of planets from the sun. Comparing rates and relationships in scientific data. Comparing linear relationships and systems of equations in scientific data. Writing number sentences for conservation of energy of a system. Collaborating with peers to define or describe an issue in society and how to evaluate solutions and running tests of solutions and change designs as needed. Constructing scientific arguments for how uneven distributions of earth's mineral, energy, groundwater resources are the result of past and current geoscience processes (e.g. metal ores, volcanic activity, soil weathering, rock deposits, and mining by humans). Solving chemistry problems. Understanding the movements of the planets. Solving equations when writing computer programs and figuring out algorithms. Comparing linear relationships and systems of equations in scientific data.

Career and Skill Connections

• Advertising • Analysis • Anthropology • Archeology • Architecture • Arts • Astronomy • Atmospheric science • Aviation • Banking/finance • Biology • Bookkeeping • Botany • Business • Carpentry • Chemistry • Choreography • Computer programming • Conservation science • Construction • Counseling • Culinary arts • Ecology	• Economist • Education • Electrician • Engineering • Environmental restoration • Epidemiology • Event planning • Floral design • Food science • Forensics • Forestry • Fundraising • Geology • Health science • HVAC • Information technology • Insurance • Landscaping • Law enforcement • Machinist • Management • Marketing	• Mechanics • Medicine • Microbiology • Mining • Physical therapy • Physics • Plumbing • Policy analysis • Ranching/farming • Sales • Sociology • Software development • Soil science • Statistics • Technician • Technology • Transportation • Travel agent • Veterinary • Video game design • Web development • Zoology

Grade	CCSS Domain	CCSS Cluster
8	Expressions and Equations	Analyze and solve linear equations and pairs of simultaneous linear equations.

Cluster Standard: 8.EE.C.7

Standard	Standards for Mathematical Practice
8.EE.C.7: Solve linear equations in one variable. ● 8.EE.C.7.A: Solve linear equations in one variable. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers). ● 8.EE.C.7.B: Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	● **SMP 1:** Make sense of problems and persevere in solving them. ○ Teacher and Student Actions ● **SMP 2:** Reason abstractly and quantitatively. ○ Teacher and Student Actions ● **SMP 3:** Construct viable arguments and critique the reasoning of others. ○ Teacher and Student Actions ● **SMP 4**: Model with mathematics. ○ Teacher and Student Actions ● **SMP 5**: Use appropriate tools strategically. ○ Teacher and Student Actions ● **SMP 6:** Attend to precision. ○ Teacher and Student Actions ● **SMP 7**: Look for and make use of structure. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students analyze, solve, and interpret linear equations and systems of linear equations.	● Combine like terms. ● Expand an equation using the distributive property. ● Solve one step equations, two step equations and multi-step (including equations where you must combine like terms and expand using the distributive property). ● Use inverse operations to solve equations. ● Determine whether an equation will have one solution (x=a), no solution (a=b) or infinite solutions (a=a) by simplifying the equation. (a and b are numbers).
DOK	**Blooms**

2-3	Apply, Evaluate

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Understand that linear equations can have one solution, infinitely many solutions, or no solutions, and give examples of each.
- Analyze and interpret linear equations and systems of linear equations.
- Understand that if two equations are equivalent then they have the same solution, and equations can be transformed into simpler forms while maintaining their original solution(s).
- Understand that performing inverse operations on both sides of an equation maintains its equality, and this process is used to isolate a variable.

Procedural Skill and Fluency:

- Solve linear equations in one variable using inverse operations.
- Show whether a linear equation has one solution, infinitely many solutions, or no solutions by successively transforming the given equation into simpler forms until an equivalent equation of the form x = a, a = a, or a = b results.
- Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

1. Solve 5x + 1 = 2x + 7 in two ways: symbolically, the way you usually do with equations, and also with pictures of a balance. Show how each step you take symbolically is shown in the pictures.

2. Solve the equation 4x = x + 1 using pictures and symbols. Discuss any issues that arise.

3. What issues arise when you try to solve the equation 2 = 2x − 4 using pictures? Do the same issues arise when you solve this equation symbolically?

4. Make up a linear equation that has one solution. Prove that it has one solution.

5. Make up a linear equation that has infinitely many solutions. Prove that it has many solutions.

6. What would happen if you solved the equation with infinitely many solutions with pictures? How is this different from an equation that has no solutions?

7. Make up a linear equation that has no solutions. Prove that it has one solution.

8. Use pictures to show why the following solution is incorrect:

$$\frac{2x}{2} + 4 = \frac{10}{2}$$
$$x + 4 = 5$$
$$-4 \quad -4$$
$$x = 1$$

9. Solve each equation below. Show your work.

a. -6x + 3x = 21 b. 12x + 10 + 2x = 38 c. 4x + 1 - 6x = 5 d. 2(x + 3) + 5x = 27

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Multi-step problems:** When an expression has several steps, sometimes students forget to follow the order of operations, struggle with moving from step to step, forget terms, combining like terms, applying the distributive property, etc.
- **Inverse operations:** Students may mistakenly believe that they can perform any operation on one side of the equation without affecting the other side's equality, not understanding that they must perform the same operation on both sides to maintain the equality of the equation. They may not understand which operations are inverse of each other

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and	**Pre-teaching** In previous classes, learners worked to develop understanding of the powers of 10 and the placement of the decimal when multiplying or dividing by powers of 10, write and evaluate numerical expressions involving whole number exponents, use ratio, rate reasoning, and unit rates, compute unit rates and recognize and represent proportional relationships, use variables to write expressions and equations and apply the properties of operations to generate equivalent expressions, and solve equations, including those that involve real world problems. Students worked with linear equations, isolating a single variable using	**Pre-teaching** Consider using standard 6.EE.A.1, which provides a foundation for work in this cluster. In 6.EE.A.1, students write and evaluate numerical expressions involving whole-number exponents. Also consider using standard 6.EE.B.5, which also provides a foundation for work in this cluster. In 6.EE.B.5, students learn that solving an equation or inequality is a process of determining which value(s) (if any) make the equation or inequality true. Also consider using standard 7.RP.A.2, which also provides a foundation for work in this cluster. In

highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships.

Engagement

Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort

inverse operations.

Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as evaluating numbers to a given exponent, evaluating expressions written in scientific notation, writing and solving expressions and equations in one variable, finding area to solve multi-step problems, interpreting graphs (especially comparing rates), calculating rates, using ratio reasoning to solve problems, generating equivalent expressions (especially using the distributive property), and comparing products. The language of this 8th grade cluster is completely new, but the skills needed for success began in 6th grade. Previously, slope is referred to as rate, unit rate, and the constant of proportionality, and the constant of proportionality is structured in the form $y = kx$, so shifting students away from this and to the form $y = bx$ or even $y = mx + b$ will require a shift in language and terminology.

If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

Re-teaching

Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at

7.RP.A.2, students recognize and represent proportional relationships between quantities.

Students must understand how to write and evaluate numerical expressions using exponents and need to understand the concept exponents as representing repeated multiplication or division.

If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

Re-teaching

Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster exploring different strategies for working with radicals, square and cube roots, numbers in scientific notation, and exponents. Give students opportunities to compare numbers written differently to gain an understanding of relative sizes written in forms they are unfamiliar with.

and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.	revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Clarify different representations for students by allowing them to compare and contrast tables, graphs, ordered pairs, and equations. Take time to practice solving linear equations, especially when working with rational coefficients.	

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/35/408/408

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● develop understanding of the	In 8th grade, learners ● use squares and square roots	In future classes, learners ● use properties of exponents

powers of 10 and the placement of the decimal when multiplying or dividing by powers of 10 • write and evaluate numerical expressions involving whole number exponents • use ratio, rate reasoning, and unit rates • compute unit rates and recognize and represent proportional relationships • use variables to write expressions and equations and apply the properties of operations to generate equivalent expressions • solve equations, including those that involve real world problems	and cubes and cube roots when working with irrational numbers and volume • compare properties of functions given a table, a graph, or an equation • use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept • use equations to graph linear and proportional relationships	to rewrite expressions and extend their knowledge of integer exponents to rational exponents • understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line) • create, solve, and rewrite equations, inequalities, and systems of equations (include equations arising from linear, exponential, and quadratic functions) • construct a viable argument to justify a solution method

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is

games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers:
https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- Describe when you would choose to use each strategy (elimination, substitution, and graphing) to solve a system of equations. Why? Is there always a better choice?
- Determine which strategy (elimination, substitution, or graphing, would be the most effective to solve this pair of simultaneous linear equations. Is one strategy always a better choice?
- Where might you see a single solution in real life? What about no solution? Multiple solutions?
- What does it mean for linear equations to have one solution, many solutions, or no solution?

Cross-Curricular Connections

Arts: Following the mathematical series of musical rhythms to learn the basic rhythms of dance. Calculating shutter speed, focal length, lighting angles, and exposure time.

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

Science: Representing the large distances of planets from the sun. Comparing rates and relationships in scientific data. Comparing linear relationships and systems of equations in scientific data. Writing number sentences for conservation of energy of a system. Collaborating with peers to define or describe an issue in society and how to evaluate solutions and running tests of solutions and change designs as needed. Constructing scientific arguments for how uneven distributions of earth's mineral, energy, groundwater resources are the result of past and current geoscience processes (e.g. metal ores, volcanic activity, soil weathering, rock deposits, and mining by humans). Solving chemistry problems. Understanding the movements of the planets. Solving equations when writing computer programs and figuring out algorithms. Comparing linear relationships and systems of equations in scientific data.

Career and Skill Connections

- Advertising
- Analysis
- Anthropology
- Archeology
- Architecture
- Arts
- Astronomy
- Atmospheric science
- Aviation
- Banking/finance
- Biology
- Bookkeeping
- Botany
- Business
- Carpentry
- Chemistry
- Choreography
- Computer programming
- Conservation science
- Construction
- Counseling
- Culinary arts
- Ecology
- Economist
- Education
- Electrician
- Engineering
- Environmental restoration
- Epidemiology
- Event planning
- Floral design
- Food science
- Forensics
- Forestry
- Fundraising
- Geology
- Health science
- HVAC
- Information technology
- Insurance
- Landscaping
- Law enforcement
- Machinist
- Management
- Marketing
- Mechanics
- Medicine
- Microbiology
- Mining
- Physical therapy
- Physics
- Plumbing
- Policy analysis
- Ranching/farming
- Sales
- Sociology
- Software development
- Soil science
- Statistics
- Technician
- Technology
- Transportation
- Travel agent
- Veterinary
- Video game design
- Web development
- Zoology

Grade	CCSS Domain	CCSS Cluster
8	Expressions and Equations	Analyze and solve linear equations and pairs of simultaneous linear equations.

Cluster Standard: 8.EE.C.8

Standard	Standards for Mathematical Practice

8.EE.C.8: Analyze and solve pairs of simultaneous linear equations.
• 8.EE.C.8.A: Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
• 8.EE.C.8.B: Solve systems of two linear equations in two variables algebraically and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.
• 8.EE.C.8.C: Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.
	• **SMP 1:** Make sense of problems and persevere in solving them.
	  ○ Teacher and Student Actions
	• **SMP 2:** Reason abstractly and quantitatively.
	  ○ Teacher and Student Actions
	• **SMP 3:** Construct viable arguments and critique the reasoning of others.
	  ○ Teacher and Student Actions
	• **SMP 4**: Model with mathematics.
	  ○ Teacher and Student Actions
	• **SMP 5**: Use appropriate tools strategically.
	  ○ Teacher and Student Actions
	• **SMP 6:** Attend to precision.
	  ○ Teacher and Student Actions
	• **SMP 7**: Look for and make use of structure.
	  ○ Teacher and Student Actions

Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students analyze, solve, and interpret linear equations and systems of linear equations.	• Calculate two linear equations with two variables in a real-world problem. • Calculate the value of two variables from two linear equations either algebraically or graphically. • Graph two equations and estimate solutions. • Analyze and solve systems of two linear equations with two variables in real-world problems. • Solve systems of two linear equations in two variables algebraically and/or graphically. • Estimate solutions by graphing the equations. • Solve simple cases by inspection.

DOK	Blooms
1-2	Apply, Analyze

Procedural and Conceptual Understanding and Application

Conceptual Understanding:

- Analyze simultaneous linear equations.
- Interpret simultaneous linear equations.
- Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs.
- Understand that points of intersection between graphs or equations satisfy both equations simultaneously.
- Solve simple cases of systems of equations by inspection, understanding that 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.
- Understand that the solution to a system of linear equations can be a single point, all points, or no points.
- Understand and explain why parallel lines have no intersection, while coincident lines have infinitely many intersections.

Procedural Skill and Fluency:

- Solve pairs of simultaneous linear equations.
- Solve systems of two linear equations in two variables algebraically (using methods such as substitution, elimination, or matrix methods) and estimate solutions by graphing the equations.
- Solve mathematical problems leading to two linear equations in two variables.
- Identify whether a system of equations has no solution or infinitely many solutions by examining the coefficients and constants.

Application:

- Solve real-world problems leading to two linear equations in two variables.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Kimi and Jordan are each working during the summer to earn money in addition to their weekly allowance, and they are saving all their money. Kimi earns $9 an hour at her job, and her allowance is $10 per week. Jordan earns $7.50 an hour, and his allowance is $16 per week.

1. Complete the two tables shown here:

Number of hours worked in a week, h	0	1	2	3	4	5	6	7
Kimi's weekly total savings, K								

Number of hours worked in a week, h	0	1	2	3	4	5	6	7
Jordan's weekly total savings, J								

2. Write an equation that can be used to calculate the total of Kimi's allowance and job earnings at the end of one week given the number of hours she works.

3. Write an equation that can be used to calculate the total of Jordan's allowance and job earnings at the end of one week given the number of hours worked.

4. Sketch the graphs of your two equations on one pair of axes.

5. Is there a value for *h* for which Kimi and Jordan will save the same amount of money? How do you know? Justify your answer using the table, the graph, and using algebra.

6. Consider the equation 5x − 2y = 3. If possible, find a second linear equation to create a system of equations that has:

a. Exactly 1 solution b. Exactly 2 solutions c. No solutions d. Infinitely many solutions

7. Consider the points (5, 10) and (12, 24). Would a line that goes through these points intersect with a line going through the points (-2, 8) and (-5, 20)?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Terms:** Students may confuse the slope and y-intercept.
- **Intersections:** Students might not understand that the way two lines intersect or do not intersect shows the number of solutions for a system of equations. They may also struggle to interpret an intersection point/the number of intersection points in the context of a real-world problem.
- **Solving a system of equations with substitution:** Students might solve an equation by substituting in only one equation in the system, or solve for one variable and then try and solve for that variable again.
- **Solving a system of equations with elimination:** Students might try to use elimination without eliminating a variable, might eliminate one variable and then try and solve for that variable again, or might not set up the equations so that one variable can be eliminated.
- **Number of solutions:** Students often struggle to understand what one solution vs. no solution vs. infinitely many solutions mean, and how they are represented (e.g. $x = 0$, $5 \neq 0$, and $0 = 0$).
- **Inverse operations:** Students may mistakenly believe that they can perform any operation on one side of the equation without affecting the other side's equality, not understanding that they must perform the same operation on both sides to maintain the equality of the equation. They may not understand which operations are inverse of each other

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language,	**Pre-teaching** In previous classes, learners worked to develop understanding of the powers of 10 and the placement of the decimal when multiplying or dividing by powers of 10, write and	**Pre-teaching** Consider using standard 6.EE.A.1, which provides a foundation for work in this cluster. In 6.EE.A.1, students write and evaluate numerical expressions involving whole-number

different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships.

Engagement

Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible

evaluate numerical expressions involving whole number exponents, use ratio, rate reasoning, and unit rates, compute unit rates and recognize and represent proportional relationships, use variables to write expressions and equations and apply the properties of operations to generate equivalent expressions, and solve equations, including those that involve real world problems. Students worked with linear equations, isolating a single variable using inverse operations.

Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as evaluating numbers to a given exponent, evaluating expressions written in scientific notation, writing and solving expressions and equations in one variable, finding area to solve multi-step problems, interpreting graphs (especially comparing rates), calculating rates, using ratio reasoning to solve problems, generating equivalent expressions (especially using the distributive property), and comparing products. The language of this 8th grade cluster is completely new, but the skills needed for success began in 6th grade. Previously, slope is referred to as rate, unit rate, and the constant of proportionality, and the constant of proportionality is structured in the form $y = kx$, so shifting students away from this and to the form $y = bx$ or even $y = mx + b$ will require a shift in language and terminology.

If students have unfinished learning leading into this standard, consider ways to provide targeted pre-

exponents.

Also consider using standard 6.EE.B.5, which also provides a foundation for work in this cluster. In 6.EE.B.5, students learn that solving an equation or inequality is a process of determining which value(s) (if any) make the equation or inequality true.

Also consider using standard 7.RP.A.2, which also provides a foundation for work in this cluster. In 7.RP.A.2, students recognize and represent proportional relationships between quantities.

Students must understand how to write and evaluate numerical expressions using exponents and need to understand the concept exponents as representing repeated multiplication or division.

If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

Re-teaching

Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster exploring different strategies for working with radicals, square and cube roots, numbers in

timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to	teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Clarify different representations for students by allowing them to compare and contrast tables, graphs, ordered pairs, and equations. Take time to practice solving linear equations, especially when working with rational coefficients.	scientific notation, and exponents. Give students opportunities to compare numbers written differently to gain an understanding of relative sizes written in forms they are unfamiliar with.

monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/35/411/411

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners • develop understanding of the powers of 10 and the placement of the decimal when multiplying or dividing by powers of 10 • write and evaluate numerical expressions involving whole number exponents • use ratio, rate reasoning, and unit rates • compute unit rates and recognize and represent proportional relationships • use variables to write expressions and equations and apply the properties of operations to generate equivalent expressions • solve equations, including those that involve real world problems	In 8th grade, learners • use squares and square roots and cubes and cube roots when working with irrational numbers and volume • compare properties of functions given a table, a graph, or an equation • use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept • use equations to graph linear and proportional relationships	In future classes, learners • use properties of exponents to rewrite expressions and extend their knowledge of integer exponents to rational exponents • understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line) • create, solve, and rewrite equations, inequalities, and systems of equations (include equations arising from linear, exponential, and quadratic functions) • construct a viable argument to justify a solution method

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:
- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.
Suggested Student Discourse Questions		
Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf		

- Describe when you would choose to use each strategy (elimination, substitution, and graphing) to solve a system of equations. Why? Is there always a better choice?
- Determine which strategy (elimination, substitution, or graphing, would be the most effective to solve this pair of simultaneous linear equations. Is one strategy always a better choice?
- Where might you see a single solution in real life? What about no solution? Multiple solutions?
- What does it mean for linear equations to have one solution, many solutions, or no solution? What does each look like when graphed? In a table?

Cross-Curricular Connections

Arts: Following the mathematical series of musical rhythms to learn the basic rhythms of dance. Calculating shutter speed, focal length, lighting angles, and exposure time.

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

Science: Representing the large distances of planets from the sun. Comparing rates and relationships in scientific data. Comparing linear relationships and systems of equations in scientific data. Writing number sentences for conservation of energy of a system. Collaborating with peers to define or describe an issue in society and how to evaluate solutions and running tests of solutions and change designs as needed. Constructing scientific arguments for how uneven distributions of earth's mineral, energy, groundwater resources are the result of past and current geoscience processes (e.g. metal ores, volcanic activity, soil weathering, rock deposits, and mining by humans). Solving chemistry problems. Understanding the movements of the planets. Solving equations when writing computer programs and figuring out algorithms. Comparing linear relationships and systems of equations in scientific data.

Career and Skill Connections

- Advertising
- Analysis
- Anthropology
- Archeology
- Architecture
- Arts
- Astronomy
- Atmospheric science
- Aviation
- Banking/finance
- Biology
- Bookkeeping
- Botany
- Business
- Carpentry
- Chemistry
- Choreography
- Computer programming
- Conservation science
- Construction
- Economist
- Education
- Electrician
- Engineering
- Environmental restoration
- Epidemiology
- Event planning
- Floral design
- Food science
- Forensics
- Forestry
- Fundraising
- Geology
- Health science
- HVAC
- Information technology
- Insurance
- Landscaping
- Law enforcement
- Machinist
- Mechanics
- Medicine
- Microbiology
- Mining
- Physical therapy
- Physics
- Plumbing
- Policy analysis
- Ranching/farming
- Sales
- Sociology
- Software development
- Soil science
- Statistics
- Technician
- Technology
- Transportation
- Travel agent
- Veterinary
- Video game design

• Counseling • Culinary arts • Ecology	• Management • Marketing	• Web development • Zoology

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards for Mathematical Practice
    - Connections to procedural, conceptual understanding, and application
    - Sample assessment items
    - Common misconceptions
    - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
    - Vertical alignment
    - Culturally and Linguistically Responsive Instruction (CLRI)
    - Suggested student discourse questions
    - Cross-curricular and career/skill connections
- A Student Discourse Guide

Helpful links:

- Lesson-planning tool from Pathways 2 Careers (click here to sign up with your district email if you don't already have an account)
- Focus by Grade Level from Achieve the Core
- Coherence Map from Achieve the Core

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Define, evaluate, and compare functions.
  - 8.F.A.1
  - 8.F.A.2
  - 8.F.A.3
- Use functions to model relationships between quantities.
  - 8.F.B.4
  - 8.F.B.5

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)

Grade	*CCSS Domain*	*CCSS Cluster*
8	Functions	Define, evaluate, and compare functions.

Cluster Standard: 8.F.A.1

Standard	Standards for Mathematical Practice
8.F.A.1: Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.	● **SMP 2**: Reason abstractly and quantitatively. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students understand that a function is a rule that takes an input and produces only one output; therefore, functions occur when there is exactly one y-value associated with any x-value. Students identify functions from equations, graphs, and tables/ordered pairs and are not expected to use the function notation f(x) at this level. This standard requires students to clarify the definitions of key terms including function, input, output, y-value, and x-value.	● Know and flexibly use the terms function, input, and output. ● Analyze tables and graphs by interpreting their relationships as functions. ● Understand that a function is a rule that states each input has exactly one output, not just how to recognize them. ● Understand that each function produces a graph. ● Formulate and defend opinion on whether a table or graph is a function or not with use of counterexamples.
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

While you may see other aspects of rigor, the focus area of this standard is conceptual understanding.

Conceptual Understanding:

- Understand that a function is a rule or relationship that assigns exactly one output to each input.
- Understand that each function produces a graph, and the graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
- Understand and explain the meaning of function, input, output, y-value, and x-value in terms of working with functions represented by equations, graphs, and tables/ordered pairs.
- Analyze tables and graphs by interpreting their relationships as functions.

- Formulate and defend opinion on whether a table or graph is a function or not with use of counterexamples.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

A certain business keeps a database of information about its customers. Let *C* be the rule which assigns to each customer shown in the table his or her home phone number. Let *P* be the rule which assigns to each phone number in the table above, the customer name associated with it.

Customer Name	Home Phone Number
Heather Baker	3105100091
Mike London	3105200256
Sue Green	3234132598
Bruce Swift	3234132598
Michelle Metz	2138061124

1. Is *C* a function? Explain your reasoning.

2. Is *P* a function? Explain your reasoning.

3. Explain why a business might want to use a person's social security number as a way to identify a particular customer instead of their phone number.

4. Graph the data represented in this table. Explain whether each data set is a function or not using the table and your graph.

	Input value						
Data set	-1	0	1	2	2	3	4
a	1	1	1	1	1	1	1
b	-1	0	1	2	2	3	4
c	-5	0	5	10	15	15	20
d	1	2	1	2	2	1	1

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Vocabulary:** Students may struggle with new terms, such as function, input, output, x-value, and y-value.
- **Repeated inputs/outputs:** Students may struggle to grasp the concept that inputs and outputs can repeat and still fall under the definition of a function as long as each input has a single output. This can get more

complicated when you are using contextual problems and students have different experiences (e.g. having two houses, taking more than one math class, etc.), so be open to their observations. It can also be complicated by real-world data for which the point is to come up with a function that models the data (e.g. in a table or graph), even if the data might not necessarily be a function.

- **Function rules:** Students might assume that every function has to have an algebraic rule that can be ascribed to it.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships.	**Pre-teaching** In previous classes, learners worked to analyze proportional relationships and use them to solve real world and mathematical problems and to solve real-world and mathematical problems using numerical and algebraic expressions and equations. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as interpreting common real-world contexts for tasks (e.g. cell phone plans) and writing expressions and equations to represent them. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at	**Pre-teaching** Consider using standard 7.RP.A.2, which provides a foundation for work in this cluster. In 7.RP.A.2, students define, evaluate and compare equations in different forms, such as tables, graphs, and equations. 7.RP.A.2 also provides a foundation for working with functions to model the relationship between quantities. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster on real-world problems, including defining, interpreting, and comparing functions.

Engagement Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before	revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Give students practice defining, evaluating, and comparing functions. Have students use functions to model relationships and critique different approaches and solutions to these problems. Graphs and tables are everywhere in the study of functions, but students need to be able to distinguish the algebraic function from its other representations. Consider delivering a mini-lesson on algebraic functions and their graphs and tables, and the similarities and differences in the information they provide about the function.	

students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/36/417/417

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● analyze proportional relationships and use them to solve real world and mathematical problems ● solve real-world and mathematical problems using numerical and algebraic expressions and equations	In 8th grade, learners ● connect expressions and linear equations to linear relationships of functions ● analyze graphs of functional relationships ● construct functions to model relationships between two quantities ● graph proportional relationships ● interpret the unit rate as the slope of the graph ● interpret equations in the form of y = mx + b as defining a linear function ● understand that a function is a rule that assigns to each input exactly one output	In future classes, learners ● interpret functions that arise in application in terms of the context ● apply the concept of a function with use of function notation ● interpret functions that arise in application in terms of the context

	• compare properties of two functions each represented in a different way	

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

students about the math they use in their careers or crafts.	with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- How do you know whether a relationship between two quantities is a function or not?
- How is your strategy for determining whether a relation is a function similar to a classmate's? How is it different? Can you both be correct?
- How can you connect functions to vending machines?
- What are inputs and outputs and how do they relate to functions?
- How can variables be used to represent inputs and outputs? Give examples.
- What does a function look like when graphed?

Cross-Curricular Connections

Arts: Adjusting paint parts to create a certain shade/quantity of paint. Following the mathematical series of musical rhythms to learn the basic rhythms of dance.

Gym: Keeping score in a game (for every touchdown, you get x amount of points).

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

Science: Working with constant speed/average speed. Examining scientific data and predicting the effect of a change in one variable on another. Reading scientific charts and graphs. Solving equations when writing computer programs and figuring out algorithms.

Social Studies: Looking at distance and time. Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations have different average life spans.

Career and Skill Connections

• Accounting auditing • Animal training • Art • Aviation • Banking/finance • Bookkeeping • Botany • Business • Computer programming • Construction • Data science • Ecology • Education	• Engineer • Environmental science • Event planning • Food science • Genetics • Health science • Hydrology • Information technology • Insurance • Machinist • Management • Marketing • Masonry	• Mechanics • Medicine • Pest control • Ranching/farming • Sales • Soil science • Tailor • Technician • Technology • Transportation • Veterinary • Wildlife biology

Grade	CCSS Domain	CCSS Cluster
8	Functions	Define, evaluate, and compare functions.

Cluster Standard: 8.F.A.2

Standard	Standards for Mathematical Practice
8.F.A.2: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.	● **SMP 4**: Model with mathematics. ○ Teacher and Student Actions ● **SMP 5:** Use appropriate tools strategically. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students understand that a function is a rule that takes an input and produces only one output; therefore, functions occur when there is exactly one y-value associated with any x-value. Students identify functions from equations, graphs, and tables/ordered pairs, and are not expected to use the function notation f(x) at this level. This standard requires students to clarify the definitions of key terms including function, input, output, y-value, and x-value.	● Determine the slope and the y intercept from an equation, a table, a graph, and a verbal description. ● Explain orally and in writing that slope represents rate of change and y-intercept represents initial value or starting value. ● Understand how to generate additional ordered pairs for a function. ● Compare the properties of a graph, an equation, a table, and verbal descriptions given a real world linear situation.
DOK	**Blooms**
1-2	Analyze, Evaluate

Procedural and Conceptual Understanding and Application

While you may see some procedural skill and fluency, the focus areas of this standard are conceptual understanding and application.

Conceptual Understanding:

- Understand that a function is a rule that takes an input and produces only one output; therefore, functions occur when there is exactly one y-value associated with any x-value.

- Understand and explain the meaning of function, input, output, y-value, and x-value in terms of working with functions represented by equations, graphs, and tables/ordered pairs.
- Understand and explain orally and in writing that slope represents rate of change and y-intercept represents initial value or starting value.

Application:

- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- Understand how to generate additional ordered pairs for a function.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Sam wants to take his MP3 player and his video game player on a car trip. An hour before they plan to leave, he realized that he forgot to charge the batteries last night. At that point, he plugged in both devices so they can charge as long as possible before they leave. Sam knows that his MP3 player has 40% of its battery life left and that the battery charges by an additional 12 percentage points every 15 minutes. His video game player is new, so Sam doesn't know how fast it is charging, but he recorded the battery charge for the first 30 minutes after he plugged it in.

time charging (minutes)	0	10	20	30
video game player battery charge (%)	20	32	44	56

1. Write a function that represents the charging speed of each device. Graph your equations.
2. What is the y-intercept and rate of change for the MP3 player? Explain how you know.
3. What is the y-intercept and rate of change for the video game player? Explain how you know.
4. If Sam's family leaves as planned, what percent of the battery will be charged for each of the two devices when they leave?
5. How much time would Sam need to charge the battery 100% on both devices?
6. Which device charges more quickly? Explain how you know.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Vocabulary:** Students may struggle with new terms, such as function, input, output, x-value, and y-value.
- **Comparing functions:** Students might not understand that they need to modify functions or compute certain values (like rate of change, starting point, etc.), perhaps by changing the form of a given function (e.g. turning a table into a graph or writing a rule for a table).
- **Rate of change:** Students may struggle to compute the rate of change, depending on the forms of the functions given. For example, when looking at a function represented in a table, students may mistakenly

focus on individual values rather than considering the overall rate of change.

- **Verbal descriptions:** Students may struggle to interpret verbal descriptions of functions accurately.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in	**Pre-teaching** In previous classes, learners worked to analyze proportional relationships and use them to solve real world and mathematical problems and to solve real-world and mathematical problems using numerical and algebraic expressions and equations. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as interpreting common real-world contexts for tasks (e.g. cell phone plans) and writing expressions and equations to represent them. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain	**Pre-teaching** Consider using standard 7.RP.A.2, which provides a foundation for work in this cluster. In 7.RP.A.2, students define, evaluate and compare equations in different forms, such as tables, graphs, and equations. 7.RP.A.2 also provides a foundation for working with functions to model the relationship between quantities. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster on real-world problems, including defining, interpreting, and comparing functions.

which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **<u>Action and Expression</u>** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure	mistakes and find new, more efficient ways of solving problems. Give students practice defining, evaluating, and comparing functions. Have students use functions to model relationships and critique different approaches and solutions to these problems. Graphs and tables are everywhere in the study of functions, but students need to be able to distinguish the algebraic function from its other representations. Consider delivering a mini-lesson on algebraic functions and their graphs and tables, and the similarities and differences in the information they provide about the function.	

students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/36/418/418

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● analyze proportional relationships and use them to solve real world and mathematical problems ● solve real-world and mathematical problems using numerical and algebraic expressions and equations	In 8th grade, learners ● connect expressions and linear equations to linear relationships of functions ● analyze graphs of functional relationships ● construct functions to model relationships between two quantities ● graph proportional relationships ● interpret the unit rate as the slope of the graph ● interpret equations in the form of y = mx + b as defining a linear function ● understand that a function is a rule that assigns to each input exactly one output ● compare properties of two functions each represented in a different way	In future classes, learners ● interpret functions that arise in application in terms of the context ● apply the concept of a function with use of function notation ● interpret functions that arise in application in terms of the context

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

	encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	

Suggested Student Discourse Questions

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- What are inputs and outputs and how do they relate to functions?
- How can variables be used to represent inputs and outputs? Give examples.
- How can a graph, table, ordered pair, or an algebraic rule help describe the relationship between two variables?

Cross-Curricular Connections

Arts: Adjusting paint parts to create a certain shade/quantity of paint. Following the mathematical series of musical rhythms to learn the basic rhythms of dance.

Gym: Keeping score in a game (for every touchdown, you get x amount of points).

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

Science: Working with constant speed/average speed. Examining scientific data and predicting the effect of a change in one variable on another. Reading scientific charts and graphs. Solving equations when writing computer programs and figuring out algorithms.

Social Studies: Looking at distance and time. Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations have different average life spans.

Career and Skill Connections

• Accounting auditing • Animal training • Art • Aviation • Banking/finance • Bookkeeping • Botany	• Engineer • Environmental science • Event planning • Food science • Genetics • Health science • Hydrology	• Mechanics • Medicine • Pest control • Ranching/farming • Sales • Soil science • Tailor

• Business • Computer programming • Construction • Data science • Ecology • Education	• Information technology • Insurance • Machinist • Management • Marketing • Masonry	• Technician • Technology • Transportation • Veterinary • Wildlife biology

Grade	CCSS Domain	CCSS Cluster
8	Functions	Define, evaluate, and compare functions.

Cluster Standard: 8.F.A.3

Standard	Standards for Mathematical Practice
8.F.A.3: Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ (s squared) giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.	● **SMP 1:** Make sense of problems and persevere in solving them. ○ Teacher and Student Actions ● **SMP 7:** Look for and make use of structure. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students understand that a function is a rule that takes an input and produces only one output; therefore, functions occur when there is exactly one y-value associated with any x-value. Students identify functions from equations, graphs, and tables/ordered pairs and are not expected to use the function notation f(x) at this level. This standard requires students to clarify the definitions of key terms including function, input, output, y-value, and x-value.	● Understand that a linear function has a constant rate of change called slope and will produce a line on a graph. ● Understand that a nonlinear function does not have a constant rate of change and will not produce a line on a graph.
DOK	**Blooms**
1-2	Understand, Apply, Analyze

Procedural and Conceptual Understanding and Application

While you may see some application, the focus areas of this standard are conceptual understanding and procedural skill and fluency.

Conceptual Understanding:

- Understand and interpret the equation y = mx + b as defining a linear function, whose graph is a straight line due to its constant rate of change.
- Understand that functions without a constant rate of change are nonlinear and produce graphs that are not straight lines.
- Understand that a function is a rule that takes an input and produces only one output; therefore, functions

occur when there is exactly one y-value associated with any x-value.
- Understand and explain the meaning of function, input, output, y-value, and x-value in terms of working with functions represented by equations, graphs, and tables/ordered pairs.

Procedural Skill and Fluency:
- Give examples of functions that are not linear.
- Distinguish between linear and nonlinear functions by examining their graphs.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

1. Decide which of the following points are on the graph of the function y = 2x + 1.

a. (0,1) b. (2,5) c. (12,2) d. (2,−1) e. (−1,−1) f. (0.5,1).

2. Find 3 more points on the graph of the function.

3. Find five points that are on the graph of the function $y = 2x^2 + 1$.

4. Plot the points in the coordinate plane. Is this a linear function? Support your conclusion.

5. Graph the first function. List as many differences between the two functions as you can.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Vocabulary:** Students may struggle with new terms, such as function, input, output, x-value, and y-value.
- **Nonlinear functions:** Students might mistakenly assume that functions cannot be curves because of confusion about the vertical line test. They might also mistakenly classify a linear function as nonlinear if given points at non-constant intervals, or classify nonlinear functions as linear if they do not look at more than one set of points for the rate of change.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

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Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows,	**Pre-teaching** In previous classes, learners worked to analyze proportional relationships and use them to solve real world and mathematical problems and to solve real-world and mathematical problems using numerical and	**Pre-teaching** Consider using standard 7.RP.A.2, which provides a foundation for work in this cluster. In 7.RP.A.2, students define, evaluate and compare equations in different forms, such as tables, graphs, and equations.

labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for	algebraic expressions and equations. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as interpreting common real-world contexts for tasks (e.g. cell phone plans) and writing expressions and equations to represent them. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Give students practice defining, evaluating, and comparing functions. Have students use functions to model relationships and critique different approaches and solutions to these problems. Graphs and tables are everywhere in the study of functions, but students need to be able to distinguish the algebraic function from its other representations. Consider delivering a mini-lesson on algebraic functions and their graphs and tables, and the similarities and	7.RP.A.2 also provides a foundation for working with functions to model the relationship between quantities. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster on real-world problems, including defining, interpreting, and comparing functions.

transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post	differences in the information they provide about the function.	

visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/36/418/419

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners • analyze proportional relationships and use them to solve real world and mathematical problems • solve real-world and mathematical problems using numerical and algebraic expressions and equations	In 8th grade, learners • connect expressions and linear equations to linear relationships of functions • analyze graphs of functional relationships • construct functions to model relationships between two quantities • graph proportional relationships • interpret the unit rate as the slope of the graph • interpret equations in the form of y = mx + b as defining a linear function • understand that a function is a rule that assigns to each input exactly one output • compare properties of two functions each represented in a different way	In future classes, learners • interpret functions that arise in application in terms of the context • apply the concept of a function with use of function notation • interpret functions that arise in application in terms of the context

Culturally and Linguistically Responsive Instruction

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- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:
- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.
Suggested Student Discourse Questions		
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- How do you know whether a relationship between two quantities is a function or not?
- What are inputs and outputs and how do they relate to functions?
- How can variables be used to represent inputs and outputs? Give examples.
- How can a graph help describe the relationship between two variables?
- Can you find the error in the work? Why do you think it is an error? Can you explain your thinking?
- What type of data could we collect in everyday life that would show a linear relationship? What about a quadratic relationship?
- Are all linear equations functions? Why or why not? Can you provide a counterexample?

Cross-Curricular Connections

Arts: Adjusting paint parts to create a certain shade/quantity of paint. Following the mathematical series of musical rhythms to learn the basic rhythms of dance.

Gym: Keeping score in a game (for every touchdown, you get x amount of points).

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

Science: Working with constant speed/average speed. Examining scientific data and predicting the effect of a change in one variable on another. Reading scientific charts and graphs. Solving equations when writing computer programs and figuring out algorithms.

Social Studies: Looking at distance and time. Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations have different average life spans.

Career and Skill Connections

- Accounting auditing
- Animal training
- Art
- Aviation
- Banking/finance
- Bookkeeping
- Botany
- Business
- Computer programming
- Construction
- Data science
- Ecology
- Education
- Engineer
- Environmental science
- Event planning
- Food science
- Genetics
- Health science
- Hydrology
- Information technology
- Insurance
- Machinist
- Management
- Marketing
- Masonry
- Mechanics
- Medicine
- Pest control
- Ranching/farming
- Sales
- Soil science
- Tailor
- Technician
- Technology
- Transportation
- Veterinary
- Wildlife biology

Grade	CCSS Domain	CCSS Cluster
8	Functions	Use functions to model relationships between quantities.

Cluster Standard: 8.F.B.4

Standard	Standards for Mathematical Practice
8.F.B.4: Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	● **SMP 4**: Model with mathematics. ○ Teacher and Student Actions ● **SMP 7:** Look for and make use of structure. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students determine and interpret the rate of change and the initial value to construct a linear model. They use a real-world situation to sketch a graph and use a graph to write a verbal description of a real-world situation.	● Write the function for a linear relationship between two quantities. ● Identify the rate of change ● Identify the slope of the function from two points (x,y), from a graph and a table. ● Interpret the rate of change (slope) and initial value of a linear function from a table, graph, equation or verbal description. ● Calculate the slope of a line using the rise over run ratio.
DOK	**Blooms**
1-3	Apply, Analyze

Procedural and Conceptual Understanding and Application

While you may see some application, the focus areas of this standard are conceptual understanding and procedural skill and fluency.

Conceptual Understanding:

- Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

- Interpret the rate of change (slope) and initial value of a linear function from a table, graph, equation or verbal description.

Procedural Skill and Fluency:

- Construct a function to model a linear relationship between two quantities.
- Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph.
- Calculate the slope of a line using the rise over run ratio.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

A car is traveling down a long, steep hill. The elevation *E* above sea level (in feet) of the car when it is *d* miles from the top of the hill is shown in the table.

d	0	1	2	3	4	5	6
E	7500	7250	7000	6750	6500	6250	6000

1. Write a function to represent the elevation of the car above sea level and graph it.

2. What are the x- and y-intercepts of the function? What do they mean in the context of the moving car?

3. What is the slope of the function? What does it mean in the context of the moving car?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Vocabulary:** Students may struggle with new terms, such as function, input, output, x-value, and y-value.
- **Rate of change:** Students may struggle to compute the rate of change, depending on the forms of the functions given. For example, when looking at a function represented in a table, students may mistakenly focus on individual values rather than considering the overall rate of change.
- **Verbal descriptions:** Students may struggle to interpret verbal descriptions of functions accurately.
- **Key Features:** Students might struggle to identify and interpret key features such as rate of change, starting point, and intercepts. This is especially true in real-world contexts where a graph might describe the speed of a car going down a hill, and students misinterpret the slope as elevation. Students might also mix up the x- and y-intercepts, or the rate of change and starting point.
- **Independent and dependent variables:** In some contexts, the independent and dependent variables are fairly obvious. In cases where they are not and either variable can be independent, students might struggle to understand that there are cases in which one variable does not determine the other, but rather we are creating a way to compare two things.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)		
Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and	**Pre-teaching** In previous classes, learners worked to analyze proportional relationships and use them to solve real world and mathematical problems and to solve real-world and mathematical problems using numerical and algebraic expressions and equations. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as interpreting common real-world contexts for tasks (e.g. cell phone plans) and writing expressions and equations to represent them. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Give students practice defining,	**Pre-teaching** Consider using standard 7.RP.A.2, which provides a foundation for work in this cluster. In 7.RP.A.2, students define, evaluate and compare equations in different forms, such as tables, graphs, and equations. 7.RP.A.2 also provides a foundation for working with functions to model the relationship between quantities. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster on real-world problems, including defining, interpreting, and comparing functions.

materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut	evaluating, and comparing functions. Have students use functions to model relationships and critique different approaches and solutions to these problems. Graphs and tables are everywhere in the study of functions, but students need to be able to distinguish the algebraic function from its other representations. Consider delivering a mini-lesson on algebraic functions and their graphs and tables, and the similarities and differences in the information they provide about the function.	

materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/36/418/422

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● analyze proportional relationships and use them to solve real world and mathematical problems ● solve real-world and mathematical problems using numerical and algebraic expressions and equations	In 8th grade, learners ● connect expressions and linear equations to linear relationships of functions ● analyze graphs of functional relationships ● construct functions to model relationships between two quantities ● graph proportional relationships ● interpret the unit rate as the slope of the graph ● interpret equations in the form of y = mx + b as defining a linear function ● understand that a function is a rule that assigns to each input exactly one output ● compare properties of two functions each represented in a different way	In future classes, learners ● interpret functions that arise in application in terms of the context ● apply the concept of a function with use of function notation ● interpret functions that arise in application in terms of the context

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.p

- df
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

	with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- What are inputs and outputs and how do they relate to functions?
- How can variables be used to represent inputs and outputs? Give examples.
- How can a graph, table, ordered pair, or an algebraic rule help describe the relationship between two variables?
- Are all linear equations functions? Why or why not? Can you provide a counterexample?
- How can you find the rate of change and starting value for your function? Will that strategy always work?
- What do the rate of change and starting value mean in the context of this problem? How can you tell?

Cross-Curricular Connections

Arts: Adjusting paint parts to create a certain shade/quantity of paint. Following the mathematical series of musical rhythms to learn the basic rhythms of dance.

Gym: Keeping score in a game (for every touchdown, you get x amount of points).

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

Science: Working with constant speed/average speed. Examining scientific data and predicting the effect of a change in one variable on another. Reading scientific charts and graphs. Solving equations when writing computer programs and figuring out algorithms.

Social Studies: Looking at distance and time. Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations have different average life spans.

Career and Skill Connections

• Accounting auditing • Animal training • Art • Aviation • Banking/finance • Bookkeeping	• Engineer • Environmental science • Event planning • Food science • Genetics • Health science	• Mechanics • Medicine • Pest control • Ranching/farming • Sales • Soil science

• Botany • Business • Computer programming • Construction • Data science • Ecology • Education	• Hydrology • Information technology • Insurance • Machinist • Management • Marketing • Masonry	• Tailor • Technician • Technology • Transportation • Veterinary • Wildlife biology

Grade	CCSS Domain	CCSS Cluster
8	Functions	Use functions to model relationships between quantities.

Cluster Standard: 8.F.B.5

Standard	Standards for Mathematical Practice
8.F.B.5: Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	● **SMP 3:** Construct viable arguments and critique the reasoning of others. ○ Teacher and Student Actions ● **SMP 4**: Model with mathematics. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students determine and interpret the rate of change and the initial value to construct a linear model. They use a real-world situation to sketch a graph and use a graph to write a verbal description of a real-world situation.	● Interpret linear and nonlinear graphs. ● Describe the relationships between two quantities (linear, nonlinear, increasing or decreasing). ● Sketch graphs of linear and nonlinear functions. ● Analyze the sketches of linear and nonlinear functions.
DOK	**Blooms**
1-3	Analyze, Create

Procedural and Conceptual Understanding and Application

While you may see other aspects of rigor, the focus area of this standard is conceptual understanding.

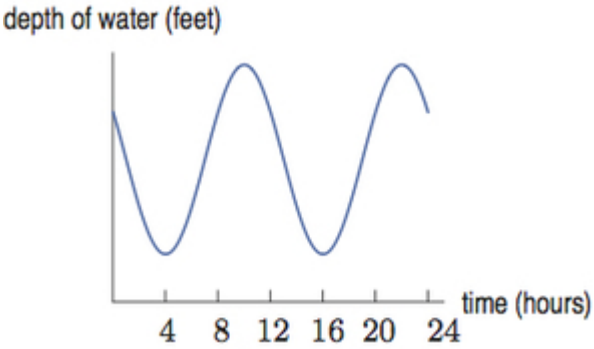
Conceptual Understanding:

- Describe qualitatively the functional relationship between two quantities by analyzing a graph for key features and graph type (linear vs, nonlinear).
- Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
- Interpret a given rate of change and initial value in order to construct a linear model.
- Use a real-world scenario to sketch a graph and use a graph to write a verbal description of a real-world scenario.
- Interpret linear and nonlinear graphs.
- Understand that in linear functions the relationship between variables forms a straight line, and in nonlinear functions this relationship makes a graph that is not a straight line.

Assessment Items

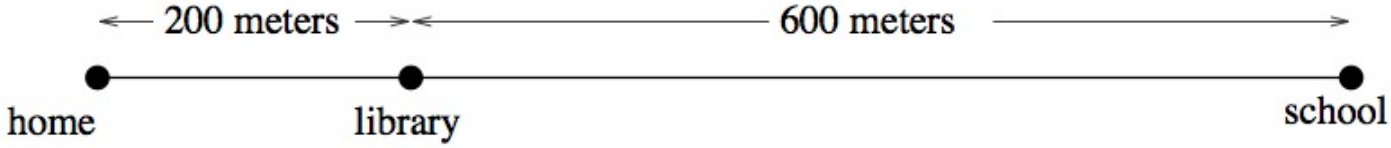
When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

The figure below gives the depth of the water at Montauk Point, New York, for a day in November.



1. How many high tides took place on this day?
2. How many low tides took place on this day?
3. How much time elapsed in between high tides?
4. When was the tide the lowest? The highest?

Nina rides her bike from her home to school passing by the library on the way, and traveling at a constant speed for the entire trip. (See map below.)



5. Sketch a graph of Nina's distance from school as a function of time.
6. Sketch a graph of Nina's distance from the library as a function of time.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Vocabulary:** Students may struggle with new terms, such as function, input, output, x-value, and y-value.
- **Verbal descriptions:** Students may struggle to interpret verbal descriptions of functions accurately.
- **Key Features:** Students might struggle to identify and interpret key features such as rate of change, starting point, and intercepts. This is especially true in real-world contexts where a graph might describe the speed of a car going down a hill, and students misinterpret the slope as elevation. Students might also mix up the x- and y-intercepts, or the rate of change and starting point.
- **Nonlinear Functions:** Students might incorrectly label nonlinear functions as linear if they only consider the overall trend of the graph instead of analyzing whether the relationship between the variables forms a straight line. Students might also equate nonlinear functions with randomness, failing to recognize that nonlinear relationships can exhibit specific patterns or behaviors.
- **Overgeneralizing Trends:** Students might overgeneralize trends observed in specific regions of a graph to the entire graph, leading to inaccuracies in their qualitative descriptions of the functional relationship between the variables.
- **Ignoring Discontinuities:** Students might overlook discontinuities in the graph, such as vertical asymptotes or

holes, which can significantly affect the qualitative features of the function.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in	**Pre-teaching** In previous classes, learners worked to analyze proportional relationships and use them to solve real world and mathematical problems and to solve real-world and mathematical problems using numerical and algebraic expressions and equations. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as interpreting common real-world contexts for tasks (e.g. cell phone plans) and writing expressions and equations to represent them. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain	**Pre-teaching** Consider using standard 7.RP.A.2, which provides a foundation for work in this cluster. In 7.RP.A.2, students define, evaluate and compare equations in different forms, such as tables, graphs, and equations. 7.RP.A.2 also provides a foundation for working with functions to model the relationship between quantities. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster on real-world problems, including defining, interpreting, and comparing functions.

which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure	mistakes and find new, more efficient ways of solving problems. Give students practice defining, evaluating, and comparing functions. Have students use functions to model relationships and critique different approaches and solutions to these problems. Graphs and tables are everywhere in the study of functions, but students need to be able to distinguish the algebraic function from its other representations. Consider delivering a mini-lesson on algebraic functions and their graphs and tables, and the similarities and differences in the information they provide about the function.	

students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/36/418/424

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● analyze proportional relationships and use them to solve real world and mathematical problems ● solve real-world and mathematical problems using numerical and algebraic expressions and equations	In 8th grade, learners ● connect expressions and linear equations to linear relationships of functions ● analyze graphs of functional relationships ● construct functions to model relationships between two quantities ● graph proportional relationships ● interpret the unit rate as the slope of the graph ● interpret equations in the form of y = mx + b as defining a linear function ● understand that a function is a rule that assigns to each input exactly one output ● compare properties of two functions each represented in a different way	In future classes, learners ● interpret functions that arise in application in terms of the context ● apply the concept of a function with use of function notation ● interpret functions that arise in application in terms of the context

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

	encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- What are inputs and outputs and how do they relate to functions?
- How can variables be used to represent inputs and outputs? Give examples.
- What does the graph show about the two variables? What do the variables represent in the context of the problem?
- What type of data could we collect in everyday life that would show a linear relationship? What about a quadratic relationship?
- What does each section of the graph you drew represent?

Cross-Curricular Connections

Arts: Adjusting paint parts to create a certain shade/quantity of paint. Following the mathematical series of musical rhythms to learn the basic rhythms of dance.

Gym: Keeping score in a game (for every touchdown, you get x amount of points).

Literature: Understanding the arithmetic behind poetry (the meter, number of words to include in a line, and the effect that certain rhythms have on the reader). Planning reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. Using linear, logical thinking to write more clearly and logically.

Science: Working with constant speed/average speed. Examining scientific data and predicting the effect of a change in one variable on another. Reading scientific charts and graphs. Solving equations when writing computer programs and figuring out algorithms.

Social Studies: Looking at distance and time. Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations have different average life spans.

Career and Skill Connections

• Accounting auditing • Animal training • Art • Aviation	• Engineer • Environmental science • Event planning • Food science	• Mechanics • Medicine • Pest control • Ranching/farming

• Banking/finance • Bookkeeping • Botany • Business • Computer programming • Construction • Data science • Ecology • Education	• Genetics • Health science • Hydrology • Information technology • Insurance • Machinist • Management • Marketing • Masonry	• Sales • Soil science • Tailor • Technician • Technology • Transportation • Veterinary • Wildlife biology

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards for Mathematical Practice
    - Connections to procedural, conceptual understanding, and application
    - Sample assessment items
    - Common misconceptions
    - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
    - Vertical alignment
    - Culturally and Linguistically Responsive Instruction (CLRI)
    - Suggested student discourse questions
    - Cross-curricular and career/skill connections
- A Student Discourse Guide

Helpful links:

- Lesson-planning tool from Pathways 2 Careers (click here to sign up with your district email if you don't already have an account)
- Focus by Grade Level from Achieve the Core
- Coherence Map from Achieve the Core

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Understand congruence and similarity using physical models, transparencies, or geometry software.
  - 8.G.A.1
  - 8.G.A.2
  - 8.G.A.3
  - 8.G.A.4
  - 8.G.A.5
- Understand and apply the Pythagorean Theorem.
  - 8.G.B.6
  - 8.G.B.7
  - 8.G.B.8
- Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres
  - 8.G.C.9

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)

Grade	CCSS Domain	CCSS Cluster
8	Geometry	Understand congruence and similarity using physical models, transparencies, or geometry software.

Cluster Standard: 8.G.A.1

Standard	Standards for Mathematical Practice
8.G.A.1: Verify experimentally the properties of rotations, reflections, and translations: ● 8.G.A.1.A: Lines are taken to lines, and line segments to line segments of the same length. ● 8.G.A.1.B: Angles are taken to angles of the same measure. ● 8.G.A.1.C: Parallel lines are taken to parallel lines.	● **SMP 4:** Model with mathematics. ○ Teacher and Student Actions ● **SMP 5:** Use appropriate tools strategically. ○ Teacher and Student Actions ● **SMP 6:** Attend to precision. ○ Teacher and Student Actions ● **SMP 7**: Look for and make use of structure. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students describe and apply translations, rotations, reflections, and dilations to understand congruent and similar figures. Students explain and understand angle relationships.	● Construct transformations by using models, transparencies or geometry software, and develop an understanding of the relationship of the original to its image. ● Analyze the relationships between corresponding sides and corresponding angles of the original figure to its image. ● Translate figures, given a set of rules, on the coordinate plane. ● Evaluate and describe transformations. ● Accurately transform figures on the coordinate plane using rotations, translations, reflections, and the correct notation. ● Identify transformations performed to transform an image to the original.
DOK	**Blooms**
3-4	Analyze, Evaluate, Create
Procedural and Conceptual Understanding and Application	
While you may see other aspects of rigor, the focus area of this standard is conceptual understanding.	

Conceptual Understanding:

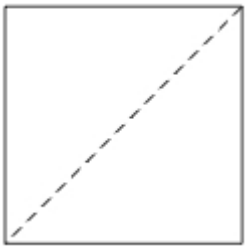
- Experimentally verify properties of geometric transformations, including rotations, reflections, and translations.
- Understand how lines, line segments, angles, and parallel lines are transformed under these operations.
- Understand why lengths of line segments and measures of angles are preserved during transformations, with lines being mapped to lines and angles to angles of the same measure.
- Understand that transformations are operations that move geometric figures in the plane without altering their shape or size.
- Visualize how geometric figures change under rotations, reflections, and translations, and how these transformations affect the relationships between geometric elements.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

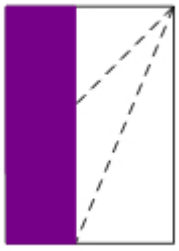
This task examines the mathematics behind an origami construction of a rectangle whose sides have the ratio $(\sqrt{2}:1)$. Such a rectangle is called a silver rectangle.

1. Beginning with a square piece of paper, first fold and unfold it leaving the diagonal crease as shown here.

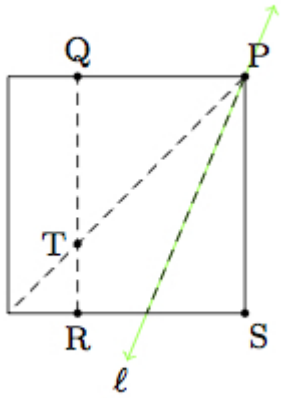


2. Next fold the bottom right corner up to the diagonal.

3. After unfolding then fold the left hand side of the rectangle over to the crease from the previous fold.



4. Here is a picture, after the last step has been unfolded, with all folds shown and some important points marked. In the picture T is the reflection of S about ℓ.



5. Suppose s is the side length of our square. Show that |PT| = s.

6. Show that △PQT is a 45-45-90 isosceles triangle.

7. Calculate |PQ| and conclude that PQRS is a silver rectangle

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Transformations:** Students might struggle to visualize what happens to a geometric shape after a reflection, rotation, or translation, which technology or manipulatives can help them do. Students might also struggle to differentiate between the different types of transformations, especially their effect on different properties of the shape, e.g. length and angle measurements.
- **Center of Rotation*:*** Students may overlook the fact that the center of rotation is a fixed point, and all points rotate around it, resulting in incorrect experimental verifications.
- **Slides*:*** Students might confuse translations with simple slides, failing to recognize that translations involve moving every point of a figure in the same direction by the same distance.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + Universal***	***Layer 2*** ***Core + Targeted***	***Layer 3*** ***Core + Targeted + Intensive***
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and	**Pre-teaching** In previous classes, learners worked to draw, construct, and describe geometric figures (such as angles and polygons) and their relationships, solve real-life and mathematical problems involving angle measure, graph points in a coordinate system and find the horizontal or vertical distance between two points in a coordinate system, draw polygons in	**Pre-teaching** Consider using standard 7.G.A.2, which provides a foundation for work in this cluster. In 7.G.A.2, students sketch, draw, and compose geometric shapes, laying the foundation for the practice of geometric deduction. Also consider using standard 7.G.B.6, which also provides a foundation for work in this cluster. In 7.G.B.6,

tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk.	a coordinate system when given vertices, find the area of squares and circles, find the volumes of right rectangular prisms, and solve real-world problems involving area and volume. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as similarity and congruence, connecting their meanings in our daily lives to their geometrical implications. Students should already be very familiar with triangles, so take time to revisit the types of triangles, angles of a triangle, and stamp that the Pythagorean theorem is only applicable to right triangles. Students will also benefit from reviewing exponents, squares, and square roots. Review cylinders, cones, and spheres and allow students to consider what the volume of these shapes look like and how their formulas connect to other shapes. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with	students review what they have already learned about triangles to connect to right triangles specifically. Also consider using standard 8.EE.A.2, which also provides a foundation for work in this cluster. In 8.EE.A.2, students use square root and cubed root symbols in order to represent solutions to equations, which will be useful when working with volume formulas. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster working with physical models, transparencies, geometry software, manipulatives, and other physical models. Spend time comparing and contrasting dilations, rotations, reflections, and translations. Give students time to practice using the Pythagorean theorem and various volume formulas.

Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.	common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Use physical models, transparencies, and geometry software to help students make the math of congruence and similarity more visual. Use manipulatives for different types of triangles. Spend ample time on real-world problems.	
Vertical Alignment		

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/37/425/425

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● draw, construct, and describe geometric figures (such as angles and polygons) and their relationships ● solve real-life and mathematical problems involving angle measure ● graph points in a coordinate system and find the horizontal or vertical distance between two points in a coordinate system ● draw polygons in a coordinate system when given vertices ● find the area of squares and circles ● find the volumes of right rectangular prisms ● solve real-world problems involving area and volume	In 8th grade, learners ● use square root symbols to represent solutions and approximate square root values ● use cube root symbols to represent solutions and approximate cube root values	In future classes, learners ● prove theorems about triangles ● use the Pythagorean Theorem to solve problems and discover other mathematical relationships ● use geometric shapes and their measurements to describe objects and solve design problems

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:
- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
● Consider options for learning from your families and communities the cultural and linguistic ways this	● Instruction should begin with conceptual understanding that allows students to contribute their informal	● Scaffold tasks and amplify language so students can make their own meaning, especially when cognates

mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- Compare and contrast rotations, reflections, and translations.
- How could you use another group's strategy to check to make sure your solution is reasonable?
- How is your strategy different from the one shown? How is it similar?
- What questions or comments do you have for your fellow students?
- What video games do you play that use transformation?

- What happens to lines when they undergo transformations? What about line segments? Angles? Parallel lines?

Cross-Curricular Connections

Arts: Making/interpreting geometric and architectural drawings. Crafting art pieces based on understanding basic geometric formulas. Using a 3-D glass shape to create sand art and calculating the amount of sand needed to create their art piece.

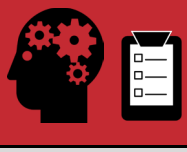
Literature: Using linear, logical thinking to write more clearly and logically. Researching a famous mathematician that has a known proof of the Pythagorean Theorem and writing an essay about the proof.

Science: Modeling the solar system at scale. Solving equations when writing computer programs and figuring out algorithms.

Career and Skill Connections

- Architecture
- Arts
- Atmospheric science
- Aviation
- CAD Programmer
- Carpentry
- Cartography
- Chemistry
- Computer programming
- Construction
- Criminal investigation
- Culinary arts
- Education
- Electrician
- Engineering
- Fashion design
- Fencing
- FIlm editing
- Film/show set design
- Firefighting
- Floor laying
- Forestry
- Geology
- Graphic design
- Historian
- Illustrator
- Industrial design
- Interior design/decoration
- Landscaping
- Machinist
- Maintenance
- Management
- Masonry
- Mechanical drafting
- Mechanics
- Metal fabrication/metalworking
- Model-making
- Optometry
- Photography
- Physical therapy
- Plumbing
- Publishing
- Ranching/farming
- Real estate
- Robotics
- Roofing
- Special effects animation
- Surveying
- Technician
- Technologist
- Urban/regional planning
- Veterinary
- Web design

Grade	*CCSS Domain*	*CCSS Cluster*
8	Geometry	Understand congruence and similarity using physical models, transparencies, or geometry software.



Cluster Standard: 8.G.A.2

Standard	Standards for Mathematical Practice
8.G.A.2: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	● **SMP 4**: Model with mathematics. ○ Teacher and Student Actions ● **SMP 5:** Use appropriate tools strategically. ○ Teacher and Student Actions ● **SMP 6**: Attend to precision. ○ Teacher and Student Actions

	● **SMP 7:** Look for and make use of structure. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students describe and apply translations, rotations, reflections, and dilations to understand congruent and similar figures. Students explain and understand angle relationships.	● Identify congruent figures by describing a sequence of rotations, translations or reflections that map one figure onto another. ● Effectively describe the series of transformations verbally or in writing. ● Create congruent figures by applying a series of transformations (use correct notation) ● Understand that a series of rotations, translations or reflections preserves the size and shape of the figure (congruence).
DOK	**Blooms**
1-2	Understand, Apply, Create

Procedural and Conceptual Understanding and Application

While you may see some application, the focus areas of this standard are conceptual understanding and procedural skill and fluency.

Conceptual Understanding:

- Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations.
- Describe and apply translations, rotations, reflections, and dilations to understand congruent and similar figures.
- Explain and understand angle relationships.
- Understand how to apply a sequence of rotations, reflections, and translations to transform one figure into another congruent figure.

Procedural Skill and Fluency:

- Describe a sequence that exhibits the congruence between two given congruent figures orally and in writing.
- Create congruent figures by applying a series of transformations using correct notation.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

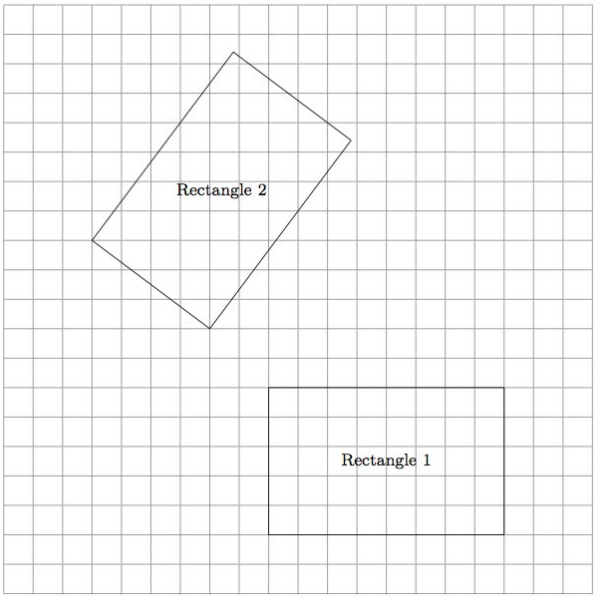
1. Line segments AB and CD have the same length. Describe a sequence of reflections that exhibits a congruence between them.

2. Here is a picture of two rectangles with the same length and width.

a. Show that the rectangles are congruent by finding a translation followed by a rotation which maps one of the rectangles to the other.

b. Explain why the congruence of the two rectangles can not be shown by translating Rectangle 1 to Rectangle 2.

c. Can the congruence of the two rectangles be shown with a single reflection? Explain.



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Common Misconceptions

- **Transformations:** Students might struggle to visualize what happens to a geometric shape after a reflection, rotation, or translation, which technology or manipulatives can help them do. Students might also struggle to differentiate between the different types of transformations, especially their effect on different properties of the shape, e.g. length and angle measurements.
- **Center of Rotation*:*** Students may overlook the fact that the center of rotation is a fixed point, and all points rotate around it, resulting in incorrect experimental verifications.
- **Slides*:*** Students might confuse translations with simple slides, failing to recognize that translations involve moving every point of a figure in the same direction by the same distance.
- **Similarity*:*** Students might mistakenly believe that two figures are congruent if they have the same shape but not necessarily the same size, confusing congruence with similarity.
- **Symmetry*:*** Students may think that if two figures exhibit symmetry or mirror images of each other, they must be congruent, overlooking the possibility of asymmetrical congruent figures.
- **Sequence of transformations:** Students might assume that any sequence of rotations, reflections, and translations will result in congruent figures rather than understanding that the order and combination of transformations matters.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning		
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materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut	ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Use physical models, transparencies, and geometry software to help students make the math of congruence and similarity more visual. Use manipulatives for different types of triangles. Spend ample time on real-world problems.	extra time during and after work within this cluster working with physical models, transparencies, geometry software, manipulatives, and other physical models. Spend time comparing and contrasting dilations, rotations, reflections, and translations. Give students time to practice using the Pythagorean theorem and various volume formulas.

materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/37/429/429

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● draw, construct, and describe geometric figures (such as angles and polygons) and their relationships ● solve real-life and mathematical problems involving angle measure ● graph points in a coordinate system and find the horizontal or vertical distance between two points in a coordinate system ● draw polygons in a coordinate system when given vertices ● find the area of squares and circles ● find the volumes of right rectangular prisms ● solve real-world problems involving area and volume	In 8th grade, learners ● use square root symbols to represent solutions and approximate square root values ● use cube root symbols to represent solutions and approximate cube root values	In future classes, learners ● prove theorems about triangles ● use the Pythagorean Theorem to solve problems and discover other mathematical relationships ● use geometric shapes and their measurements to describe objects and solve design problems

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- df
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

	with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	

Suggested Student Discourse Questions

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- Compare and contrast rotations, reflections, and translations.
- What is congruence?
- How could you use another group's strategy to check to make sure your solution is reasonable?
- How is your strategy different from the one shown? How is it similar?
- What questions or comments do you have for your fellow students?
- What video games do you play that use transformation?
- What happens to lines when they undergo transformations? What about line segments? Angles? Parallel lines?

Cross-Curricular Connections

Arts: Making/interpreting geometric and architectural drawings. Crafting art pieces based on understanding basic geometric formulas. Using a 3-D glass shape to create sand art and calculating the amount of sand needed to create their art piece.

Literature: Using linear, logical thinking to write more clearly and logically. Researching a famous mathematician that has a known proof of the Pythagorean Theorem and writing an essay about the proof.

Science: Modeling the solar system at scale. Solving equations when writing computer programs and figuring out algorithms.

Career and Skill Connections

• Architecture • Arts • Atmospheric science • Aviation • CAD Programmer • Carpentry • Cartography • Chemistry • Computer programming • Construction • Criminal investigation • Culinary arts • Education	• Film/show set design • Firefighting • Floor laying • Forestry • Geology • Graphic design • Historian • Illustrator • Industrial design • Interior design/decoration • Landscaping • Machinist • Maintenance	• Model-making • Optometry • Photography • Physical therapy • Plumbing • Publishing • Ranching/farming • Real estate • Robotics • Roofing • Special effects animation • Surveying • Technician

• Electrician • Engineering • Fashion design • Fencing • FIlm editing	• Management • Masonry • Mechanical drafting • Mechanics • Metal fabrication/metalworking	• Technologist • Urban/regional planning • Veterinary • Web design

Grade	CCSS Domain	CCSS Cluster
8	Geometry	Understand congruence and similarity using physical models, transparencies, or geometry software.

Cluster Standard: 8.G.A.3

Standard	Standards for Mathematical Practice

8.G.A.3: Describe the effect of dilations, translations, rotations, and reflections on two dimensional figures using coordinates.	• **SMP 4**: Model with mathematics.
	  ○ Teacher and Student Actions
	• **SMP 5:** Use appropriate tools strategically.
	  ○ Teacher and Student Actions
	• **SMP 6**: Attend to precision.
	  ○ Teacher and Student Actions
	• **SMP 7:** Look for and make use of structure.
	  ○ Teacher and Student Actions

Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students describe and apply translations, rotations, reflections, and dilations to understand congruent and similar figures. Students explain and understand angle relationships.	• Identify the image of a figure on a coordinate grid given a scale factor and center of dilation. • Create a dilation of a polygon on a square grid given a scale factor and center of dilation. • Describe (orally) a figure on a coordinate grid and its image under a dilation, using coordinates to refer to points. • Draw and label a diagram of a line segment rotated 90 degrees clockwise or counterclockwise about a given center. • Generalize (orally and in writing) the process to reflect any point in the coordinate plane. • Identify (orally and in writing) coordinates that represent a transformation of one figure to another. • Determine and describe a series of transformations from a preimage to an image. • Recognize the relationship between the original coordinates and the coordinates of the image and understand that rotations, reflections and translations follow a specific pattern on the coordinate plane. • Recognize that you can use coordinates to find the scale factor of a dilation.

DOK	Blooms
1-2	Understand

Procedural and Conceptual Understanding and Application

While you may see other aspects of rigor, the focus area of this standard is conceptual understanding.

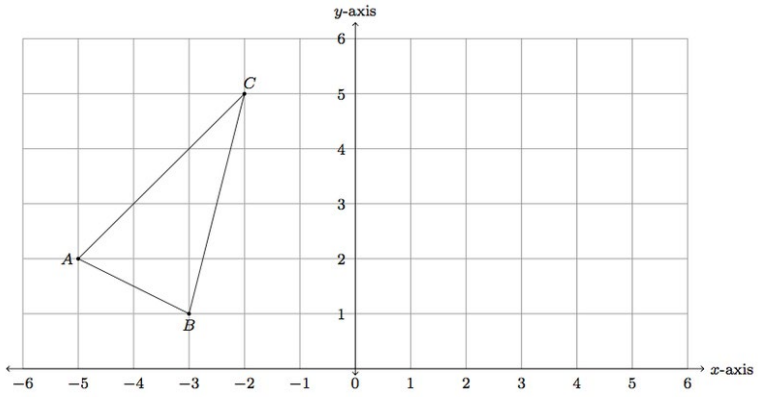
Conceptual Understanding:

- Describe the effect of dilations, translations, rotations, and reflections on two dimensional figures using coordinates.
- Describe and apply translations, rotations, reflections, and dilations to understand congruent and similar figures.
- Explain and understand angle relationships.
- Describe a figure on a coordinate grid and its image under a dilation, using coordinates to refer to points.
- Explain the process to reflect any point in the coordinate plane.
- Determine and describe a series of transformations from a preimage to an image.
- Recognize the relationship between the original coordinates and the coordinates of the image and understand that rotations, reflections, and translations follow a specific pattern on the coordinate plane.
- Recognize that coordinates can be used to find the scale factor of a dilation.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

1. Shown here is a picture of a triangle on a coordinate grid.



   a. Draw the reflection of △ABC over the line x = −2. Label the image of A as A′, the image of B as B′ and the image of C as C′.

   b. Draw the reflection of △A′B′C′ over the line x = 2. Label the image of A′ as A″, the image of B′ as B″ and the image of C′ as C″.

   c. What single rigid transformation of the plane will map △ABC to △A″B″C″? Explain.

2. Consider the triangle shown below on the coordinate grid.

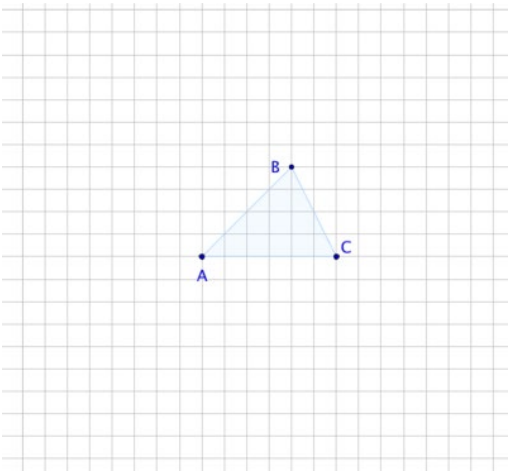
   a. Draw a dilation of ABC with:
   I. Center A and scale factor 2. II. Center B and scale factor 3. III. Center C and scale factor 12.

b. For each dilation, answer the following questions:

I. By what factor do the base and height of the triangle change? Explain.

II. By what factor does the area of the triangle change? Explain.

III. How do the angles of the scaled triangle compare to the original? Explain.



You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Transformations:** Students might struggle to visualize what happens to a geometric shape after a reflection, rotation, or translation, which technology or manipulatives can help them do. Students might also struggle to differentiate between the different types of transformations, especially their effect on different properties of the shape, e.g. length and angle measurements.
- **Center of Rotation*:*** Students may overlook the fact that the center of rotation is a fixed point, and all points rotate around it, resulting in incorrect experimental verifications.
- **Slides*:*** Students might confuse translations with simple slides, failing to recognize that translations involve moving every point of a figure in the same direction by the same distance.
- **Similarity*:*** Students might mistakenly believe that two figures are congruent if they have the same shape but not necessarily the same size, confusing congruence with similarity.
- **Symmetry*:*** Students may think that if two figures exhibit symmetry or mirror images of each other, they must be congruent, overlooking the possibility of asymmetrical congruent figures.
- **Sequence of transformations:** Students might assume that any sequence of rotations, reflections, and translations will result in congruent figures rather than understanding that the order and combination of transformations matters.
- **Coordinates:** Students might mix up their x- and y-coordinates and/or the x- and y-axes.
- **Dilations:** Students might struggle to understand why a scale factor over 1 creates a larger image, while a scale factor less than one makes a smaller image.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + Universal*	*Layer 2* *Core + Targeted*	*Layer 3* *Core + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual	**Pre-teaching** In previous classes, learners worked to draw, construct, and describe	**Pre-teaching** Consider using standard 7.G.A.2, which provides a foundation for work

strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships.

Engagement

Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a

geometric figures (such as angles and polygons) and their relationships, solve real-life and mathematical problems involving angle measure, graph points in a coordinate system and find the horizontal or vertical distance between two points in a coordinate system, draw polygons in a coordinate system when given vertices, find the area of squares and circles, find the volumes of right rectangular prisms, and solve real-world problems involving area and volume.

Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as similarity and congruence, connecting their meanings in our daily lives to their geometrical implications. Students should already be very familiar with triangles, so take time to revisit the types of triangles, angles of a triangle, and stamp that the Pythagorean theorem is only applicable to right triangles. Students will also benefit from reviewing exponents, squares, and square roots. Review cylinders, cones, and spheres and allow students to consider what the volume of these shapes look like and how their formulas connect to other shapes.

If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

in this cluster. In 7.G.A.2, students sketch, draw, and compose geometric shapes, laying the foundation for the practice of geometric deduction.

Also consider using standard 7.G.B.6, which also provides a foundation for work in this cluster. In 7.G.B.6, students review what they have already learned about triangles to connect to right triangles specifically.

Also consider using standard 8.EE.A.2, which also provides a foundation for work in this cluster. In 8.EE.A.2, students use square root and cubed root symbols in order to represent solutions to equations, which will be useful when working with volume formulas.

If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

Re-teaching

Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster working with physical models, transparencies, geometry software, manipulatives, and other physical models. Spend time comparing and contrasting dilations, rotations, reflections, and translations. Give students time to

subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to	**Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Use physical models, transparencies, and geometry software to help students make the math of congruence and similarity more visual. Use manipulatives for different types of triangles. Spend ample time on real-world problems.	practice using the Pythagorean theorem and various volume formulas.

templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/37/431/431

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners • draw, construct, and describe geometric figures (such as angles and polygons) and their relationships • solve real-life and mathematical problems involving angle measure • graph points in a coordinate system and find the horizontal or vertical distance between two points in a coordinate system • draw polygons in a coordinate system when given vertices • find the area of squares and circles • find the volumes of right rectangular prisms • solve real-world problems involving area and volume	In 8th grade, learners • use square root symbols to represent solutions and approximate square root values • use cube root symbols to represent solutions and approximate cube root values	In future classes, learners • prove theorems about triangles • use the Pythagorean Theorem to solve problems and discover other mathematical relationships • use geometric shapes and their measurements to describe objects and solve design problems

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:
- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.
Suggested Student Discourse Questions		

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- Compare and contrast rotations, reflections, and translations.
- How could you use another group's strategy to check to make sure your solution is reasonable?
- How is your strategy different from the one shown? How is it similar?
- What questions or comments do you have for your fellow students?
- What video games do you play that use transformation?
- What happens to lines when they undergo transformations? What about line segments? Angles? Parallel lines?
- How is dilation of a figure different from translation, reflection, or rotation of a figure?

Cross-Curricular Connections

Arts: Making/interpreting geometric and architectural drawings. Crafting art pieces based on understanding basic geometric formulas. Using a 3-D glass shape to create sand art and calculating the amount of sand needed to create their art piece.

Literature: Using linear, logical thinking to write more clearly and logically. Researching a famous mathematician that has a known proof of the Pythagorean Theorem and writing an essay about the proof.

Science: Modeling the solar system at scale. Solving equations when writing computer programs and figuring out algorithms.

Career and Skill Connections

- Architecture
- Arts
- Atmospheric science
- Aviation
- CAD Programmer
- Carpentry
- Cartography
- Chemistry
- Computer programming
- Construction
- Criminal investigation
- Culinary arts
- Education
- Electrician
- Engineering
- Fashion design
- Fencing
- FIlm editing
- Film/show set design
- Firefighting
- Floor laying
- Forestry
- Geology
- Graphic design
- Historian
- Illustrator
- Industrial design
- Interior design/decoration
- Landscaping
- Machinist
- Maintenance
- Management
- Masonry
- Mechanical drafting
- Mechanics
- Metal fabrication/metalworking
- Model-making
- Optometry
- Photography
- Physical therapy
- Plumbing
- Publishing
- Ranching/farming
- Real estate
- Robotics
- Roofing
- Special effects animation
- Surveying
- Technician
- Technologist
- Urban/regional planning
- Veterinary
- Web design

Grade	*CCSS Domain*	*CCSS Cluster*
8	Geometry	Understand congruence and similarity using physical models, transparencies, or geometry software.

Cluster Standard: 8.G.A.4

Standard	Standards for Mathematical Practice
8.G.A.4: Understand that a two-dimensional figure is similar to another if the second can be obtained from the	● **SMP 4**: Model with mathematics. ○ Teacher and Student Actions

first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	● **SMP 5:** Use appropriate tools strategically. ○ Teacher and Student Actions ● **SMP 6**: Attend to precision. ○ Teacher and Student Actions ● **SMP 7:** Look for and make use of structure. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students describe and apply translations, rotations, reflections, and dilations to understand congruent and similar figures. Students explain and understand angle relationships.	● Understand the concept of similar figures. ● Conclude that a two-dimensional figure is similar to another by describing a sequence of translations, rotations, reflections and dilations that will map the original figure onto the image (vice-versa). ● Express their understanding verbally and in written form. ● Create similar figures given a sequence of transformations.
DOK	**Blooms**
1-4	Understand, Apply, Create

Procedural and Conceptual Understanding and Application

While you may see some application, the focus areas of this standard are conceptual understanding and procedural skill and fluency.

Conceptual Understanding:

- Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations.
- Interpret and describe translations, rotations, reflections, and dilations to understand congruent and similar figures.
- Explain and understand angle relationships.

Procedural Skill and Fluency:

- Describe a sequence that exhibits the similarity between two given similar two-dimensional figures.
- Apply translations, rotations, reflections, and dilations to understand congruent and similar figures.
- Create similar figures given a sequence of transformations.

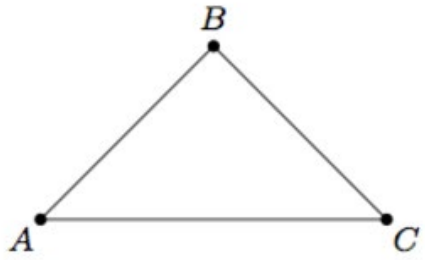
Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

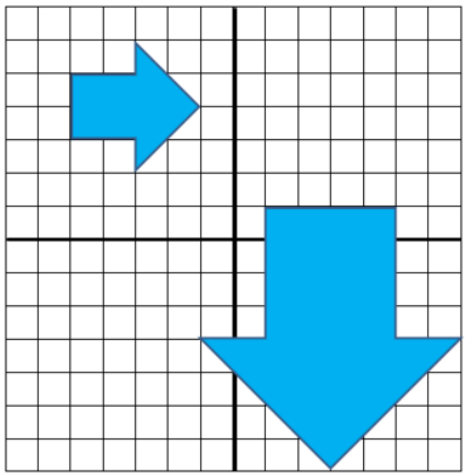
In triangle ABC below, ∠B is a right angle and |AB|=|BC|.

1. Draw a line segment joining one of the vertices of △ABC to the opposite side so that it divides △ABC into two triangles which are both similar to △ABC.

2. Explain, using rigid motions and dilations, why the triangles are similar.



3. Determine, using rotations, translations, reflections, and/or dilations, whether the two polygons below are similar. The intersection of the dark lines on the coordinate plane represents the origin (0,0) in the coordinate plane.



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values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or	shapes. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Use physical models, transparencies, and geometry software to help students make the math of congruence and similarity more visual. Use manipulatives for different types of triangles. Spend ample time on real-world problems.	revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster working with physical models, transparencies, geometry software, manipulatives, and other physical models. Spend time comparing and contrasting dilations, rotations, reflections, and translations. Give students time to practice using the Pythagorean theorem and various volume formulas.

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Vertical Alignment

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https://tools.achievethecore.org/coherence-map/8/37/432/432

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● draw, construct, and describe geometric figures (such as angles and polygons) and their relationships ● solve real-life and mathematical problems involving angle measure ● graph points in a coordinate system and find the horizontal or vertical distance between two points in a coordinate system ● draw polygons in a coordinate system when given vertices ● find the area of squares and circles ● find the volumes of right rectangular prisms ● solve real-world problems involving area and volume	In 8th grade, learners ● use square root symbols to represent solutions and approximate square root values ● use cube root symbols to represent solutions and approximate cube root values	In future classes, learners ● prove theorems about triangles ● use the Pythagorean Theorem to solve problems and discover other mathematical relationships ● use geometric shapes and their measurements to describe objects and solve design problems

Culturally and Linguistically Responsive Instruction		
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Consider these questions as you plan for instruction that is culturally and linguistically responsive: • How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? • How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics.	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

	• Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	

Suggested Student Discourse Questions

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- Compare and contrast rotations, reflections, and translations.
- What is congruence?
- How could you use another group's strategy to check to make sure your solution is reasonable?
- How is your strategy different from the one shown? How is it similar?
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- How is dilation of a figure different from translation, reflection, or rotation of a figure?

Cross-Curricular Connections

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Science: Modeling the solar system at scale. Solving equations when writing computer programs and figuring out algorithms.

Career and Skill Connections

• Architecture • Arts • Atmospheric science • Aviation • CAD Programmer • Carpentry • Cartography • Chemistry	• Film/show set design • Firefighting • Floor laying • Forestry • Geology • Graphic design • Historian • Illustrator	• Model-making • Optometry • Photography • Physical therapy • Plumbing • Publishing • Ranching/farming • Real estate

• Computer programming • Construction • Criminal investigation • Culinary arts • Education • Electrician • Engineering • Fashion design • Fencing • FIlm editing	• Industrial design • Interior design/decoration • Landscaping • Machinist • Maintenance • Management • Masonry • Mechanical drafting • Mechanics • Metal fabrication/metalworking	• Robotics • Roofing • Special effects animation • Surveying • Technician • Technologist • Urban/regional planning • Veterinary • Web design

Grade	CCSS Domain	CCSS Cluster
8	Geometry	Understand congruence and similarity using physical models, transparencies, or geometry software.

Cluster Standard: 8.G.A.5

Standard	Standards for Mathematical Practice
8.G.A.5: Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.	● **SMP 3**: Construct viable arguments and critique the reasoning of others. ○ Teacher and Student Actions ● **SMP 4**: Model with mathematics. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students describe and apply translations, rotations, reflections, and dilations to understand congruent and similar figures. Students explain and understand angle relationships.	● Use informal arguments to establish facts about the angles created when parallel lines are cut by a transversal. ● Apply their knowledge of angle relationships to reason about parallel lines. ● Identify exterior and interior angles of triangles. ● Apply their knowledge to determine if two triangles are similar. ● Use the angle-angle criterion for similarity of triangles. ● Determine if two triangles are similar or not and explain how they know.
DOK	**Blooms**
2	Apply

Procedural and Conceptual Understanding and Application

While you may see other aspects of rigor, the focus area of this standard is conceptual understanding.

Conceptual Understanding:

- Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the

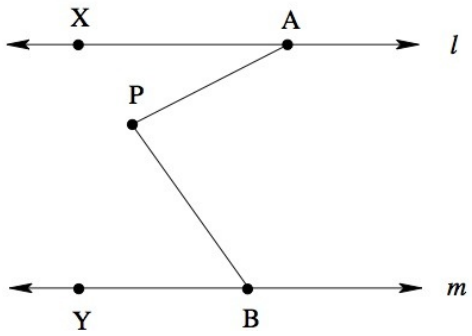
angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

- Describe and apply translations, rotations, reflections, and dilations to understand congruent and similar figures.
- Explain and understand angle relationships to reason about parallel lines.
- Apply knowledge to determine if two triangles are similar.
- Determine and explain whether two triangles are similar or not.
- Understand the Triangle Angle Sum Theorem and Exterior Angle Theorem.
- Understand and use the Angle-Angle (AA) criterion for similarity.

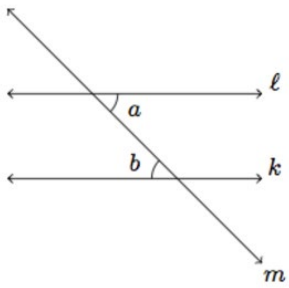
Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

1. In the picture below, lines l and m are parallel. The measure of angle ∠PAX is 31∘, and the measure of angle ∠PBY is 54∘. What is the measure of angle ∠APB?



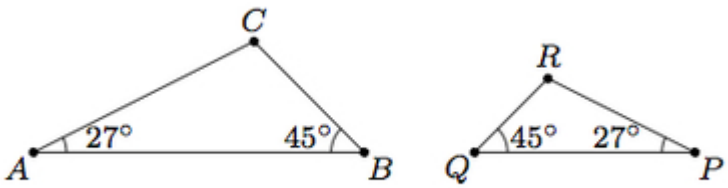
2. In the picture below, ℓ and k are parallel lines.



a. Show that angle a is congruent to angle b using rigid motions.

b. Which other angles, made by the intersection of ℓ and m or by the intersection of k and m, are congruent to a? Explain using rigid motions.

3. Triangles ABC and PQR below share two pairs of congruent angles as marked.



a. Explain, using dilations, translations, reflections, and/or rotations, why △PQR is similar to △ABC.

b. Are angles C and R congruent?

c. Can you show the similarity in part a without using a reflection? What about without using a dilation? Explain.

d. Suppose DEF and KLM are two triangles with m(∠D)=m(∠K) and m(∠E)=m(∠L). Are triangles DEF and KLM similar?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Transformations:** Students might struggle to visualize what happens to a geometric shape after a reflection, rotation, or translation, which technology or manipulatives can help them do. Students might also struggle to differentiate between the different types of transformations, especially their effect on different properties of the shape, e.g. length and angle measurements.
- **Similarity*:*** Students might mistakenly believe that two figures are congruent if they have the same shape but not necessarily the same size, confusing congruence with similarity.
- **Vocabulary:** Students might struggle to understand all of the new vocabulary, such as various angle types, parallel lines, transversal, and the angle-angle criterion for similarity of triangles, especially if they are taught to memorize all of the terms rather than understand what they mean.
- **Similarity criteria:** Students might incorrectly apply similarity criteria, such as AAA, to establish triangle similarity, without recognizing that AAA alone is not sufficient to prove similarity.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + Universal*	*Layer 2* *Core + Targeted*	*Layer 3* *Core + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating	**Pre-teaching** In previous classes, learners worked to draw, construct, and describe geometric figures (such as angles and polygons) and their relationships,	**Pre-teaching** Consider using standard 7.G.A.2, which provides a foundation for work in this cluster. In 7.G.A.2, students sketch, draw, and compose geometric

displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they	solve real-life and mathematical problems involving angle measure, graph points in a coordinate system and find the horizontal or vertical distance between two points in a coordinate system, draw polygons in a coordinate system when given vertices, find the area of squares and circles, find the volumes of right rectangular prisms, and solve real-world problems involving area and volume. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as similarity and congruence, connecting their meanings in our daily lives to their geometrical implications. Students should already be very familiar with triangles, so take time to revisit the types of triangles, angles of a triangle, and stamp that the Pythagorean theorem is only applicable to right triangles. Students will also benefit from reviewing exponents, squares, and square roots. Review cylinders, cones, and spheres and allow students to consider what the volume of these shapes look like and how their formulas connect to other shapes. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching**	shapes, laying the foundation for the practice of geometric deduction. Also consider using standard 7.G.B.6, which also provides a foundation for work in this cluster. In 7.G.B.6, students review what they have already learned about triangles to connect to right triangles specifically. Also consider using standard 8.EE.A.2, which also provides a foundation for work in this cluster. In 8.EE.A.2, students use square root and cubed root symbols in order to represent solutions to equations, which will be useful when working with volume formulas. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster working with physical models, transparencies, geometry software, manipulatives, and other physical models. Spend time comparing and contrasting dilations, rotations, reflections, and translations. Give students time to practice using the Pythagorean theorem and various volume

complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-	Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Use physical models, transparencies, and geometry software to help students make the math of congruence and similarity more visual. Use manipulatives for different types of triangles. Spend ample time on real-world problems.	formulas.

assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/37/436/436

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● draw, construct, and describe geometric figures (such as angles and polygons) and their relationships ● solve real-life and mathematical problems involving angle measure ● graph points in a coordinate system and find the horizontal or vertical distance between two points in a coordinate system ● draw polygons in a coordinate system when given vertices ● find the area of squares and circles ● find the volumes of right rectangular prisms ● solve real-world problems involving area and volume	In 8th grade, learners ● use square root symbols to represent solutions and approximate square root values ● use cube root symbols to represent solutions and approximate cube root values	In future classes, learners ● prove theorems about triangles ● use the Pythagorean Theorem to solve problems and discover other mathematical relationships ● use geometric shapes and their measurements to describe objects and solve design problems

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:
- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating

mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.
Suggested Student Discourse Questions		
Consider this resource for student discourse from Pathways2Careers:		

https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- What do all triangles have in common? How can they be different from each other?
- What is a transversal? What happens to parallel lines when they are cut by a transversal?
- How could you use another group's strategy to check to make sure your solution is reasonable?
- How is your strategy different from the one shown? How is it similar?
- What questions or comments do you have for your fellow students?
- What happens to lines when they undergo transformations? What about line segments? Angles? Parallel lines?

Cross-Curricular Connections

Arts: Making/interpreting geometric and architectural drawings. Crafting art pieces based on understanding basic geometric formulas. Using a 3-D glass shape to create sand art and calculating the amount of sand needed to create their art piece.

Literature: Using linear, logical thinking to write more clearly and logically. Researching a famous mathematician that has a known proof of the Pythagorean Theorem and writing an essay about the proof.

Science: Modeling the solar system at scale. Solving equations when writing computer programs and figuring out algorithms.

Career and Skill Connections

- Architecture
- Arts
- Atmospheric science
- Aviation
- CAD Programmer
- Carpentry
- Cartography
- Chemistry
- Computer programming
- Construction
- Criminal investigation
- Culinary arts
- Education
- Electrician
- Engineering
- Fashion design
- Fencing
- FIlm editing
- Film/show set design
- Firefighting
- Floor laying
- Forestry
- Geology
- Graphic design
- Historian
- Illustrator
- Industrial design
- Interior design/decoration
- Landscaping
- Machinist
- Maintenance
- Management
- Masonry
- Mechanical drafting
- Mechanics
- Metal fabrication/metalworking
- Model-making
- Optometry
- Photography
- Physical therapy
- Plumbing
- Publishing
- Ranching/farming
- Real estate
- Robotics
- Roofing
- Special effects animation
- Surveying
- Technician
- Technologist
- Urban/regional planning
- Veterinary
- Web design

Grade	*CCSS Domain*	*CCSS Cluster*
8	Geometry	Understand and apply the Pythagorean Theorem.

Cluster Standard: 8.G.B.6

Standard	Standards for Mathematical Practice
8.G.B.6: Explain a proof of the Pythagorean Theorem and its converse.	● **SMP 3**: Construct viable arguments and critique the reasoning of others. ○ Teacher and Student Actions ● **SMP 4**: Model with mathematics. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students explore the relationships between sides of a right triangle to understand the formula $\mathbf{a^2 + b^2 = c^2}$ **.** They solve problems applying the Pythagorean Theorem.	● Model a proof of the Pythagorean Theorem and verbally or in written form explain the proof. ● Understand the converse of the Pythagorean Theorem and be able to apply it to any triangle to prove it is or is not a right triangle.
DOK	**Blooms**
2-4	Apply, Evaluate

Procedural and Conceptual Understanding and Application

While you may see other aspects of rigor, the focus area of this standard is conceptual understanding.

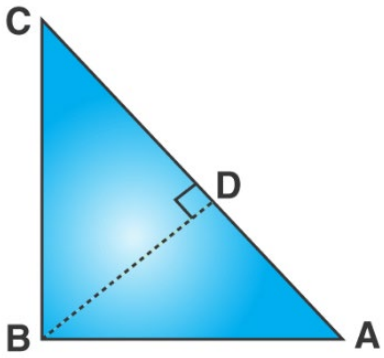
Conceptual Understanding:

- Model and explain a proof of the Pythagorean Theorem and its converse.
- Understand the formula $\mathbf{a^2 + b^2 = c^2}$ by exploring the relationships between sides of a right triangle.
- Solve problems applying the Pythagorean Theorem.
- Understand the converse of the Pythagorean Theorem and be able to apply it to any triangle to prove it is or is not a right triangle.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

1. Explain how the triangle below can be used to prove the Pythagorean Theorem.



2. A Pythagorean triple (a,b,c) is a set of three positive whole numbers which satisfy the equation $a^2 + b^2 = c^2$.Many ancient cultures used simple Pythagorean triples such as (3,4,5) in order to accurately construct right angles: if a triangle has sides of lengths 3, 4, and 5 units, respectively, then the angle opposite the side of length 5 units is a right angle.

a. State the Pythagorean Theorem and its converse.

b. Explain why this practice of constructing a triangle with side-lengths 3, 4, and 5 to produce a right angle uses the converse of the Pythagorean Theorem.

c. Explain, in this particular case, why the converse of the Pythagorean Theorem is true.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Pythagorean Theorem:** Students might struggle to apply this theorem if they are taught to memorize it only and do not understand what the various terms represent. They may mistakenly believe that the Pythagorean Theorem can be applied to any triangle, not just right triangles. Students might also confuse the Pythagorean Theorem with similarity criteria for triangles, such as the Angle-Angle (AA) criterion, leading to errors in applying the theorem to establish similarity.
- **Converse of the Pythagorean Theorem:** Especially if students have only a superficial understanding of the Pythagorean THeorem, students might struggle to understand and apply the converse of the theorem or even think that it says the exact same thing in new terms.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + Universal*	*Layer 2* *Core + Targeted*	*Layer 3* *Core + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating	**Pre-teaching** In previous classes, learners worked to draw, construct, and describe geometric figures (such as angles and polygons) and their relationships,	**Pre-teaching** Consider using standard 7.G.A.2, which provides a foundation for work in this cluster. In 7.G.A.2, students sketch, draw, and compose geometric

displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships.

Engagement

Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they solve real-life and mathematical problems involving angle measure, graph points in a coordinate system and find the horizontal or vertical distance between two points in a coordinate system, draw polygons in a coordinate system when given vertices, find the area of squares and circles, find the volumes of right rectangular prisms, and solve real-world problems involving area and volume.

Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as similarity and congruence, connecting their meanings in our daily lives to their geometrical implications. Students should already be very familiar with triangles, so take time to revisit the types of triangles, angles of a triangle, and stamp that the Pythagorean theorem is only applicable to right triangles. Students will also benefit from reviewing exponents, squares, and square roots. Review cylinders, cones, and spheres and allow students to consider what the volume of these shapes look like and how their formulas connect to other shapes.

If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

Re-teaching

shapes, laying the foundation for the practice of geometric deduction.

Also consider using standard 7.G.B.6, which also provides a foundation for work in this cluster. In 7.G.B.6, students review what they have already learned about triangles to connect to right triangles specifically.

Also consider using standard 8.EE.A.2, which also provides a foundation for work in this cluster. In 8.EE.A.2, students use square root and cubed root symbols in order to represent solutions to equations, which will be useful when working with volume formulas.

If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content.

Re-teaching

Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster working with physical models, transparencies, geometry software, manipulatives, and other physical models. Spend time comparing and contrasting dilations, rotations, reflections, and translations. Give students time to practice using the Pythagorean theorem and various volume

complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-	Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Use physical models, transparencies, and geometry software to help students make the math of congruence and similarity more visual. Use manipulatives for different types of triangles. Spend ample time on real-world problems.	formulas.

assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/37/437/437

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● draw, construct, and describe geometric figures (such as angles and polygons) and their relationships ● solve real-life and mathematical problems involving angle measure ● graph points in a coordinate system and find the horizontal or vertical distance between two points in a coordinate system ● draw polygons in a coordinate system when given vertices ● find the area of squares and circles ● find the volumes of right rectangular prisms ● solve real-world problems involving area and volume	In 8th grade, learners ● use square root symbols to represent solutions and approximate square root values ● use cube root symbols to represent solutions and approximate cube root values	In future classes, learners ● prove theorems about triangles ● use the Pythagorean Theorem to solve problems and discover other mathematical relationships ● use geometric shapes and their measurements to describe objects and solve design problems

Culturally and Linguistically Responsive Instruction

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- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:
- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating

mathematical identities as capable mathematicians that can use mathematics within school and society?		
Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.
Suggested Student Discourse Questions		
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https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- Is the Pythagorean theorem the only strategy we can use to solve triangle problems?
- What ways can we check if our answer is correct or reasonable?
- Where can we see the use of triangles in the real world?
- What is the Pythagorean theorem and what is it used to find?

Cross-Curricular Connections

Arts: Making/interpreting geometric and architectural drawings. Crafting art pieces based on understanding basic geometric formulas. Using a 3-D glass shape to create sand art and calculating the amount of sand needed to create their art piece.

Literature: Using linear, logical thinking to write more clearly and logically. Researching a famous mathematician that has a known proof of the Pythagorean Theorem and writing an essay about the proof.

Science: Modeling the solar system at scale. Solving equations when writing computer programs and figuring out algorithms.

Career and Skill Connections

- Architecture
- Arts
- Atmospheric science
- Aviation
- CAD Programmer
- Carpentry
- Cartography
- Chemistry
- Computer programming
- Construction
- Criminal investigation
- Culinary arts
- Education
- Electrician
- Engineering
- Fashion design
- Fencing
- FIlm editing
- Film/show set design
- Firefighting
- Floor laying
- Forestry
- Geology
- Graphic design
- Historian
- Illustrator
- Industrial design
- Interior design/decoration
- Landscaping
- Machinist
- Maintenance
- Management
- Masonry
- Mechanical drafting
- Mechanics
- Metal fabrication/metalworking
- Model-making
- Optometry
- Photography
- Physical therapy
- Plumbing
- Publishing
- Ranching/farming
- Real estate
- Robotics
- Roofing
- Special effects animation
- Surveying
- Technician
- Technologist
- Urban/regional planning
- Veterinary
- Web design

Grade	*CCSS Domain*	*CCSS Cluster*
8	Geometry	Understand and apply the Pythagorean Theorem.
	Cluster Standard: 8.G.B.7	
Standard		**Standards for Mathematical Practice**

8.G.B.7: Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	● **SMP 7:** Look for and make use of structure. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students explore the relationships between sides of a right triangle to understand the formula $a^2 + b^2 = c^2$. They solve problems applying the Pythagorean Theorem.	● Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. ● Solve problems where they must apply the Pythagorean Theorem.
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

While you may see some conceptual understanding, the focus areas of this standard are procedural skill and fluency and application.

Procedural Skill and Fluency:

- Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in mathematical problems in two and three dimensions.

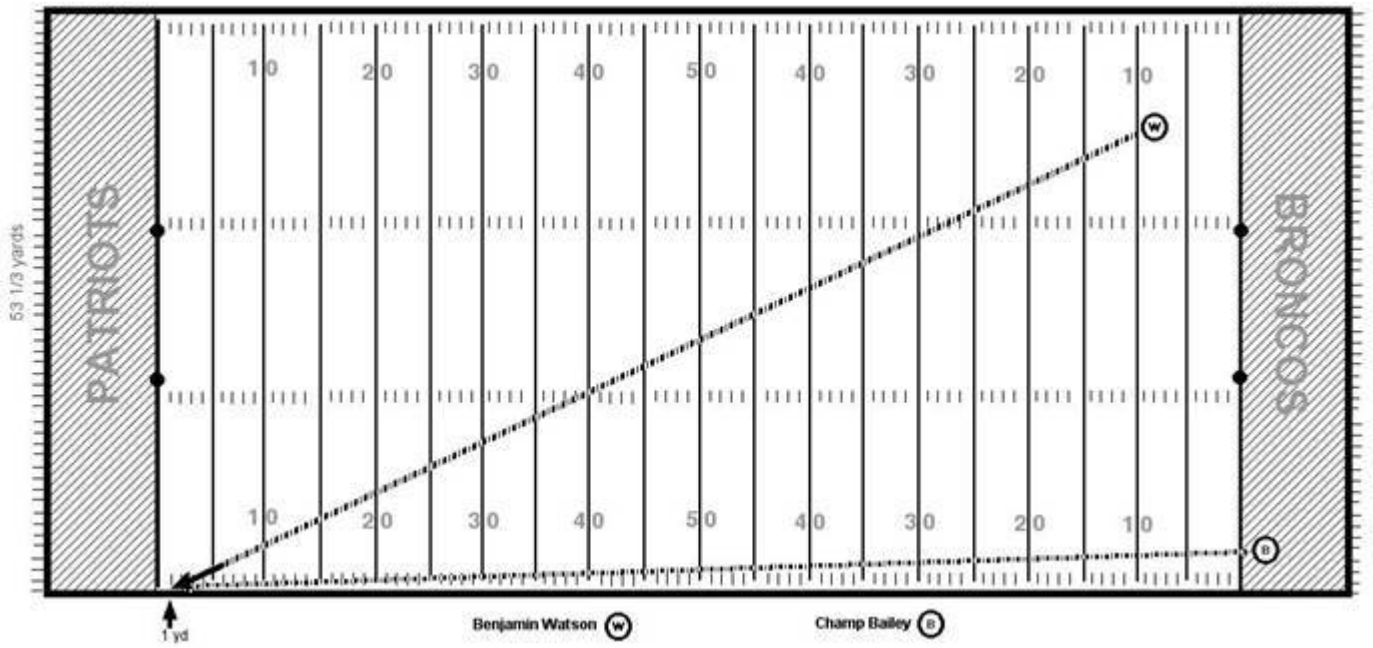
Application:

- Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world problems in two and three dimensions.
- Understand and use the formula $a^2 + b^2 = c^2$ to explore the relationships between sides of a right triangle.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

During the 2005 Divisional Playoff game between The Denver Broncos and The New England Patriots, Bronco player Champ Bailey intercepted Tom Brady's pass around the goal line (see the circled B). He ran the ball nearly all the way to other goal line. Ben Watson of the New England Patriots (see the circled W) chased after Champ and tracked him down just before the other goal line. In the image below, each hash mark is equal to one yard, and the field is 53⅓ yards wide.

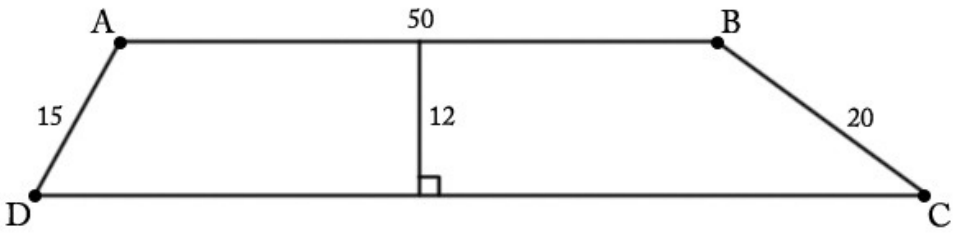


1. How can you use the diagram and the Pythagorean Theorem to find approximately how many yards Ben Watson ran to track down Champ Bailey?

2. Use the Pythagorean Theorem to find approximately how many yards Watson ran in this play.

3. Which player ran further during this play? By approximately how many more yards?

4. Quadrilateral ABCD is a trapezoid, AD=15, AB=50, BC=20, and the altitude is 12. What is the area of the trapezoid?



You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Pythagorean Theorem:** Students might struggle to apply this theorem if they are taught to memorize it only and do not understand what the various terms represent. They may mistakenly believe that the Pythagorean Theorem can be applied to any triangle, not just right triangles. Students might also confuse the Pythagorean Theorem with similarity criteria for triangles, such as the Angle-Angle (AA) criterion, leading to errors in applying the theorem to establish similarity. They may assume that it can be applied directly to find the lengths of sides in three-dimensional figures, neglecting the need for additional considerations in three-dimensional geometry. Students may also assume that if a triangle is not given, they cannot use a triangle to solve problems (e.g. in the problems above).
- **Units*:*** Students might forget to include units when solving problems using the Pythagorean Theorem, leading

to errors in the interpretation of results in real-world contexts.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + Universal*	*Layer 2* *Core + Targeted*	*Layer 3* *Core + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in	**Pre-teaching** In previous classes, learners worked to draw, construct, and describe geometric figures (such as angles and polygons) and their relationships, solve real-life and mathematical problems involving angle measure, graph points in a coordinate system and find the horizontal or vertical distance between two points in a coordinate system, draw polygons in a coordinate system when given vertices, find the area of squares and circles, find the volumes of right rectangular prisms, and solve real-world problems involving area and volume. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as similarity and congruence, connecting their meanings in our daily lives to their geometrical implications. Students should already be very familiar with triangles, so take time to revisit the types of triangles, angles of a triangle, and stamp that the Pythagorean theorem is only applicable to right triangles. Students will also benefit from reviewing exponents, squares, and square roots. Review cylinders, cones, and spheres and allow students to consider what the volume of these shapes look like and how their formulas connect to other shapes.	**Pre-teaching** Consider using standard 7.G.A.2, which provides a foundation for work in this cluster. In 7.G.A.2, students sketch, draw, and compose geometric shapes, laying the foundation for the practice of geometric deduction. Also consider using standard 7.G.B.6, which also provides a foundation for work in this cluster. In 7.G.B.6, students review what they have already learned about triangles to connect to right triangles specifically. Also consider using standard 8.EE.A.2, which also provides a foundation for work in this cluster. In 8.EE.A.2, students use square root and cubed root symbols in order to represent solutions to equations, which will be useful when working with volume formulas. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and

which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure	If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Use physical models, transparencies, and geometry software to help students make the math of congruence and similarity more visual. Use manipulatives for different types of triangles. Spend ample time on real-world problems.	examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster working with physical models, transparencies, geometry software, manipulatives, and other physical models. Spend time comparing and contrasting dilations, rotations, reflections, and translations. Give students time to practice using the Pythagorean theorem and various volume formulas.

students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/37/440/440

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners • draw, construct, and describe geometric figures (such as angles and polygons) and their relationships • solve real-life and mathematical problems involving angle measure • graph points in a coordinate system and find the horizontal or vertical distance between two points in a coordinate system • draw polygons in a coordinate system when given vertices • find the area of squares and circles • find the volumes of right rectangular prisms • solve real-world problems involving area and volume	In 8th grade, learners • use square root symbols to represent solutions and approximate square root values • use cube root symbols to represent solutions and approximate cube root values	In future classes, learners • prove theorems about triangles • use the Pythagorean Theorem to solve problems and discover other mathematical relationships • use geometric shapes and their measurements to describe objects and solve design problems

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

	encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- Is the Pythagorean theorem the only strategy we can use to solve triangle problems?
- What ways can we check if our answer is correct or reasonable?
- Where can we see the use of triangles in the real world?
- What is the Pythagorean theorem and what is it used to find?

Cross-Curricular Connections

Arts: Making/interpreting geometric and architectural drawings. Crafting art pieces based on understanding basic geometric formulas. Using a 3-D glass shape to create sand art and calculating the amount of sand needed to create their art piece.

Literature: Using linear, logical thinking to write more clearly and logically. Researching a famous mathematician that has a known proof of the Pythagorean Theorem and writing an essay about the proof.

Science: Modeling the solar system at scale. Solving equations when writing computer programs and figuring out algorithms.

Career and Skill Connections

• Architecture • Arts • Atmospheric science • Aviation • CAD Programmer • Carpentry • Cartography • Chemistry • Computer programming • Construction • Criminal investigation • Culinary arts • Education • Electrician • Engineering	• Film/show set design • Firefighting • Floor laying • Forestry • Geology • Graphic design • Historian • Illustrator • Industrial design • Interior design/decoration • Landscaping • Machinist • Maintenance • Management • Masonry	• Model-making • Optometry • Photography • Physical therapy • Plumbing • Publishing • Ranching/farming • Real estate • Robotics • Roofing • Special effects animation • Surveying • Technician • Technologist • Urban/regional planning

• Fashion design • Fencing • FIlm editing	• Mechanical drafting • Mechanics • Metal fabrication/metalworking	• Veterinary • Web design

Grade	*CCSS Domain*	*CCSS Cluster*
8	Geometry	Understand and apply the Pythagorean Theorem.

Cluster Standard: 8.G.B.8

Standard	Standards for Mathematical Practice
8.G.B.8: Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	● **SMP 6**: Attend to precision. ○ Teacher and Student Actions ● **SMP 7:** Look for and make use of structure. ○ Teacher and Student Actions

Clarification Statement	Students Who Demonstrate Understanding Can...
Students explore the relationships between sides of a right triangle to understand the formula $a^2 + b^2 = c^2$. They solve problems applying the Pythagorean Theorem.	• Apply the Pythagorean Theorem to find the distance between two points on a coordinate system. • Recognize the diagonal line is the hypotenuse and the vertical and horizontal legs that connect are the legs. • Solve real-world problems using the Theorem as a strategy. • Explain solution strategies using correct mathematical vocabulary.
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

While you may see other aspects of rigor, the focus area of this standard is procedural skill and fluency.

Procedural Skill and Fluency:

- Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
- Solve problems applying the Pythagorean Theorem.
- Recognize the diagonal line is the hypotenuse and the vertical and horizontal legs that connect are the legs.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

1. Plot the points (5,3), (-1,1), and (2,-3) in the coordinate plane and find the lengths of the three segments connecting the points.

2. Find the distance between (5,9) and (-4,2) without plotting the points.

3. If (u,v) and (s,t) are two distinct points in the plane, what is the distance between them? Explain how you know.

4. Does your answer to #2 agree with your calculations in #1 and #2? Explain.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Pythagorean Theorem:** Students might struggle to apply this theorem if they are taught to memorize it only and do not understand what the various terms represent. They may mistakenly believe that the Pythagorean Theorem can be applied to any triangle, not just right triangles. Students might also confuse the Pythagorean Theorem with similarity criteria for triangles, such as the Angle-Angle (AA) criterion, leading to errors in applying the theorem to establish similarity. They may assume that it can be applied directly to find the lengths of sides in three-dimensional figures, neglecting the need for additional considerations in three-dimensional geometry. Students may also assume that if a triangle is not given, they cannot use a triangle to solve problems (e.g. in the problems above).
- **Units*:*** Students might forget to include units when solving problems using the Pythagorean Theorem, leading to errors in the interpretation of results in real-world contexts.
- **Distance formula:** Students might struggle to understand where the distance formula comes from and should not just be taught to memorize it without any underlying conceptual development.
- **Coordinates:** Students might mix up their x- and y-coordinates and/or the x- and y-axes. Students might also forget to account for negative coordinates when finding the differences between x-coordinates and y-coordinates, leading to errors in calculating distances.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 ***Core Instruction + Universal***	***Layer 2*** ***Core + Targeted***	***Layer 3*** ***Core + Targeted + Intensive***
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming	**Pre-teaching** In previous classes, learners worked to draw, construct, and describe geometric figures (such as angles and polygons) and their relationships, solve real-life and mathematical problems involving angle measure, graph points in a coordinate system and find the horizontal or vertical distance between two points in a coordinate system, draw polygons in a coordinate system when given vertices, find the area of squares and circles, find the volumes of right rectangular prisms, and solve real-world problems involving area and volume. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as similarity and congruence, connecting their meanings in our daily lives to their geometrical implications. Students	**Pre-teaching** Consider using standard 7.G.A.2, which provides a foundation for work in this cluster. In 7.G.A.2, students sketch, draw, and compose geometric shapes, laying the foundation for the practice of geometric deduction. Also consider using standard 7.G.B.6, which also provides a foundation for work in this cluster. In 7.G.B.6, students review what they have already learned about triangles to connect to right triangles specifically. Also consider using standard 8.EE.A.2, which also provides a foundation for work in this cluster. In 8.EE.A.2, students use square root and cubed root symbols in order to represent solutions to equations, which will be useful when working with volume formulas. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-

connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning	should already be very familiar with triangles, so take time to revisit the types of triangles, angles of a triangle, and stamp that the Pythagorean theorem is only applicable to right triangles. Students will also benefit from reviewing exponents, squares, and square roots. Review cylinders, cones, and spheres and allow students to consider what the volume of these shapes look like and how their formulas connect to other shapes. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Use physical models, transparencies, and geometry software to help students make the math of congruence and similarity more visual. Use manipulatives for different types of triangles. Spend ample time on real-world problems.	teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster working with physical models, transparencies, geometry software, manipulatives, and other physical models. Spend time comparing and contrasting dilations, rotations, reflections, and translations. Give students time to practice using the Pythagorean theorem and various volume formulas.

about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/37/441/441

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● draw, construct, and describe geometric figures (such as angles and polygons) and their relationships ● solve real-life and mathematical problems involving angle measure ● graph points in a coordinate system and find the	In 8th grade, learners ● use square root symbols to represent solutions and approximate square root values ● use cube root symbols to represent solutions and approximate cube root values	In future classes, learners ● prove theorems about triangles ● use the Pythagorean Theorem to solve problems and discover other mathematical relationships ● use geometric shapes and their measurements to describe objects and solve

horizontal or vertical distance between two points in a coordinate system • draw polygons in a coordinate system when given vertices • find the area of squares and circles • find the volumes of right rectangular prisms • solve real-world problems involving area and volume		design problems

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive

towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- Is the Pythagorean theorem the only strategy we can use to solve triangle problems?
- What ways can we check if our answer is correct or reasonable?
- Where can we see the use of triangles in the real world?
- What is the Pythagorean theorem and what is it used to find?

Cross-Curricular Connections

Arts: Making/interpreting geometric and architectural drawings. Crafting art pieces based on understanding basic geometric formulas. Using a 3-D glass shape to create sand art and calculating the amount of sand needed to create their art piece.

Literature: Using linear, logical thinking to write more clearly and logically. Researching a famous mathematician that has a known proof of the Pythagorean Theorem and writing an essay about the proof.

Science: Modeling the solar system at scale. Solving equations when writing computer programs and figuring out algorithms.

Career and Skill Connections

• Architecture	• Film/show set design	• Model-making

• Arts • Atmospheric science • Aviation • CAD Programmer • Carpentry • Cartography • Chemistry • Computer programming • Construction • Criminal investigation • Culinary arts • Education • Electrician • Engineering • Fashion design • Fencing • FIlm editing	• Firefighting • Floor laying • Forestry • Geology • Graphic design • Historian • Illustrator • Industrial design • Interior design/decoration • Landscaping • Machinist • Maintenance • Management • Masonry • Mechanical drafting • Mechanics • Metal fabrication/metalworking	• Optometry • Photography • Physical therapy • Plumbing • Publishing • Ranching/farming • Real estate • Robotics • Roofing • Special effects animation • Surveying • Technician • Technologist • Urban/regional planning • Veterinary • Web design

Grade	CCSS Domain	CCSS Cluster
8	Geometry	Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Cluster Standard: 8.G.C.9

Standard	Standards for Mathematical Practice

8.G.C.9: Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	• **SMP 1:** Make sense of problems and persevere in solving them.
	  ○ Teacher and Student Actions
	• **SMP 3**: Construct viable arguments and critique the reasoning of others.
	  ○ Teacher and Student Actions
	• **SMP 6**: Attend to precision.
	  ○ Teacher and Student Actions

Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students know and apply the volume formulas of a cylinder, cone, and a sphere.	• Write formulas from memory for finding the volume of cones, spheres, and cylinders. • Know that these formulas are special equations

	that are specific in use. ● Make connections between the 3-D figures and their formulas. ● Use formulas to calculate volumes of cones, cylinders and spheres. ● Explain the relationship in their volumes. ● Apply the formulas to solve real world application problems related to volume.
DOK	**Blooms**
1-3	Understand, Apply, Evaluate

Procedural and Conceptual Understanding and Application

You should see all three aspects of rigor in this standard.

Conceptual Understanding:

- Understand that volume is the amount of space enclosed by a 3-D figure.
- Know that the formulas for the volumes of cones, cylinders, and spheres are special equations that are specific in use and explain when to apply them, making connections between the 3-D figures and their formulas.
- Explain the relationship in the volume of cones, cylinders, and spheres.

Procedural Skill and Fluency:

- Know the formulas for the volumes of cones, cylinders, and spheres, including the significance of each component in the formulas, and use them to solve mathematical problems.
- Identify the significance of each component in the

Application:

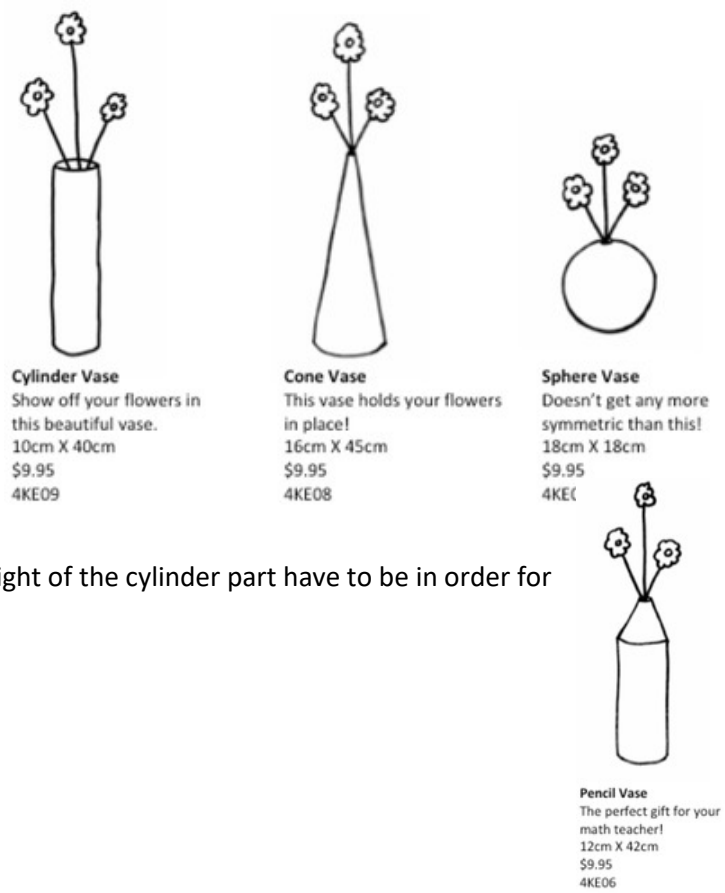
- Know the formulas for the volumes of cones, cylinders, and spheres, including the significance of each component in the formulas, and use them to solve real-world problems, being mindful of units when present.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

David's sister's birthday is in a few weeks and he would like to buy her a new vase to keep fresh flowers in her house. She often forgets to water her flowers and needs a vase that holds a lot of water. In a catalog there are three vases available and he wants to purchase the one that holds the most water. The first vase is a cylinder with diameter 10 cm

and height 40 cm. The second vase is a cone with base diameter 16 cm and height 45 cm. The third vase is a sphere with diameter 18 cm.



1. Which vase should he purchase?
2. How much more water does the largest vase hold than the smallest vase?
3. Suppose the diameter of each vase decreases by 2 cm. Which vase would hold the most water?
4. The vase company designs a new vase that is shaped like a cylinder on bottom and a cone on top. The catalog states that the width is 12 cm and the total height is 42 cm. What would the height of the cylinder part have to be in order for the total volume to be 1224π cm^3?
5. Design your own vase with composite shapes, determine the volume, and write an ad for the catalog.

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Formulas:** Students might struggle to apply formulas, especially for complex shapes, if they have been asked to memorize formulas without understanding where their components come from.
- **Units*:*** Students might forget to include units when solving problems using formulas leading to errors in the interpretation of results in real-world contexts,
- **Vocabulary:** Students might struggle with all of the names of shapes, as well as terms such as 𝜋 and volume vs. surface area.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + Universal*	*Layer 2* *Core + Targeted*	*Layer 3* *Core + Targeted + Intensive*

Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to	**Pre-teaching** In previous classes, learners worked to draw, construct, and describe geometric figures (such as angles and polygons) and their relationships, solve real-life and mathematical problems involving angle measure, graph points in a coordinate system and find the horizontal or vertical distance between two points in a coordinate system, draw polygons in a coordinate system when given vertices, find the area of squares and circles, find the volumes of right rectangular prisms, and solve real-world problems involving area and volume. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as similarity and congruence, connecting their meanings in our daily lives to their geometrical implications. Students should already be very familiar with triangles, so take time to revisit the types of triangles, angles of a triangle, and stamp that the Pythagorean theorem is only applicable to right triangles. Students will also benefit from reviewing exponents, squares, and square roots. Review cylinders, cones, and spheres and allow students to consider what the volume of these shapes look like and how their formulas connect to other shapes. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments.	**Pre-teaching** Consider using standard 7.G.A.2, which provides a foundation for work in this cluster. In 7.G.A.2, students sketch, draw, and compose geometric shapes, laying the foundation for the practice of geometric deduction. Also consider using standard 7.G.B.6, which also provides a foundation for work in this cluster. In 7.G.B.6, students review what they have already learned about triangles to connect to right triangles specifically. Also consider using standard 8.EE.A.2, which also provides a foundation for work in this cluster. In 8.EE.A.2, students use square root and cubed root symbols in order to represent solutions to equations, which will be useful when working with volume formulas. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster working with physical models, transparencies, geometry software, manipulatives, and other physical models. Spend time comparing and

their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support	Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Use physical models, transparencies, and geometry software to help students make the math of congruence and similarity more visual. Use manipulatives for different types of triangles. Spend ample time on real-world problems.	contrasting dilations, rotations, reflections, and translations. Give students time to practice using the Pythagorean theorem and various volume formulas.

independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/37/442/442

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● draw, construct, and describe geometric figures (such as angles and polygons) and their relationships ● solve real-life and mathematical problems involving angle measure ● graph points in a coordinate system and find the horizontal or vertical distance between two points in a coordinate system ● draw polygons in a coordinate system when given vertices ● find the area of squares and circles ● find the volumes of right rectangular prisms ● solve real-world problems involving area and volume	In 8th grade, learners ● use square root symbols to represent solutions and approximate square root values ● use cube root symbols to represent solutions and approximate cube root values	In future classes, learners ● prove theorems about triangles ● use the Pythagorean Theorem to solve problems and discover other mathematical relationships ● use geometric shapes and their measurements to describe objects and solve design problems

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

	achievement.	

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- How could you use estimation to check and make sure your solution is reasonable?
- Why is it important to know the volume or surface area of items?
- What real-life items are shaped like a cylinder? A cone? A sphere?
- What is the relationship between area and volume?

Cross-Curricular Connections

Arts: Making/interpreting geometric and architectural drawings. Crafting art pieces based on understanding basic geometric formulas. Using a 3-D glass shape to create sand art and calculating the amount of sand needed to create their art piece.

Literature: Using linear, logical thinking to write more clearly and logically. Researching a famous mathematician that has a known proof of the Pythagorean Theorem and writing an essay about the proof.

Science: Modeling the solar system at scale. Solving equations when writing computer programs and figuring out algorithms.

Career and Skill Connections

- Architecture
- Arts
- Atmospheric science
- Aviation
- CAD Programmer
- Carpentry
- Cartography
- Chemistry
- Computer programming
- Construction
- Criminal investigation
- Culinary arts
- Education
- Electrician
- Engineering
- Fashion design
- Fencing
- FIlm editing
- Film/show set design
- Firefighting
- Floor laying
- Forestry
- Geology
- Graphic design
- Historian
- Illustrator
- Industrial design
- Interior design/decoration
- Landscaping
- Machinist
- Maintenance
- Management
- Masonry
- Mechanical drafting
- Mechanics
- Metal fabrication/metalworking
- Model-making
- Optometry
- Photography
- Physical therapy
- Plumbing
- Publishing
- Ranching/farming
- Real estate
- Robotics
- Roofing
- Special effects animation
- Surveying
- Technician
- Technologist
- Urban/regional planning
- Veterinary
- Web design

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards for Mathematical Practice
    - Connections to procedural, conceptual understanding, and application
    - Sample assessment items
    - Common misconceptions
    - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
    - Vertical alignment
    - Culturally and Linguistically Responsive Instruction (CLRI)
    - Suggested student discourse questions
    - Cross-curricular and career/skill connections
- A Student Discourse Guide

Helpful links:

- Lesson-planning tool from Pathways 2 Careers (click here to sign up with your district email if you don't already have an account)
- Focus by Grade Level from Achieve the Core
- Coherence Map from Achieve the Core

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Know that there are numbers that are not rational and approximate them by rational numbers.
  - 8.NS.A.1
  - 8.NS.A.2

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)

Grade	CCSS Domain	CCSS Cluster
8	The Number System	Know that there are numbers that are not rational and approximate them by rational numbers.

Cluster Standard: 8.NS.A.1

Standard	Standards for Mathematical Practice

8.NS.A.1: Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually and convert a decimal expansion which repeats eventually into a rational number.	• **SMP 2:** Reason abstractly and quantitatively.
	  ○ Teacher and Student Actions
	• **SMP 6**: Attend to precision.
	  ○ Teacher and Student Actions
	• **SMP 7**: Look for and make use of structure.
	  ○ Teacher and Student Actions
	• **SMP 8**: Look for and express regularity in repeated reasoning.
	  ○ Teacher and Student Actions

Clarification Statement	**Students Who Demonstrate Understanding Can...**
Expand knowledge of numbers to include irrational numbers. Convert decimals to rational numbers. Use a number line to approximate, compare, and order rational and irrational numbers.	• Classify numbers as rational or irrational. • Understand that every number has a decimal expansion. • Explain that an irrational number is a decimal that does not terminate or repeat, it cannot be written in the form a/b, where b cannot be equal to zero. • Identify and explain that a rational number of repeats or terminates. • Explain what a rational number is and give examples. • Explain what an irrational number is and give an example.
DOK	**Blooms**
1-2	Understand

Procedural and Conceptual Understanding and Application

While you may see some application, the focus areas of this standard are conceptual understanding and procedural skill and fluency.

Conceptual Understanding:

- Understand that every number can be represented as a decimal.
- Understand and explain how to differentiate between a rational and an irrational number.
- Explain what rational and irrational numbers are and give examples of each.
- Give examples of rational and irrational numbers.

Procedural Skill and Fluency:

- Convert decimals to rational numbers.
- Use a number line to approximate, compare, and order rational and irrational numbers.
- Classify numbers as rational or irrational.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

1. Decide whether each of the following numbers is rational or irrational. If it is rational, explain how you know.

   a. $0.33\underline{3}$ b. $\sqrt{4}$ c. $\sqrt{2} = 1.414213\ldots$ d. 1.414213

   e. $\pi = 3.141592\ldots$ f. 11 g. $\frac{1}{7} = 0.\underline{142857}$ h. $12.3456565\underline{656}$

2. Tiffany is working on some math problems. She knows that 3 thirds equals 1, so $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$. She also knows that $\frac{1}{3}=0.333\ldots$ where the 3's go on forever. She adds them up as decimals and gets 0.999.

$$\begin{array}{r} 0.333\ldots \\ 0.333\ldots \\ +0.333\ldots \\ \hline 0.999\ldots \end{array}$$

"I just added up the tenths, then the hundredths, then the thousands, and so on. What went wrong?"

a. Write 0.999… in the form of a fraction $\frac{a}{b}$ where a and b are whole numbers. Are Tiffany's calculations consistent with what you find? Explain.

b. Use Tiffany's idea of adding decimals to write $\frac{1}{3}+\frac{1}{6}$ as a repeating decimal. Can this also be written as a terminating decimal?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Real Number System:** Students might struggle with understanding relationships of the subsets of the Real

Number System.

- **Rational numbers:** Some students may think some rational numbers in decimal form repeat three or more digits and students mislabel them as irrational because they do not divide far enough to see the pattern or repeating digits
- **Irrational numbers:** Students might struggle to understand what rational numbers are, and may assume that every decimal can be converted into a fraction.
- **Rational vs. irrational numbers:** Students might have trouble differentiating between the two types of numbers.
- **Non-terminating decimals:** Students might mistakenly believe that any decimal that does not terminate is irrational, e.g. 0.3333.... is a repeating decimal and represents the rational number $\frac{1}{3}$. Students might also struggle to internalize what a non-terminating decimal number actually is and what it represents, especially a number that would normally be rounded up (e.g. 0.99999…).
- **Non-repeating decimals:** Students may think that all non-repeating decimals are irrational, but some rational numbers also have non-repeating decimals, e.g. $\frac{1}{6}$ = 0.1666...
- **Converting a decimal to a fraction:** Students might struggle with the process of converting decimals to fractions.
- **Vocabulary:** Students might struggle with learning the various new vocabulary terms and symbols associated with this standard.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 ***Core Instruction + UDL***	***Layer 2*** ***Core + UDL + Targeted***	***Layer 3*** ***Core + UDL + Targeted + Intensive***
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply	**Pre-teaching** In previous classes, learners worked to round decimals to any place value, place rational numbers on a number line, and convert rational numbers to decimals using long division. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as multiplication (especially with signed numbers), the distributive property, division, converting rational numbers to decimals using long division, and interpreting quotients in real-world contexts. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-	**Pre-teaching** Consider using standard 7.NS.A.2, which provides a foundation for work in this cluster. In 7.NS.A.2, students apply and deepen their understanding of multiplication and division of fractions to their work with rational numbers and learn more about working rational numbers. Teachers can help students develop the concept of performing operations on irrational numbers, but students first need a firm foundation in working with rational numbers. They must also be able to extend their knowledge of fractions and be able to convert fractions to decimals and vice versa. If students have unfinished learning leading into this standard, consider

background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals.	teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems.	ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster working with the Real Number System. Teachers can help students understand the subsets of the Real Number System by displaying them in a Venn Diagram, and students can use manipulatives such as boxes that fit inside one another to represent those subsets. Adding examples of numbers in the subsets can further help with the concept.

Action and Expression Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/38/443/443

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● round decimals to any place value ● place rational numbers on a number line and convert rational numbers to decimals	In 8th grade, learners ● use square root and cube root symbols to encounter irrational numbers	In future classes, learners ● extend their knowledge of irrational numbers to complex numbers ● use rational exponents

using long division		

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

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- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

	achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- Name, compare, and contrast two numbers, one rational and one irrational. What do you notice about each number? How do you know you have one rational and one irrational number?
- How can you identify rational and irrational numbers?
- What type of numbers do we predominantly see around us–rational or irrational?
- Which would you prefer to be called 'rational thinker' or an 'irrational thinker'? Explain why.
- What are the subsets of rational numbers and how are they related to each other?

Cross-Curricular Connections

Literature: Using linear, logical thinking to write more clearly and logically.

Science: Representing collected data in different forms of rational and irrational numbers.

Career and Skill Connections

• Archeology • Arts • Banking/finance • Business • Computer programming	• Culinary arts • Education • Electrician • Engineering • Information technology	• Management • Mechanics • Metalworking • Video game design • Web design

Grade	CCSS Domain	CCSS Cluster
8	The Number System	Know that there are numbers that are not rational and approximate them by rational numbers.

Cluster Standard: 8.NS.A.2

Standard	Standards for Mathematical Practice
8.NS.A.2: Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π2). For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.	● **SMP 2:** Reason abstractly and quantitatively. ○ Teacher and Student Actions ● **SMP 6**: Attend to precision. ○ Teacher and Student Actions ● **SMP 7**: Look for and make use of structure. ○ Teacher and Student Actions ● **SMP 8**: Look for and express regularity in repeated reasoning. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Expand knowledge of numbers to include irrational numbers. Convert decimals to rational numbers. Use a number line to approximate, compare, and order rational and irrational numbers.2	● Approximate square roots ● Plot square roots on the number line. ● Express thinking in writing about how to approximate values and locations on a number line.
DOK	**Blooms**
1-2	Understand, Apply

Procedural and Conceptual Understanding and Application

While you may see some application, the focus areas of this standard are conceptual understanding and procedural skill and fluency.

Conceptual Understanding:

- Understand that irrational numbers can be approximated by rational numbers and explain how to get better approximations.
- Explain how to approximate the rational value of irrational numbers (including square roots) in order to compare the values of irrational numbers and locate them approximately on a number line diagram.

Procedural Skill and Fluency:

- Approximate the rational value of irrational numbers (including square roots) in order to compare the values of irrational numbers, locate them approximately on a number line diagram, and estimate the value of an expression that includes an irrational number.
- Convert decimals to rational numbers.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Without using your calculator, label approximate locations for the following numbers on the number line.

a.

π

b.

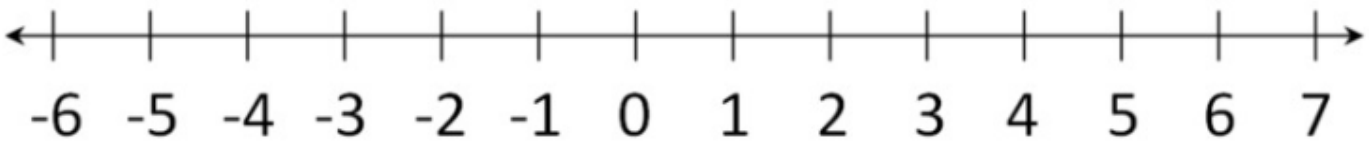
$-(\frac{1}{2} \times \pi)$

c.

$2\sqrt{2}$

d.

$\sqrt{17}$



You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Real Number System:** Students might struggle with understanding relationships of the subsets of the Real Number System.
- **Irrational numbers:** Students might struggle to understand what rational numbers are and may assume that

every decimal can be converted into a fraction.

- **Converting a decimal to a fraction:** Students might struggle with the process of converting decimals to fractions.
- **Reasoning about numbers:** Students may struggle with prior knowledge of products, squares, and square roots of benchmark numbers in order to compare numbers, particularly numbers that are very similar to others but slightly larger or smaller.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning (UDL)

Layer 1 *Core Instruction + UDL*	*Layer 2* *Core + UDL + Targeted*	*Layer 3* *Core + UDL + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships.	**Pre-teaching** In previous classes, learners worked to round decimals to any place value, place rational numbers on a number line and convert rational numbers to decimals using long division. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as multiplication (especially with signed numbers), the distributive property, division, converting rational numbers to decimals using long division, and interpreting quotients in real-world contexts. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at	**Pre-teaching** Consider using standard 7.NS.A.2, which provides a foundation for work in this cluster. In 7.NS.A.2, students apply and deepen their understanding of multiplication and division of fractions to their work with rational numbers and learn more about working rational numbers. Teachers can help students develop the concept of performing operations on irrational numbers, but students first need a firm foundation in working with rational numbers. They must also be able to extend their knowledge of fractions and be able to convert fractions to decimals and vice versa. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending

Engagement Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before	revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems.	time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster working with the Real Number System. Teachers can help students understand the subsets of the Real Number System by displaying them in a Venn Diagram, and students can use manipulatives such as boxes that fit inside one another to represent those subsets. Adding examples of numbers in the subsets can further help with the concept.

students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/38/443/445

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● round decimals to any place value ● place rational numbers on a number line and convert rational numbers to decimals using long division	In 8th grade, learners ● use square root and cube root symbols to encounter irrational numbers	In future classes, learners ● extend their knowledge of irrational numbers to complex numbers ● use rational exponents

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:
- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- Name, compare, and contrast two numbers, one rational and one irrational. What do you notice about each number? How can you use the rational number to help determine how big the irrational number is?
- How can we modify an irrational number to make it easier to estimate?
- What are the subsets of rational numbers and how are they related to each other?

Cross-Curricular Connections

Literature: Using linear, logical thinking to write more clearly and logically.

Science: Representing collected data in different forms of rational and irrational numbers.

Career and Skill Connections

- Archeology
- Arts
- Banking/finance
- Business
- Computer programming
- Culinary arts
- Education
- Electrician
- Engineering
- Information technology
- Management
- Mechanics
- Metalworking
- Video game design
- Web design

The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

The NMIS is a teacher-influenced tool, designed to provide instructional planning support at the programmatic level for districts and instructional level for teachers. Its foundation stems from the vision and mission of the PED and came into existence to assure that students in NM will be engaged in a culturally and linguistically responsive educational system that meets the social, emotional, and academic needs of ALL students. This is also rooted in the belief that all students must have access to on-grade-level standards, focusing on acceleration. The purpose of this tool is to help educators understand each of the grade level standards and how those standards connect to the students' overall preparation for college and career readiness.

Standards are defined as the most critical prerequisite skills and knowledge. This document is color-coded to reflect both anchor and priority standards. Though previous emphasis was placed on priority standards to address lost learning due to COVID-19, New Mexico teachers should note that moving forward, while priority standards allow for acceleration of learning, all standards should be addressed in instruction throughout the school year.

In this guide you will find:

- A breakdown of each of the grade level standards within the cluster, including:
    - Standards for Mathematical Practice
    - Connections to procedural, conceptual understanding, and application
    - Sample assessment items
    - Common misconceptions
    - Planning for a Multi-layer System of Support (MLSS) and Universal Design for Learning (UDL)
    - Vertical alignment
    - Culturally and Linguistically Responsive Instruction (CLRI)
    - Suggested student discourse questions
    - Cross-curricular and career/skill connections
- A Student Discourse Guide

Helpful links:

- Lesson-planning tool from Pathways 2 Careers (click here to sign up with your district email if you don't already have an account)
- Focus by Grade Level from Achieve the Core
- Coherence Map from Achieve the Core

Key		
	Priority Standard	Priority standards, as identified by NMPED, are denoted with red highlighting. Priority standards are the most critical prerequisite skills and knowledge a student needs. This does not mean that these are only standards required to be taught, just these are the standards that will allow for the acceleration the students of New Mexico need during this time.
	*Conceptual Understanding*	Conceptual Understanding standards help students build a deep understanding of the **how** and **why** of mathematics.
	*Application*	Application standards help students identify the appropriate concepts and skills to tackle **novel real-world problems.**
	*Procedural Skill and Fluency*	Procedural standards help students develop **efficiency** and **accuracy** in computations.

Standards Breakdown

- Investigate patterns of association in bivariate data.
  - 8.SP.A.1
  - 8.SP.A.2
  - 8.SP.A.3
  - 8.SP.A.4

Student Discourse Guide

- Purposeful, rich classroom discourse offers students the opportunity to express their ideas, thinking, and to critique the reasoning of others in a variety of ways (writing, drawing, verbal). Purposeful implementation of classroom discourse allows students to activate funds of knowledge and to refine their mathematical understanding. When students have frequent opportunities for discourse, they find various paths to solutions and reveal knowledge or misunderstandings to educators. The process also allows educators to honor students' culture, lived experiences and evolving math identities.
- Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels, O'Connor, and Resnick, 2008)

Grade	CCSS Domain	CCSS Cluster
8	Statistics and Probability	Investigate patterns of association in bivariate data.

Cluster Standard: 8.SP.A.1

Standard	Standards for Mathematical Practice
8.SP.A.1: Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	● **SMP 2**: Reason abstractly and quantitatively. ○ Teacher and Student Actions ● **SMP 4:** Model with mathematics. ○ Teacher and Student Actions ● **SMP 5:** Use appropriate tools strategically. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students construct scatter plots and interpret patterns focusing on linear association. They construct two-way tables and interpret relationships using relative frequencies.	● Construct a Scatter Plot using two sets of quantitative data. ● Identify outliers and clusters in a scatter plot. ● Determine if there is a linear or nonlinear association in a scatter plot; determine if a linear association is positive or negative. ● Explain what the different patterns mean in different contexts. ● Describe the patterns and associations they see between two quantities.
DOK	**Blooms**
1-2	Apply

Procedural and Conceptual Understanding and Application

While you may see some application, this cluster of standards focuses on conceptual understanding and procedural skills and fluency.

Conceptual Understanding:

- Interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities (focusing on linear association, whether positive or negative).
- Interpret positive and negative associations in context.
- Describe patterns such as clustering, outliers, positive or negative, linear, and nonlinear associations.
- Interpret relationships using relative frequencies.

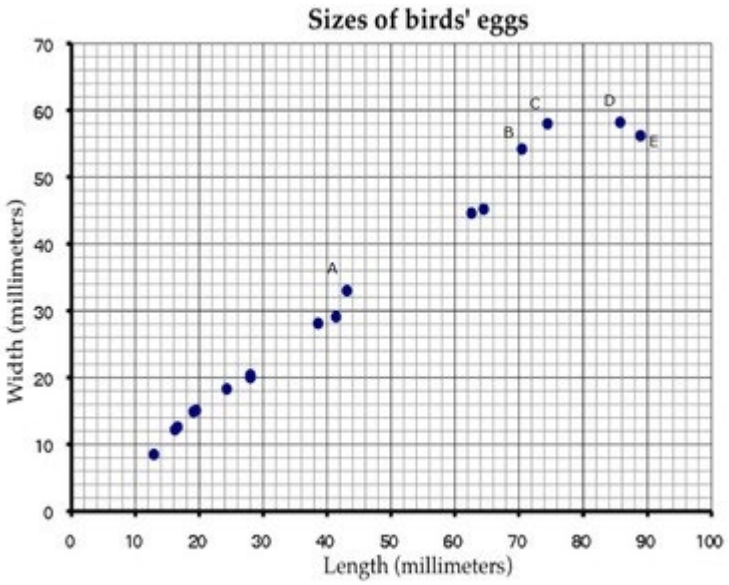
Procedural Skill and Fluency:

- Construct scatter plots for bivariate measurement data to investigate patterns of association between two quantities (focusing on linear association, whether positive or negative).
- Construct two-way tables using relative frequencies.
- Identify outliers and clusters in a scatter plot.
- Identify positive vs. negative associations.

Assessment Items

When available, you should use your locally selected or created high-quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

This scatter diagram shows the lengths and widths of the eggs of some American birds.



1. A biologist measured a sample of one hundred Mallard duck eggs and found they had an average length of 57.8 millimeters and average width of 41.6 millimeters. Use an X to mark a point that represents this on the scatter diagram.

2. What does the graph show about the relationship between the lengths of birds' eggs and their widths?

3. Another sample of eggs from similar birds has an average length of 35 millimeters. If these bird eggs follow the trend in the scatter plot, about what width would you expect these eggs to have, on average?

4. Describe the differences in shape of the two eggs corresponding to the data points marked C and D in the plot.

5. Which of the eggs A, B, C, D, and E has the greatest ratio of length to width? Explain how you decided.

6. Do taller people tend to have bigger hands? To investigate this question, each student in your class should measure his or her hand span (in cm) and height (in inches). Record these values in the table below.

7. Create a clearly labeled graph that displays the relationship between height and hand span.

8. Based on the graph, how would you answer the question about whether taller people tend to have bigger hands?

9. Based on your graph, would you describe the relationship between hand span and height as linear or nonlinear? Explain your choice.

10. Do you notice any areas of clustering? What about outliers?

Student	Hand Span (cm)	Height (inches)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Linear relationships:** Students may struggle to read and interpret graphs. For graphs displaying a roughly linear relationship, students may struggle to understand that data points do not have to fall directly on a line. When a scatter plot shows no association, students may struggle and need examples of data that may have no association (length of a person's hair and his or her final grade in mathematics).
- **Graphing:** Students may struggle with scaling the x- and y-axes so that the data is visible but not misleading. They may also think that the line of best fit must go through at least some of the data points on the scatter plot.
- **Clustering:** Students might mistakenly believe that whenever data points cluster together, there must be a

strong relationship between the two variables.

- **Outliers:** Some students may assume that outliers should always be disregarded as errors or anomalies.
- **Nonlinear relationships:** Students might think that all relationships between two variables must be linear and ignore nonlinear relationships or regard them as having no association.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + Universal*	*Layer 2* *Core + Targeted*	*Layer 3* *Core + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement**	**Pre-teaching** In previous classes, learners worked to plot points in a coordinate grid. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as studying patterns represented in scatter plots and in contexts described verbally. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Students should spend time examining tasks from a different perspective, especially using straight lines to	**Pre-teaching** Consider using standard 5.G.A.2, which provides a foundation for work in this cluster. In 5.G.A.2, students interpret real-word problems and produce a graph based on information gathered from the problem. It is essential that students develop awareness of how real-world information is represented visually and how visual representations relate to each other. Give students time to really dive into graphs and interpret them rather than jumping in and explicitly explaining to them what a graph represents. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive

Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support	model relationships, because this leaves room for discussion amongst peers and shows us how students have arrived at their conclusions about a data set.	extra time during and after work within this cluster constructing, reading, and interpreting scatter plots. Give them time to describe relationships using statistical vocabulary, fitting a model to data, assess the fit of a model, and using and interpreting equations. Students should practice reading, interpreting, and constructing two-way tables and calculating and interpreting relative frequencies.

discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/39/448/448

Previous Learning	*Current Learning*	*Future Learning*
• Plot points in a coordinate grid.	• Construct an equation or a function to model a linear relationship. • Determine and interpret the slope and y-intercept	• Compute and interpret the correlation coefficient. • Distinguish between correlation and causation. **(HS.S-ID.C.9)** • Represent two variables on a scatter plot and describe how they are related. (**HS.S-ID.B.6**) • Construct, interpret, and summarize data in a two-way table.

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

	achievement.	

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- What does each point on a scatter plot represent?
- What does clustering in a scatter plot represent?
- Why are outliers important?
- How could you use a different classmate's strategy to check to make sure your solution is reasonable?
- How can we use the equation of the line of best fit to make a prediction about the data?
- What two types of data in real life will show positive correlation? Negative correlation? No correlation?
- How does the amount of data we collect have an impact on the correlation?

Cross-Curricular Connections

Literature: Using linear, logical thinking to write more clearly and logically.

Science: Examining biological characteristics of a sample. Making an argument about the growth and development of organisms. Developing models and identifying components and describing relationships between components. Modeling data and describing how the data can be used to create designs through testing and modification. Identifying limitations of models. Engaging in the Engineering Design Process. Modeling genetic information and sexual reproduction results and using Punnett squares. Creating scatterplots of bivariate data. Using simulations to generate data that can be used to modify a proposed object, tool, or process. Conducting experiments in connection with NGSS science standards, collecting bivariate data, representing that data in a two-way table, and hypothesizing correlations between the two variables. Reading scientific charts and graphs. Analyzing scientific studies.

Social Studies: Sampling populations and analyzing data. Studying trends in areas such as species populations, the stock market, or gross domestic product. Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations have different average life spans. Understanding statistical information on populations.

Career and Skill Connections

• Advertising • Aeronautics • Agriculture • Analysis • Aviation • Banking/finance • Botanist • Coaching • Counseling • Data science • Ecology	• Economist • Education • Engineering • Gardening • Health science • Information technology • Law • Machinist • Management • Mechanic • Medicine	• Park ranger • Political science • Psychology • Ranching/farming • Sales • Statistics • Technician • Transportation • Urban planning • Veterinary

Grade	CCSS Domain	CCSS Cluster
8	Statistics and Probability	Investigate patterns of association in bivariate data.

Cluster Standard: 8.SP.A.2

Standard	Standards for Mathematical Practice
8.SP.A.2: Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	● **SMP 3:** Construct viable arguments and critique the reasoning of others. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students construct scatter plots and interpret patterns focusing on linear association. They construct two-way tables and interpret relationships using relative frequencies.	● Construct a trend line and justify its placement among the data. ● Model real-world linear relationships on a graph. ● Use a trend line to determine whether a set of paired data has a linear association, nonlinear association or no association. ● Determine whether the association is positive or negative, strong or weak. ● Justify whether a trend line is a good fit or not. ● Explain orally and/or inwriting the meaning of the trend line.
DOK	**Blooms**
1-2	Apply

Procedural and Conceptual Understanding and Application

While you may see other aspects of rigor, the focus area of this standard is conceptual understanding.

Conceptual Understanding:

- Know that straight lines are widely used to model relationships between two quantitative variables.
- Informally fit a straight line to scatter plots that suggest a linear association and justify its placement.
- Informally assess the fit of a straight line to a scatter plot by judging the closeness of the data points to the line.

- Construct scatter plots and interpret patterns, focusing on linear association
- Construct two-way tables and interpret relationships using relative frequencies.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

1. Is there an association between the weight of an animal's body and the weight of the animal's brain? Make a scatterplot using the following data.

2. Do there appear to be outliers in this data? Which animals appear to be outliers? Explain how you identified these outliers.

3. Removing the outliers from the data set, make a new scatterplot of the remaining animal body and brain weights.

4. Does there appear to be a relationship between body weight and brain weight? If yes, write a brief description of the relationship.

5. Take a piece of uncooked spaghetti and use that spaghetti to informally fit a line to the data. Attempt to place your line so that the vertical distances from the points to the line are as small as possible.

6. How well does the spaghetti line appear to fit the data? Explain.

Animal	Body Weight (kg)	Brain Weight (g)
Mountain beaver	1.35	8.1
Cow	465	423
Grey wolf	36.33	119.5
Goat	27.66	115
Guinea pig	1.04	5.5
Asian elephant	2547	4603
Donkey	187.1	419
Horse	521	655
Potar monkey	10	115
Cat	3.3	25.6
Giraffe	529	680
Gorilla	207	406
Human	62	1320
African elephant	6654	5712
Rhesus monkey	6.8	179
Kangaroo	35	56
Golden hamster	0.12	1
Mouse	0.023	0.4
Rabbit	2.5	12.1
Sheep	55.5	175
Jaguar	100	157
Chimpanzee	52.16	440
Mole	0.122	3
Pig	192	180

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Linear relationships:** Students may struggle to read and interpret graphs. For graphs displaying a roughly linear relationship, students may struggle to understand that data points do not have to fall directly on a line. When a scatter plot shows no association, students may struggle and need examples of data that may have no association (length of a person's hair and his or her final grade in mathematics).

- **Graphing:** Students may struggle with scaling the x- and y-axes so that the data is visible but not misleading. They may also think that the line of best fit must go through at least some of the data points on the scatter plot, or that there is only one correct straight line that can be fitted to a scatter plot.
- **Clustering:** Students might mistakenly believe that whenever data points cluster together, there must be a strong relationship between the two variables.
- **Outliers:** Some students may assume that outliers should always be disregarded as errors or anomalies.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + Universal*	*Layer 2* *Core + Targeted*	*Layer 3* *Core + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships.	**Pre-teaching** In previous classes, learners worked to plot points in a coordinate grid. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as studying patterns represented in scatter plots and in contexts described verbally. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Students should spend time examining tasks	**Pre-teaching** Consider using standard 5.G.A.2, which provides a foundation for work in this cluster. In 5.G.A.2, students interpret real-word problems and produce a graph based on information gathered from the problem. It is essential that students develop awareness of how real-world information is represented visually and how visual representations relate to each other. Give students time to really dive into graphs and interpret them rather than jumping in and explicitly explaining to them what a graph represents. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with

Engagement Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before	from a different perspective, especially using straight lines to model relationships, because this leaves room for discussion amongst peers and shows us how students have arrived at their conclusions about a data set.	common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster constructing, reading, and interpreting scatter plots. Give them time to describe relationships using statistical vocabulary, fitting a model to data, assess the fit of a model, and using and interpreting equations. Students should practice reading, interpreting, and constructing two-way tables and calculating and interpreting relative frequencies.

students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/39/451/451

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● plot points in a coordinate grid	In 8th grade, learners ● construct an equation or a function to model a linear relationship ● determine and interpret the slope and y-intercept	In future classes, learners ● compute and interpret the correlation coefficient ● distinguish between correlation and causation ● represent two variables on a scatter plot and describe how ● they are related ● construct, interpret, and summarize data in a two-way table

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

	low expectations and low achievement.	

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- What does each point on a scatter plot represent?
- What does clustering in a scatter plot represent?
- Why are outliers important?
- How could you use a different classmate's strategy to check to make sure your solution is reasonable?
- How can we use the equation of the line of best fit to make a prediction about the data?
- What two types of data in real life will show positive correlation? Negative correlation? No correlation?
- How can you choose the best line to fit a scatter plot? What should you try to do?
- How can you measure whether your line is a good fit for the scatterplot? Can you come up with another line and make an argument for why it is better?
- How does the amount of data we collect have an impact on the correlation?

Cross-Curricular Connections

Literature: Using linear, logical thinking to write more clearly and logically.

Science: Examining biological characteristics of a sample. Making an argument about the growth and development of organisms. Developing models and identifying components and describing relationships between components. Modeling data and describing how the data can be used to create designs through testing and modification. Identifying limitations of models. Engaging in the Engineering Design Process. Modeling genetic information and sexual reproduction results and using Punnett squares. Creating scatterplots of bivariate data. Using simulations to generate data that can be used to modify a proposed object, tool, or process. Conducting experiments in connection with NGSS science standards, collecting bivariate data, representing that data in a two-way table, and hypothesizing correlations between the two variables. Reading scientific charts and graphs. Analyzing scientific studies.

Social Studies: Sampling populations and analyzing data. Studying trends in areas such as species populations, the stock market, or gross domestic product. Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations have different average life spans. Understanding statistical information on populations.

Career and Skill Connections

• Advertising • Aeronautics • Agriculture • Analysis • Aviation • Banking/finance • Botanist • Coaching • Counseling	• Economist • Education • Engineering • Gardening • Health science • Information technology • Law • Machinist • Management	• Park ranger • Political science • Psychology • Ranching/farming • Sales • Statistics • Technician • Transportation • Urban planning

• Data science • Ecology	• Mechanic • Medicine	• Veterinary

Grade	CCSS Domain	CCSS Cluster
8	Statistics and Probability	Investigate patterns of association in bivariate data.

Cluster Standard: 8.SP.A.3

Standard	Standards for Mathematical Practice
8.SP.A.3: Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.	● **SMP 2**: Reason abstractly and quantitatively. ○ Teacher and Student Actions ● **SMP 6**: Attend to precision. ○ Teacher and Student Actions
Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students construct scatter plots and interpret patterns focusing on linear association. They construct two-way tables and interpret relationships using relative frequencies.	● Use linear models to make predictions from data in a scatterplot (trend line) in context. ● Interpret the slope and intercept for a trend line in the context of a real-world problem. ● Write the linear equation for a trend line. ● Analyze and interpret the meaning of the slope and y- intercept in a linear model from data in a scatterplot. ● Make predictions from the line.
DOK	**Blooms**
1-3	Apply, Analyze

Procedural and Conceptual Understanding and Application

While you may see some application, this cluster of standards focuses on conceptual understanding and procedural skills and fluency.

Conceptual Understanding:

- Interpret patterns focusing on linear association.
- Interpret relationships using relative frequencies.
- Use linear models to make predictions from data in a scatterplot (trend line) in the context of a real-world problem.
- Interpret the slope and intercept for a trend line in the context of a real-world problem.

- Make predictions from the trend line.

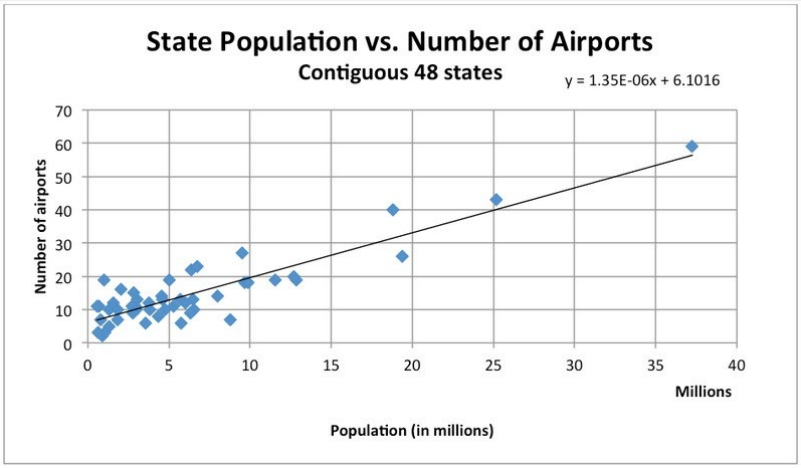
Application:

- Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.
- Construct scatter plots, focusing on linear association.
- Construct two-way tables.
- Use linear models to make predictions from data in a scatterplot (trend line).
- Write the linear equation for a trend line.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

The scatter plot below shows the relationship between the number of airports in a state and the population of that state according to the 2010 Census. Each dot represents a single state.



1. How would you characterize the relationship between the number of airports in a state and the state's population? Explain your thinking.

2. LaToya writes a function to model the relationship between the number of airports, where y and the population in a state, x: $y = (1.35 \times 10^{-6})x + 6.1$

a. What does 1.35×10^{-6} represent in the context of the problem of airports vs. populations?

b. What does the number 6.1 mean in the context of airports vs. populations?

3. How many airports does LaToya's model predict for a state with a population of 30 million people?

You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Linear relationships:** Students may struggle to read and interpret graphs. For graphs displaying a roughly linear relationship, students may struggle to understand that data points do not have to fall directly on a line. When a scatter plot shows no association, students may struggle and need examples of data that may have no association (length of a person's hair and his or her final grade in mathematics).
- **Graphing:** Students may struggle with scaling the x- and y-axes so that the data is visible but not misleading. They may also think that the line of best fit must go through at least some of the data points on the scatter

plot, or that there is only one correct straight line that can be fitted to a scatter plot.

- **Clustering:** Students might mistakenly believe that whenever data points cluster together, there must be a strong relationship between the two variables.
- **Outliers:** Some students may assume that outliers should always be disregarded as errors or anomalies.
- **Linear models:** Students may incorrectly interpret the y-intercept of a linear model as a data point on the graph rather than as the value of the dependent variable when the independent variable is zero. They may also struggle to understand the slope in the context of the data and need extra help defining the two variables and understanding how they work together as y over x. Students may struggle to substitute values for x and y into the equation to answer questions in the context of the problem.

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + Universal*	*Layer 2* *Core + Targeted*	*Layer 3* *Core + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed	**Pre-teaching** In previous classes, learners worked to plot points in a coordinate grid. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as studying patterns represented in scatter plots and in contexts described verbally. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain	**Pre-teaching** Consider using standard 5.G.A.2, which provides a foundation for work in this cluster. In 5.G.A.2, students interpret real-word problems and produce a graph based on information gathered from the problem. It is essential that students develop awareness of how real-world information is represented visually and how visual representations relate to each other. Give students time to really dive into graphs and interpret them rather than jumping in and explicitly explaining to them what a graph represents. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending

outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or	mistakes and find new, more efficient ways of solving problems. Students should spend time examining tasks from a different perspective, especially using straight lines to model relationships, because this leaves room for discussion amongst peers and shows us how students have arrived at their conclusions about a data set.	time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster constructing, reading, and interpreting scatter plots. Give them time to describe relationships using statistical vocabulary, fitting a model to data, assess the fit of a model, and using and interpreting equations. Students should practice reading, interpreting, and constructing two-way tables and calculating and interpreting relative frequencies.

nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/39/453/453

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● plot points in a coordinate grid	In 8th grade, learners ● construct an equation or a function to model a linear relationship ● determine and interpret the slope and y-intercept	In future classes, learners ● compute and interpret the correlation coefficient ● distinguish between correlation and causation ● represent two variables on a scatter plot and describe how ● they are related ● construct, interpret, and summarize data in a two-way table

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Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media,	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

	they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- What does each point on a scatter plot represent?
- What does clustering in a scatter plot represent?
- Why are outliers important?
- How could you use a different classmate's strategy to check to make sure your solution is reasonable?
- How can we use the equation of the line of best fit to make a prediction about the data?
- What two types of data in real life will show positive correlation? Negative correlation? No correlation?
- How can you choose the best line to fit a scatter plot? What should you try to do?
- How can you measure whether your line is a good fit for the scatterplot? Can you come up with another line and make an argument for why it is better?
- What does the slope of this line mean in the context of the situation?
- How does the amount of data we collect have an impact on the correlation?

Cross-Curricular Connections

Literature: Using linear, logical thinking to write more clearly and logically.

Science: Examining biological characteristics of a sample. Making an argument about the growth and development of organisms. Developing models and identifying components and describing relationships between components. Modeling data and describing how the data can be used to create designs through testing and modification. Identifying limitations of models. Engaging in the Engineering Design Process. Modeling genetic information and sexual reproduction results and using Punnett squares. Creating scatterplots of bivariate data. Using simulations to generate data that can be used to modify a proposed object, tool, or process. Conducting experiments in connection with NGSS science standards, collecting bivariate data, representing that data in a two-way table, and hypothesizing correlations between the two variables. Reading scientific charts and graphs. Analyzing scientific studies.

Social Studies: Sampling populations and analyzing data. Studying trends in areas such as species populations, the stock market, or gross domestic product. Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations have different average life spans. Understanding statistical information on populations.

Career and Skill Connections

• Advertising • Aeronautics • Agriculture • Analysis • Aviation	• Economist • Education • Engineering • Gardening • Health science	• Park ranger • Political science • Psychology • Ranching/farming • Sales

• Banking/finance • Botanist • Coaching • Counseling • Data science • Ecology	• Information technology • Law • Machinist • Management • Mechanic • Medicine	• Statistics • Technician • Transportation • Urban planning • Veterinary

Grade	CCSS Domain	CCSS Cluster
8	Statistics and Probability	Investigate patterns of association in bivariate data.

Cluster Standard: 8.SP.A.4

Standard	Standards for Mathematical Practice

8.SP.A.4: Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?	• **SMP 1:** Make sense of problems and persevere in solving them.
	  ○ Teacher and Student Actions
	• **SMP 2**: Reason abstractly and quantitatively.
	  ○ Teacher and Student Actions
	• **SMP 3:** Construct viable arguments and critique the reasoning of others.
	  ○ Teacher and Student Actions
	• **SMP 4:** Model with mathematics.
	  ○ Teacher and Student Actions
	• **SMP 6**: Attend to precision.
	  ○ Teacher and Student Actions
	• **SMP 8:** Look for and express regularity in repeated reasoning.
	  ○ Teacher and Student Actions

Clarification Statement	**Students Who Demonstrate Understanding Can...**
Students construct scatter plots and interpret patterns focusing on linear association. They construct two-way tables and interpret relationships using relative frequencies.	• Create two-way frequency tables to display data. • Collect categorical data on two variables • Analyze and interpret the data in two-way frequency tables. • Calculate relative frequencies and describe possible associations between the variables.
DOK	**Blooms**
2-4	Apply, Create

Procedural and Conceptual Understanding and Application

You should see all three aspects of rigor in this standard.

Conceptual Understanding:

- Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.
- Interpret a two-way table summarizing data on two categorical variables collected from the same subjects.

Procedural Skill and Fluency:

- Construct a two-way table summarizing data on two categorical variables collected from the same subjects.
- Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Application:

- Collect categorical data on two variables, construct a two-way table summarizing the data, and interpret the data, using relative frequencies in some cases.

Assessment Items

When available, you should use your locally selected or created high quality instructional materials. However, the following are example assessment items you can use if you don't have local instructional materials available.

Is there an association between whether a student plays a sport and whether he or she plays a musical instrument? To investigate these questions, each student in your class should answer the following two questions:

- Do you play a sport? (yes or no)
- Do you play a musical instrument? (yes or no)

1. Record the answers in the table..
2. Summarize the data into a clearly labeled table.
3. Of those students who play a sport, what proportion play a musical instrument?
4. Of those students who do not play a sport, what proportion play a musical instrument?
5. Based on the class data, do you think there is an association between playing a sport and playing an instrument?
6. Create a graph that would help visualize the association, if any, between playing a sport and playing a musical instrument.

Student	Sport?	Musical Instrument?
1		
2		
3		
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You can find the task above, as well as others aligned to this standard, here.

Common Misconceptions

- **Bivariate categorical tables:** Students may struggle to understand how to display bivariate categorical data in a table. They might have trouble interpreting and creating tables that model data. They might especially struggle with identifying the two variables that each value represents.
- **Relative frequencies:** Students might have trouble interpreting and calculating relative frequencies. They might especially struggle with identifying the information that each value represents and what the resulting value means in context, including if a value implies correlation or not.
- **Equal frequencies:** Students may assume that if the frequencies in each cell of a two-way table are similar, the two variables are independent of each other.
- **Associations:** Students may incorrectly believe that if two variables show an association in a two-way table, there must be a directional relationship between them (e.g., one variable is influencing the other).

Planning for Multi-Layer System of Support (MLSS) & Universal Design for Learning

Layer 1 *Core Instruction + Universal*	*Layer 2* *Core + Targeted*	*Layer 3* *Core + Targeted + Intensive*
Representation Teachers can reduce barriers and leverage students' individual strengths by presenting content using multiple modalities and annotating displays with specific language, different colors, shading, arrows, labels, notes, diagrams, drawings, etc. Support the use of vocabulary, mathematical notation, and symbols with charts, pictures, diagrams, and tables, and use translations, descriptions, movement, and images to support unfamiliar words or phrases. Present problems or contexts in multiple ways, using diagrams, drawings, pictures, media, tables, graphs, and other mathematical representations, and highlight connections between different mathematical representations to make patterns and properties explicit. Activate or supply background knowledge to build connections to prior understandings and experiences and maximize transfer and generalization by naming	**Pre-teaching** In previous classes, learners worked to plot points in a coordinate grid. Students might benefit from opportunities to review vocabulary terms, and you should take the time to introduce new vocabulary. Students might need to review key concepts and skills such as studying patterns represented in scatter plots and in contexts described verbally. If students have unfinished learning leading into this standard, consider ways to provide targeted pre-teaching support prior to the start of the unit to ensure that students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address	**Pre-teaching** Consider using standard 5.G.A.2, which provides a foundation for work in this cluster. In 5.G.A.2, students interpret real-word problems and produce a graph based on information gathered from the problem. It is essential that students develop awareness of how real-world information is represented visually and how visual representations relate to each other. Give students time to really dive into graphs and interpret them rather than jumping in and explicitly explaining to them what a graph represents. If students have unfinished learning leading into this standard, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. Students should spend most of their time accessing their

connections to previous examples, inviting students to identify important details or features to remember. Provide reading accommodations as needed, as well as blank or partially-completed outlines, graphic organizers, or representations, to emphasize key ideas and relationships. **Engagement** Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Provide access to a variety of tools, strategies, and materials designed to help students self-motivate and become more independent. Leverage curiosity and students' existing interests and invite students to name connections to their own lived experiences. Provide choice by inviting students to decide which problem to start with, select a subset of problems to complete, which strategy to use, the order they complete a task, etc. Use visible timers and alerts to prepare for transitions, and chunk tasks into more manageable parts and check in with students to provide feedback and encouragement after each chunk. Differentiate the degree of difficulty or complexity by starting with accessible values. Periodically revisit community norms and provide group feedback that encourages collaboration and community. Provide ongoing feedback that helps students maintain sustained effort and persistence during a task and encourage self-reflection and identification of personal goals. **Action and Expression** Throughout the curriculum, students should be invited to share both their understanding and their reasoning	misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Give students time to find and explain mistakes and find new, more efficient ways of solving problems. Students should spend time examining tasks from a different perspective, especially using straight lines to model relationships, because this leaves room for discussion amongst peers and shows us how students have arrived at their conclusions about a data set.	current grade-level content. **Re-teaching** Examine assessments for evidence of lingering misconceptions. To address misconceptions, consider spending time on a mini-lesson aimed at revisiting student thinking and examining sample work with common mistakes being made. Students may benefit from intensive extra time during and after work within this cluster constructing, reading, and interpreting scatter plots. Give them time to describe relationships using statistical vocabulary, fitting a model to data, assess the fit of a model, and using and interpreting equations. Students should practice reading, interpreting, and constructing two-way tables and calculating and interpreting relative frequencies.

about mathematical ideas with others. Offer flexibility and choice with the ways students demonstrate and communicate their understanding and invite students to explain their thinking verbally or nonverbally with manipulatives, drawings, diagrams. Provide independent think time before students engage with others or responses are discussed and support discourse with sentence frames or visible language displays. Ensure students have enough time to complete tasks and provide extra time if needed, as well as pre-cut materials, assistive tools, devices, and software. Support fluency with graduated levels of support or practice, applying and gradually releasing scaffolds to support independent learning. Support the development of organizational skills in problem-solving with access to templates, rubrics, and checklists and provide opportunities for self-assessment and enable students to monitor their own progress. Post visible goals, objectives, and schedules.		

Vertical Alignment

Consider using this coherence map to help guide your planning
https://tools.achievethecore.org/coherence-map/8/39/455/455

Previous Learning	*Current Learning*	*Future Learning*
In previous classes, learners ● plot points in a coordinate grid	In 8th grade, learners ● construct an equation or a function to model a linear relationship ● determine and interpret the slope and y-intercept	In future classes, learners ● compute and interpret the correlation coefficient ● distinguish between correlation and causation ● represent two variables on a scatter plot and describe how ● they are related ● construct, interpret, and summarize data in a two-way

		table

Culturally and Linguistically Responsive Instruction

Consider these resources for vocabulary from Pathways2Careers:

- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Glossary.pdf
- https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Vocabulary%20Graphic%20Organizer.pdf

Consider these questions as you plan for instruction that is culturally and linguistically responsive:

- How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
- How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Validate and Affirm	*Build and Bridge*	*Linguistic Vocabulary Support*
• Consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can learn about how probability is connected to games that their family enjoys playing and discuss whether the probability makes the games more or less interesting. • Students can also discuss sampling procedures and whether statistics show bias towards certain groups of people. • Students can also talk with their families about the type of mathematics and logical thinking the people in their family use when working. • Consider inviting community members to talk with students about the math they use in their careers or crafts.	• Instruction should begin with conceptual understanding that allows students to contribute their informal knowledge and any background information they might have. • When new learning begins with procedures, it privileges those with strong prior familiarity with school mathematics and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. • Students should be allowed to meaningfully apply their learning to meaningful situations and contexts that are relevant to living in the real world. • Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and	• Scaffold tasks and amplify language so students can make their own meaning, especially when cognates exist. • Provide opportunities and supports for helping students to describe their mathematical thinking to others clearly, whether that is orally, visually, or in writing. • Use tools and strategies such as sentence stems, time for brainstorming, and communication in students' home languages. • Provide opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class) whenever possible. • Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.

	achieve in mathematics. • Unfortunately, the reverse is also true; when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.	

Suggested Student Discourse Questions

Consider this resource for student discourse from Pathways2Careers: https://engage.pathway2careers.com/api/staticcontent/lms/materials/P2CMath/P2C%20Math%20Academic%20Conversation%20Cards.pdf

- What does each point on a scatter plot represent?
- How can we represent the date from a scatter plot in a table?
- What does each box in the table represent?
- Explain what this row of boxes represents in the context of this situation.
- What is a relative frequency? What does it mean in the context of this situation?
- How can we use relative frequencies to determine possible association?
- Why are outliers important?
- How does the amount of data we collect have an impact on the correlation?

Cross-Curricular Connections

Literature: Using linear, logical thinking to write more clearly and logically.

Science: Examining biological characteristics of a sample. Making an argument about the growth and development of organisms. Developing models and identifying components and describing relationships between components. Modeling data and describing how the data can be used to create designs through testing and modification. Identifying limitations of models. Engaging in the Engineering Design Process. Modeling genetic information and sexual reproduction results and using Punnett squares. Creating scatterplots of bivariate data. Using simulations to generate data that can be used to modify a proposed object, tool, or process. Conducting experiments in connection with NGSS science standards, collecting bivariate data, representing that data in a two-way table, and hypothesizing correlations between the two variables. Reading scientific charts and graphs. Analyzing scientific studies.

Social Studies: Sampling populations and analyzing data. Studying trends in areas such as species populations, the stock market, or gross domestic product. Reviewing charts and graphs that provide historical data or information on ethnic groups. Understanding how the elevation of an area affects its population or charting the extent to which different populations have different average life spans. Understanding statistical information on populations.

Career and Skill Connections

• Advertising • Aeronautics	• Economist • Education	• Park ranger • Political science

• Agriculture • Analysis • Aviation • Banking/finance • Botanist • Coaching • Counseling • Data science • Ecology	• Engineering • Gardening • Health science • Information technology • Law • Machinist • Management • Mechanic • Medicine	• Psychology • Ranching/farming • Sales • Statistics • Technician • Transportation • Urban planning • Veterinary

Section 3: Resources, References, and Glossary

Resources

Evidence-Based Resources	English Learner Resources	MLSS Resources	Mathematics Standard Resources
What Works Clearinghouse Best Evidence Encyclopedia Evidence for Every Students Succeeds Act Evidence in Education Lab	World-Class Instructional Design and Assessment (WIDA) Standards USCALE Language Routines for Mathematics English Language Development Standards Spanish Language Development Standards	NM Multi-Layered System of Supports (MLSS) Universal Design for Learning Guidelines Achieve the Core: Instructional Routines for Mathematics Project Zero Thinking Routines	Focus by Grade Level and Widely Applicable Prerequisites High school Coherence Map College-and Career Ready Math Shifts Fostering Math Practices: Routines for the Mathematical Practices

Planning Guidance for Multi-Layered Systems of Support: Core Instruction[9]

Core Instructional Planning must reflect and leverage scientific insights into how humans learn in order to ensure all students are ready for success, thus the following guidance for optimizing teaching and learning is grounded in the **Universal Design Learning (UDL) Framework**

Key design questions, planning actions, and potential strategies are provided below, with respect to guidance for minimizing barriers to learning and optimizing (1) universal ACCESS to learning experiences, (2) opportunities for students to BUILD their understanding of the **Learning Goal**, and (3) INTERNALIZATION of the Learning Goal.

Optimizing Universal ACCESS to Learning Experiences	
ENGAGEMENT ? How will you provide multiple options for recruiting interest?	**Recruiting Student Interest:** ? What do you anticipate in the range of student interest for this lesson? ➢ Plan for options for recruiting student interest: ❒ provide choice (e.g. sequence or timing of task completion) ❒ set personal academic goals ❒ provide contextualized examples connected to their lives ❒ support culturally relevant connections (i.e home culture) ❒ create socially relevant tasks ❒ provide novel & relevant problems to make sense of complex ideas in creative ways

[9] Adapted from: CAST (2018). *Universal Design for Learning Guidelines version 2.2*. Retrieved from http://udlguidelines.cast.org

	❒ provide time for self-reflection about content & activities ❒ create accepting and supportive classroom climate ❒ utilize **instructional routines** to involve all students
REPRESENTATION [?] How will you reduce barriers to perceiving the information presented in this lesson?	**Perception:** [?] What do you anticipate about the range in how students will perceive information presented in this lesson? ➢ Plan for different modalities and formats to reduce barriers to learning: ❒ display information in a flexible format to vary perceptual features ❒ offer alternatives for auditory information ❒ offer alternatives for visual information
ACTION & EXPRESSION [?] How will the learning for students provide a variety of methods for navigation to support access?	**Physical Action:** [?] What do you anticipate about the range in how students will physically navigate and respond to the learning experience? ➢ Plan a variety of methods for response and navigation of learning experiences by offering alternatives to: ❒ requirements for rate, timing, speed, and range of motor action with instructional materials, manipulatives, and technologies ❒ physically indicating selections ❒ interacting with materials by hand, voice, keyboard, etc.

Opportunities for Students to BUILD their Understanding

ENGAGEMENT [?] How will the learning for students provide options for sustaining effort and persistence?	**Sustaining Effort & Persistence:** [?] What do you anticipate about the range in student effort? ➢ Plan multiple methods for attending to student attention and affect by: ❒ prompting learners to explicitly formulate or restate learning goals ❒ displaying the learning goals in multiple ways ❒ using prompts or scaffolds for visualizing desired outcomes ❒ engaging assessment discussions of what constitutes excellence ❒ generating relevant examples with students that connect to their cultural background and interests ❒ providing alternatives in the math representations and scaffolds ❒ creating cooperative groups with clear goals, roles, responsibilities ❒ providing prompts to guide when and how to ask for help ❒ supporting opportunities for peer interactions and supports (e.g. peer tutors) ❒ constructing communities of learners engaged in common interests ❒ creating expectations for group work (e.g., rubrics, norms, etc.) ❒ providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge ❒ providing feedback that: ❒ emphasizes effort, improvement, and achieving a standard rather than on relative performance ❒ is frequent, timely, and specific ❒ is informative rather than comparative or competitive

	❒ models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success
REPRESENTATION ? How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners?	**Language & Symbols:** ? What do you anticipate about the range of student background experience and vocabulary? ➢ Plan multiple methods for attending to linguistic and nonlinguistic representations of mathematics to ensure universal clarity by: ❒ pre-teaching vocabulary and symbols in ways that promote connection to the learners' experience and prior knowledge ❒ graphic symbols with alternative text descriptions ❒ highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to structure ❒ embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) ❒ embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) ❒ highlighting structural relations or make them more explicit ❒ making connections to previously learned structures ❒ making relationships between elements explicit (e.g., highlighting the transition words in an argument, links between ideas, etc.) ❒ allowing the use of text-to-speech and automatic voicing with digital mathematical notation (math ml) ❒ allowing flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) ❒ clarification of notation through lists of key terms ❒ making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf ❒ linking key vocabulary words to definitions and pronunciations in both dominant and heritage languages ❒ defining domain-specific vocabulary (e.g., "map key" in social studies) using both domain-specific and common terms ❒ electronic translation tools or links to multilingual web glossaries ❒ embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc) ❒ presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) ❒ making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams
ACTION & EXPRESSION ? How will the learning provide multiple	**Expression & Communication:** ? What do you anticipate about the range in how students will express their thinking in the learning environment? ➢ Plan multiple methods for attending to the various ways in which students can express knowledge, ideas, and concepts by providing:

modalities for students to easily express knowledge, ideas, and concepts in the learning environment?	❒ options to compose in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video ❒ use of social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) ❒ flexibility in using a variety of problem solving strategies ❒ spell or grammar checkers, word prediction software ❒ text-to-speech software, human dictation, recording ❒ calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper ❒ sentence starters or sentence strips ❒ concept mapping tools ❒ Computer-Aided-Design (CAD) or mathematical notation software ❒ virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) ❒ multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches) ❒ multiple examples of novel solutions to authentic problems ❒ different approaches to motivate, guide, feedback or inform students of progress towards fluency ❒ scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) ❒ differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners)

Optimizing INTERNALIZATION of the Learning Goal

ENGAGEMENT ⍰ How will the design of the learning strategically support students to effectively cope and engage with the environment?	**Self-Regulation:** ⍰ What do you anticipate about barriers to student engagement? ➢ Plan to address barriers to engagement by promoting healthy responses and interactions, and ownership of learning goals: ❒ metacognitive approaches to frustration when doing mathematics ❒ increase length of on-task orientation through distractions ❒ frequent self-reflection and self-reinforcements ❒ address subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") ❒ offer devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the math practices for the purpose of monitoring and improving ❒ use activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely
REPRESENTATION ⍰ How will the learning support transforming accessible information into usable knowledge	**Comprehension:** ⍰ What do you anticipate about barriers to student comprehension? ➢ Plan to address barriers to comprehension by intentionally building connections to prior understandings and experiences, relating meaningful information to learning goals,

that is accessible for future learning and decision-making?	providing a process for meaning making of new learning, and applying learning to new contexts: ❒ incorporate explicit opportunities for review and practice ❒ note-taking templates, graphic organizers, concept maps ❒ scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) ❒ explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) ❒ opportunities over time to revisit key ideas and connections ❒ make explicit cross-curricular connections ❒ highlight key elements in tasks, graphics, diagrams, formulas ❒ outlines, graphic organizers, unit organizer routines, concept organizer routines, and concept mastery routines to emphasize key ideas and relationships ❒ multiple examples & non-examples ❒ cues and prompts to draw attention to critical features ❒ highlight previously learned skills that can be used to solve unfamiliar problems ❒ options for organizing and possible approaches (tables and representations for processing mathematical operations) ❒ interactive representations that guide exploration and new understandings ❒ introduce graduated scaffolds that support information processing strategies ❒ tasks with multiple entry points and optional pathways ❒ "Chunk" information into smaller elements ❒ remove unnecessary distractions unless essential to learning goal ❒ anchor instruction by linking to and activating relevant prior knowledge (e.g., using visual imagery, concept anchoring, or concept mastery routines) ❒ pre-teach critical prerequisite concepts via demonstration or representations ❒ embed new ideas in familiar ideas and contexts (e.g., use of analogy, metaphor, drama, music, film, etc.) ❒ advanced organizers (e.g., KWL methods, concept maps) ❒ bridge concepts with relevant analogies and metaphors
ACCESS ACTION & EXPRESSION ⍰ How will the learning for students support the development of executive functions to allow them to take advantage of their environment?	**Executive Functions:** ⍰ What do you anticipate about barriers to students demonstrating what they know? ➢ Plan to address barriers to demonstrating understanding by providing opportunities for students to set goals, formulate plans, use tools and processes to support organization and memory, and analyze their growth in learning and how to build from it: ❒ prompts and scaffolds to estimate effort, resources, difficulty ❒ models and examples of process and product of goal-setting ❒ guides and checklists for scaffolding goal-setting ❒ post goals, objectives, and schedules in an obvious place ❒ embed prompts to "show and explain your work" ❒ checklists and project plan templates for understanding the problem, prioritization, sequences, and schedules of steps ❒ embed coaches/mentors to demonstrate think-alouds of process ❒ guides to break long-term goals into short-term objectives ❒ graphic organizers/templates for organizing information & data ❒ embed prompts for categorizing and systematizing ❒ checklists and guides for note-taking ❒ asking questions to guide self-monitoring and reflection ❒ showing representations of progress (e.g., before and after photos, graphs/charts showing progress, process portfolios)

	❒ prompt learners to identify type of feedback or advice they seek ❒ templates to guide self-reflection on quality & completeness ❒ differentiated models of self-assessment strategies (e.g., role-playing, video reviews, peer feedback) ❒ assessment checklists, scoring rubrics, and multiple examples of annotated student work/performance examples

Planning Guidance for Culturally and Linguistically Responsive Instruction[10]

In order to ensure our students from marginalized cultures and languages view themselves as confident and competent learners and doers of mathematics within and outside of the classroom, educators must intentionally plan ways to counteract the negative or missing images and representations that exist in our curricular resources. The guiding questions below support the design of lessons that validate, affirm, build, and bridge home and school culture for learners of mathematics:

> **Validate/Affirm:** How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
>
> **Build/Bridge:** How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

In addition, Aguirre and her colleagues[11] define **mathematical identities** as the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives. Many students see themselves as "not good at math" and approach math with fear and lack of confidence. Their identity, developed through earlier years of schooling, has the potential to affect their school and career choices.

Five Equity-Based Mathematics Teaching Practices[12]

> **Go deep with mathematics.** Develop students' conceptual understanding, procedural fluency, and problem solving and reasoning.
>
> **Leverage multiple mathematical competencies.** Use students' different mathematical strengths as a resource for learning.
>
> **Affirm mathematics learners' identities.** Promote student participation and value different ways of contributing.

[10] This resource relied heavily on the work of: Hollie, S. (2011). Culturally and linguistically responsive teaching and learning. Teacher Created Materials. (see also, https://www.culturallyresponsive.org/vabb)

[11] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices. Reston, VA: National Council of Teachers of Mathematics (p. 14).

[12] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc. (p.6). (adapted from Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013) (p. 43).

Challenge spaces of marginality. Embrace student competencies, value multiple mathematical contributions, and position students as sources of expertise.

Draw on multiple resources of knowledge (mathematics, language, culture, family). Tap students' knowledge and experiences as resources for mathematics learning.

The following lesson design strategies support Culturally and Linguistically Responsive Instruction, specific examples for each cluster of standards can be found in part 2 of the document. These were adapted from the Promoting Equity section of the Taking Action series published by NCTM.[13]

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.

Mathematical Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher.

Modifying Mathematical Tasks: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)."

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics.

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages.

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we

[13] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians.

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others."

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time."

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students.

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Glossary[14]

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (- 3/4) = (- 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[15]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. See: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances
from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten.
For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

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[14] Glossary and tables taken from: Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/
[15] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[16] See also: median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.
Integer. A number expressible in the form a or -a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line.
Also known as a dot plot.[17]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[18] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 Ã· 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 Ã— 4/3 = 4/3 Ã— 3/4 = 1.

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[16] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

[17] Adapted from Wisconsin Department of Public Instruction, op. cit.

[18] To be more precise, this defines the arithmetic mean.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin,
selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also*: uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form *a/b* or – *a/b* for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also*: terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[19]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar
model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

[19] Adapted from Wisconsin Department of Public Instruction, op. cit.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also*: median, first quartile, interquartile range.

Table 1: Common addition and subtraction.[1]

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 =5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now?5-2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?? -2 = 3
	TOTAL UNKNOWN	ADDEND UNKNOWN	BOTH ADDENDS UNKNOWN[2]
PUT TOGETHER / TAKE APART[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5-3 = ?	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? 5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1, 5 = 2 + 3, 5 = 3 + 2
COMPARE	DIFFERENCE UKNOWN	BIGGER UNKNOWN	SMALLER UNKNOWN
	("How many more?" version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2: Common multiplication and division situations.[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	3 x ? = 18, and 18 ÷ 3 = ?	? x 6 = 18, and 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	a x b = ?	a x ? = p and p ÷ a = ?	? x b = p, and p ÷ b = ?

[1]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3: The properties of operations.

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	(a +b) + c = a + (b+c)
Commutative property of addition	a + b = b + a

Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists *-a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) x c = a x (b x c)
Commutative property of multiplication	a x b = b x a
Multiplicative identity property 1	a x 1 = 1 x a = a
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that a x $1/a$ = $1/a$ x a = 1
Distributive property of multiplication over additions	a x (b + c) = a x b + a x c

Table 4: The properties of equality.

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a.
Symmetric property of equality	If a = b, then b = a.
Transitive property of equality	If a = b and b = c , then a = c.
Addition property of equality	If a = b, then a +c = b + c.
Subtraction property of equality	If a = b then a – c = b – c.
Multiplication property of equality	If a = b, then a x c = b x c.
Division property of equality	If a = b and $c \neq 0$, then a ÷ c = b ÷ c.
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality.

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b.
If a > b and b > c then a > c.
If a > b, b < a.
If a > b, then -a <-b.
If a > b, then a ± c > b ± c.
If a > b and c > 0, then a x c > b x c.
If a > b and c < 0, then a x c < b x c.
If a > b and c > 0, then a ÷ c > b ÷ c.
If a > b and c < 0, then a ÷ c < b ÷ c.